B

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(1) Binomial Distribution

If we perform a series of independent trials such that for each trial p is the probability of a success and q that of a failure, then the prob. of r successes in a series of n trials is given by

$$^{n}C_{r}\, p^{r} q^{n-r},$$

where r takes any integral value from 0 to n.

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$$ q^n,\ {}^nC_1\, p q^{n-1},\ {}^nC_2\, p^2 q^{n-2},\ \ldots\ {}^nC_r\, p^r q^{n-r}\ \ldots,\ p^n. $$

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The prob. of the number of successes so obtained is ... distribution for the simple ...

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The prob. of the number of successes so obtained is called the binomial distribution for the simple ~~reason~~ reasons that the probabilities are the successive terms in the expansion of the binomial $(q+p)^n$.

$\therefore$ The sum of the prob.

$$q^n + {}^nC_1\, p\, q^{n-1} + {}^nC_2\, p^2 q^{n-2} + \ldots + p^n = (p+q)^n = 1$$

$\therefore$ The sum of the pro... $+ \cdots + p^n = (p+q)^n = 1$

$= q^n + {}^nC_1\, p\, q^{n-1} + {}^nC_2\, p^2 q^{n-2} + \cdots$

② Constants of the Binomial distribution:

The moment generating function about the origin is

$$M_0(t) = E(e^{tx}) = \sum {}^nC_x\, p^x q^{n-x} e^{tx}$$

$$= \sum {}^nC_x (pe^t)^x q^{n-x} = (q+pe^t)^n$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(q-p)^2}{npq} = \frac{(1-2p)^2}{npq}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3 + \frac{1-6pq}{npq}$$

mean $= np$, standard deviation $= \sqrt{npq}$

skewness $= \dfrac{(1-2p)}{\sqrt{npq}}$, kurtosis $= \beta_2$

Obs: The skewness is positive for $p < \frac{1}{2}$ and negative for $p > \frac{1}{2}$. when $p = \frac{1}{2}$, the skewness is zero, i.e the probability curve of the binomial distribution will be symmetrical (bell-shaped).

As n the number of trials increase indefinitely, $\beta_1 \to 0$ & $\beta_2 \to 0$.

(3) Binomial frequency distribution:

If n independent trials constitute one experiment and this experiment be repeated N times, then the frequency of r successes is $N\,{}^{n}C_{r}\,p^{r}q^{n-r}$.

The possible number of successes together with their expected frequencies constitute the binomial frequency distribution.

(4) Application of Binomial distribution:

This distribution is applied to problems concerning:

frequency distribution.

(A) Application of Binomial distribution :

This distribution is applied to problems concerning :

(i) Number of defectives in a sample from production line ;

(ii) Estimation of reliability of systems,

(iii) Number of rounds fired from a gun hitting a target.

(iv) Radar detection.

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The prob. that a pen manufactured by a company will be defective is $1/10$. ③

If 12 such pens are manufactured, find the prob. that

(a) exactly two will be defective.

(b) atleast two will " "

(c) none " "

Solution:

(b) atleast two will " "

(c) none " "

Solution :

The Prob. of a defective pen is $\frac{1}{10} = 0.1$

$\therefore$ The prob. of a non-defective pen is $1 - 0.1 = 0.9$

(a) The Prob. that exactly two will be defective.

$$= {}^{12}C_2 (0.1)^2 (0.9)^{10} = 0.2301$$

(b) The Prob. that at least two will be defective.

$= 1 - ($Prob. that either none or one ~~on~~ is ~~non~~ defect

(a) The Prob. that exactly two will be defective.

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(b) The Prob. that at least two will be defective.

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$$= 1 - \left[{}^{12}C_0 (0.9)^{12} + {}^{12}C_1 (0.1)(0.9)^{11} \right] = 0.3412.$$

(c) The Prob. that none will be defective.

$$= {}^{12}C_0 (0.9)^{12} = 0.2833.$$

~~Ex - 26.3~~

Ex - 26.41 The following data are the number of

~~Ex - 26.3~~

Ex - 26.41 The following data are the number of seeds germinating out of 10 on damp filter paper for 80 sets of seeds. Fit a binomial distribution to these data:

x:	0	1	2	3	4	5	6	7	8	9	10
$f(x)$:	6	20	28	12	8	6	0	0	0	0	0

Solution :-

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Solution :-

Here $n = 10$ & $N = \Sigma f_i = 80$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{20+56+36+32+30}{80} = \frac{174}{80} = 2.175$$

Now, the mean of a binomial distribution $= np$

i.e $np = 10p = 2.175 \therefore p = 0.2175, q = 1 - p = 0.78.$

Hence the binomial distribution to be fitted is

$$N(q+p)^n = 80(0.7825 + 0.2175)^{10}$$

$$\quad {}^{10}\quad {}^{10} \quad (\quad)^{9}(0.2175)^{1} + {}^{10}C(0.7825$$

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Hence the binomial distribution to be fitted is

$$N(q+p)^n = 80\,(0.7825 + 0.2175)^{10}$$

$$= 80 \cdot {}^{10}C_0\,(0.7825)^{10} + 80 \cdot {}^{10}C_2\,(0.7825)^{9}\,(0.2175)^{1} + {}^{10}C_2\,(0.782\ldots$$

$$(0.2175)^2 + \ldots$$

$$- - - - + {}^{80}C_9\,(0.7825)^1\,(0.2175)^9 + {}^{80}C_{10}\,(0.2175)$$

$$= 6.885 + 19.13 + 23.94 + \cdots + 0.0007 + 0.00002$$

$\therefore$ The successive terms in the expression give the expected or theoretical frequencies which are

x:	0	1	2	3	4	5	6	7	8	9	1
$f(x)$:	6.9	19.1	24.0	17.8	8.6	2.9	0.7	0.1	0	0	[illegible]

$$= 6.885 + 19.13 + 23.94 + \cdots + 0.0007 + 0.00002$$

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sson Distribution :

It is a distribution related to the prob. of events which are extremely rare, but which have a large number of independent opportunities for occurrence.

This distribution can be derived as a limiting case of the binomial distribution by making n very large and p very small, keeping np fixed ($=m$, say).

The prob. of r successes in a binomial distribution is

$$P(r) = {}^nC_r\, p^r q^{n-r} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{}\, p^r q^{n-r}$$

very large ...

The prob. of r successes in a binomial distribution is

$$P(r) = {}^nC_r\, p^r q^{n-r} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{\lfloor r}\, p^r q^{n-r}$$

$$= \frac{np(np-p)(np-2p)\cdots(np-(r-1)p)}{\lfloor r}\,(1-p)^{n-r}$$

$$(1-p)^{n-r} \to \left(1-\frac{np}{n}\right) \to \left(1-\frac{m}{n}\right)^{n-r}$$

As $n \to \infty$, $p \to 0$, $(np = m)$, we have

$$P(r) = \frac{m^r}{\lfloor r} \lim_{n\to\infty} \frac{\left(1-\frac{m}{n}\right)^n}{\left(1-\frac{m}{n}\right)^r}.$$

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$= \frac{np(np-p)(np-2p)\cdots}{\lfloor r}$

$1-\frac{np}{n}$

$\left(1-\frac{m}{n}\right)^{n-r}$

As $n \to \infty$, $p \to 0$, $(np = m)$, we have

$$P(x) = \frac{m^r}{\lfloor r} \lim_{n\to\infty} \frac{\left(1-\frac{m}{n}\right)^n}{\left(1-\frac{m}{n}\right)^r}$$

$$= \frac{m^r}{\lfloor r} e^{-m}$$

so that the probabilities of $0, 1, 2, ---, r,$ ——— success in a Poisson distribution are given by ———

e^{-m}, me^{-m}, $\frac{m^2 e^{-m}}{}$, ———, $\frac{m^x e^{-m}}{}$, ———

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5 / 21 162% Find

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so that the probabilities of 0, 1, 2, ---, r, —— success in a Poisson distribution are given by ——

$$e^{-m},\ m e^{-m},\ \frac{m^2 e^{-m}}{\lfloor 2},\ ——,\ \frac{m^r e^{-m}}{\lfloor r},\ ——$$

✲ The sum of these probabilities is unity as it should be

② Constants of the Poisson distribution:

These constants can easily be derived from the corresponding constants of the binomial distribution simply by making $n \to \infty$, $p \to 0$, $(q \to 1)$ and noting that $np = m$

$$\text{Mean} = Lt(np) = m$$

$$\mu_2 = Lt(npq) = m\,Lt(q) = m$$

$\therefore$ Standard deviation $= \sqrt{m}$

Also $\mu_3 = m$, $\mu_4 = m + 3m^2$

(8) Applications of Poisson distribution :

This distribution is applied to problems concerning

(i) Arrival pattern of 'defective vehicles in a workshop', 'patients in a hospital' or 'telephone calls'

(ii) Demand pattern for certain spare parts,

(iii) Number of fragments from a shell hitting a target.

(iv) Spatial distribution of bomb hits.

26.42 If the prob. of a bad reaction from (7) a certain injection is 0.001, determine the chance that out of 2,000 individuals more than two will get a bad reaction.

<u>Solution:</u> It follows a Poisson distribution as the Prob. of occurrence is very small.

$$\text{Mean } m = np = 2000(0.001) = 2$$

Prob. that more than 2 will get a bad reaction.

... reaction

solution : It follows a Poisson ...

the Prob. of occurrence is very small.

Mean $m = np = 2000(0.001) = 2$

Prob. that more than 2 will get a bad reaction.

= 1 − [Prob. that no one gets a bad reaction

+ Prob. that one gets a bad reaction

+ Prob. that two get bad reaction].

$$= 1 - \left\{ e^{-m} + \frac{m}{\lfloor 1} e^{-m} + \frac{m^2 e^{-m}}{\lfloor 2} \right\} = 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2} \right]$$

+ Prob. that two get bad reaction]

$$= 1 - \left\{ e^{-m} + \left[\frac{m}{\lfloor 1} e^{-m} + \frac{m^2 e^{-m}}{\lfloor 2} \right] \right. = 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2} \right]$$

$$= 1 - \frac{5}{e^2} = 0.32$$

Qn Fit a Poisson distribution to the set of observation.

x:	0	1	2	3	4
f:	122	60	15	2	1

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Solution

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{60+30+6+4}{200} = 0.5$$

$\therefore$ Mean of Poisson distribution i.e $m = 0.5$, Hence the theoretical frequency for r success ⑧

is $\dfrac{N e^{-m} (m)^r}{\lfloor r} = \dfrac{200\, e^{-0.5} (0.5)^r}{\lfloor r}$, where $r = 0, 1, 2, 3, 4$

$\therefore$ The theoretical frequencies are

x:	0	1	2	3	4
f:	121	61	15	2	0

$(\because e^{-.5} = 0.61)$

(4)

istribution Function :

The distribution function $F(x)$ of the discrete variate X is defined by.

$$F(x) = P(X \le x) = \sum_{i=1}^{x} p(x_i)$$ where x is any integer.

The distribution funⁿ is also sometimes called ~~com~~ cumulative distribution funⁿ.

Ex- ~~26~~ 28

Ex. ~~26~~ 28

A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of success.

Sol^n: Prob. of a success $= \frac{2}{6} = \frac{1}{3}$,

Prob. of failure $= 1 - \frac{1}{3} = \frac{2}{3}$.

Prob. of no success = Prob of all failures

$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

one success & 2 failure

Prob. of failure $= 1 - \frac{}{3} = \frac{}{3}$.

Prob. of no success = Prob of all failures

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

Prob. of one success & 2 failure

$$= {}^3C_1 \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

Prob. of two success and one failure

$$= {}^3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

Prob. of three successes $= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

$$= {}^3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{\quad}{3} = \frac{\quad}{9}$$

Prob. of three successes $= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

Now

x_i	$= 0$	1	2	3
p_i	$= \frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

$\therefore$ mean $\mu = \Sigma p_i x_i = 0 + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$