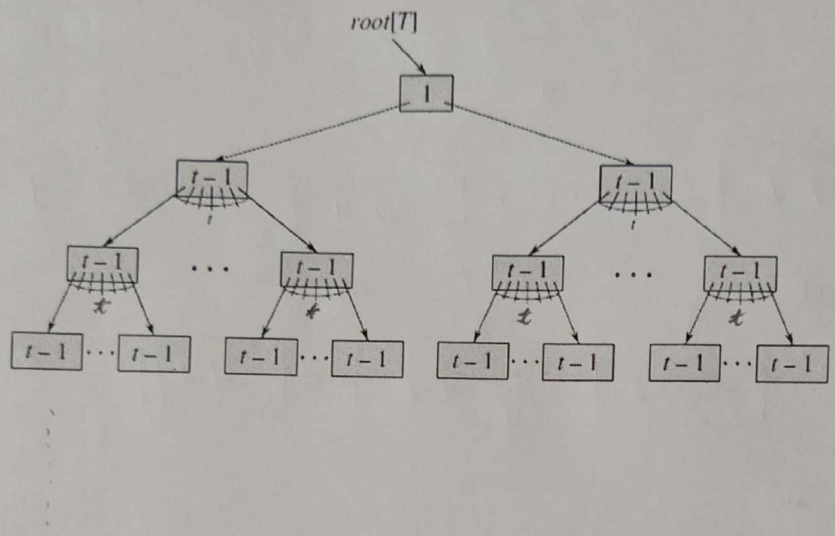
eg (1) Maximum & minimum no. of keys in a B-tree of height h

For minimum no. of keys,



depth	number of nodes
0	1
1	2
2	$2t$
3	$2t^2$
⋮	⋮
h	$2t^{h-1}$

Therefore, total min. no. of keys at height h will be :→

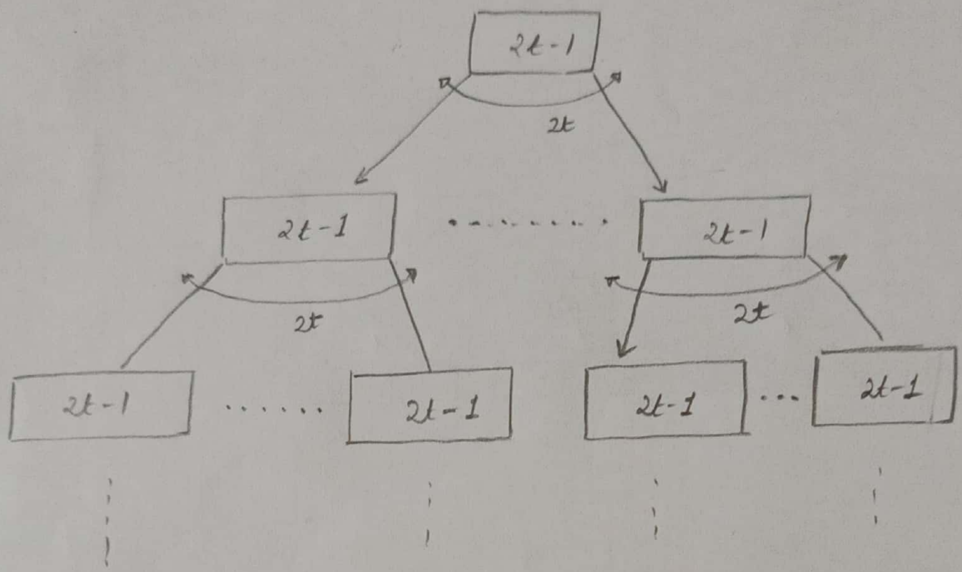
$$\Rightarrow \underbrace{1}_{\text{keys at root}} + (t-1)\{\underbrace{2 + 2t + 2t^2 + \cdots 2t^{h-1}}_{\text{no. of nodes till height } h}\}$$

$$\Rightarrow 1 + (t-1)\left\{\frac{2(t^h - 1)}{(t-1)}\right\}$$

$$\Rightarrow 2t^h - 1$$

$$\boxed{n \geq 2t^h - 1}$$

for max. no. of keys,



Depth	No. of nodes
0	1
1	$2t$
2	$(2t)^2$
⋮	⋮
h	$(2t)^h$

So, total maximum no of keys at height h will be :→

$$\Rightarrow \underbrace{(2t-1)}_{\text{at root}} + (2t-1)\underbrace{\{2t + (2t)^2 + \cdots (2t)^h\}}_{\text{no. of nodes till height } h}$$

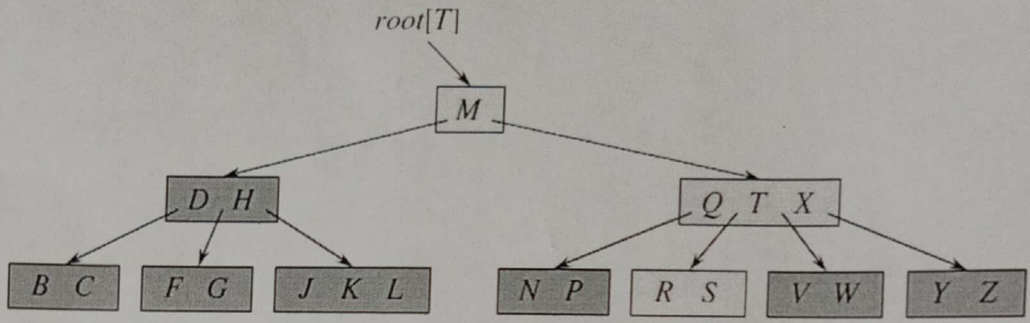
$$\Rightarrow (2t-1) + (2t-1)\left\{\frac{2t\left((2t)^h-1\right)}{(2t-1)}\right\}$$

$$\Rightarrow (2t-1) + \left\{(2t)^{h+1} - 2t\right\}$$

$$\Rightarrow (2t)^{h+1} - 1$$

$$\boxed{n \le (2t)^{h+1} - 1}$$

Eg (2) For what values of t is the following tree a legal B-tree?



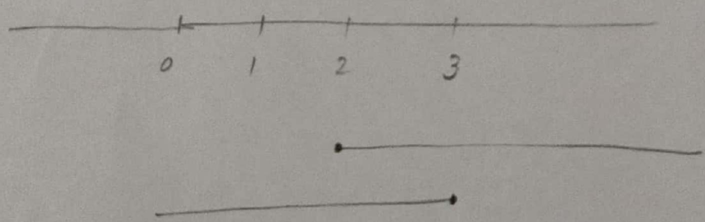
Sol^n

Here no. of keys of each node is either 2 or 3 (except root)

We know that $(t-1) \le n \le (2t-1)$

Therefore, $t-1 \le 2$ and $2t-1 \ge 3$

$t \le 3$ and $t \ge 2$



$t = 2, 3$

Eg (3) show all legal B-trees of min. degree 2 that represent {1,2,3,4,5}

Sol^n

Here $t = 2$

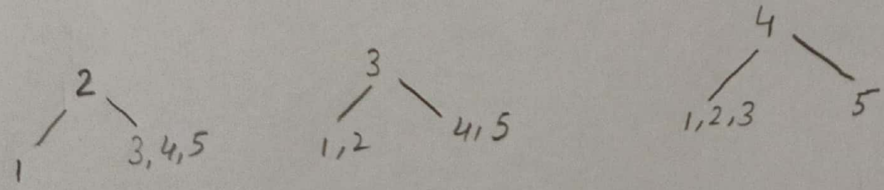
min no. of keys $(2-1) = 1$ }

max no of keys $(2\times 2-1) = 3$ } for any node

$\Rightarrow$ { min. no. of children = 2 ; max. no of children = 4 } (for any internal node)

→ all keys can't be in a single node, since max. no. of keys allowed are only 3.

→



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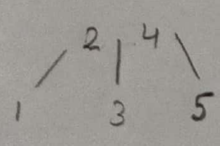

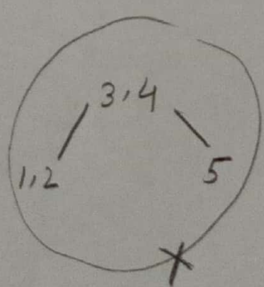

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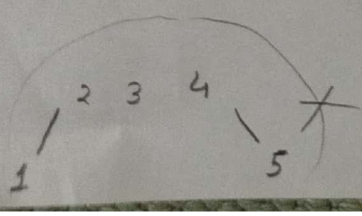

* As we know that

$n \geq 2t^h - 1$

$\Rightarrow 5 \geq 2t^h - 1$

$\Rightarrow 3 \geq t^h$

$\Rightarrow 3 \geq 2^h$ {$t = 2$

so only values of h are $h = 0, 1$

Eg(4) show all legal B-trees of min. degree 3 that represent $\{1, 2, 3, 4, 5\}$

Sol^n

Here $t = 3$

min no of keys $(3-1) = 2$

max no. of keys $(2\times3-1) = 5$

$$\Rightarrow \left\{ \begin{array}{ll} \text{min no. of children} & : 3 \\ \text{max} \longrightarrow ,, \text{———} & : 6 \end{array} \right\}$$