

Write-Up

How I Structured the Code

The code is organized around a modular `MobiusStrip` class. Upon initialization, the class takes in the radius R , strip width w , and resolution n , then generates a 3D mesh using the standard parametric equations of a Möbius strip. The core functionalities are split into methods:

- `_compute_mesh()` creates the (X, Y, Z) grid.
 - `plot()` visualizes the surface in 3D using Matplotlib.
 - `surface_area()` estimates the total surface area numerically.
 - `edge_length()` calculates the total length of both edges.
- This structure ensures clean separation between geometry generation, visualization, and numerical analysis.

How I Approximated Surface Area

To estimate the surface area, I used a numerical technique based on the surface integral:

$$A = \iint_D \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| du dv = \iint_D \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| du dv$$

Instead of computing symbolic derivatives, I used finite differences to approximate the partial derivatives at each point. I then computed the cross product of these tangent vectors and integrated the magnitude over the full domain $(u \in [0, 2\pi], v \in [-w/2, w/2])$ using `scipy.integrate.dblquad`.

Any Challenges I Faced

One challenge was maintaining numerical accuracy in the surface area calculation while keeping performance reasonable. Finite difference methods are sensitive to step size, so I had to carefully balance between precision and speed. Another subtle challenge was ensuring that the Möbius strip rendered correctly with the expected half-twist and did not suffer from mesh overlap or orientation errors.

Visualization:

