Experiment 12

Matrix Chain Multiplication (MCM) problem

Compatible: No. of columns of A must equal the no. of rows of B.

If A is $p \times q$ matrix and B is a $q \times r$ matrix, the resulting matrix C is a $p \times r$ matrix

Pseudocode

```
MATRIX-MULTIPLY(A,B)
 if columns[A] ≠ rows[B]
  then error "incompatible dimensions"
  else for i ← 1 to rows[A]
        do for j ← 1 to columns[B]
          do c[i,j] ← 0
           for R ← 1 to columns[A]
             do C[i,j] ← c[i,j] + A[i,R] · B[R,j]
      return C
```

* Time to compute C is dominated by the no. of scalar multiplications in last second line, which is pqr.

Time Complexity :- $O(n^3)$

Auxiliary Space :- $O(n^2)$

0 (1,1)	15750 (1,2)	7875 (1,3)	9375 (1,4)	11875 (1,5)	15125 (1,6)
	0 (2,2)	2625 (2,3)	4375 (2,4)	7125 (2,5)	10500 (2,6)
		0 (3,3)	750 (3,4)	2500 (3,5)	5375 (3,6)
			0 (4,4)	1000 (4,5)	3500 (4,6)
				0 (5,5)	5000 (5,6)
					0 (6,6)

P_0	P_1	P_2	P_3	P_4	P_5	P_6
30	35	15	5	10	20	25

A_1 A_2 A_3 A_4 A_5 A_6

Recurrence Relation

$$P(n) = \begin{cases} 1 & \text{if } n = 1 \\ \sum_{k=1}^{n-1} P(k)P(n-k) & \text{if } n \geq 2 \end{cases}$$

Recurrence Sol^n

$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \leq k < j} \{ m[i,k] + m[k+1,j] + p_{i-1} p_k p_j \} & \text{if } i < j \end{cases}$$

* $m[1,1], m[2,2], m[3,3], m[4,4], m[5,5], m[6,6] = 0$

* $m[1,2] = m[1,1] + m[2,2] + p_0 p_1 p_2$
$= 0 + 0 + 30 \times 35 \times 15 = 15750$

similary $m[2,3] = 2625$
$m[3,4] = 750$
$m[4,5] = 1000$
$m[5,6] = 5000$

* $m[1,3] = \min \begin{cases} m[1,1] + m[2,3] + p_0 p_1 p_3 \\ m[1,2] + m[3,3] + p_0 p_2 p_3 \end{cases} = \begin{cases} 7875 \\ 18000 \end{cases}$ ✓

similary, $m[2,4] = 4375$
$m[3,5] = 2500$
$m[4,6] = 3500$

* $m[1,4] = \min \begin{cases} m[1,1] + m[2,4] + p_0 p_1 p_4 \\ m[1,2] + m[3,4] + p_0 p_2 p_4 \\ m[1,3] + m[4,4] + p_0 p_3 p_4 \end{cases} = \begin{cases} 14875 \\ 20250 \\ 9375 \end{cases}$ ✓

similarly, $m[2,5] = 7125$
$m[3,6] = 5375$

* $m[1,5] = \min \begin{cases} m[1,1] + m[2,5] + p_0 p_1 p_5 \\ m[1,2] + m[3,5] + p_0 p_2 p_5 \\ m[1,3] + m[4,5] + p_0 p_3 p_5 \\ m[1,4] + m[5,5] + p_0 p_4 p_5 \end{cases} = \begin{cases} 28125 \\ 27250 \\ 11875 \\ 15375 \end{cases}$ ✓

similarly, $m[2,6] = 10500$

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$$* \quad m[1,6] = \min \begin{cases} m[1,1] + m[2,6] + p_0 p_1 p_6 \\ m[1,2] + m[3,6] + p_0 p_2 p_6 \\ m[1,3] + m[4,6] + p_0 p_3 p_6 \\ m[1,4] + m[5,6] + p_0 p_4 p_6 \\ m[1,5] + m[6,6] + p_0 p_5 p_6 \end{cases} \Rightarrow \underline{15125}$$

(required ans.)
(Minimum cost)