```
# Problem 3

import math
import numpy as np
from sklearn.model_selection import train_test_split

# Calculate MSE given y_pred and y
def MSE(y_pred, y):
    a = y.shape[0]
    return (1/(2 * a)) * np.sum(np.square(y_pred - y))

# Calculate the MSE given the prediction parameters and actual y value
def calculate_error(x, w, b, y):
    y_pred = x.dot(w) + b
    return MSE(y_pred, y)

# Loop through the samples based on the batch_size hyperparameter
def batch_loop(x, y, size):
    if(len(x) == len(y)):
        random_x = x[np.random.permutation(x.shape[0])]
        random_y = y[np.random.permutation(y.shape[0])]
        for i in np.arange(0, y.shape[0], size):
            yield random_x[i:i + size], random_y[i:i + size]

# Perform gradient_descent using the prediction parameters, actual y,
learning rate, and regularized term
def gradient_descent(x, y, b, w, learn_rate, alpha = 0.1):
    a = y.shape[0]
    y_pred = x.dot(w) + b

    # Store the derivatives of w and b
    d_of_w = (1/a) * x.T.dot(y_pred - y)
    d_of_b = (1/a) * np.sum(y_pred - y)
    # Add the regularized term to the derivative of w
    d_of_w += ((alpha/a) * w)
    # Update parameters with gradient descent
    w = w - (learn_rate * d_of_w)
    b = b - (learn_rate * d_of_b)

    return w, b

# Perform SGD
def stochastic_gradient_descent(x, y, b, w, learn_rate: float,
num_of_epoch: int, batch_size: int, dataset_size: int, alpha):

    for n in range(num_of_epoch - 1):
        for mini_batch_x, mini_batch_y in batch_loop(x, y,
batch_size):
            w, b = gradient_descent(mini_batch_x, mini_batch_y, b,
w, learn_rate, alpha)
```

```
    return w, b

# Find the lowest error by performing SGD
def find_lowest_error(x, y, learn_rate, num_of_epoch, batch_size,
alpha):
    dataset_size = len(y)
    m = np.expand_dims(a=y, axis=-1)
    w = np.random.rand(x.shape[1]) * np.sqrt(1/(x.shape[1] +
m.shape[1]))
    b = np.random.rand(m.shape[1])
    w_trained, b_trained = stochastic_gradient_descent(x, y, b, w,
learn_rate, num_of_epoch, batch_size, dataset_size, alpha)
    return w_trained, b_trained

# Implementation of grid_search to tune our hyperparameters by looping
through the various values we have for each
def grid_search():
    hyperparameters = {
        "learn_rate": [0.1, 0.01, 0.001, 0.0001],
        "num_of_epoch": [5, 10, 15, 20],
        "batch_size": [10, 20, 50, 100],
        "alpha": [0.75, 0.5, 0.25, 0.1]
    }
    for a in range(len(hyperparameters["num_of_epoch"])):
        for b in range(len(hyperparameters["batch_size"])):
            for c in range(len(hyperparameters["learn_rate"])):
                for d in range(len(hyperparameters["alpha"])):
                        yield hyperparameters["num_of_epoch"]
[a], hyperparameters["batch_size"][b], hyperparameters["learn_rate"]
[c], hyperparameters["alpha"][d]

# This function will train the age regressor based on various
hyperparameters provided in the grid_search() function
def train_age_regressor():

    # Load data
    starting_x_tr = np.reshape(np.load("age_regression_Xtr.npy"), (-
1, 48*48))
    starting_y_tr = np.load("age_regression_ytr.npy")
    x_te = np.reshape(np.load("age_regression_Xte.npy"), (-1, 48*48))
    y_te = np.load("age_regression_yte.npy")
    x_tr, x_val, y_tr, y_val = train_test_split(starting_x_tr,
starting_y_tr, train_size=0.8)

    # Initialize best hyperparameter values as the worst they could
be
    best_error = 1000000
    best_num_of_epoch = -1
    best_batch_size = -1
    best_learn_rate = -1
    best_alpha = -1

    # Loop through each combination of the hyperparameters in
grid_search() to find the best combination to minimize MSE
    for num_of_epoch, batch_size, learn_rate, alpha in grid_search():

        w_trained, b_trained = find_lowest_error(x_tr, y_tr,
learn_rate, num_of_epoch, batch_size, alpha)

        # Calculate the MSE from the validation dataset
        error = calculate_error(x_val, w_trained, b_trained, y_val)
        print("train/validation unregularized MSE: ", error)

        # Store the hyperparameters that led to reduced error in
the following variables
        if error < best_error:
            best_error = error
            best_learn_rate = learn_rate
            best_num_of_epoch = num_of_epoch
            best_batch_size = batch_size
            best_alpha = alpha

    # Finally, calculate the error using the trained weights and
biases
    error = calculate_error(x_te, w_trained, b_trained, y_te)
    print("\n")
    print("Results of training:")
    print("best error from validation dataset: ", best_error)
    print("best learning rate: ", best_learn_rate)
    print("best number of epochs: ", best_num_of_epoch)
    print("best batch size: ", best_batch_size)
    print("best reg term: ", best_alpha)
    print("unregularized MSE from test dataset: ", error)

    return w_trained, b_trained

def main():

    print("Problem 3 Output:")
    w_output, b_output = train_age_regressor()

if __name__ == '__main__':
    main()
```

```
Problem 3 Output:
train/validation unregularized MSE:  nan

c:\Users\rakes\anaconda3\Lib\site-packages\numpy\core\
fromnumeric.py:86: RuntimeWarning: overflow encountered in reduce
  return ufunc.reduce(obj, axis, dtype, out, **passkwargs)
```

```
C:\Users\rakes\AppData\Local\Temp\ipykernel_11520\3979377061.py:36:
RuntimeWarning: invalid value encountered in subtract
  w = w - (learn_rate * d_of_w)
C:\Users\rakes\AppData\Local\Temp\ipykernel_11520\3979377061.py:37:
RuntimeWarning: invalid value encountered in subtract
  b = b - (learn_rate * d_of_b)

train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  170.21064791764127
train/validation unregularized MSE:  135.47496519515798
train/validation unregularized MSE:  150.15603253858993
train/validation unregularized MSE:  144.32203983141477
train/validation unregularized MSE:  136.35303705553747
train/validation unregularized MSE:  136.89091361137616
train/validation unregularized MSE:  137.40048043523007
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train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
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train/validation unregularized MSE:  nan
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train/validation unregularized MSE:  133.48721249704104
train/validation unregularized MSE:  135.72377069380403
train/validation unregularized MSE:  136.49086007970536
train/validation unregularized MSE:  136.12489959989404
train/validation unregularized MSE:  135.82999072190552
train/validation unregularized MSE:  135.80759045601127
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  inf

C:\Users\rakes\AppData\Local\Temp\ipykernel_11520\3979377061.py:10:
RuntimeWarning: overflow encountered in square
  return (1/(2 * a)) * np.sum(np.square(y_pred - y))
```

```
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  135.59989915424987
train/validation unregularized MSE:  135.38186982678764
train/validation unregularized MSE:  135.21050328093162
train/validation unregularized MSE:  135.74104086290356
train/validation unregularized MSE:  137.22904685927196
train/validation unregularized MSE:  136.1779473062348
train/validation unregularized MSE:  136.56620118664006
train/validation unregularized MSE:  136.90164715080323
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  inf
train/validation unregularized MSE:  8.752355009211058e+194
train/validation unregularized MSE:  7.300769064454156e+194
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train/validation unregularized MSE:  8.312149399119605e+194
train/validation unregularized MSE:  136.12378814775857
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train/validation unregularized MSE:  nan
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train/validation unregularized MSE:  134.92615134781076
train/validation unregularized MSE:  132.49832879195498
train/validation unregularized MSE:  144.82989083349224
train/validation unregularized MSE:  134.69786031123263
train/validation unregularized MSE:  135.86181118116647
train/validation unregularized MSE:  135.7498864444108
train/validation unregularized MSE:  134.4781895597993
train/validation unregularized MSE:  135.9777643322185
train/validation unregularized MSE:  nan
```

```
C:\Users\rakes\AppData\Local\Temp\ipykernel_11520\3979377061.py:28:
RuntimeWarning: overflow encountered in add
  y_pred = x.dot(w) + b
```

```
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
train/validation unregularized MSE:  nan
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train/validation unregularized MSE:  136.04411424209536
train/validation unregularized MSE:  136.27094595207345
train/validation unregularized MSE:  135.5147804150093
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train/validation unregularized MSE:  135.75219972793278
train/validation unregularized MSE:  135.44818788652464
train/validation unregularized MSE:  135.8371500736761


Results of training:
best error from validation dataset:  126.77897172963536
best learning rate:  0.001
best number of epochs:  20
best batch size:  10
best reg term:  0.1
unregularized MSE from test dataset:  139.17092912962255
```

```python
# Problem 4c

import numpy as np

def update_b_mae(X, y, w, b):

  # Calculate the predictions.
  predictions = X.dot(w) + b

  # Calculate the signs of the errors.
  errors = np.sign(predictions - y)

  # Update the bias term.
  b -= np.mean(errors)
  return b

def update_w_mae(X, y, w, b):

  # Calculate the predictions
  y_pred = X.dot(w) + b

  # Calculate the gradients
  grad_w = X.T.dot(np.sign(y_pred - y))

  # Update the weight vector
  w_new = w - grad_w

  return w_new

# Test both functions with initial values for X, y, w, and b
X = np.array([[1, 2], [3, 4], [5, 6]])
y = np.array([3, 7, 11])
w = np.array([1, 2])
b = 0
# Update both parameters.
b_new = update_b_mae(X, y, w, b)
w_new = update_w_mae(X, y, w, b)
print("Problem 4c Output:")
print(b_new)
print(w_new)
```

```
Problem 4c Output:
-1.0
[ -8 -10]
```