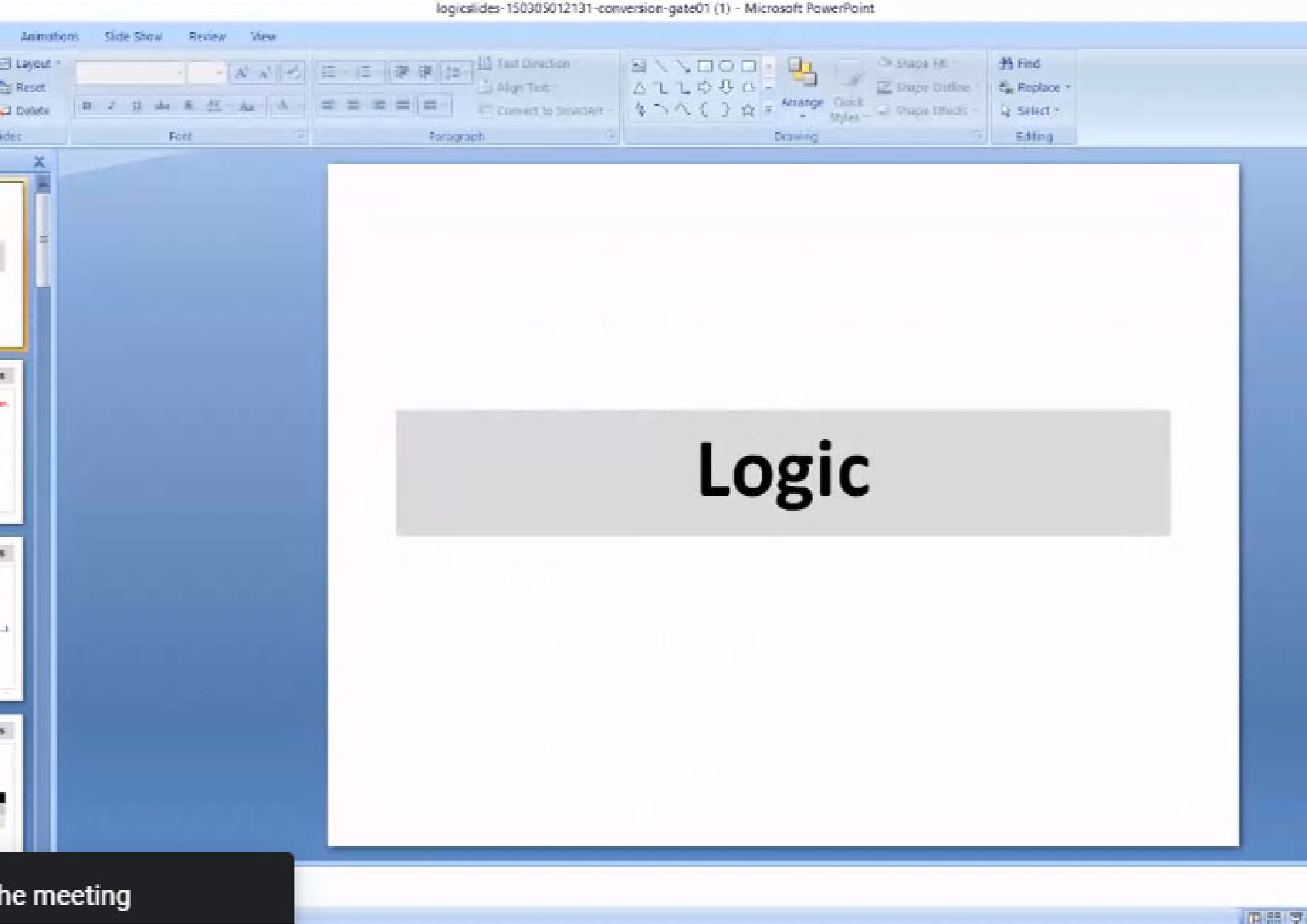

logicslides-150305012131-conversion-gate01 (1) - Microsoft PowerPoint
Animations
Slide Show
Review
View
Layout
Reset
Delete
Font
Paragraph
Drawing
Editing
Find
Replace
Select
Arrange
Logic
he meeting

Propositions and Logical Operations

Definition: A statement or proposition is a **declarative sentence** that is either **true** (T) or **false** (F), but **not both**.

Example: Which of the following are statements?

a. It is raining.

b. $2 + 3 = 5$

c. Do you speak English?

d. $3 - x = 5$

e. Take two aspirins.

Click to add notes

Propositions and Logical Operations

- The letters p, q, r are denote propositional variables
  - p: I am teaching.
  - q: $3 \times 23 = 70$
- Compound statements: propositional variables combined by logical connectives (*and*, *or*, *if … then*, …):
  - p and q
  - p or q
  - If p then q

Propositions and Logical Operations

Definition: If p is a statement, the negation of p is the statement *not p*, denoted by $\sim p$ (sometimes, $\neg p$, $\bar{p}$).

$\sim p$: "It is not the case that p".

Example:

- p: $2 + 3 > 1$ $\quad \sim \boldsymbol{p}$
- q: It is cold. $\quad \sim \boldsymbol{p}$

p	$\sim p$
T	F
F	T

Propositions and Logical Operations

Definition: If p and q are statements, the conjunction of p and q is the compound statement *p and q*, denoted by $p \wedge q$.

Example:

- p: It is raining. q: It is cold.
- p: $2 < 3$. q: $-3 < -2$.
- $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Click to add notes

Propositions and Logical Operations

Definition: If p and q are statements, the disjunction of p and q is the compound statement p *or* q, denoted by $p \vee q$.

Example:

- p: It is raining. q: It is cold.
- p: $2 < 3$. q: $-3 < -2$.
- $p \vee q$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Click to add notes

logicslides-150305012131-conversion-gate01 (1) - Microsoft PowerPoint

Propositions and Logical Operations

A compound statement may have many components:

$$p \vee (q \wedge (\sim (p \wedge r)))$$

Example: Make a truth table for $(p \wedge q) \vee \sim p$.

p	q	$p \wedge q$	$\sim p$	$\vee$
T	T			
T	F			
F	T			
F	F			

Click to add notes

Propositions and Logical Operations

Definition: If p and q are statements, the compound statement "if p then q", denoted $p \Rightarrow q$, is called a conditional statement or implication.

The statement p is called antecedent or hypothesis and q is called the consequent or conclusion.

Example:

- p: I am hungry. q: I will eat.
- p: It is snowing. q: $3 + 2 = 5$.
- $p \Rightarrow q$.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Click to add notes

Propositions and Logical Operations

Definition: If $p \Rightarrow q$ is an implication, then

- the converse of $p \Rightarrow q$ is the implication $q \Rightarrow p$
- the inverse of $p \Rightarrow q$ is the implication $\sim p \Rightarrow \sim q$
- the contrapositive of $p \Rightarrow q$ is the implication $\sim q \Rightarrow \sim p$

Example: $p \Rightarrow q$: "If it is raining then I get wet" then

$$q \Rightarrow p, \qquad \sim p \Rightarrow \sim q, \qquad \sim q \Rightarrow \sim p?$$

logicslides-150305012131-conversion-gate01 (1) - Microsoft PowerPoint

Propositions and Logical Operations

Definition: If p and q are statements, the compound statement "p if and only if q", denoted $p \Leftrightarrow q$ is called an **equivalence** or **biconditional**.

Example:

- p: $3 > 2$. q: $0 < 3 - 2$.
- p: It is snowing. q: $3 + 2 = 5$.
- $p \Leftrightarrow q$.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Click to add notes

Propositions and Logical Operations

Example: Compute the truth table of the statement

$$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$$

p	q	$p \Rightarrow q$	$\sim q$	$\sim p$	$\sim q \Rightarrow \sim p$	$\Leftrightarrow$
T	T					
T	F					
F	T					
F	F					

Click to add notes

Propositions and Logical Operations

Definition: A statement that is true for all possible values of its propositional variables is called a tautology.

Definition: A statement that is false for all possible values of its propositional variables is called a contradiction or an absurdity.

Definition: A statement that can be either true or false for all possible values of its propositional variables is called contingency.

logicslides-150305012131-conversion-gate01 (1) - Microsoft PowerPoint

Propositions and Logical Operations

Definition: A statement that is true for all possible values of its propositional variables is called a tautology.

Definition: A statement that is false for all possible values of its propositional variables is called a contradiction or an absurdity.

Definition: A statement that can be either true or false for all possible values of its propositional variables is called contingency.

Click to add notes

English (United States)

Propositions and Logical Operations

Definition: We say that the statements p and q are logically equivalent (or simply equivalent), denoted by $p \equiv q$, if $p \Leftrightarrow q$ is tautology.

Example: Show that

- $(p \vee q) \equiv (q \vee p)$
- $(p \Rightarrow q) \equiv (\sim p \vee q)$

Propositions and Logical Operations

Definition: We say that the statements p and q are logically equivalent (or simply equivalent), denoted by $p \equiv q$, if $p \Leftrightarrow q$ is tautology.

Example: Show that

- $(p \vee q) \equiv (q \vee p)$
- $(p \Rightarrow q) \equiv (\sim p \vee q)$

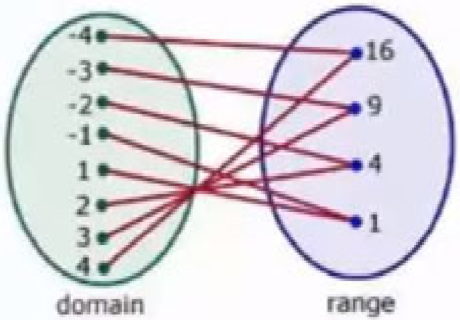
Types of functions

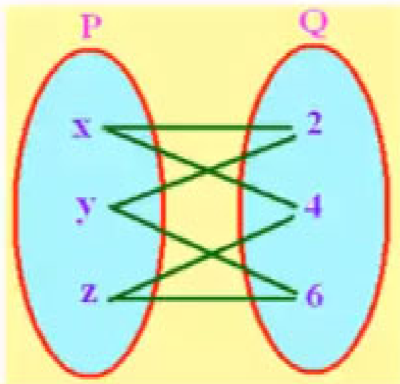
Let f be a function from X to Y then f is called

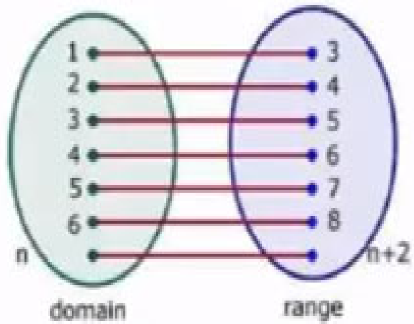
(i) One-one function (or injection) iff different elements of X have different images in Y i.e. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$ or $\forall x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

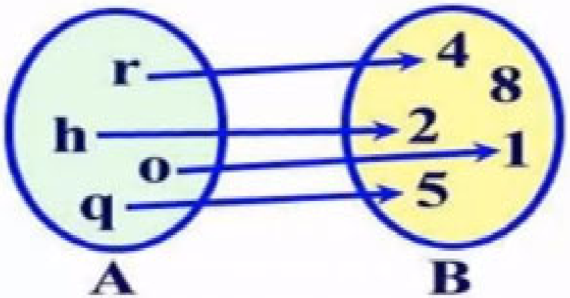
(ii) Many-one function iff two or more elements of X have same image in Y, i.e., f is not one-one. (iii) Onto (or surjection) iff each element of Y is the image of at least one element of X, i.e., iff range of f = Y.

(iv) Into function iff there exists at least one element in Y which is not the image of any element of X, i.e., iff range of f is a proper subset of Y. One-one correspondence (or bijection) iff f is both one-one and onto.

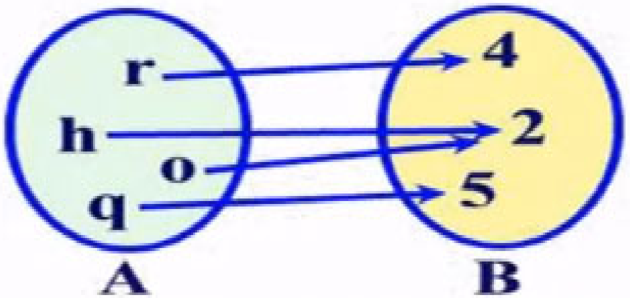






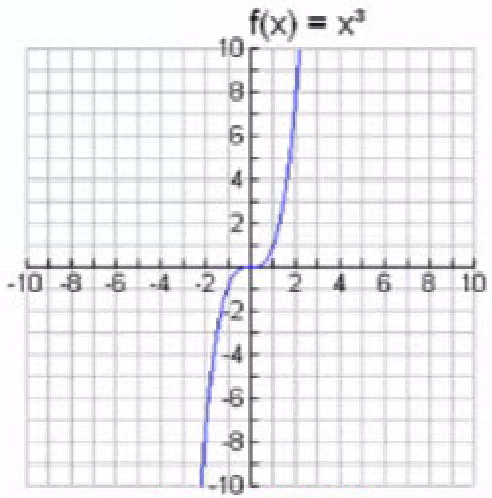


"One-to-One"

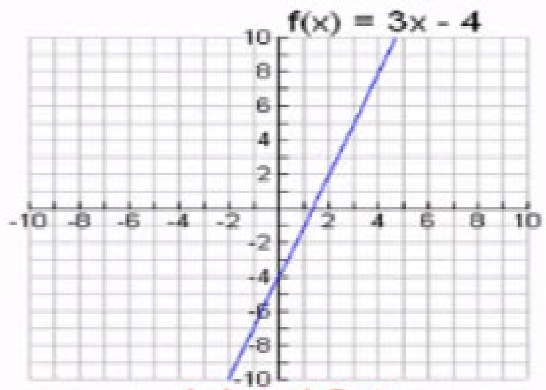


NOT "One-to-One"

A function f from A to B is called one-to-one (or 1-1) if whenever $f(a) = f(b)$ then $a = b$. No element of B is the image of more than one element in A.



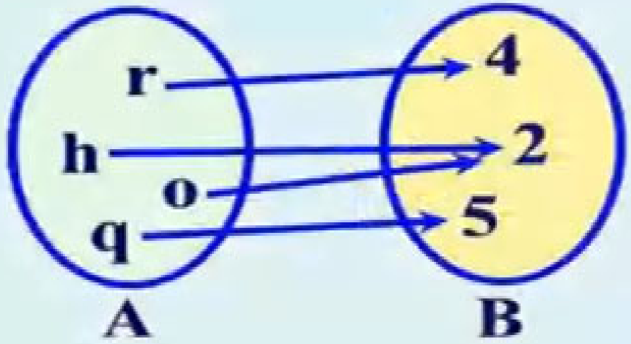
1-1 and ONTO



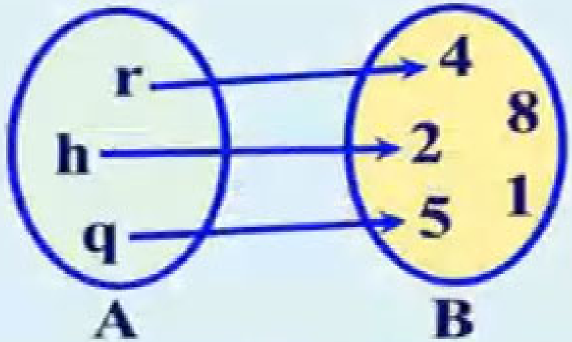
1-1 and Onto

The function $f(x) = x^3$ is One-to-One.

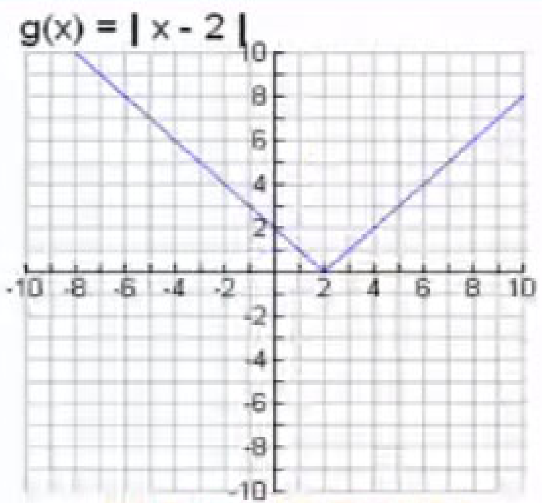
Click to add notes



"Onto"
(all elements in B are used)



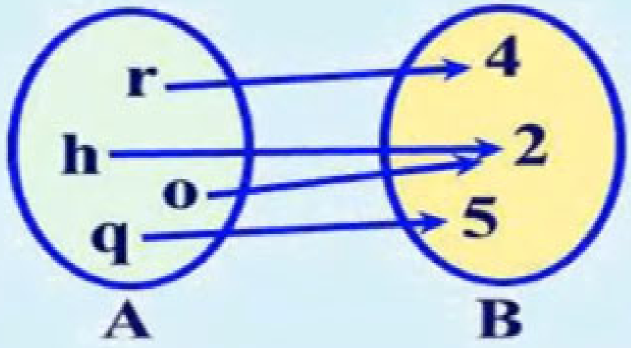
NOT "Onto"
(the 8 and 1 in Set B are not used)



NOT 1-1, NOT ONTO

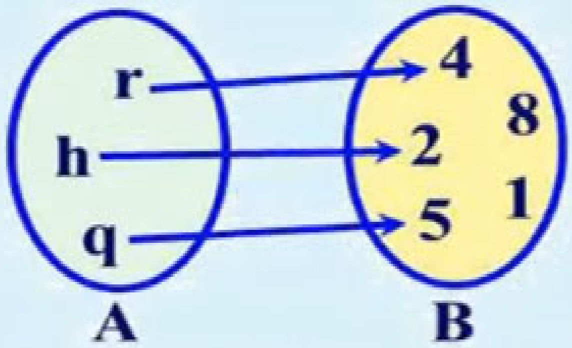
A function f from A to B is called onto if for all b in B there is an a in A such that $f(a) = b$. All elements in B are used.

The function $g(x) = |x - 2|$
NOT One-to-One.



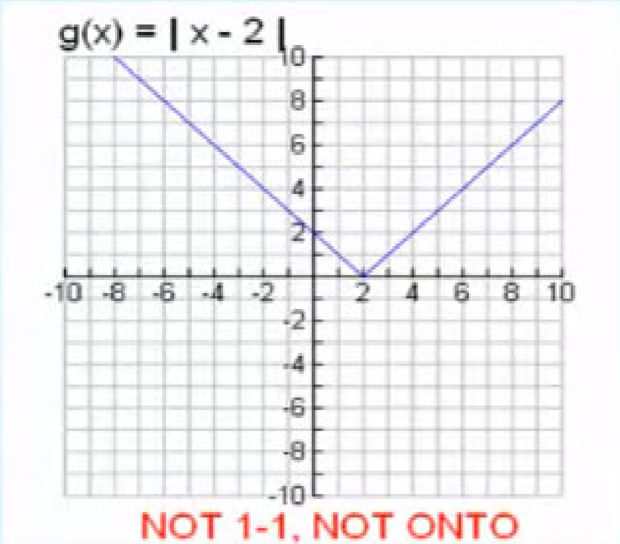
"Onto"

(all elements in B are used)



NOT "Onto"

(the 8 and 1 in Set B are not used)



A function *f* from A to B is called onto if for all *b* in B there is an *a* in A such that *f* (*a*) = *b*. All elements in B are used.

The function $g(x) = |x - 2|$
NOT One-to-One.

discretemathematics-100623140020-phpapp02 - Microsoft PowerPoint

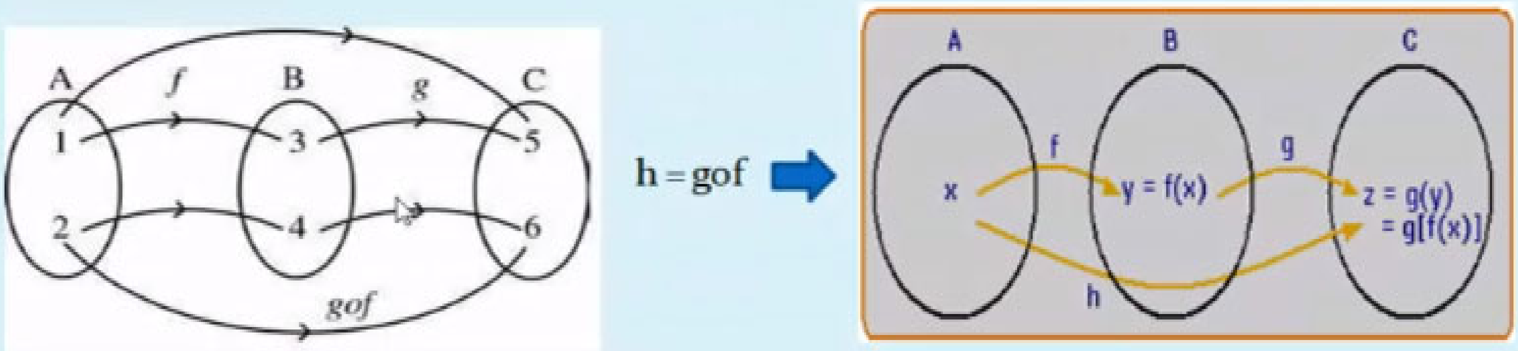
Example (ii): Let $A=R-\{3\}$ and $B=R-\{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = (x-2)/(x-3)$. Is f one-one and onto? Justify your answer.

Solution: The function is one-one. Let $x_1, x_2 \varepsilon A$ be such that $f(x_1)=f(x_2) \Rightarrow$
$(x_1-2)/(x_1-3) = (x_2-2)/(x_2-3) \Rightarrow x_1x_2 - 2x_1 - 3x_2 + 6 = x_1x_2 - 3x_1 - 2x_2 + 6$ then
$-2x_1-3x_2 = -3x_1-2x_2 \Rightarrow x_1 = x_2$, Hence f is one-one.
We shall find the range of the function, Let $y=f(x)$, then $y=(x-2)/(x-3) \Rightarrow$
$xy-3y=x-2$ then $x=(3y-2)/(y-1)$
but $x \varepsilon R$. $(3y-2)/(y-1) \varepsilon R$ it implies $y \neq 1$. Range of $f = R-\{1\}=B$. Thus the range of the function f= Codomain of f. Therefore f is onto. We can say it bijective.

Click to add notes

The composition of two functions is also called the resultant of two functions or function of a function. The function on the right acts first and the function on the left acts second, reversing English reading order. We remember the order by reading the notation as "g of f". The order is important, because rarely do we get the same result both ways.

Let $f : A \to B$ and $g : B \to C$ be two functions . Then the composition of f and g , denoted by gof, is defined as the function $gof : A \to C$ given by **$gof(x) = g(f(x))$, $\forall x \in A$.**



Click to add notes

Remark: If A and B are (non-empty) finite sets containing m and n elements respectively, then

(i) **the number of functions from A to B** $= n^m$ or $n(B)^{n(A)}$.

Remark : If f and g are functions such that gof is defined and

(i) If gof is 1-1 , then f and g both necessarily 1-1.

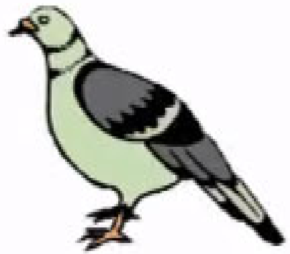
(ii) If gof is onto , then f and g both need not be necessarily onto. It can be verified in general that gof is 1-1 implies that f is 1-1. Similarly, gof is onto implies that g is onto.

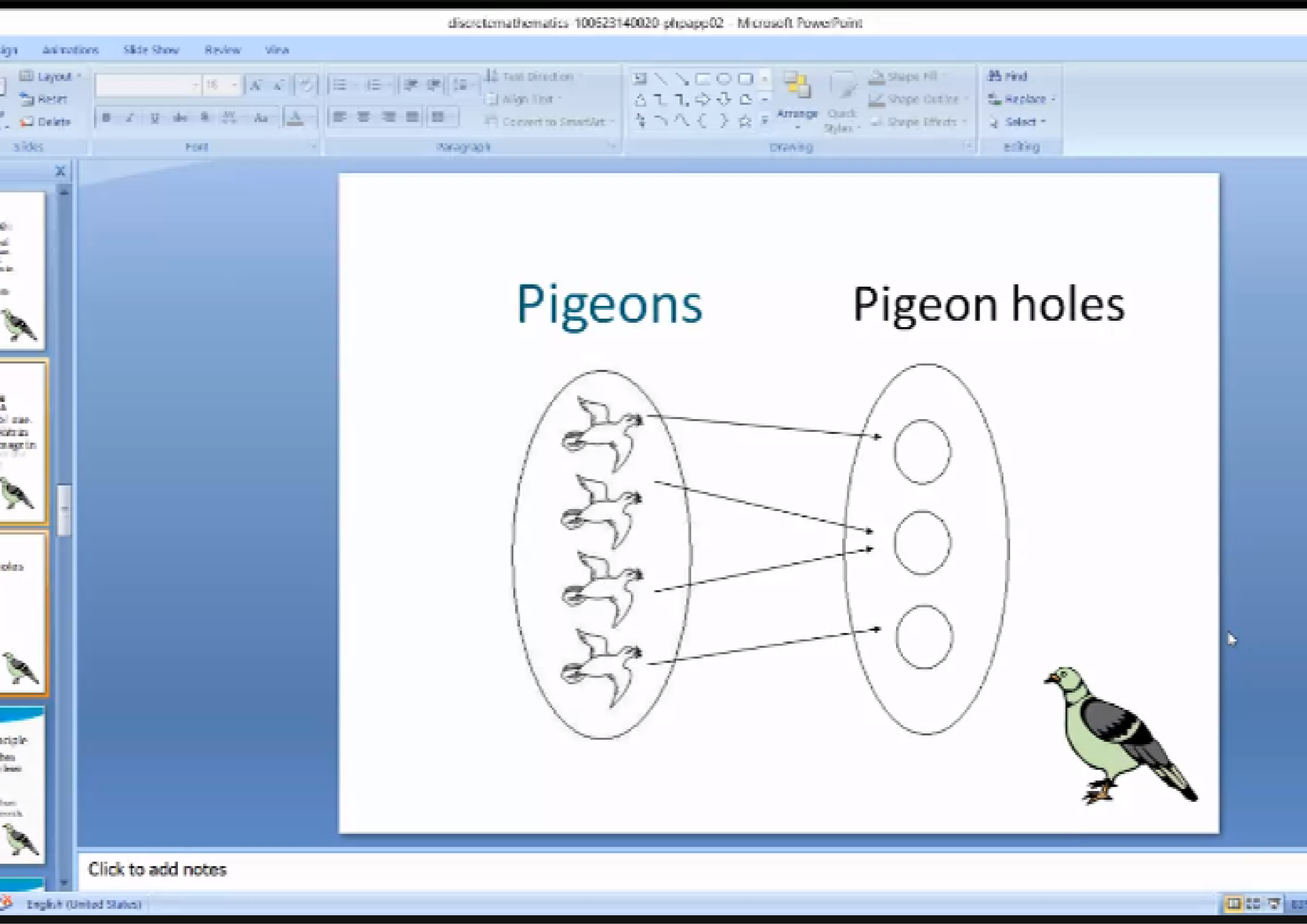
Examples : 1. $f : \{1,2,3,4\} \rightarrow \{1,2,3,4,5,6\}$ defined as $f(x) = x \ \forall x$ and
$g : \{1,2,3,4,5,6\} \rightarrow \{1,2,3,4,5,6\}$ as $g(x) = x$, for $x=1,2,3,4$
and $g(5)= g(6) = 5$. then $gof(x)= x \ \forall x$, which shows that gof is 1-1 but g is not 1-1.

2. $f : \{1,2,3,4\} \rightarrow \{1,2,3,4\}$ and $g : \{1,2,3,4\} \rightarrow \{1,2,3\}$ defined as
$f(1) =1$, $f(2) = 2$, $f(3)=f(4) =3$, $g(1)=1$, $g(2)=2$ and $g(3)=g(4)=3$. It shows
that gof is onto but f is not onto.

The Pigeonhole Principle

- Suppose a flock of pigeons fly into a set of pigeonholes. If there are more pigeons than pigeonholes, then there must be at least 1 pigeonhole that has more than one pigeon in it.
- If n items are put into m pigeonholes with n > m, then at least one pigeonhole must contain more than one item.



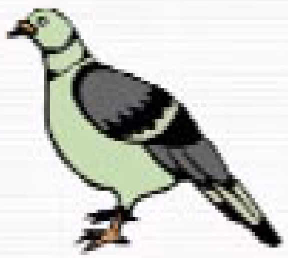

discretemathematics 100523140020 phpapp02 - Microsoft PowerPoint
Animations
Slide Show
Review
View
Layout
Reset
Delete
Slides
Font
Paragraph
Drawing
Editing
Arrange
Find
Replace
Select
Pigeons
Pigeon holes
Click to add notes
English (United States)

Generalized Pigeonhole Principle

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects...!

For Example:

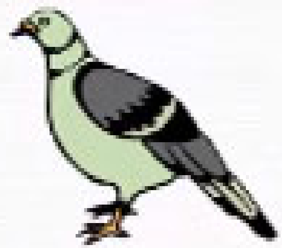
Among any 100 people there must be at least $\lceil 100/12 \rceil = 9$ who were born in the same month.



gupta has left the meeting

Problem Statement

Show that if any 5 no from 1 to 8 arechosen then two of them will add to 9.



How to Solve...?

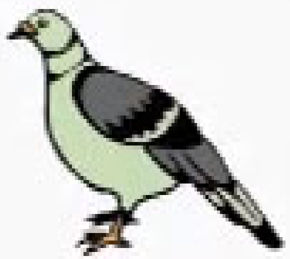
Construct 4 different sets each containing two numbers that add upto 9.

$A_1=\{1,8\}$

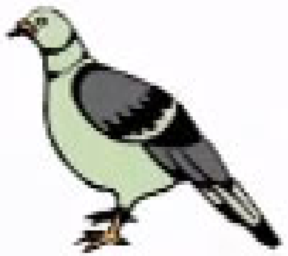
$A_2=\{2,7\}$

$A_3=\{4,5\}$

$A_4=\{3,6\}$



- Each of the 5 no's chosen must belong to oneof thesesets, since thereare only 4 sets the pigeon hole principle states that two of the chosen numbers belong to the samesets.
- These numbers add up to9.



Click to add notes

What do you mean bycardinality...?

Cardinality

Cardinality of two Sets

Two sets A and B have the same cardinality if and only if there is a one-to-one correspondence from A to B i.e. there is a function from A to B that is one-to-one and onto...!

How Cardinality is defined...?

There are two approaches to cardinality –

- one which compares sets directly using bijections and injections
- and another which usescardinal numbers.

Cardinal Numbers:

No. of elements in a set is called cardinal number.

$$0, 1, 2, 3, \cdots, n, \cdots; \aleph_0, \aleph_1, \aleph_2, \cdots, \aleph_\alpha, \cdots$$

Click to add notes

How Cardinality is defined...?

There are two approaches to cardinality –

- one which compares sets directly using bijections and injections
- and another which uses cardinal numbers.

Cardinal Numbers:

No. of elements in a set is called cardinal number.

$$0, 1, 2, 3, \cdots, n, \cdots; \aleph_0, \aleph_1, \aleph_2, \cdots, \aleph_\alpha, \cdots$$

Click to add notes

Properties of Cardinality

- **Reflexive property of cardinality;** A has the same cardinality as A.
- **Symmetric property of cardinality;** If A has the same cardinality as B then B has the same cardinality as A.
- **Transitive property of cardinality;** If A has the same cardinality as B and B has the same cardinality as C then A has the same cardinality as C…

Cardinality of an Infinite Set

Infinite Set:

A set is said to be infinite if it is equivalent to its propersubset.

Countable Set

A set A is said to be countable if

- it is either finite

OR

- its Denumerable

Denumerable: If a set is equivalent to the set of natural numbers N then it is called denumerable set.

Countable Properties

- Every subset of N is countable.
- S is countable if and only if $|S| \leq |N|$.
- Any subset of a countable set is countable.
- Any image of a countable set is countable.

Conclusion

Pigeonhole Principle says that there can't exist 1-1 correspondence between two sets those have different cardinality. But we saw that two sets which have 1-1 correspondence have the same cardinality.