Proposition

①

A statement, or a proposition, is a declarative sentence that is either true or false, but not both.

Def^n Let p be a statement. The negation of p, written $\neg p$, is the statement obtained by negating statement p.

It follows that the truth values of p and $\neg p$ are opposite.

p : 2 is positive

$\neg p$: It is not the case that 2 is positive

p	$\neg p$

Def^n Let p be a statement. The negation of p, written $\sim p$, is the statement obtained by negating statement p.

It follows that the truth values of p and $\sim p$ are opposite

p: 2 is positive

$\sim p$: It is not the case that 2 is positive

p	$\sim p$
T	F
F	T

Conjunction p: 2 is an even integer

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Find

Conjuction p: 2 is an even integer

q: 7 divides 14.

r: 2 is an even integer & 7 divides 4.

r is combination of p & q.

'$\wedge$' is called and.

p, q statements.

The truth table of $p \wedge q$

p	q	$p \wedge q$
	T	T

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r: ...

r is combination of p, ...

'$\wedge$' is called and.

p, q statements.

The truth table of $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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Disjunction :

Given two statements p & q

p 'or' q

p : 2 is an integer.

q : 3 is greater than 5.

Then we can form the stat

r : 2 is an integer or 3 is greater than 5.

'$\vee$'

p	q	$p \vee q$
		T

p or q

p : 2 is an integer
q : 3 is greater than 5.

Then we can form the stat

r : 2 is an integer or 3 is greater than 5.

'∨'

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication.

"~~It~~ if it is cold then I will wear a Jacket".

Eg "If ~~it~~ I get bonus, then I will buy a car".

Def^n "if "p then q" is a statement called as implication, or a condition, written $p \to q$.

3

The statement $p \to q$ is also read as

The statement $p \to q$ is also read as

p implies q

or

p is sufficient for q

or

q if p,

or

q whenever p.

p	q	$p \to q$

q if p,

or

q whenever p.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T.

☆

i) Converse $q \to p$ is converse of $p \to q$

... of $p \to q$

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i) ~~Converse~~ $q \rightarrow p$ is converse of $p \rightarrow q$

(ii) $\sim p \rightarrow \sim q$. is inverse of $p \rightarrow q$

(iii) $\sim q \rightarrow \sim p$ is contrapositive of $p \rightarrow q$.

Biimplication or biconditional of statements p & q.

$p \leftrightarrow q$ ("p if and only if q")

may also read as "p is necessary and sufficient for q")

p	q	$p \leftrightarrow q$

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(iii) $\sim q \rightarrow$

Biimplication or biconditional of statements of p & q.

$p \leftrightarrow q$ ("p if and only if q")

may also read as ("p is necessary and sufficient for q")

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

statement (Formula)

p & q.

we constructed the statements.

negation $\neg p$, conjunction $p \wedge q$,

disjunction $p \vee q$,

implication $p \to q$, and biimplication $p \leftrightarrow q$

The symols $\neg$, $\wedge$, $\vee$, $\to$, and $\leftrightarrow$ are called logical connectives.

~~Here~~

symbols p, q, r, ——

implication $p \rightarrow q$, and biimplication $p \leftrightarrow q$

The symbols $\sim$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ are called logical connectives.

~~Here~~ Henceforth, we use the symbols p, q, r, —— ~~connectives~~ called statement variables.

p	q	$(p \vee q)$	$\sim(p \vee q)$	$q \wedge p$	A
T	T	T	F	T	T
T	F	T	F	F	T

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p	q	$(p \vee q)$	$\sim(p \vee q)$	$q \wedge p$	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

p	q	r	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \to r$
T	T	T	F	F	F T
T	T	F	F	F	T
[illegible]	[illegible]	[illegible]	F	F	T

p	q	r	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \to r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	F	T

③ ①

5 - Properties of Binary Relations:

Let R be a binary relation on A. R is said to be a reflexive Relation if (a, a) is in R for every a in A. In other words, in a reflexive relation, every element in A is related to itself.

~~For~~.

Let R be a binary relation on A. R is said to be a symmetric relation if (a, b) in R implies that (b, a) is also in R.

Let R be a binary relation on A. R is said to be an antisymmetric relation if (a, b) in R implies that (b, a) is not in R unless a = b.

... (a, b) & (b, a) are in R, then

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A is related to itself.

~~Sor~~

Let R be a binary relation on A. R is said to be a symmetric relation if (a,b) in R implies that (b,a) is also in R.

Let R be a binary relation on A. R is said to be an antisymmetric relation if (a,b) in R implies that (b,a) is not in R unless a=b.

In other words, if both (a,b) & (b,a) are in R, then it must be the case that a=b

Let R be a binary relation on A. R is said to be ...itive relation if (a,c) is in R whenever both

In other words, if ...
it must be the case that $a=b$

Let R be a binary relation on A. R is said to be a transitive relation if (a,c) is in R whenever both (a,b) and (b,c) are in R,

for Ex, let $A = \{a, b, c\}$ and
$X = \{(a,a), (a,b), (a,c), (b,c)\}$.
we note that X is a transitive relation.

Let R be a binary relation on A. The transitive extension of R, denoted R_1, is a binary relation on A such that R_1 contains R, and moreover, if (a,b) and (b,c)

for Ex, let

$X = \{(a,a), (a,b), (a,c), (b,c)\}$.

we note that X is a transitive relation.

Let R be a binary relation on A. The transitive extension of R, denoted R_1, is a binary relation on A such that R_1 contains R, and moreover, if (a,b) and (b,c) are in R, then (a,c) is in R_1.

Equivalence Relations & Partitions

A binary relation might have one or more of the fo[llowing] properties: reflexivity, symmetry, antisymmetry, an[d] transitivity.

A binary relation on a set is said to be an equivalence relation. if it is reflexive, symmetric & transitive.

Partial ordering Relations and Lattices.

A binary relation is said to be partial ordering ... & transitive

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Partial ordering Relations and lattice

A binary relation is said to be partial ordering relation if it is reflexive, anti symmetric and transitive

Let A be a set of positive integers, and let R be a binary relation on A such that (a, b) is in R if a divides b. Since any integer divides itself, R is a reflexive relation, since if a divides b means b does not divide a unless a=b, R is an ~~on~~ antisymmetric relation. Since if a divides c, R is a ... then a divides relation if it is ...

Let A be a set of positive integers, and let R be a binary relation on A such that (a, b) is in R if a divides b. Since any integer divides itself, R is a reflexive relation, since if a divides b means b does not divide a unless a = b, R is an ~~an~~ antisymmetric relation. Since if a divides b and b divides c, then a divides c, R is a transitive relation.

Consequently, R is a partial ordering relation b and b divides c, ...

transitive relation.

Consequently, R is a partial ordering relation

Ex → In a partial ordering relation two objects are related if one of them is smaller (larger) than, or inferior (superior

Like

Let $(A, \leq)$ be a partially ordered set.

Partition of a set :

A partition of a set A is a set of non-empty subsets of A denoted $\{A_1, A_2, ---, A_m\}$ such that the union of A_i's is equal to A and the intersection of A_i & A_j is empty for any distinct A_i and A_j.

In other words: a partition of a set is a division of the elements in the set into disjoint ~~set~~ subsets.

These subsets are also called blocks of the partition.

For Ex: Let

These subsets are also called blocks ...

<u>For Ex:</u> let

$$A = \{a, b, c, d, e, f, g\}$$

then $\{\{a\}, \{b, c, d\}, \{e, f\}, \{g\}\}$ is a partition of A.

From an equivalence relation on A, we can define a partition of A — so that every two ... in a block are related and any ... blocks are not.

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of A.

From an equivalence relation on A, we can define a partition of A — so that every two elements in a block are related and any two elements in different blocks are not.

This partition is said to be the partition induced by the equivalence relation, and the blocks in the partition are called the equivalence

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Conversely, from a partition of a set A we can define an equivalence relation on A so that every two elements in the same block of the partition are related, and any two elements in different blocks are not related.

For Ex: (i) Let A be a set of people and R be a binary relation on A such that (a, b) is in R if and only if a and b have the same family name.

We note that R is an equivalence relation which ... equivalence classes

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For Ex: (i) Let A be a set of people and R be a binary relation on A such that (a, b) is in R if and only if a and b have the same family name.

We note that R is an equivalence relation which induces a partition of A where the equivalence classes are families.

(ii) $a \equiv b \pmod{n}$ is also an equivalence class.

Note (i) $R_1 \cap R_2$ relations is always an equivalence

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∴ (ii) ~~X~~ $a \equiv b \pmod{n}$ is also an equivalence class.

Note (i) $R_1 \cap R_2$ intersection relations is always an equivalence relation.)

(ii) union of two equivalence relations is always a reflexive and symmetric relation.

P.132

4.24 (a) Let R be an equivalence relation on ~~A set~~ a set A. Let $\{A_1, A_2, \ldots A_k\}$ be a set of subsets of A such that $A_i \neq A_j$ for $i \neq j$ and such that combined. in one of the subsets

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(ii) union of
a reflexive and sym

P.132
4.24 (a) Let R be an equivalence relation on ~~a set~~ a set A. Let $\{A_1, A_2, \ldots, A_k\}$ be a set of subsets of A such that $A_i \neq A_j$ for $i \neq j$ and such that a and b are combined in one of the subsets if and only if the ordered pair (a, b) is in R.
Show that $\{A_1, A_2, \ldots, A_k\}$ is a partition of A.

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Let $\{A_1, A_2, \ldots, A_k\}$ be a partition of a set A. We define a binary relation R on A such that an ordered pair (a, b) is in R if and only if a and b are in the same block of the partition. Show that R is an equivalence relation.

★ Functions and the ~~Pigeonhol~~ PIGEONHOLE PRINCIPLE

A binary relation R from A to B is said to be a function ... there is a unique element ... from A

Functions and the ~~Pigeonhol~~ PIGEONHOLE PRINCIPLE

A binary relation R from A to B is said to be a function
if for every element a in A, there is a unique element
b in B so that (a,b) is in R. for a function R from A
to B, instead of writting $(a,b) \in R$. for a function R
from A to B, instead of writting $(a,b) \in R$, we also
use the notation $R(a) = b$, where b is called the image
of a. The set A is called the domain of the function R.
and the set B is called the range of the function
R. ... of a function is but a

to B, instead of writting (a,b)∈R ...

from A to B, instead of writting $(a,b) \in R$, we also use the notation $R(a) = b$, where b is called the image of a. The set A is called the domain of the function R. and the set B is called the range of the function R.

The ~~notation~~ notion of a function is but a formalization of the notion of associating or assigning an element in the range to each of the elements in the domain.

For EX: Let A be a set of houses and B be a set of ~~colours~~ colors.

... is an assignment

assigning an element in the

of the elements in the domain.

for Ex: Let A be a set of houses and B be a set of ~~colours~~ colors.

Then a function from A to B is an assignment of colors for painting the houses.

A function from A to B is said to be an onto function if every element of B is the imag[e] of one or more elements of A.

One to one function

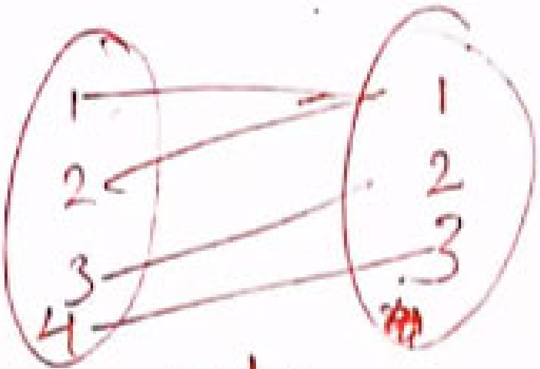
A function from A to B is said to be a one-to-one function if no two elements of A have the same image.

one-to-one onto function:

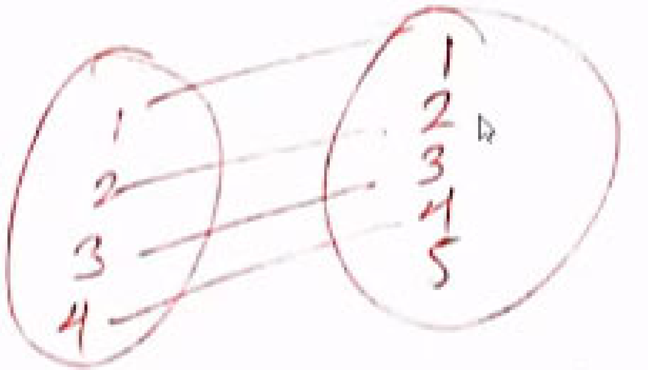
... it is both an onto and a one to one function

A function from A to B is said ...
function if no two elements of A have the same image.

one-to-one onto function:
If it is both an onto and a one to one function



onto



onto to one

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~~Pigeon~~ Pigeonhole:

A well-known proof techniques in mathematics is the socalled pigeonhole principle, also known as the shoe box argument or Dirichlet drawer principle.

In an informal way the pigeonhole principle says that if there are "many" pigeons and

"a few" ... must be some pigeonhole

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A well-known proof techniques in mathematics is the socalled pigeonhole principle, also known as the Shoe box argument or Dirichlet drawer principle.

(7)

In an informal way the pigeonhole principle says that if there are "many" pigeons and "a few" pigeonholes, then there must be some pigeonhole occupied by two or more pigeons.

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...istribution function :

The distribution function $F(x)$ of the discrete variate X is defined by.

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$$ where x is any integer.

The distribution fun^n is also ~~sometimes~~ called ~~com~~ cumulative distribution fun^n.

Ex. ~~26~~ 28

... A success is 'getting 1 or 6'

Ex. ~~20~~ 28

A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of success.

Sol^n: Prob. of a success $= \frac{2}{6} = \frac{1}{3}$,

Prob. of failure $= 1 - \frac{1}{3} = \frac{2}{3}$.

Prob. of no success = Prob of all failures

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

... one success & 2 failure

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Prob. of two success and one failure

$$= {}^3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

Prob. of three successes $= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

Now

x_i	$= 0$	1	2	3
p_i	$= \frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

$\therefore$ mean $\mu = \sum p_i x_i = 0 + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$

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Prob. of two success and one failure

$$= {}^3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

Prob. of three successes $= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

Now

x_i	$= 0$	1	2	3
p_i	$= \frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

$\therefore$ mean $\mu = \Sigma p_i x_i = 0 + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$

$\therefore$ mean $\mu = \sum p_i x_i = 0 + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$

Also $\sum p_i x_i^2 = 0 + \frac{4}{9} + \frac{8}{9} + \frac{9}{27} = \frac{5}{3}$

$\therefore$ variance $\sigma^2 = \sum p_i x_i^2 - \mu^2 = \frac{5}{3} - 1 = \frac{2}{3}$