PROBABILITY & STATISTICS

Permutations :- Is an arrangement of objects.

Ex: In how many way 3-digit no can be formed using the digits (1, 2, 3, 4, 5) with and without repetations of digits.

Sol: (i) without repetation :

$$\underset{\text{O}\ \text{O}\ \text{O}}{5\times4\times3} \Rightarrow \frac{(5\times4\times3)\times(2\times1)}{(2\times1)} = \frac{5!}{2!} , \frac{5!}{(5-3)!} = {}^5P_3$$

(ii) with repeation:

$$\underset{\text{O}\ \text{O}\ \text{O}}{5\times5\times5} \Rightarrow 5^3$$

Without Repetation: ${}^nP_r$	With Repetation: n^r

Ex: In how many ways 5 persons can be arranged in a row.

Sol: $\underset{\text{O}\ \text{O}\ \text{O}\ \text{O}\ \text{O}}{5\times4\times3\times2\times1} = \frac{(5\times4\times3\times2\times1)}{(5-5)!} = {}^5P_5 = 5!$

Ex: In how many ways 10 books can be distributed to 15 students.

Sol: $\underset{\text{O}\ \text{O}\ \text{O}}{15\times15\times15}$ . . . upto 10^{th} book : 15^{10}

Ex: In how many ways 10 books can be distributed to 15 students so that no student can take more than 1 book:

Sol: $\underset{\text{O}\ \text{O}\ \text{O}}{15\times14\times13}$ . . . upto 10^{th} book = ${}^{15}P_{10}$

Ex: In how many ways 7 car accidents occured in a ~~days~~ week.

Sol: $\underset{\text{O}\ \text{O}\ \text{O}\ \text{O}\ \text{O}\ \text{O}\ \text{O}}{7\times7\times7\times7\times7\times7\times7} = 7^7$

Ex: 3 boys and 3 girls are standing random. (i) In how many ways we can arrange them:

(ii) All girls stand together

(iii) Boys & girls stand alternatively.

(iv) No 2 boys stands together.

Sol: $3B + 3G = 6G$

(i) $\underset{6}{O}\ \underset{5}{O}\ \underset{4}{O}\ \underset{3}{O}\ \underset{2}{O}\ \underset{1}{O} = 6!$ ($6 \times 5 \times 4 \times 3 \times 2 \times 1$)

(ii) $\overset{4}{B_1} \times \overset{3}{B_2} \times \overset{2}{B_3} \times \overset{1}{\underbrace{(G_1 G_2 G_3)}_{3!}} = 4! \times 3!$

(iii) $\overbrace{B_1 G_1 B_2 \cancel{B} G_2 B_3 G_3}^{3!}$ ($3!$) $= 3! \times 3!$

$+$

$\overbrace{G_1 B_1 G_2 B_2 G_3 B_3}^{3!}$ ($3!$) $= 3! \times 3!$

$\Big\}\ 3! \times 3! + 3! \times 3!$

(iv) ~~$B_1 G_1 B_2 G_2 B_3 G_3$~~

$\_\ G_1\ \_\ G_2\ \_\ G_3\ \_ = 3! \times {}^4P_3$

- No. of circular permutation of 'n' objects is $(n-1)!$

Ex: In how many ways 5 persons can be sitting around a round table for dinner?

Sol: $(5-1)! = 4!$

* No. of circular arrangement of 'n' objects either in clockwise or anti-clockwise $= \dfrac{(n-1)!}{2}$

Ex: In how many ways a garland maker can stich the flower to make a garland with 10 different flowers.

Sol: $\frac{(10-1)!}{2} = \frac{9!}{2}$

· Combinations: Selection of objects:

* No. of selection of 'r' objects from 'n' objects

$$^nC_r = \frac{^nP_r}{r!}$$

- $^nC_0 = {}^nC_n = 1$
- $^nC_1 = {}^nC_{n-1} = n$
- $^nC_r = {}^nC_{n-r}$

Ex: Out of 4 boys and 3 girls, a commity of 3 members is to form.

(i) In how many ways we can form a commity.

(ii) Commity should contain atleast one girl.

Sol: $4B + 3G = 7$

(i) $^7C_3 = \frac{7\times6\times5}{3\times2\times1} = 35$ ways

(ii) Atleast 1 girls: $^3C_1 \times {}^4C_2$

2 girl: $^3C_2 \times {}^4C_1$

3 girl: $^3C_3 \times {}^4C_0$

$(^3C_1 \times {}^4C_2) + (^3C_2 + {}^4C_1) + (^3C_3 \times {}^4C_0)$

$= (3\times6) + (3\times4) + (1\times1)$

$= 31$

EXPERIMENT

- Deterministic Experiment
- Random Experiment

• Deterministic Experiments

→ It is an experiment or observation in which we can predict the outcome.

Ex: 2+3=5 , Delhi is capital of India, Boiling point of H_2O is $100°c$ etc.

• Random Experiment:

→ It is an experiment or observation in which outcome is not predictable.

Ex: Tossing a coin, throwing a dice, hitting a target. etc

• Sample Space:

→ Set of all possible outcomes in a Random experiment is said to be Sample Space (S).

→ Some Examples of a Sample Space:

• Coins:

* If a single coin is tossed: S: {H, T} → n(S) = 2
* If 2 coins are tossed: S: {HH, HT, TH, TT} → n(S) = 4
* If 'n' coins are tossed: S: {...} → n(S) = 2^n

• Dice :

* If a single dice is rolled: S: {1,2,3,4,5,6} → n(S) = 6
* If 2 dices are rolled: S: { 1 1, 2 2, 3 3, 4 4, 5 5, 6 6 } → n(S) = 6^2

middle
↑

Sum of 2 dices can be: 2 3 4 5 6 7 8 9 10 11 12

for sum ($2 \le r \le 7$)

Number of ways: ${}^{r-1}C_{2-1} = {}^{r-1}C_1$

# of count/ways are same
(1,1) ; (6,6)

Sum	No. of ways	Sum
2	1	12
3	2	11
4	3	10
5	4	9
6	5	8
7	6	

* If 3 dices are rolled $\longrightarrow n(S) = 6^3$

Sum: 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

No. of ways are same

No. of way for sum 'r' $(3 \leq r \leq 8)$ $\ {}^{r-1}C_2$

for sum 9 and 12 = 25 ways

for sum 10 and 11 = 27 ways.

sum (r)	No. of ways
$3 \leq r \leq 8$	$\frac{(r-1)(r-2)}{2}$
9 & 12	25
10 & 11	27
$13 \leq r \leq 18$	$\frac{(19-r)(20-r)}{2}$

* If 4 dices are rolled $\rightarrow n(S) = 6^4$

Sum: 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

No. of ways are same

for sum $(4 \leq r \leq 14)$

${}^{r-1}C_3$

Ex: If a card is drawn from a pack of 52 cards then $n(S) = {}^{52}C_1 = 52$

- 52 cards
  - 26 black
    - 13 spades
    - 13 clubs
  - 26 Red
    - 13 diamonds
    - 13 Hearts

- J, Q, K, A $\longrightarrow$ Special Cards (16)
- J, Q, K $\longrightarrow$ Face Cards (12)
- J $\longrightarrow$ Knave (4)

- Event : Every outcome of a sample space is called an event (E)

  Ex: If a die is rolled $S = \{1, 2, 3, 4, 5, 6\}$
  - $\rightarrow$ getting a number 5 is an event $E = \{5\}$
  - $\rightarrow$ getting an even number is an event $E = \{2, 4, 6\}$

Probability :

- The probability of an event 'E' of a sample space 'S' is :-

$$* \quad \boxed{P(E) = \frac{\text{No. of the favourable cases of E in S}}{\text{No. of elements in 'S'}} = \frac{n(E)}{n(S)}}$$

$$* \quad \boxed{0 \leq P(E) \leq 1}$$

* If $P(E) = 0$, then 'E' is called impossible event.

$P(E) = 1$, then 'E' is called Sure Event.

* $P(S) = 1$

* $P(\phi) = 0$

* $P(\bar{E}) = 1 - P(E)$

* $0 \leq P(\bar{E}) \leq 1$

* If $E_1 \cap E_2 = \phi$ then the events are mutually exclusive.

i.e If E_1, E_2 are mutually exclusive, then $E_1 \cap E_2 = \phi$

$\Rightarrow P(E_1 \cap E_2) = P(\phi) = 0$,

~~* If a coin~~

Ex: If a coin is tossed getting head and getting tail are mutually exclusive.

i.e $E_1 = \{H\}$, $E_2 = \{T\}$, clearly $(E_1 \cap E_2) = \phi$

* $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ [Addition Theorem for Two Event]

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$ (If $E_1 \cap E_2 = \phi$

* $P(E_1 \cap \bar{E_2}) = P(E_1) - P(E_1 \cap E_2)$

Q: If 3 coins are tossed simultaneously find the probability of

(i) Getting atleast one head

(ii) atmost one head

(iii) no head.

Sol: $S = \{HHH, HHT, \ldots TTT\} \longrightarrow n(S) = 2^3$

(i) $P(\text{atleast one head}) = \frac{7}{8}$

(or)

$P(\text{atleast one head}) = 1 - P(\text{none})$

$= 1 - P(\text{no head})$

$= 1 - \frac{1}{8} = \frac{7}{8}$

(ii) $P(\text{atmost one head}) = P(\text{no head or one head})$

$= \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}$

(iii) $P(\text{no head}) = \frac{1}{8}$

GATE Q If 4 coins are tossed simultaneously, find the change of getting atleast one head and atleast 1 tail.

Sol: $P(\text{atleast one head and atleast one tail}) = \frac{14}{2^4} = \frac{14}{16} = \frac{7}{8}$

HHHH ×
TTTT ×

Q If dice are rolled simultaneously, find the probability of:

(i) Getting same number

(ii) Sum is neither 8 not 9

(iii) Sum is perfect square

Sol: (i) $P(\text{same number}) = \frac{6}{36} = \frac{1}{6}$

(ii) $P(\text{sum is neither 8 nor 9}) = 1 - P(\text{sum is 8 or 9})$

$= 1 - \frac{5+4}{36}$

$= 1 - \frac{9}{36} = \frac{3}{4} = 0.75$

(iii) $P(\text{sum of perfect square}) = P(\text{sum 4 or 9})$

$= \frac{3+4}{36} = \frac{7}{36}$

Q If 3 dices are rolled simultaneously, find the probability of getting sum on 3 dices is 9 or 16.

Sol. Sum 9: No. of ways $= 25$

Sum 16: $\frac{(19-r)(20-r)}{2} = 6$

(or)

Sum 5: $\frac{(r-1)(r-2)}{2} = 6$

(or)

${}^{r-1}C_2$

$P(\text{Sum 9 or 16}) = \frac{25+6}{6^3} = \frac{31}{216}$

Q If 4 dice are rolled, find the chance of getting

(i) Sum is 19

(ii) Sum is 21 or 22

Sol:

No. of ways of sum 19 = Sum of 9

here $r=9 \Rightarrow$ No. of ways $= {}^{r-1}C_3 = {}^{8}C_3 = 56$

(i) $P(\text{Sum } 19) = \frac{56}{6^4} = \frac{56}{1296}$

(ii) $P(\text{Sum } 21 \text{ or Sum } 22) = \frac{20+10}{6^4} = \frac{30}{1296}$

No. of ways for Sum 21 = No. of ways for Sum 7

here $r=7 \Rightarrow$ No. of ways $= {}^{6}C_3 = 20$

No. of ways for Sum 22 = No. of way for Sum 6

here $r=6 \Rightarrow$ No. of ways $= {}^{5}C_3 = 10$

Q If a card is drawn from a pack of 52 cards, find the probability that

(i) card is special card or a spade

(ii) card is diamond or a heart.

(iii) card is Red or a King.

Sol: (i) $P(\text{Special card or a spade}) = \frac{{}^{16}C_1 + {}^{13}C_1 - {}^{4}C_1}{{}^{52}C_1} = \frac{16+13-4}{52} = \frac{25}{52}$

(ii) $P(\text{Diamond or heart}) = \frac{{}^{13}C_1 + {}^{13}C_1}{{}^{52}C_1} = \frac{26}{52} = \frac{1}{2}$

(iii) $P(\text{Red or King}) = \frac{{}^{26}C_1 + {}^{4}C_1 - {}^{2}C_1}{{}^{52}C_1} = \frac{26+4-2}{{}^{52}C_1} = \frac{28}{52} = \frac{7}{13}$

Q Cards are dealting one by one till 'ACE' appears without replacement. Find the probability that 'ACE' appears in 4th draw.

Sol: $P(\text{'ACE' appears in 4th time}) = \frac{{}^{48}C_1}{{}^{52}C_1} * \frac{{}^{47}C_1}{{}^{51}C_1} * \frac{{}^{46}C_1}{{}^{50}C_1} * \frac{{}^{4}C_1}{{}^{49}C_1}$

$= \frac{48}{52} * \frac{47}{51} * \frac{46}{50} * \frac{4}{49}$

Q If A bag contains 5 red balls, 4 Yellow balls and 3 white balls all with different sizes. If 2 balls are selected at random. find the probability that

(i) Both are of same colour.

(ii) Both are of different colour.

Sol:
(i) P(both are same color) $= \frac{{}^5C_2 + {}^4C_2 + {}^3C_2}{{}^{12}C_2} \Rightarrow \frac{19\times 2}{12\times 11} = \frac{19}{66}$

(ii) P(both are ~~same~~ different colour) = 1 − P(same colour)

$= 1 - \frac{19}{66}$

$= \frac{47}{66}$

$\frac{{}^5C_1\,{}^4C_1 + {}^4C_1\,{}^3C_1 + {}^5C_1\,{}^3C_1}{{}^{12}C_2}$

Q Two tickets are selected randomly from 40 tickets numbered 1, 2, ... to 40. Find the probability that selected tickets are not consecutive.

Sol: $n(S) = {}^{40}C_2 = 20\times 39$

favourable cases for consecutive = $\{(1,2),(2,3),(3,4),\ldots,(39,40)\} \rightarrow n(E) = 39$

P(Two are not consecutive) = 1 − P(Two are consecutives)

$= 1 - \frac{39}{20\times 39} \Rightarrow 1 - \frac{1}{20}$

$= \frac{19}{20}$

Q An integer is selected randomly from first 200 positive integers. Find the probability that selected integers is divisible by 6 or 8.

Sol: 1, 2, 3, ..., 200

$n(S) = {}^{200}C_1 = 200$

6	8	6 & 8
6	8	24
12	16	48
18	24	⋮
⋮	⋮	
$\frac{200}{6} = 33$	$\frac{200}{8} = 25$	$\frac{200}{LCM(6,8)} = 8$

P(no. divisible by 6 or 8)

$= \frac{33+25-8}{200}$

$= \frac{1}{4} = 0.25$

Q A permutation is selected randomly from 10! permutations formed by the number 1, 2, 3, . . . , 10. Find the probability, 2 appears as a first even no. than any other even numbers.

Sol: $n(S) = {}^{(10)!}C_1 = (10)!$

favourable Case:

□ □ □ □ □ □ □ □ □ □

→ 9! ways (2 in 1^{st} place)

→ $5 \times 8!$ ways (2 in 2^{nd} place)

→ $5 \times 4 \times 7!$ ways (2 in 3^{rd} place)

→ $5 \times 4 \times 3 \times 6!$ ways (2 in 4^{th} place)

→ $(5 \times 4 \times 3 \times 2) \times 5!$ ways (2 in 5^{th} place)

→ $5! \times 4!$ ways (2 in 6^{th} place)

∴ Require Probability is

$$P(E) = \frac{(9!) + (5 \times 8!) + (5 \times 4 \times 7!) + (5 \times 4 \times 3 \times 6!) + (5 \times 4 \times 3 \times 2) \times 5! + (5 \times 4 \times 3 \times 2 \times 1) \times 4!}{(10)!}$$

Q A cow is tied to a pole with 100 meters long rope. Find the probability that at some point of time cow is atleast 60 metes aways from the pole.

Sol:

P(cow is atleast 60 m away from pole) = 1 − P(cow is within 60 m from starting point)

$$= 1 - \frac{\text{Area of small circle}}{\text{Area of big circle}}$$

$$= 1 - \frac{\pi(60)(60)}{\pi(100)(100)} = 1 - \frac{36}{100}$$

$$= \frac{64}{100} = \frac{16}{25}$$

Q A man is ~~th~~ alternatively throwing a coin ~~continuously~~ and a dice, starting with coin. Find the probability that he gets 'Head' first before he throw 5 or 6 on die.

Sol: $P(\text{head for coin}) = \frac{1}{2}$ $\qquad P(5 \text{ or } 6 \text{ die}) = \frac{2}{6} = \frac{1}{3}$

$P(\text{failure for coin}) = \frac{1}{2}$ $\qquad P(\text{failure on die}) = 1 - \frac{1}{3} = \frac{2}{3}$

	Coin	Die
1st trail	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{3}$
2nd trail	$\left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{1}{2}$	$\left(\frac{1}{2}\right)^2 + \frac{2}{3} \times \frac{1}{3}$
3rd trail	$\left(\frac{1}{2} \times \frac{2}{3}\right)^2 \times \frac{1}{2}$	

$\therefore \dots$ (Infinite way)

$$P(\text{head first } 5 \text{ or } 6) = \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 \times \frac{2}{3} + \dots$$

$$= \frac{1}{2}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{3}}\right] \qquad \because \left(1 + r + r^2 + \dots = \frac{1}{1-r}\right)$$

$$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4} = \underline{0.75}$$

- **Independent Events:**
  - Events in a sample space are said to be independent if the happing of one does not depends on happening on others.

    Ex: (i) If you perform same experiment repeated no of time all outcomes are independent.

    (ii) ~~Attem~~ Hitting a target by 3 persons are independent event.

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Note : Mutually Exclusive events are not independent.

Conditional Probability:

• If E_1, E_2 are two events of a sample space then the conditional probability of E_1 after the occurance of E_2 is

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}, \quad (P(E_2) \neq 0)$$

$$\Rightarrow \boxed{P(E_1 \cap E_2) = P\left(\frac{E_1}{E_2}\right) \cdot P(E_2)}$$ [Multiplication Theorem on Probability]

* If E_1, E_2 are independent then $P\left(\frac{E_1}{E_2}\right) = P(E_1)$, then

$$\cancel{P(E_1 \cap E_2)} \quad \boxed{P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)}$$ [from multiplication Theorem]

* If E_1, E_2 are independent then $\bar{E}_1$, $\bar{E}_2$ are also independent.

$$P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \cdot P(\bar{E}_2)$$

* Observation:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$= P(E_1) + P(E_2)$. [If E_1, E_2 are mutally exclusive]

$= P(E_1) + P(E_2) - P(E_1) \cdot P(E_2)$ [If E_1 E_2 are independent event]

Q In a city, 40% of ppl are eating chappati, 25% eating rice, 15% eating both chappati's and rice in ~~lunch~~ lunch time. A person is selected at random and observed that he is eating rice. Find the probability that he is not eating chappati's.

Sol: $P(C) = \frac{40}{100} = 0.4$

$P(R) = \frac{25}{100}$

$P(B) = \frac{15}{100} = P(C \cap R)$

$$P\left(\frac{\bar{C}}{R}\right) = \frac{P(\bar{C} \cap R)}{P(R)} = \frac{P(R) - P(C \cap R)}{P(R)} = 1 - \frac{P(C \cap R)}{P(R)}$$

$$= 1 - \frac{15}{25} = \frac{10}{25} = 0.4$$

Q Page number of 100 pages book are ~~pp~~ printed as 00, 01, 02, . . . , 99. Sum of two digits ~~one~~ one each page is denoted with X and product is Y. Find the probability for selected X = 9 if Y = 0

Sol: $$P\left(\frac{X=9}{Y=0}\right) = \frac{P(X=9 \cap Y=0)}{P(Y=0)} = \frac{\frac{2}{100}}{\frac{19}{100}} = \frac{2}{19}$$

Product Y = 0

00, 01, 02, ... 09, 10, 20, 30, ... 90 $\Big\}$ 19

Sum X = 9 & Y = 0

09, 90 $\Big\}$ ⇒ 2

Q If a die is rolled 3 times and the sum is noted. Find the probability that 5 appears 3rd time if the sum is found to be 16.

Sol: $$P\left(\frac{5 \text{ appears } 3^{rd} \text{ time}}{\text{sum} = 16}\right) = \frac{P(5 \text{ on } 3^{rd} \text{ time} \cap \text{sum} = 16)}{P(\text{sum} = 16)} =$$

$$\Rightarrow \frac{\frac{2}{6^3}}{\frac{6}{6^3}} = \frac{1}{3}$$

No. of ways sum = 16

$= \frac{(19-r)(20-r)}{2}$

$= 6$

sum = 16 & 5 appears on 3rd time

6 5 5

5 6 5

Q. In a village every family have 5 children and the chance of the child is girl is 0.5. Find the probability that randomly selected family has atleast one boy if that family is already having atleast one girl.

Sol: P(child is girl) = 0.5

P(child is boy) = 0.5

P(atleast one girl) = 1 - P(no. girl)

= 1 - P(all boys)

= $1-((0.5)^5)$

= $\frac{31}{32}$

$$P\left(\frac{\text{atleast one boy}}{\text{atleast one girl}}\right) = \frac{P(\text{atleast one boy} \cap \text{atleast one girl})}{P(\text{atleast one girl})}$$

$$= \frac{1 - P(\text{all boys or girls})}{31/32}$$

$$= \frac{1-\frac{2}{32}}{31/32} = \frac{30/32}{31/32}$$

$$= 30/31$$

Q. If $P(A)=1$, $P(B)=\frac{1}{2}$, find $P(A/B)$, $P(B/A)$.

Sol. $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{1/2}{1/2} = 1$

$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{1/2}{1} = \frac{1}{2}$

$P(A) = 1 \Rightarrow A = S$

$A\cap B = B$

$P(A\cap B) = P(B) = 1/2$

Q The chance of A can solve the problem is 1/2, the chance of B can solve it is 1/3, C can solve the problem 1 out of 4 trails (1/4). Find the probability that atleast can solve the problem.

Sol: $P(A) = \frac{1}{2} \Rightarrow P(\bar{A}) = \frac{1}{2}$

$P(B) = \frac{1}{3} \Rightarrow P(\bar{B}) = \frac{2}{3}$

$P(C) = \frac{1}{4} \Rightarrow P(\bar{C}) = \frac{3}{4}$

$P(\text{atleast one}) = P(A \cup B \cup C)$

$= 1 - P(\text{none})$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$ [These are independent events]

$= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= \frac{3}{4}$

Q The chance of 'A' telling truth is 70%, chance of 'B' telling truth is 60%. Find the probability that:

(i) Atleast one of them is lying.

(ii) Exactly one is lying.

Sol. $P(A) = \frac{7}{10} \Rightarrow P(\bar{A}) = \frac{3}{10}$

$P(B) = \frac{6}{10} \Rightarrow P(\bar{B}) = \frac{4}{10}$

These are independent events

(i) $P(\text{atleast one is lying}) = P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

$= 1 - P(A) \cdot P(B)$

$= 1 - \frac{7}{10} \cdot \frac{6}{10}$

$= \frac{58}{100} = 58\%$

$= P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) + P(\bar{B}) - P(\bar{A}) \cdot P(\bar{B})$

$= 0.3 + 0.4 - 0.3 \times 0.4$

$= 0.7 - 0.12$

$= 0.58$

(ii) $P(\text{exactly one is lying}) = P(\bar{A} \cap B) + P(A \cap \bar{B})$ [One is truth at the same time (∩) other is lying]

$= P(\bar{A}) \cdot P(B) + P(A) \cdot P(\bar{B})$

$= 0.3 \times 0.6 + 0.7 \times 0.4$

$= 0.46$

Q 4 hydralic gates are operating independently, the chance of failure of each gate is 0.2. If the gates 1, 2 and 3 are already failed. Find the chance of failure of 4th gate also.

Sol: Since all are independent events so chance of failure of 4th gate = $\underline{0.2}$

$$P(G_1) = P(G_2) = P(G_3) = P(G_4) = 0.2$$

$$P\left(\frac{G_4}{G_1 \cap G_2 \cap G}\right) = \frac{P(G_4 \cap (G_1 \cap G_2 \cap G_3))}{P(G_1 \cap G_2 \cap G_3)} = \frac{P(G_4) \cap \cancel{P(G_1 \cap G_2 \cap G_3)}}{\cancel{P(G_1 \cap G_2 \cap G_3)}}$$

$$= P(G_4) = \underline{0.2}$$

Q If A, B are independent events such that $A \cup B = 0.5$ & $P(A) = P(B)$; find $P(A)$

Sol: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad (P(A \cup B) = 0.5)$

$$0.5 = P(A) + P(A) - P(A) \cdot P(A)$$

$$0.5 = 2P(A) - |P(A)|^2 \quad (\because P(A) = P(B))$$

$$|P(A)|^2 - 2P(A) + 0.5 = 0$$

$$2P(A)^2 - 4P(A) + 1 = 0$$

$$\therefore P(A) = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

Only valid root is:

$$P(A) = \frac{2-\sqrt{2}}{2}$$

$$= \underline{0.29}$$

Bayes's Theorem: (Theorem of causes):

Statement: If $E_1, E_2, \ldots E_n$ are mutually exclusive events of a sample space 'S' suchthat $\sum_{i=1}^{n} E_i = S$. Let A be any arbitary event of 'S' such that $A \subseteq \bigcup_{i=1}^{n} E_i$. Then the conditional probability of E_1 after the occurance of A is

$$\boxed{P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(A/E_i)\cdot P(E_i)}{\sum P(A/E_i)\cdot P(E_i)}}$$

$\left[\frac{\text{for one event}}{\text{All event}}\right]$

Example: In a college 25% of the boys and 10% of the girls studying mathematics. Girls constitute 60% of whole body. If a student is selected at random and found to be studying mathematics, find the probability that the student is a girl. Find the probability of students studying mathematics (in %).

Sol: $P(G) = 60\% = 0.6$ $\quad P\left(\frac{MB}{B}\right) = 0.25 = 25\%$

$P(B) = 40\% = 0.4$ $\quad P\left(\frac{MG}{G}\right) = 10\% = 0.1$

(i) $P\left(\frac{G}{M}\right) = ? = \dfrac{P(G \cap M)}{P(M)} = \dfrac{P\left(\frac{M}{G}\right)\cdot P(G)}{P\left(\frac{M}{G}\right)\cdot P(G) + P\left(\frac{M}{B}\right)\cdot P(B)}$

$$= \frac{0.1 \times 0.6}{0.1 \times 0.6 + 0.25 \times 0.4}$$

$$= \frac{3}{8}$$

(ii) $P(M) = P\left(\frac{M}{G}\right)\cdot P(G) + P\left(\frac{M}{B}\right)\cdot P(B) = 0.16 \Rightarrow 16\%$

Q A chemical test is carried out to detect a certain disease. 15% of those positive and 12% of those negative, reports are incorrect. The chance that a patient receives positive report is 0.01. Find the probability that the report given to him is incorrect.

Sol: Given, $P(\frac{I}{P}) = 15\% = 0.15$ $\quad P(P) = 0.01$

$P(\frac{I}{N}) = 12\% = 0.12$ $\quad P(N) = 1 - 0.01 = 0.99$

$$\Rightarrow P(\text{Incorrect}) = P(P \cap I) + P(N \cap I)$$

$$= P(\tfrac{I}{P}) \cdot P(P) + P(\tfrac{I}{N}) \cdot P(N)$$

$$= 0.15 \times 0.01 + 0.12 \times 0.99$$

$$= 0.12$$

* Statistics:

- The term "statistics" refer to the collection of data, analysis of data and interpretion of data.
- In the process of analysing the data we can measure the center of the data using mean, mode, median.
- Ther term dispersion refers to the distance of the data points scattered away from the centre.
- We can measure the dispersion of the data using Range, Quartile deviation (Q.D), Mean deviation (M.D), Standard deviation (S.D).

Measure of the centre of the data:

(i) Mean:

- Mean for individual Observations
- Mean for Grouped data

• Mean for individual Observations:

• If $x_1, x_2, \ldots x_n$ are individual observations then

$$\text{Mean } (\mu) = \text{Average} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

Example: Marks of a student in 4 subjects are 90, 80, 60, 50

$$\text{Avg marks} = \frac{90 + 80 + 60 + 50}{4} = 70$$

• Mean for Grouped data:

x	x_1	x_2	$\ldots$	x_n
no. of observations	f_1	f_2	$\ldots$	f_n

$$\text{Mean } (\mu) = \frac{\Sigma xf}{\Sigma f}$$

Example:

x (weight of students)	60	70	50
f (no. of students)	10	15	15

$$\text{Mean weight of students} = \frac{\Sigma xf}{\Sigma f}$$

$$= \frac{(60 \times 10) + (70 \times 15) + (50 \times 15)}{10 + 15 + 15}$$

$$= \frac{600 + 1050 + 750}{40} = \frac{240}{4} = 60$$

(ii) Mode:

- If the data points are uncountable we can measure the centre of the data using mode.

* • Mode is the most repeated value in the observations.

• Mode for Individual Observation:

Example: Find the mode of the following data:

3, 7, 6, 11, 4, 11, 6, 6, 7, 6, 5, 4, 6, 6, 5, 6, 6

Here 6's are repeated more no. of time.

∴ Mode = 6

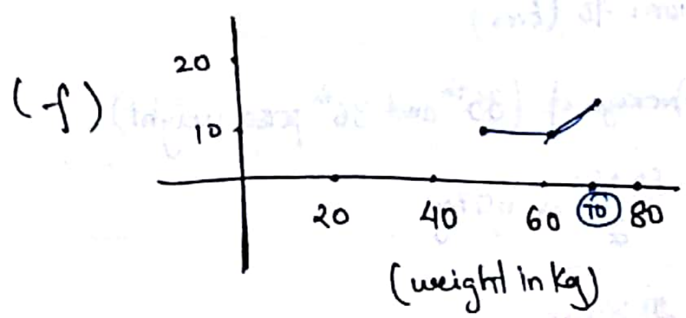
• Mode for Grouped data:

Example: Find the mode of following data:

x (weight in Kgs)	f (no. of students)
60	10
70	15
50	10

Repeated more no. of time ← 70

∴ mode = 70



*Mode is a point of maxima

(iii) Median:

- Median is the value that divides entire distribution into two equal parts. To do that first it should be in ascending or descending order.

- Measure of median:
  - arrange the data points in ascending or descending order.
  - If the no of observations are odd then median is the exact middle observation.
  - If the no. of observations are even then median is the average of middle 2 observation.

- Median for individual observation:

Example: Find the median of : 3, 4, 11, 8, 6 (odd)

Sol: Ascending : 3, 4, 6, 8, 11

Median

Example: Find the median of : 3, 10, 8, 6, 4, 7 (even)

Sol: Ascending : 3 4 6 7 8 10

Median = $\frac{6+7}{2} = 6.5$

- Median for Grouped data:

Example:

x (weight in kgs)	f (no. of students)
50	10
60	40
70	20

If the x's is not in order. make it. But it is already ascending order

Sol: Total no. of observation: 70 (Even)

Median = Average of (35th and 36th person weight)

$= \frac{60+60}{2} \Rightarrow 60 \text{ Kg}$

$\boxed{\text{Empirical Mode} = 3 \text{ Median} - 2 \text{ Mean}}$

* This doesn't give exact value. Only use if data is not given but median & mean is given.

Measure Of Dispersion:

(i) Variance:

- Varience for individual observation:
  - If $x_1, x_2 \ldots x_n$ are 'n' observation and μ is mean.

Then
$$\boxed{\text{variance } \sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$$

$$\boxed{\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}}$$

- Varience for Grouped data:

x	x_1	x_2	$\ldots$	x_n
f	f_1	f_2	$\ldots$	f_n

$$\text{Varience}(\sigma^2) = \frac{f_1(x_1-\mu)+f_2(x_2-\mu)+\ldots+f_n(x_n-\mu)}{f_1+f_2+\ldots+f_n}$$

$$\boxed{\sigma^2 = \frac{\sum_{i=1}^{n} f_i\,(x_i-\mu)}{\sum_{i=1}^{n} f_i}}$$

$$\boxed{\text{Standard } \cancel{\text{variance}}\text{ deviation } (\sigma) = \sqrt{\text{varience}}}$$

Example: find the Standard deviation of following data:

34, 35, 35, 16

Sol: Mean $(\mu) = \frac{34+35+35+16}{4} = \frac{120}{4} = 30$

$$\text{Varience}(\sigma^2) = \frac{(34-30)^2+(35-30)^2+(35-30)^2+(16-30)^2}{4} = \frac{262}{4} = 65.5$$

$$\text{Standard Deviation}(\sigma) = \sqrt{65.5} = 8.07$$

$$\boxed{6\,Q.D = 5\,M.D = 4\,S.D}$$

* This gives the exact values.

Random Variable:

• Random variable is an function defined from $S: \to R$, where S - sample space corresponding to the given experiment and R real no. set.

• It is denoted $X: S \to R$

• Random variables are of 2 types:

(i) Discrete Random variable.

(ii) Continuous Random variable.

• **Discrete Random Variable and its distribution:**

• If a Random variable 'X' gives countable no. of values [Discrete] that Random variable is called Discrete Random Variable.

• The corresponding distribution is

x	$x_1 \quad x_2 \ldots x_n$
$p(x)$	$p(x_1)\ p(x_2) \ldots p(x_n)$

Here $p(x)$ is called discrete probability mass function.

* $\sum_{i=1}^{n} p(x) = 1$

* $P(x) \geq 0$

* Mean $= \dfrac{\sum x\,p(x)}{\sum p(x)} = \sum x\,p(x) \quad (\because \sum p(x) = 1)$

$$\boxed{\therefore \text{Mean } (\mu) = \text{Expected value of } X\ (E(x)) = \sum x\,p(x)}$$

* Varience $\sigma^2 = \dfrac{\sum (x-\mu)^2 p(x)}{\sum p(x)} \qquad (\because \sum p(x) = 1)$

$= \sum (x-\mu)^2 \cdot p(x)$

$= \sum x^2 p(x) - 2\mu \sum x\,p x + \mu^2 \sum p(x).$

$= \sum x^2 p(x) - 2\mu^2 + \mu^2$

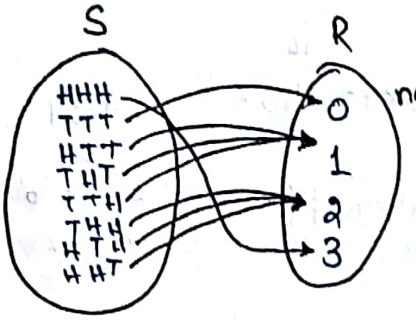
Varience $\sigma^2 = V(x) = \sum x^2 p(x) - \mu^2$

$$\boxed{\therefore V(x) = E(x^2) - [E(x)]^2}$$

$S.D = \sqrt{\text{Varience}}$

Example: If a coin is tossed 3 times. Find the expected no. of Heads. Find S.D and $P(0 < X < 3)$.

Sol:



x (no. of heads)	0	1	2	3
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

(i) Expected no. of heads $E(x) = \sum x\,p(x) = (0\times\frac{1}{8}) + (1\times\frac{3}{8}) + (2\times\frac{3}{8}) + (3\times\frac{1}{8}) = \frac{12}{8} = 1.5$

Varience $V(X) = \sum x^2 p(x) - \mu^2$

$= (0^2\times\frac{1}{8}) + (1^2\times\frac{3}{8}) + (2^2\times\frac{3}{8}) + (3^2\times\frac{1}{3}) - (\frac{3}{2})^2$

$= 3 - \frac{9}{4} = \frac{3}{4}$

(ii) S.D $(\sigma) = \sqrt{3/4} = \frac{\sqrt{3}}{2}$

(iii) Now, $P(0 < X < 3) = P(X=1) + P(X=2)$

$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4} = 0.75$

* Q A player wins ₹1000 if prime no. comes as an outcome in a single throw of die, he wins ₹1500 if composite no. comes as an outcome. Otherwise he has to pay ₹6000. What is the expected gain of a players?

Sol.

x (gain in ₹)	$+1000$	$+1500$	-6000
$P(x)$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

Expected gain of a player $= E(X) = \sum x \cdot p(x)$

$= 1000\times\frac{3}{6} + 1500\times\frac{2}{6} + (-6000)\times\frac{1}{6}$

$= 0$

* Q A question paper consisting of 150 MCQ each with 4 options. Every correct answer carries 1 Marks and each wrong answer carries $\frac{1}{4}$ negative mark. If a student attempt every question. Find the expected marks of a student.

Sol: for 1 question:

x(marks)	1	-1/4
P(x)	1/4	3/4

Expected marks for 1 question $= \Sigma x p(x)$

$= \frac{1}{16}$

$\therefore$ Expected marks of a student $= 150 \times \frac{1}{16} = 9.37$

Q Random variable 'x' has a distribution function $P(x) = \begin{cases} 0.4 & x=0 \\ 0.6 & x=1 \end{cases}$, find varience of x.

Sol.

x	0	1
P(x)	0.4	0.6

Varience $= E(x^2) - [E(x)]^2$

$= \Sigma x^2 p(x) - (\Sigma x p(x))^2$

$= (0^2 \times 0.4 + 1^2 \times 0.6) - (0 \times 0.4 + 1 \times 0.6)^2$

$= 0.6 - 0.36$

$= 0.24$

Binomial Distribution: (Bernaulli's distribution for 'n' trials).

- If an experiment is conducted 'n' times independently and if 'p' is probability of success, 'q' is the probability of failure; then the binomial expansion,

$$(q+p)^n = {}^nC_0 \cdot q^n + {}^nC_1 \cdot p \cdot q^{n-1} + {}^nC_2 \cdot p^2 \cdot q^{n-2} + \dots + {}^nC_x \cdot p^x \cdot q^{n-x} + \dots + {}^nC_n \, p^n$$

the distribution is:

x	0	1	2	.. x	... n
P(x)	${}^nC_0 q^n$	${}^nC_1 \cdot p \cdot q^{n}$	${}^nC_2 \, p^2 \cdot q^{n-2}$	${}^nC_r p^r q^{n-r}$	${}^nC_n \cdot p^n$

* The probability of 'x' success in Binomial distribution for 'n' independent trails is $\boxed{p(x) = {}^nC_x \cdot p^x \cdot q^{n-x}}$

* Mean $(\mu) = \Sigma x \, p(x) = \sum_{x=0}^{n} x \, {}^nC_x \cdot p^x \cdot q^{n-x} = np(q+p)^{n-1} = n \cdot p (1)^{n-1}$

$\boxed{\therefore \text{Mean} = np}$

$\boxed{\text{varience} = npq}$

$\boxed{S.D = \sqrt{npq}}$

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Q If a coin is tossed 6 times, find the chance of getting Head more no. of times? (14)

Sol: $n=6$

$P(\text{head}) = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

H	T
0	6 ✗
1	5 ✗
2	4 ✗
3	3 ✗
4	2 ✓
5	1 ✓
6	0 ✓

P(head appears more no. of time)

$= P(X=4) + P(X=5) + P(X=6)$

$= {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 + {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}^6C_6 \left(\frac{1}{2}\right)^6$

$= \frac{11}{32}$

Q If a coin is tossed ~~repeatedly~~ repeatedly. find the probability that head appears 6th time in 11th toss.

Sol: $P(\text{head}) = p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

P(head appears 6th time in 11th toss) = P(5 head appears in first 10 tosses) × P(head in 11th toss)

$= \left| {}^{10}C_5 \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5 \right| \times \frac{1}{2} = \frac{{}^{10}C_5}{2^{11}}$.

Q The chance of a man takes a step forward is 0.4 and step backward is 0.6. find the probability that ~~after steps~~ he is one step ahead after 11 steps from starting point.

Sol: $P(\text{forward}) = 0.4$

$P(\text{backward}) = 0.6$

From the given data at the end of 10 steps he will be in starting point.

P(one step away from starting point at 11th step)

P(one step away from starting point at 11th step)

= P(11th step is a 6th forward step or 6th backward step)

$= {}^{11}C_6 (0.4)^6 \times (0.6)^5 + {}^{11}C_6 (0.6)^6 \times (0.4)^5$

$= {}^{11}C_6 (0.4)^5 \cdot (0.6)^5 (0.4 + 0.6)$

$= 462 \times (0.24)^5$

Q The chance of man hitting a target is $1/2$. How many times he has to fire so that ~~change~~ chance hitting a target atleast once is greater than 90%.

Sol: Given, P(hitting a target) = $1/2 \Rightarrow q = \frac{1}{2}$

also given $P(x \geq 1) > 90\% \Rightarrow 1 - P(x=0) > 0.9$

$$= \quad 0.1 > P(x=0)$$

$$\Rightarrow P(x=0) < 0.1$$

$${}^{n}C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^{n-0} < 0.1$$

$$1 \cdot 1 \cdot \left(\tfrac{1}{2}\right)^n < 0.1$$

$\therefore$ for $n=1$: $\left(\frac{1}{2}\right)^1 \not< 0.1$

$\vdots$

for $n=4$: $\left(\frac{1}{2}\right)^4 < 0.1$

$\therefore$ He has to fire atleast 4 times to satisfy the given condition.

Q The mean and variance of a binomial distribution is 6 and 4 respectively. Find 'n'.

Sol: Given: $np = 6$

$npq = 4$

$\Rightarrow 6q = 4$

$q = 2/3 \Rightarrow p = 1/3$

$\therefore np = 6$

$\frac{n}{3} = 6 \Rightarrow n = 18$

NOTE: In a binomial distribution $\boxed{\text{Varience} < \text{Mean}}$

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Poisson Distribution:

- for sufficiently large values of 'n' and for small values of ~~small~~ 'p' binomial distribution approaches to Poisson distribution i.e. ~~for binom.~~ poisson distribution is a limiting case of binomial distribution.

* The probability of 'x' successes in a poisson distribution is

$$\boxed{P(x) = \frac{e^{-\mu} \cdot \mu^x}{x!}}, \text{ where } \mu = np = \text{Mean of B.D.}$$

* $$\boxed{\text{Mean} = \sum_{x=0}^{\infty} x\, p(x) = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\mu} \cdot \mu^x}{x!} = \mu}$$

* ∴ Mean = $np = \mu$
  |||^ly varience = $\mu = np$
  S.D = $\sqrt{np}$

NOTE: for a Poisson distribution

$$\boxed{\text{Mean} = \text{Varience} = \mu = np}$$

Q The chance that a patient suffers from bad reaction of an injection is 0.001. The probability that non-of them suffers from bad reaction out of 2000 patients, find the probability that non-of them suffers from bad reaction of an injection.

Sol: P(patient suffers from bad reaction of an injection) = $0.001 = \frac{1}{1000} = p$

$n = 2000$

$\mu = np = 2000 \times \frac{1}{1000} = 2$

$\mu = 2$

P(no one suffers from bad reaction) = $P(X=0) = \frac{e^{-\mu} \cdot \mu^x}{x!} = \frac{e^{-2} \cdot 2^0}{0!} = \frac{1}{e^2}$ //

Q 2% of the computer components produced by a company are defective. If 100 items/components selected at random, find the probability that atmost 2 items are defective.

Sol: $P(\text{defective item}) = \frac{2}{100}$

$n = 100$

$\mu = n \times p = 100 \times \frac{2}{100} = 2$

$P(\text{atmost 2 defective component}) = P(X \le 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{e^{-(2)} 2^0}{0!} + \frac{e^{-2}(2)^1}{1!} + \frac{e^{-2}(2^2)}{2!}$

$= \frac{5}{e^2}$

Q On an average 240 vechiles are passing through a junction on one specific highway location in one hour duration. If the arrival of vechicles follows poisson distribution, find the probability that:-

(i) atleast two vechiles passing through the junction over 30 seconds time interval.

(ii) no vechicle is passing through the junction over 30 sec time interval.

Sol: $\mu = 240$ vech/hour i.e 60 min — 240

1 min — $\frac{240}{60} = 4$ vech

30 sec — 2 vech

$\therefore \mu = 2$ vech

6:30-1 Prob @ 309

2 to 7 30 CN @ 101

(i) $P(\text{atleast two vechiles}) = P(X \ge 2) = 1 - P(X < 2)$

$= 1 - (P(X=0) + P(X=1))$

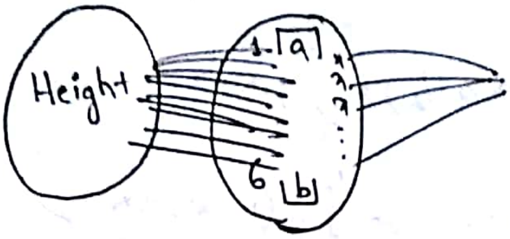
$= 1 -$

(ii) $P(\text{none}) = P(X=0) = \frac{e^{-4} \cdot \mu^0}{0!}$

Continuous Random Variable:

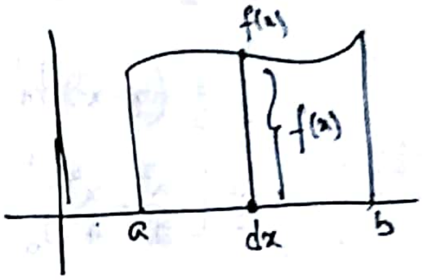
- If a random variable X gives continous values (uncountable values) which falls in the interval [a, b] is called continuous random variable.

Example: Height, weight, Time etc.



$f(x)$: Probability Density Function (P.D.F)

Here $f(x)\,dx$ = Probability at some ~~these~~ point in [a, b]



Now; $P(a \le x \le b) = \lim_{n \to \infty} \underbrace{\sum f(x)\,dx}_{n}$

- $\int_a^b f(x)\,dx$ = Area under the curve $f(x)$ b/w a and b.

* $\int_{-\infty}^{\infty} f(x)\,dx = 1$

* Mean = $\int_a^b x\, f(x)\,dx$ [Mean = $\sum x\,p(x)$]

* Varience = $\int_a^b x^2 f(x)\,dx - \mu^2$ [Varience = $\sum x^2 p(x) - \mu^2$]

* S.D = $\sqrt{\text{varience}}$

Q The p.d.f of a continuous random random X is

$$f(x) = \begin{cases} K(1-x^2) & : 0 \le x \le 1 \\ 0 & : \text{otherwise} \end{cases}$$

find (i) k
(ii) Mean
(iii) $P(X \ge 1/2)$

Sol: (i) we have $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 f(x)\,dx = 1$

$$\Rightarrow \int_0^1 k(1-x^2)\,dx = 1$$

$$\Rightarrow k\left(x-\frac{x^3}{3}\right)_0^1 = 1$$

$$k\left(1-\frac{1}{3}\right) = 1$$

$$k = 3/2$$

(ii) Mean $\mu = \int_0^1 x\,f(x)\,dx$

$$= \int_0^1 x\,\frac{3}{2}(1-x^2)\,dx$$

$$= \int_0^1 \frac{3}{2}(x-x^3)\,dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1}{2}-\frac{1}{4}\right]$$

$$= \frac{3}{2}\cdot\frac{1}{4}$$

$$= \frac{3}{8}$$

(iii) $P(x \geq 1/2) = \int_{1/2}^{\infty} f(x)\,dx$

$$= \int_{1/2}^{1} \frac{3}{2}(1-x^2)\,dx$$

$$= \frac{3}{2}\left(x-\frac{x^3}{3}\right)_{1/2}^{1}$$

$$= \frac{3}{2}\left[\left(1-\frac{1}{3}\right)-\left(\frac{1}{2}-\frac{1}{24}\right)\right]$$

$$= \frac{3}{2}\left[\frac{2}{3}-\frac{11}{24}\right]$$

$$= \frac{5}{16}$$

Q The p.d.f of a continuous random variable x is $f(x) = \begin{cases} e^{-2x} & : x \geq 0 \\ 0 & : \text{otherwise} \end{cases}$

Find $P(0 \leq x < 2)$

Sol: $P(0 \leq x < 2) = \int_0^2 f(x)\,dx = \int_0^2 e^{-2x}\,dx = \left(\frac{e^{-2x}}{-2}\right)_0^2 = -\frac{1}{2}\left(e^{-4}-e^{0}\right)$

$$= -\frac{1}{2}\left[\frac{1}{e^4}-1\right]$$

$$= \frac{1}{2}-\frac{1}{2e^4}$$

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Continuous Distribution:

• Normal Distribution: (Gaussian Distribution):

• The p.d.f of a continuous random variable X in normal distribution is

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

, where μ = mean, σ = S.D
Here n is neither large nor p is small.

• Area Property of a Normal Curve:

• $$P(\mu < X < x) = \int_{\mu}^{x} f(x)dx = \int_{\mu}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $z = \frac{x-\mu}{\sigma}$

$x = \sigma z + \mu$

$dx = \sigma\, dz$

$x : \mu \to x$

$z : 0 \to z$

$$\Rightarrow P(\mu < X < x) = P(0 < Z < z)$$

$$= \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} (\sigma\, dz)$$

$$= \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= \int_0^z \phi(z)\, dz,$$ where $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ is the probability distribution function of a standard normal variable.

• Properties of Normal Curve:

• Normal curve is bell shaped and symmetric about the line $x = \mu$.

• Mean, mode and median of a normal distribution are equal.

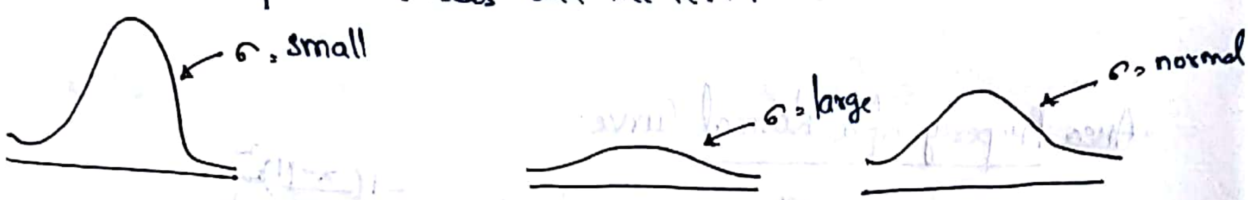
• Total area under the normal curve is Unity.

- X-axis is an asymptote of Normal curve [~~curve~~ lines ~~or~~ intersect at ∞]
- Maximum value of a Normal curve is $f(\mu)$

$$f(\mu) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{\mu-\mu}{\sigma}\right)^2}$$

$$\boxed{f(\mu) = \frac{1}{\sqrt{2\pi}\,\sigma}}$$

- If 'σ' increases $f(x)$ decreases and vice versa.

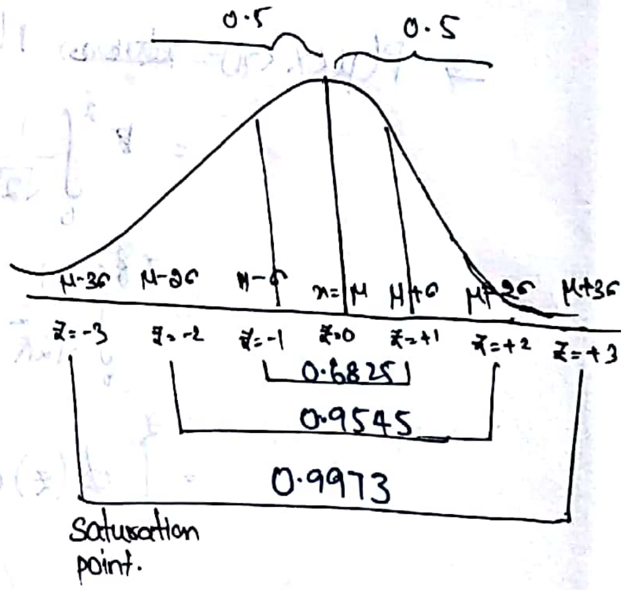


- "$x = \mu \pm \sigma$" are the point of inflections of a normal curve.

we have $z = \frac{x-\mu}{\sigma}$

when $x = \mu - \sigma$: $z = \frac{\mu - \sigma - \mu}{\sigma} = -1$

when $x = \mu + \sigma$: $z = \frac{\mu + \sigma - \mu}{\sigma} = 1$



$$P(\mu-\sigma < x < \mu+\sigma) = P(-1 < z < 1)$$

$$= 2\int_{z=0}^{1} \phi(z)\,dz$$

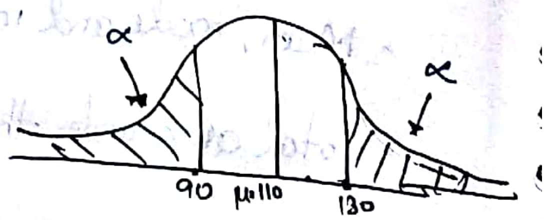
$$= 2 \times 0.3413$$

$$= \underline{\underline{0.6826}}$$

Q A random variable X is normally distributed with mean $\mu = 110$. If $P(x > 130) = \alpha$ find $P(90 < x < 130)$:

Sol: $P(90 < x < 130) = 1 - (\alpha + \alpha)$

$= \underline{\underline{1 - 2\alpha}}$



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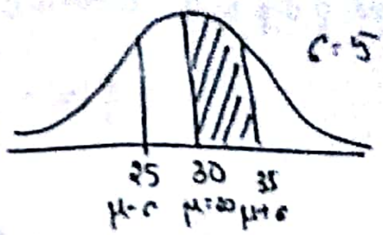
Q. A random variable X is normally distributed with mean $\mu = 30$ & $\sigma = 5$ find $(30 < x < 35)$



Sol: $P(30 < X < 35) = P\left(\dfrac{\mu - \sigma < X < \mu + \sigma}{2}\right)$

$= \dfrac{0.6826}{2}$

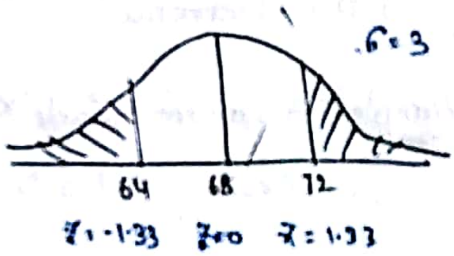
$= \underline{\underline{0.3413}}$

Q. The masses of 300 students are normally distributed with $\mu = 68$ Kg & $\sigma = 3$ Kg. Find the no. of students whose masses are

(i) greater than 72 Kg

(ii) less than 64 Kg

(iii) b/w 65 to 71 Kg (Area under the normal curve from $z=0$ to $z = 1.33$ is 0.4081)



Sol: (i) & (ii)

we have $z = \dfrac{x - \mu}{\sigma}$

when $x = 72$ then $z = \dfrac{72 - 68}{3} = \dfrac{4}{3} = 1.33$

$P(x > 72) = P(z > 1.33) = 0.5 - \int_0^{1.33} \phi(z)\,dz$

$= 0.5 - 0.4081$

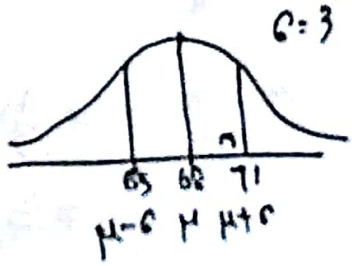
$= 0.0919$

$\therefore$ The no. of student whose masses are greater than 72 Kgs $= 0.0919 \times 300$

$= 27.57 \cong \underline{\underline{28}}$

(iii) $P(65 < x < 71) = P(\mu - \sigma < X < \mu + \sigma)$

$= 0.6826$



$\therefore$ No. of students whose mass is b/w 65 to 71 is

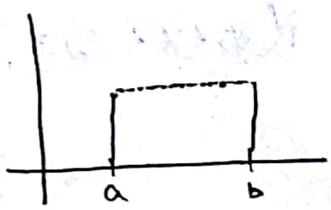
$= 0.6826 \times 300$

$= 204.7 \cong 205$

Uniform Distribution: (Rectangular Distribution)

- The p.d.f of a continuous random variable x in uniform distributions

$$f(x) = \begin{cases} \frac{1}{b-a} & : a \leq x \leq b \\ 0 & : \text{otherwise} \end{cases}$$



* Mean $= \int_a^b x f(x)\,dx = \int_a^b x \cdot \frac{1}{(b-a)}\,dx$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a}\cdot\left(\frac{b^2-a^2}{2}\right)$$

$$= \frac{a+b}{2}$$

* Variance $= \frac{(b-a)^2}{12}$

* S.D $= \sqrt{\text{varience}}$

Example: A random variable X is uniformly distributed in [1,3]. find:
(i) Mean (ii) S.D (iii) $E(x^3)$ (iv) $E(2x+1)$

Sol: (i) Mean $= \frac{a+b}{2} = \frac{1+3}{2} = 2$

(ii) S.D $= \sqrt{\frac{(3-1)^2}{12}} = \sqrt{\frac{1}{3}}$

(iii) $E(x^3) = \int_a^b x^3 f(x)\,dx = \int_1^3 x^3\left(\frac{1}{3-1}\right)dx = \frac{1}{2}\left[\frac{x^4}{4}\right]_1^3 = \frac{1}{2}\left[\frac{81-1}{4}\right] = 10$

(iv) $E(2x+1) = \int_a^b (2x+1)\, f(x)\,dx = \int_1^3 (2x+1)\left(\frac{1}{3-1}\right)\cdot dx = \frac{1}{2}\left[2\left(\frac{x^2}{2}\right)+x\right]_1^3 =$

$$= \frac{1}{2}[10]$$

$$= 5$$

Exponential Distribution:

• The p.d.f of a continuous random variable x in exponential distribution is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & : x \geq 0 ; \lambda > 0 \\ 0 & : \text{otherwise} \end{cases}$$

* Mean $= \int_0^\infty x f(x) dx = \int_0^\infty x \cdot \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$

* Varience $= \frac{1}{\lambda^2}$ * SD $= \frac{1}{\lambda}$

Example: The telephone conversation between 2 person is exponentially distributed with the p.d.f $f(x) = \begin{cases} \frac{1}{5} e^{-x/5} & : x \geq 0 \\ 0 & : \text{otherwise} \end{cases}$

Find the probability that conversation also exceed 20 minutes. Also find the mean conversation time.

Sol: (i) $P(X > 20) = \int_{20}^{\infty} f(x) dx = \int_{20}^{\infty} \frac{1}{5} e^{-x/5} dx = \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_{20}^{\infty} = -\left[e^{-\infty} - e^{-4} \right]$

$$= -\left[0 - \frac{1}{e^4} \right] = \frac{1}{e^4}$$

(ii) Mean conversation time $= \frac{1}{\lambda} = \frac{1}{(1/5)} = 5$ minutes

Property Of Random Variables:

• If X, Y are any two random variables, then   (V: Varience)

→ $E(aX \pm bY) = a E(X) \pm b E(Y)$

→ $E(aX \pm b) = a E(X) \pm b$

→ $E(\text{const. } k) = k$

→ $V(aX \pm bY) = a^2 V(X) + b^2 V(Y) \pm 2$ covarience of (X,Y), where covarience $(X,Y) = E(XY) - E(X) \cdot E(Y)$

→ $V(\text{const. } k) = 0$ → $V(aX \pm b) = a^2 V(X)$

- If X, Y are independent random variables then:
  - $\rightarrow E(XY) = E(X) \cdot E(Y)$
  - $\rightarrow$ Covariance$(X,Y) = 0 \Rightarrow [E(XY) - E(X)E(Y)]$
  - $\rightarrow V(aX \pm bY) = a^2V(X) + b^2V(Y)$

Pg 178

② Total no. of car = 10

Two are defective $\Rightarrow$ P(defective) $= \frac{1}{5} = 0.2$

P(Good car) $= 0.8$

6th car is defective $\Rightarrow$ five car already sold out and among 5 no defective or one is possible

$\therefore$ P(6th car is defective) $= \left[{}^5C_0 (0.2)^0 (0.8)^5 + {}^5C_1 (0.2)^1 (0.8)^4 \right] \times 0.2$

$= \underline{0.144 \cong 0.15}$

③ $P(A) = 0.25$, $P(B) = 0.5$ & $P(A \cap B) = 0.14$

find $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - (0.25 + 0.5 - 0.14)$

$= 1 - 0.61$

$= 0.39$

④ $P(A \cap B) = 0.2$, $P(A) + P(B) = 0.7$

$P\{(A \cap \bar{B}) \cup (B \cap \bar{A})\}$

$\Rightarrow P(A \cap \bar{B}) + P(B \cap \bar{A}) - \underbrace{P((A \cap \bar{B}) \cap (B \cap \bar{A}))}_{\phi}$

$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= 0.7 - 0.4$

$= 0.3$

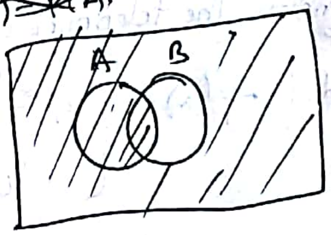
⑤ $P(A) = \frac{1}{2}$, $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$; $P(A \cup \bar{B}) = ?$

$P(\overline{A \cup B}) = \frac{1}{3}$

$P(A \cup B) = 1 - P(\overline{A \cup B})$

$= 1 - \frac{1}{3} = \frac{2}{3}$

~~P(A∪B) = P(A) = P(A)~~



only B $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$P(A \cup \bar{B}) = 1 - P(\text{only } B) = 1 - \frac{1}{6} = \frac{5}{6}$

Q6 0.29

⑦ P(String contains atleast 4 ones)

$= P(4 \text{ one} + 5 \text{ one} + 6 \text{ one})$

$= \frac{11}{32}$

⑧ $n(S) = {}^{15}C_2$

$P(\ ) = \frac{31}{105} \left(\frac{{}^4C_2 + {}^6C_2 + {}^5C_2}{{}^{15}C_2} \right)$

⑨

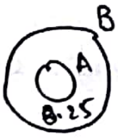
X	1	2	3	...
$P(X=j) = \frac{1}{2^j}$	$\frac{1}{2}$	$(\frac{1}{2})^2$	$(\frac{1}{2})^3$	

P(X is divisible by 3) $= \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \dots$

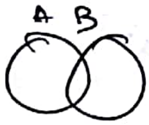
$\frac{1}{2^3}\left(1+\frac{1}{2}+\frac{1}{2^2}+\dots\right) = \frac{1}{2^3}\times\left(\frac{1}{1-\frac{1}{2^3}}\right)$

$= 1/7$

(10)



$A\cap B_{max} = 0.25$



$P(A)+P(B) = 1.05$

∴ Min value of $P(A\cap B) = 0.05$ to make it 1.

∴ $0.05 \le P(A\cap B) \le 0.25$

(11) A B C

$1/6 \quad 5/6\cdot\frac{1}{6} \quad \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$

$= \left(\frac{5}{6}\right)^{3n}\frac{1}{6}$

$= \left(\left(\frac{5}{6}\right)^0+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6+ \quad\right)\times\frac{1}{6}$

$\Rightarrow$

$= \left(1+\left(\frac{5}{6}\right)^3+\left(\left(\frac{5}{6}\right)^3\right)^2+\dots\right)\times\frac{1}{6}$

$= \frac{1}{6}\left(\frac{1}{1-\left(\frac{5}{6}\right)^3}\right) = \frac{36}{91}$

(12) $P(\text{sum } 10) = \frac{27}{6^3} = \frac{1}{8}$

(13) $P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9} \Rightarrow P(\overline{\text{sum } 5}) = \frac{8}{9}$

$P(\text{Sum } 8) = \frac{5}{36} \Rightarrow P(\overline{\text{sum } 8}) = \frac{31}{36}$

Sum 5: $\frac{1}{9}$ Sum 8: $\frac{8}{9}\times\frac{5}{36}$

$\frac{8}{9}\times\frac{31}{36}\times\frac{1}{9}$ $\quad$ $\times\frac{8}{9}\times\frac{5}{36}$

(20)

$\therefore P(\text{sum } 5) = \frac{1}{9}\left[1+\left(\frac{8}{9}\times\frac{31}{36}\right)+\left(\frac{8}{9}\times\frac{31}{36}\right)^2+\dots\right]$

$= \frac{9}{19}$

(14) Solved in example.

(15) $P\left(\frac{\text{one face is 4 } (E_1)}{\text{diff no. on dice } (E_2)}\right) = \frac{P(E_1\cap E_2)}{P(E_2)} = \frac{\frac{1}{6}\times\frac{5}{6}}{\frac{6}{6}\times\frac{5}{6}}$

$= \frac{1}{6}$

(16) $P(\text{Truth}) = P(T) = \frac{3}{4} \Rightarrow P(F) = 1-\frac{3}{4} = \frac{1}{4}$

R = Report given by a man

$P(6) = \frac{1}{6}$; $P(\text{not } 6) = \frac{5}{6}$

$P\left(\frac{\text{actually } 6}{\text{reported } 6}\right) =$

$= \frac{P\left(\frac{R}{6}\right)\times P(6)}{P\left(\frac{R}{6}\right)\times P(6)+P\left(\frac{R}{\bar{6}}\right)\times P(\bar{6})}$

$= \frac{P(T)\times P(6)}{P(T)\times P(6)+P(F)\times P(\bar{6})} = \frac{\frac{3}{4}\times\frac{1}{6}}{\frac{3}{4}\times\frac{1}{6}+\frac{1}{4}\times\frac{5}{6}}$

$= \frac{3}{8}$

~~(17) $\mu = 50$ SD = 5~~

~~$E(X^2) = ?$~~

(17) $f(x) = \begin{cases} x^2 e^{-bx} & x \ge 0 \\ 0 & \end{cases}$

mode of x = ?

$f'(x) = x^2(-b)\,e^{-bx} + e^{-bx}\,2x$

$= e^{-bx}(-bx^2+2x)$

put $f'(x) = 0$

$e^{-bx}(2x-bx^2)=0$

$2x-bx^2=0$

$= x(2-bx)=0$

$\Rightarrow x=0,\ \boxed{x=\frac{2}{b}}$ is a stationary point

from the options $\frac{2}{b}$ is a mode.

Note: $\boxed{f''\left(\frac{2}{b}\right)<0}$

(18) $\mu=50$, SD $=5$; $E(X^2)=?$

variance $=(SD)^2=25$

$V(X)=E(X^2)-[E(X)]^2$

$25=E(X^2)-(50)^2$

$\Rightarrow E(X^2)=(50)^2+25$

$E(X^2)=2525$

~~(19) n = 6~~

~~p(head) = 1/2~~

~~q = 1/2~~

~~P(First 3 heads, last 3 heads)~~

~~$={}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3=\frac{{}^6C_3}{2^6}$~~

(19)

H	T
0	3
1	2
2	1
3	0

$n=6$

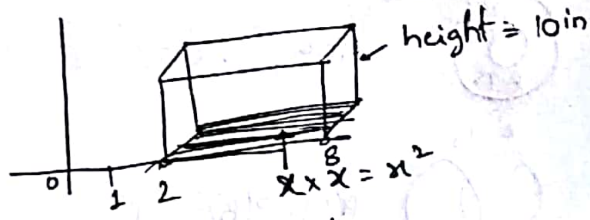
$p=\frac{1}{2},\ q=\frac{1}{2}$

$P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$={}^6C_0\left(\frac{1}{2}\right)^6+{}^6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5+{}^6C_2\left(\frac{1}{2}\right)^6+{}^6C_3\left(\frac{1}{2}\right)^6$

$=$

(20)



volume of a box $= l\times b\times h$

$= 10\times x\times x = 10x^2$

x is uniformly distributed over $[2,8]$

$f(x)=\frac{1}{8-2}=\frac{1}{6}$

Expected volume $=E(X^2)=\int_2^8 10x^2 f(x)\,dx$

$=\int_2^8 (10x^2)\,\frac{1}{6}\,dx$

$=\frac{10}{8}\left[512-8\right]^{2}$

(21) p.d.f is $f(x)=\begin{cases}2x & : 0<x<1\\ 0 & : \text{elsewhere}\end{cases}$

$P\left(\frac{x\le 1/2}{1/3\le x\le \frac{2}{3}}\right)=?$

Sol. $\frac{P\left(x\le 1/2 \cap \frac{1}{3}\le x\le\frac{2}{3}\right)}{P\left(\frac{1}{3}\le x\le\frac{2}{3}\right)}$

$=\frac{P\left(\frac{1}{3}\le x\le\frac{1}{2}\right)}{P\left(\frac{1}{3}\le x\le\frac{2}{3}\right)}=\frac{\int_{1/3}^{1/2}2x\,dx}{\int_{1/3}^{2/3}2x\,dx}$

$=\frac{(x^2)_{1/3}^{1/2}}{(x^2)_{1/3}^{2/3}}=\frac{\frac{1}{4}-\frac{1}{9}}{\frac{4}{9}-\frac{1}{9}}=\frac{\frac{5}{36}}{\frac{3}{9}}=\frac{5}{12}$

Q22 $P(\text{face } j) = Kj, \forall j, K = \text{prop. const}$

x (face j)	1	2	3	4	5	6
$P(x)$	K	2K	3K	4K	5K	6K

we have $\Sigma P(x) = 1$

$K + 2K + 3K + 4K + 5K + 6K = 1$

$21K = 1$

$K = \frac{1}{21}$ //

$P(\text{odd no of dots}) = P(1) + P(3) + P(5)$

$= \frac{1}{21} + \frac{3}{21} + \frac{5}{21}$

$= \frac{9}{21} = \frac{3}{7}$ //

(23) $f(x) = \begin{cases} 2x^{-2} & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$ $\int_a^b \text{p.d.f} = 1$

$g(x) = \begin{cases} 1 + |x| & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$\int_1^2 f(x)\,dx = \int_1^2 2x^{-2}\,dx = \left(\frac{-2}{x}\right)_1^2 = 1$

$\therefore f(x)$ is p.d.f

$\int_{-1}^{1} 1 + |x|\,dx = 2\int_0^1 (1+x)\,dx$ [$1+|x|$ is even]

$= 2\left(1 + \frac{1}{2}\right) = 3 \neq 1$

$\therefore g(x)$ is not pdf.

Only $f(x)$ is pdf, $g(x)$ is not pdf.

(24)

X	-3	6	9
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

$E(2x+1)^2 = \Sigma (2x+1)^2 P(x)$

$= (2(-3)+1)^2 \cdot \frac{1}{6} +$

$(2(6)+1)^2 \cdot \frac{1}{2} +$

$(2 \times 9 + 1)^2 \cdot \frac{1}{3}$

$\Rightarrow 209$

(25)

$f(x) = \begin{cases} \frac{1}{20} e^{-\frac{x}{20}} & x > 0 \\ 0 & x \le 0 \end{cases}$

$P(\text{atleast } 30000) = \int_{30000}^{\infty} f(x)\,dx$

$= \int_{30K}^{\infty} \frac{1}{20} e^{-x/20}\,dx$

$= e^{-\infty} - e^{-\frac{30000}{20}}$

$= -(-e^{-1500}) = e^{-1500} = \frac{1}{e^{1500}}$

(26) $P(\text{2 heads on first 9 toss}) \times P(\text{Head on } 10^{th})$

$\Rightarrow \left|{}^9C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^7\right| \times \frac{1}{2}$

$\Rightarrow \frac{{}^9C_2}{2^{10}}$

(27) $P(\text{bomb will strike}) = 50\% = p = \frac{1}{2}$

$q = \frac{1}{2}$

$P(x \geq 2) \geq 99\%$

$P(x \geq 2) \geq 0.99$

$1 - P(X < 2) \geq 0.99$

$0.01 \geq P(X < 2)$

$P(X < 2) \leq 0.01$

$\Rightarrow P(X=0) + P(X=1) \leq 0.01$

$\Rightarrow {}^nC_0\, p^0 q^{n} + {}^nC_1\, p^1 q^{n-1} \leq 0.01$

$\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n n \leq 0.01$

$\left(\frac{1}{2}\right)^n (n+1) \leq 0.01$

find n

(28) x has P.D with S.D = 2

$P\left(\frac{x=1}{x\geq 1}\right) = ?$

for a PD ⇒ mean = variance = $(SD)^2 = 4$

$\therefore \mu = 4$

$$P\left(\frac{x=1}{x\geq 1}\right) = \frac{P(x=1 \cap x\geq 1)}{P(x\geq 1)}$$

$$= \frac{P(x=1)}{P(x\geq 1)} = \frac{P(x=1)}{1-P(x=0)}$$

$$= \frac{\frac{e^{-\mu}\mu^1}{1!}}{1-\frac{e^{-\mu}\mu^0}{0!}} = \frac{e^{-\mu}\cdot\mu}{1-e^{-\mu}}$$

$\mu = 4$

$$\frac{4e^{-4}}{1-e^{-4}} = \frac{4\cancel{e^{-4}}}{\cancel{e^{-4}}(e^{4}-1)}$$

$$= \frac{4}{e^{4}-1}$$

(29) x: no hit (x follow P.D)

$P(x=0) = \frac{1}{3}$

$$\frac{e^{-\mu}\cdot\mu^0}{0!} = \frac{1}{3}$$

$e^{-\mu} = \frac{1}{3} \Rightarrow -\mu = \ln\left(\frac{1}{3}\right)$

$\mu = -\ln\left(\frac{1}{3}\right)$

P(2 or more hits) = $P(x\geq 2)$

$= 1-P(x<2)$

$= 1-[P(x=0)+P(x=1)]$

$= 1-\left[\frac{1}{3}+\frac{e^{-\mu}\mu}{1!}\right] = 0.3005$

(30) P(pages with errors) = $\frac{390}{520}$

$n = 5$ pages

$\mu = np = 5 \times \frac{390}{520} = 3.75$

P(no errors in 5 pages)

$= P(x=0) = \frac{e^{-\mu}\mu^0}{0!} = e^{-3.75}$

$= \frac{1}{e^{3.75}}$

(31) $\lambda = 1 = \mu$

$$P\left(\frac{x\geq 2}{x\leq 4}\right) = \frac{P(x\geq 2 \cap x\leq 4)}{P(x\leq 4)}$$

$$= \frac{P(2\leq x\leq 4)}{P(x\leq 4)}$$

$$= \frac{P(x=2)+P(x=3)+P(x=4)}{P(x=0)+\dots+P(x=4)}$$

$$= \frac{\frac{e^{-1}1^2}{2!}+\frac{e^{-1}\cdot 1^3}{3!}+\frac{e^{-1}\cdot 1^4}{4!}}{\frac{e^{-1}\cdot 1^0}{0!}+\frac{e^{-1}\cdot 1^1}{1!}+\frac{e^{-1}\cdot 1^2}{2!}+\frac{e^{-1}\cdot 1^3}{3!}+\frac{e^{-1}\cdot 1^4}{4!}}$$

(32) here: $n = 2000$

$p = \frac{1}{1000}$

Avg no. of cashew in each mixture $\mu = np$

$= 2000 \times \frac{1}{1000}$

$\mu = 2$

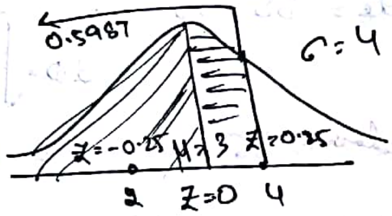
$P(X=0) = \frac{e^{-\mu}\mu^0}{0!} = e^{-2} = \frac{1}{e^2} = 0.13$

(33) $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

median = 0.5

(34) Given, $\mu = 3$

variance $= 16 \Rightarrow SD(\sigma) = 4$



$z = \frac{x-\mu}{\sigma}$

when $x = 2 \Rightarrow z = \frac{2-3}{4} = \frac{-1}{4} = -0.25$

$x = 4 \Rightarrow z = \frac{4-3}{4} = \frac{1}{4} = 0.25$

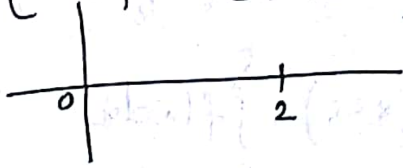
$P(2 < X < 4) = P(-0.25 < z < 0.25)$

$P(0 \le Z \le 0.25) = 0.5987 - 0.5$

$= 0.0987$

(35) $[0, 2]$ [Uniform Distribution]

$f(x) = \frac{1}{2-0} = \frac{1}{2}$



Now $P(\frac{2}{3} < x < \frac{3}{2}) = \int_{2/3}^{3/2} f(x)\,dx$

$= \int_{2/3}^{3/2} \frac{1}{2}\,dx = (3/2 - 2/3) \cdot \frac{1}{2}$

$= \frac{9-4}{6} \times \frac{1}{2} = \frac{5}{12}$

(36) $[0, 10]$

$f(x) = \frac{1}{10-0} = \frac{1}{10}$

$P((x + \frac{10}{x}) \ge 7) = P(\frac{x^2+10}{x} \ge 7)$

$= P(x^2 - 7x + 10 \ge 0)$

$= P((x-2)(x-5) \ge 0)$

$= P(X \le 2) + P(X \ge 5)$

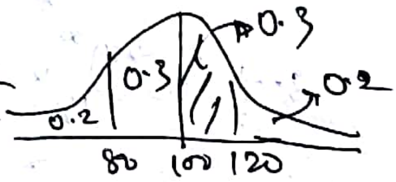
$= \int_0^2 \frac{1}{10}\,dx + \int_5^{10} \frac{1}{10}\,dx$

$\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

(37) $\mu = 100$; $P(100 < X < 120) = 0.3$

$P(X < 80) = ?$

$\therefore P(X < 80) = 0.2$



(38) Solved example.

(39) $\lambda = \frac{1}{10}$

$$f(x) = \begin{cases} \frac{1}{10} e^{-x/10} & : x \geq 0 \\ 0 & : \text{otherwise} \end{cases}$$

$$P(X<5) = \int_0^5 f(x)dx$$

$$= \frac{1}{10}\int_0^5 e^{-x/10} dx$$

$$= \frac{1}{10}(-10)\cdot\left[e^{-x/10}\right]_0^5$$

$$= (-1)(e^{-1/2} - e^{-0})$$

$$= (-1)(e^{-1/2} - 1)$$

$$= 1 - \frac{1}{e^{1/2}}$$

(40) $\frac{1}{\lambda} = 0.5 = \frac{1}{2}$

$\lambda = 2$

$$f(x) = \begin{cases} 2e^{-2x} & : x \geq 0 \\ 0 & : \text{otherwise} \end{cases}$$

~~Now $P(X\geq 2) = \int_0$~~

$$P(X > 1/2) = \int_{1/2}^{\infty} f(x)dx = \int_{1/2}^{\infty} 2\cdot e^{-2x} dx$$

$$= 2\cdot -\frac{1}{2}\left(e^{-2x}\right)_{1/2}^{\infty} = -(e^{-\infty} - e^{-1})$$

$$= \frac{1}{e}$$

(41) mean $\beta = 5$

$$\lambda = \frac{1}{\beta} = \frac{1}{5}$$

$$f(x) = \begin{cases} \frac{1}{5} e^{-x/5} & : x \geq 0 \\ 0 & : \text{otherwise} \end{cases}$$

$$P(X>8) = \int_8^{\infty} f(x)dx = \int_8^{\infty} \frac{1}{5} e^{-x/5} dx$$

$$\frac{1}{5}\cdot -5 \left. e^{-x/5} \right|_8^{\infty}$$

$$= -(e^{-\infty} - e^{-8/5})$$

$$= e^{-8/5}$$

(42) 27, 27, 35, 35, 36, 36, 40

$$\text{Mean} = \frac{27+27+35+35+36+36+40}{7}$$

$$= \frac{238}{7} = 34 = \mu$$

Mode = 35, 36

Median = 35

$$S.D. = \sqrt{\frac{\sum(x_i - \mu)^2}{n}} = 4.14$$

(43)

x	1	2	3	4	5
$P(x)$	0.1	0.2	0.4	0.2	0.1

Mean $= 1\times0.1 + 2\times0.2 + 3\times0.4 + \ldots$

$= 3 = \mu$

Mode = value of x having max $P(x)$ = 3

Median = ∴ middle is 3

$$S.D = \sqrt{\sum x^2\cdot p(x) - \mu^2} = \sqrt{1.2} = 1.09$$

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$$f(x) = \begin{cases} Kx(1-x) & : 0 \le x \le 1 \\ 0 & : \text{otherwise} \end{cases}$$

$$\int_0^1 f(x)\,dx = 1$$

$$\int_0^1 Kx(1-x) = 1$$

$$K\int_0^1 (x - x^2) = 1$$

$$K\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$$

$$K\left(\frac{1}{2} - \frac{1}{3}\right) = 1$$

$$K\left(\frac{1}{6}\right) = 1$$

(i) $\underline{K = 6}$

(ii) Mean; $\mu = \int_0^1 x f(x)\,dx$

$$= 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right) =$$

$$= \frac{6}{12} = \frac{1}{2}$$

(iii) Mode = point of maxima

$$f(x) = 6x(1-x)$$

$$= 6x - 6x^2$$

$$f'(x) = 6 - 12x$$

$$f''(x) = -12$$

put $f'(x) = 0$

$$6 - 12x = 0$$

$$\boxed{x = 1/2}$$

Now $f''(1/2) = -12 < 0$

$x = \frac{1}{2}$ is a point of maxima

i.e $x = \frac{1}{2}$ is mode

(iv) Median:

Let M is a median

then $\int_0^1 f(x)\,dx = 1$

$$\Rightarrow \int_0^M f(x)\,dx = \frac{1}{2} \text{ \& } \int_M^1 f(x)\,dx = \frac{1}{2}$$

put $\int_0^M f(x)\,dx = \frac{1}{2}$

$$\int_0^M (6x - 6x^2)\,dx = \frac{1}{2} \Rightarrow 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^M = \frac{1}{2}$$

$$\Rightarrow 6\left(\frac{M^2}{2} - \frac{M^3}{3}\right) = \frac{1}{2}$$

$$3M^2 - 2M^3 = \frac{1}{2}$$

$$2M^3 - 3M^2 + \frac{1}{2} = 0$$