Discrete Mathematics (MAIR-24)

Unit-1: Discrete Probability

Topic covered- **Set theory**

BASIC CONCEPTS

- Set
- Set of sets
- Subset and Superset
- Properties of sets
- Power set
- Equal sets
- Union of sets
- Intersection of sets
- Venn Diagram

Reference– *Elements of Discrete Mathematics* –C. L. Liu.

SET

A **set** is a collection of well defined objects, or distinct objects. The objects in the set are also called as the members/elements of the set.
For example,

- Set of all even numbers between 1 and 9 is $A = \{2,4,6,8\}$.
- Set of vowels $V=\{a, e, i, o, u\}$

- Set of all odd numbers $O=\{\ldots-3,-1,1,3,5,\ldots\}$
- Set of all natural numbers $N=\{1,2,3,\ldots\}$.

SET OF SETS

A **set of sets** is a set whose elements are itself sets. For example,

$A=\{a, b\}$, $B=\{a, b, c, \{a, b\}\}$, $C= \{\{a\}, \{b\}, \{a, b, c, \{a, b\}\}, \{a, b\}\}$
or $B=\{a, b, c, A\}$ and $C= \{\{a\}, \{b\}, B, A\}$. Here C is the set of sets.

In addition, if $D=\{\{a, b\}\}$, $E= \{\{\{a, b\}\}\}$, then

- $A\in B$, $A\in C$, $A\in C$
- $A\notin E$, $A\in D$, $D\in E$.

SUBSET and SUPERSET

A **subset** of a set is the set having all the elements belonging to the set. For example,

- $A=\{1,2,3,4\}$, $B=\{1,3,4\}$ then $B \subseteq A$. Here B is a proper subset of A (containing less elements than A), and we write $B \subset A$.

- Then A is called the **superset** of B and $A \supseteq B$. For proper superset we write $A \supset B$.
- $A=\{a, b, c\}$ $B=\{b, c\}$, $C=\{a, b\}$ $D=\{a, b, d\}$ then $B \subseteq A$, $C \subseteq A$, $C \subseteq D$, $C \nsubseteq B$, $B \nsubseteq D$.

Properties of SETS

- The empty set (null set or void set) denoted by {} or Φ contains no elements. For example, the set of all positive even numbers less than 2.
- Empty set is the subset of every set, i.e. $\Phi \subseteq A$.

- Every set is a subset of itself, i.e. $A \subseteq A$.
- For every set A, there are only two **improper subsets** , namely, A and Φ, rest of all subsets are proper.

POWER SET

The *power set* of a set A, denoted $\mathscr{P}(A)$, is the set that contains exactly all the subsets of A. Thus $\mathscr{P}(\{a, b\}) = \{\{\ \}, \{a\}, \{b\}, \{a, b\}\}$, and $\mathscr{P}(\{\ \}) = \{\{\ \}\}$. Note that for any set A, $\{\ \} \in \mathscr{P}(A)$ as well as $\{\ \} \subseteq \mathscr{P}(A)$. For example, let $A = \{$novel, published-in-1975, paperback$\}$ be the three attributes concerning the books in the library in which we are interested. Then $\mathscr{P}(A)$ is the set of all possible combinations of these attributes the books might possess, ranging from books that have none of these attributes [the empty set in $\mathscr{P}(A)$] to books that have all three of these attributes [the set A in $\mathscr{P}(A)$].

EQUAL SETS

Two sets P and Q are said to be *equal* if they contain the same collection of elements. For example, the two sets

$$P = \{x | x \text{ is an even positive integer not larger than } 10\}$$

$$Q = \{x | x = y + z \text{ where } y \in \{1, 3, 5\}, z \in \{1, 3, 5\}\}$$

are equal. In a seemingly roundabout way, we can also say that two sets P and Q are equal if P is a subset of Q, and Q is a subset of P. We shall see later that on some occasions this is a convenient way to define the equality of two sets.

UNION OF SETS

The union of two sets P and Q denoted by P∪Q is the set containing all the elements that are either in P or Q, For example,

$$\{a, b\} \cup \{c, d\} = \{a, b, c, d\}$$

$$\{a, b\} \cup \{a, c\} = \{a, b, c\}$$

$$\{a, b\} \cup \phi = \{a, b\}$$

$$\{a, b\} \cup \{\{a, b\}\} = \{a, b, \{a, b\}\}$$

INTERSECTION OF SETS

The intersection of two sets P and Q denoted by P∩Q is the set containing all the elements that are both in P and in Q, For example,

$$\{a, b\} \cap \{a, c\} = \{a\}$$

$$\{a, b\} \cap \{c, d\} = \phi†$$

$$\{a, b\} \cap \phi = \phi$$

† Two sets are said to be *disjoint* if their intersection is the empty set.

DIFFERENCE OF SETS

The *difference* of two sets P and Q, denoted $P - Q$, is the set containing exactly those elements in P that are not in Q. For example,

$$\{a, b, c\} - \{a\} = \{b, c\}$$

$$\{a, b, c\} - \{a, d\} = \{b, c\}$$

$$\{a, b, c\} - \{d, e\} = \{a, b, c\}$$

The difference of two sets P and Q denoted by P-Q is also called the ***complement*** of the set Q with respect to P. For example,

P= set of the students who appeared in an examination
Q= set of the students who passed in that examination
P-Q= set of the students who failed in that examination.

SYMMETRIC DIFFERENCE OF SETS

The *symmetric difference* of two sets P and Q, denoted $P \oplus Q$, is the set containing exactly all the elements that are in P or in Q but not in both. In other words, $P \oplus Q$ is the set $(P \cup Q) - (P \cap Q)$. For example,

$$\{a, b\} \oplus \{a, c\} = \{b, c\}$$

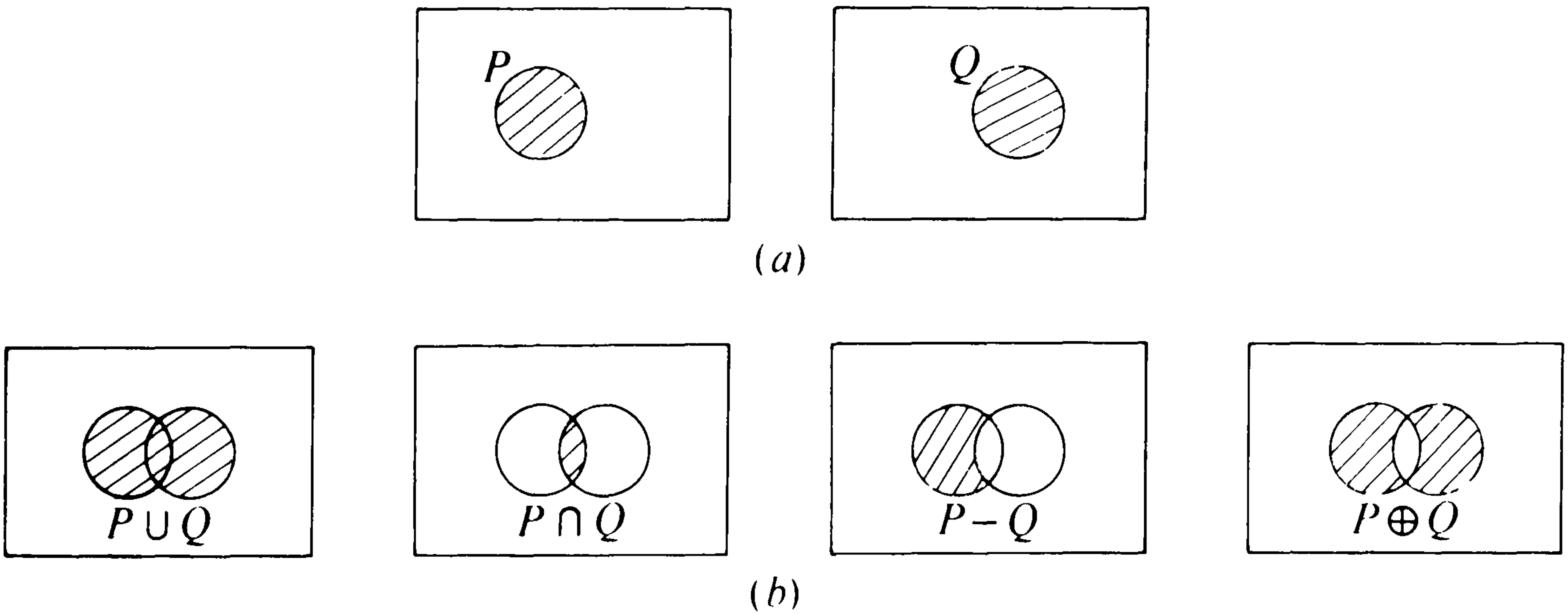
$$\{a, b\} \oplus \phi = \{a, b\}$$

$$\{a, b\} \oplus \{a, b\} = \phi$$

*Example–SYMMETRIC DIFFERENCE

If we let P denote the set of cars that have defective steering mechanisms and Q denote the set of cars that have defective transmission systems, then $P \oplus Q$ is the set of cars that have one but not both of these defects. Suppose that a student will get an A in a course if she did well in both quizzes, will get a B if she did well in one of the two quizzes, and will get a C if she did poorly in both quizzes. Let P be the set of students who did well in the first quiz and Q be the set of students who did well in the second quiz. Then $P \cap Q$ is the set of students who will get A's, $P \oplus Q$ is the set of students who will get B's, and $S - (P \cup Q)$ is the set of students who will get C's, where S is the set of all students in the course. We define $P_1 \oplus P_2 \oplus \cdots \oplus P_k$ to be the set of elements that are in an odd number of the sets $P_1, P_2, \ldots, P_k$.

Union, Intersection, Difference and symmetric difference using Venn Diagram



Thank You.