(2) (1)

26.3 Probability and set notations:

Notations:

(i) Probability of happening of events A or B is written as $P(A+B)$ or $P(A \cup B)$.

(ii) Probability of happening of both the events A and B is written as $P(AB)$ or $P(A \cap B)$.

(iii) 'Event A implies ($\Rightarrow$) event B' is expressed as $A \subset B$.

(iv) 'Events A and B are mutually exclusive' is expressed as $A \cap B = \phi$

(6) For any two events A and B,

$$P(A \cap B') = P(A) - P(A \cap B)$$

Proof : From Fig 'a',

$(A \cap B') \cup (A \cap B) = A$

Proof :- From Fig 'a',

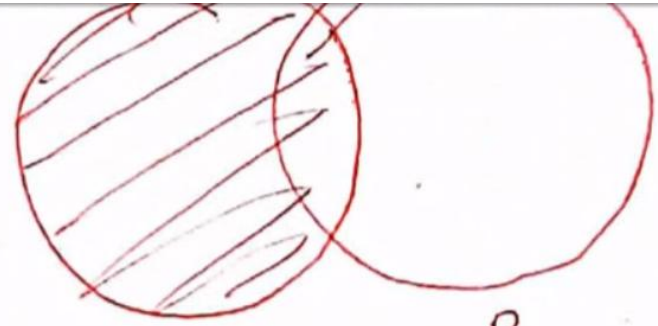

Fig. 'a'

$(A \cap B') \cup (A \cap B) = A$

$\therefore P[(A \cap B') \cup (A \cap B)] = P(A)$

$P(A \cap B') + P(A \cap B) = P(A)$

$P(A \cap B') = P(A) - P(A \cap B)$

Similarly.

$P(A' \cap B) = P(B) - P(A \cap B)$

26.4: Addition Law of Probability or Theorem of Total Probability

(1) If the probability of an event A happening as a result of a trial is $P(A)$ and the Probability of a mutually exclusive event B happening is $P(B)$, then the probability of either of the events happening as a result of the trial is $P(A+B)$ or $P(A \cup B) = P(A) + P(B)$.

Proof: Let n be the total number of equally likely cases and let m_1 be favourable to the event A and

m_2 be

m_2 be favourable to the event B. Then the number of cases favourable to A or B is $m_1 + m_2$. Hence the probability of A or B happening as a result of the trial.

$$= \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n} = P(A) + P(B).$$

(2) If A, B, are any two events (not mutually exclusive), then

$$P(A+B) = P(A) + P(B) - P(AB)$$

or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If the events A and B are any two events then, there are some outcomes which favour both A and B. If m_3 be their number, then there are included in both m_1 and m_2. Hence the total number of outcomes favouring either A or B or both is

$$m_1 + m_2 - m_3.$$

Thus the probability of occurrence of A or B or both

$$= \frac{m_1 + m_2 - m_3}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m_3}{n}.$$

$$= \frac{m_1 + m_2 - m_3}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m_3}{n}.$$

Hence $P(A+B) = P(A) + P(B) - P(AB)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Ans- When A and B are mutually exclusive

$P(AB)$ or $P(A \cap B) = 0$ and we get.

$P(A+B)$ or $P(A \cup B) = P(A) + P(B)$.

In general, for a number of mutually exclusive events.

$A_1, A_2, \dots, A_n$, we have.

$P(A_1 + A_2 + \dots + A_n)$ or $P(A_1 \cup A_2 \cup \dots \cup A_n)$
$= P(A_1) + P(A_2) + \dots + P(A_n)$

③ If A, B, C are any three events, then

$$P(A+B+C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Proof: using the above result for any two events, we have

$$P(A \cup B \cup C) = P[(A \cup B) \cup C] = P(A \cup B) + P(C) - P[(A \cup B) \cap C]$$

$$= [P(A) + P(B) - P(A \cap B)] + P(C) - P[(A \cap C) \cup (B \cap C)]$$

(Distributive law)

$$= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)]$$

$$[\because (A\cap C)\cap(B\cap C) = A\cap B\cap C]$$

$$= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$[\because A\cap C = C\cap A]$$

Ex: 26.11 In a race, the odds in favour of the four horses: H_1, H_2, H_3, H_4 are $1:4$, $1:5$, $1:6$, $1:7$ respectively. Assuming that a dead heat is not possible, find the chance that one of them wins the race.

solution: since it is not possible for all the horses to cover the same distances in the same time (a dead heat), the events are mutually exclusive.

If P_1, P_2, P_3, P_4 be the probabilities of winning of the horses H_1, H_2, H_3, H_4 respectively, then

$$P_1 = \frac{1}{1+4} = \frac{1}{5} \quad [\because \text{odds in favour of } H_1 \text{ are } 1:4]$$

$$P_2 = \frac{1}{6}, \quad P_3 = \frac{1}{7}, \quad P_4 = \frac{1}{8}.$$

$1+4$

$P_2 = \frac{1}{6}, \quad P_3 = \frac{1}{7}, \quad P_4 = \frac{1}{8}.$

Hence the chance that one of them wins

$$= P_1 + P_2 + P_3 + P_4$$

$$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{533}{840}$$

Ans

ndependent

..dependent Events

5

Two events are said to be independent, if happening or failure of one doesnot affect the happening or failure of the other. otherwise the events are said to be dependent.

for two dependent events A and B, the symbol $P(B/A)$ denotes the probability of occurrence of B, when A has already occurred. and is ... the conditional probability

$P(B/A)$ denotes the ... when A has already occurred. It is known as the conditional probability and is read as a 'Probability of B given A'.

② Multiplication law of Probability or Theorem of Compound Probability.

If the prob. of an event A happening as a result of trial is $P(A)$ and after A has happened the probability of an event B happening as a result of another trial (i.e conditional prob. of B given A) $P(B/A)$, then the Prob. of both the events ... of two trials of another trial ... Prob. of both the events

is, $P(B/A)$, then the Prob. of both trials A and B happening as a result of two trials

is $P(AB)$ or $P(A \cap B) = P(A) \cdot P(B/A)$.

(3) If the events A and B are independent, i.e.

Ex: if the happening of B doesnot depend on wheather A has happened or not, then

$$P(B/A) = P(B) \text{ and } P(A/B) = P(A)$$

$$P(AB) \text{ or } P(A \cap B) = P(A) \cdot P(B).$$

③ If the events A and B are independent, i.e

Ex: if the happening of B does not depend on wheather A has happened or not, then

$$P(B/A) = P(B) \text{ and } P(A/B) = P(A)$$

$$\therefore P(AB) \text{ or } P(A \cap B) = P(A) \cdot P(B).$$

In general, $P(A_1, A_2, \ldots, A_n)$ or $P(A_1 \cap A_2 \cap \cdots \cap A_n)$

$$= P(A_1) \cdot P(A_2) \cdots P(A_n).$$

Cor. : If P_1, P_2 be the probabilities of happening of two independent events, then

(i) The prob. that the first event happens and the second fails is $P_1(1-P_2)$.

(ii) The prob. that both events fails to happen is $(1-P_1)(1-P_2)$

(iii) The prob. that at least one of the events happen is

$(1-p_1)(1-p_2)$

(iii) The prob. that at least one of the events happens is

$1-(1-p_1)(1-p_2)$.

This is commonly known as their commulative prob.

In general, If $p_1, p_2, p_3, \text{——}, p_n$ be the chances of happening of n independent events, then their commulative prob. (i.e the chance that at least one of the events will happen) is

Ex: Two cards are drawn in succession from a pack of 52 cards. Find the chances that the first is a king and the second a queen if the first card is (i) replaced (ii) not replaced.

Sol (i) The prob. of drawing a king $= \frac{4}{52} = \frac{1}{13}$

If the card is replaced, the pack will again have 52 cards so that the prob. of drawing a queen is $\frac{1}{13}$

52 cards so that the prob. of drawing a queen is $\frac{1}{13}$

The two events being independent, the prob. of drawing both cards in succession $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$ Ans

The prob. of drawing a king $= \frac{1}{13}$ (7)

...ill have 51

The prob. of drawing a king $= \frac{1}{13}$ (7)

If the card is not replaced, the pack will have 51 cards only so that the chance of drawing a queen is

$$\frac{4}{51}.$$

Hence the prob. of drawing both cards $= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$

A

Random variable :

If a real variable X be associated with the outcome of a random experiment, then since the values which X takes depend on chance, it is called a random variable or a stochastic variable or simply a variate.

For instances, if a random experiment E consists of tossing a pair of dice, the sum X of the two numbers which turn up have the value 2, 3, 4, — 12 X is the random variable.

chance. Then

two numbers which turn up ...
depending on chance. Then X is the random variable.
It is a function whose values are real numbers
and depend on chance.

If in a random experiment, the event corresponding
to a number a occurs, then the corresponding random
variable X is said to assume the value a and
the prob. of the event is denoted by $P(X = a)$,

variable X is said ... by P(X = a).
the prob. of the event is denoted by
Similarly the prob. of the event ~~X~~ assuming any value in the interval $a < X < b$ is denoted by $P(a < X < b)$.
The Prob. of the event $X \leq c$ is written as $P(X \leq c)$.

If a random variable takes a finite set of values, it is called a discrete variate. On the other hand, if it assumes an infinite number of uncountable values, it is called a continuous variate.

§6.8 (1) Discrete Prob. Distribution :

Suppose a discrete variate X is the outcome of some experiment. If the prob. that X takes the values x_i, is

§6.8 (i) Discrete Prob. Distribution :

Suppose a discrete variate X is the outcome of some experiment. If the prob. that X takes the values x_i, is p_i, then

$$P(X = x_i) = p_i \quad \text{or} \quad p(x_i) \quad \text{for } i = 1, 2, \ldots$$

where (i) $p(x_i) \geq 0$ for all values of i, (ii) $\Sigma p(x_i) = 1$.

The set of values x_i with their probabilities p_i constitute a discrete prob. distribution of the discrete variate X.

For Ex: the discrete prob distribution for X, the sum of the numbers which turn on tossing a pair of dice is given by the following table:

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$p(x_i)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$p(x_i)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

[$\because$ There are $6 \times 6 = 36$ equally likely outcomes and therefore, each has the prob. $1/36$.

we have $X = 2$ for one outcome i.e $(1,1)$; $X = 3$ for two outcomes $(1,2)$ & $(2,1)$; $X = 4$ for three outcomes $(1,3)$, $(2,2)$ and $(3,1)$ & so on.]

Q.29: The Prob. density function of a variate X is

X:	0	1	2	3	4	5	6
$p(x)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$.

(i) Find $P(X<4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.

(ii) what will be the minimum value of k so that $P(X \leq 2) > 3$.

Solution: (i) If X is a random variable, then

$$\sum_{i=0}^{6} p(x_i) = 1 \text{ i.e } K + 3K + 5K + 7K + 9K + 11K + 13K = 1$$

$$K = \frac{1}{49}$$

$P(X \geq 5) = 11k + 13k = 24k = \frac{24}{49}$.

$P(3 < X \leq 6) = 9k + 11k + 13k = 33k = \frac{33}{49}$

(ii) $P(X \leq 2) = k + 3k + 5k = 9k > 0.3$ or $k > \frac{1}{30}$

Thus min value of $k = \frac{1}{30}$

A