# Discrete Mathematics (MAIR-24)

## Unit-1: Discrete Probability

## Topics covered-Basic Definitions and Examples

# BASIC CONCEPTS of PROBABILITY

- Experiment
- Outcome
- Sample Space
- Probability
- Events

# Experiment, Outcome and Sample Space

An experiment or trial is any procedure that can be repeated infinitely, and has a well defined set of possible outcomes, known as the sample space.

# Random Experiment

An experiment is said to be random if it has more than one possible outcome.

Example-Let a fair die be rolled, the possible outcomes are 1,2,3,4,5,6, and sample space is S ={1,2,3,4,5,6}.

# Deterministic Experiment

An experiment is said to be deterministic if it has only one outcome.

Example-Let a coin with both sides marked as head be tossed. The possible outcome is H, and the sample space is S ={H}.

# Definition of Probability

The ratio of the number of times an event occurs to the total number of trials, i.e. the number of times the activity is performed.

# *Example

Let a fair coin be tossed. The sample space is $S=\{H,T\}$, $|S|=2$.

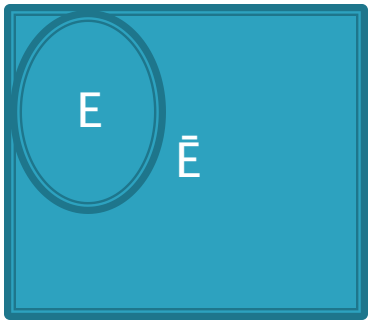
The probability of getting a Tail is ½ and that of getting a Head is ½.

# Axioms of Probability

1. $P(E) \geq 0$.
2. $P(A \cup B \cup \ldots) = P(A) + P(B) + \ldots$
   for disjoint events A,B, …
3. $P(S) = 1$.

# Properties of Probability

1. $0 \leq P(E) \leq 1$
2. Probability of an impossible event is 0, and that of a certain event is 1.
3. $P(\bar{E}) = 1 - P(E)$, where $\bar{E}$ is the set of events complementary to E.



# INDEPENDENT EVENTS

Events A and B are independent events if the occurrence of A does not affect the probability of the occurrence of B.

If A and B are independent events
$P(A \text{ and } B) = P(A).P(B)$ or

$\mathbf{P(A \cap B) = P(A).P(B).}$

# *Example

Let two fair coin be tossed. The sample space is S={H,T}, |S|=2, for each coin independently.

The probability of getting a Tail on first coin is P(A)= ½ and that of getting a Head on second coin is P(B)= ½.
Probability of getting a Tail on first coin and a Head on second coin is P(A∩B)= ½. ½ = ¼.

# DEPENDENT EVENTS

Events A and B are dependent events if they are not independent.

If A and B are dependent events
$P(A \text{ and } B) = P(A).P(B/A)$ or

**$P(A \cap B) = P(A).P(B/A).$**

# *Example

From a box of 50 balls including 10 blue balls, two are selected at random without replacement. What is the probability that the second ball is blue given that the first ball is blue?

P(B)= probability of first ball being blue =10/50=1/5

P(A/B)= probability of second ball being blue, when first ball is blue=9/49.

P(A∩B)= probability of first and second ball being blue =(1/5).(9/49)=9/245.

# MUTUALLY EXCLUSIVE EVENTS

Events A and B are mutually exclusive if they cannot occur concurrently.

If A and B are mutually exclusive,

$P(A \text{ or } B) = P(A) + P(B)$ or

$\mathbf{P(A \cup B) = P(A) + P(B)}$

# *Example

Let a fair coin be tossed once. The sample space is S={H,T}, |S|=2. Either Head may turn on or the Tail.

The probability of getting a Tail on the coin is P(A)= ½ and that of getting a Head on the coin is P(B)= ½.

Probability of getting a Tail on the coin or a Head on the coin is $P(A \cup B) = ½ + ½ = 1$. (we are sure to get a Tail or a Head.)

# NON MUTUALLY EXCLUSIVE EVENTS

If A and B are not mutually exclusive,

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

or

$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

# *Example

Let a fair dice be rolled once. The probability of getting an odd face, i.e. {1,3,5} on is P(A)= 3/6= ½, and that of getting a number less than 4, i.e. {1,2,3} is P(B)= 3/6= ½.

We have $A \cap B = \{1,3\}$, and $A \cup B = \{1,2,3,5\}$. Then $P(A \cap B) = 2/6 = 1/3$.
Probability of getting an odd face on or a number less than 4 is
$P(A \cup B) = 1/2 + 1/2 - 1/3 = 2/3$.

Also, $P(A \cup B) = 4/6 = 2/3$ (verified).

# Thank You.