

Experiment 11

Longest Common Subsequence

LCS-LENGTH(X, Y) $m \leftarrow \text{length}[X]$ $n \leftarrow \text{length}[Y]$ for $i \leftarrow 1$ to m do $c[i, 0] \leftarrow 0$ for $j \leftarrow 0$ to n do $c[0, j] \leftarrow 0$ for $i \leftarrow 1$ to m do for $j \leftarrow 1$ to n do if $x_i = y_j$ then $c[i, j] \leftarrow c[i-1, j-1] + 1$ $b[i, j] \leftarrow \nwarrow$ else if $c[i-1, j] \geq c[i, j-1]$ then $c[i, j] \leftarrow c[i-1, j]$ $b[i, j] \leftarrow \uparrow$ else $c[i, j] \leftarrow c[i, j-1]$ $b[i, j] \leftarrow \leftarrow$ return c and b

where X & Y are sequence in input i.e., $X = \langle x_1, x_2, \dots, x_m \rangle$
 $Y = \langle y_1, y_2, \dots, y_n \rangle$

* Recursive Solution

Let us define $c[i, j]$ to be the length of an LCS of the sequence X_i and Y_j . If either $i=0$ or $j=0$, one of the sequence has length 0, so the LCS has length 0.

$$c[i, j] = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } i, j > 0 \text{ and } x_i \neq y_j \end{cases}$$

Eg:- Sequence are $\langle 10010101 \rangle$ and $\langle 010110110 \rangle$

		j	0	1	2	3	4	5	6	7	8	9
		i	y_j	0	1	0	1	1	0	1	1	0
0	x_i	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	1	1	1	1	1	1	1	1	1
2	0	0	1	1	2	2	2	2	2	2	2	2
3	0	0	1	1	2	2	2	3	3	3	3	3
4	1	0	1	2	2	3	3	3	4	4	4	4
5	0	0	1	2	3	3	3	4	4	5	5	5
6	1	0	1	2	3	4	4	4	5	5	6	6
7	0	0	1	2	3	4	4	5	5	5	6	6
8	1	0	1	2	3	4	5	5	6	6	6	6

Length of LCS is 6. Some instances are $(100110), (010101), (001101), (101101), (101010), (001010), (001011)$