EI5503 CONTROL SYSTEM ANALYSIS AND DESIGN

ASSIGNMENT REPORT ON

MODELLING OF CONICAL TANK USING MATLAB - SIMULINK

Submitted by

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AIM

To derive the transfer function and state space model of a conical process tank analytically and verify the same using simulation.

ABSTRACT

The control of liquid level is mandatory in process industries. But the control of nonlinear process is complex. Many process industries use conical tanks because of its nonlinear shape which contributes better drainage for solid mixtures, slurries and viscous liquids. For example, a level well above the surface can upset the process reaction balances and damage equipment, but a level below the required set-point can also cause serious problems.

So, level control of conical tank presents a challenging task due to its non-linearity and constantly changing cross-section. The main objective is to implement the suitable controller design for conical tank system to maintain the desired level.

SYSTEM DESCRIPTION

The conical tank level process is a highly nonlinear process because of its varying cross section from bottom to top. The experimental setup is shown in Fig. 1. The parameters that vary with respect to the process variable are considered. At a fixed outlet flow rate the system is controlled and maintained at the desired level. The tank level process to be simulated is single input single output (SISO) tank system. The desired level **h** is maintained by manipulating the inlet flow rate $\mathbf{F_{in}}$ to the system. Here **h** is the controlled variable and $\mathbf{F_{in}}$ is the manipulated variable.

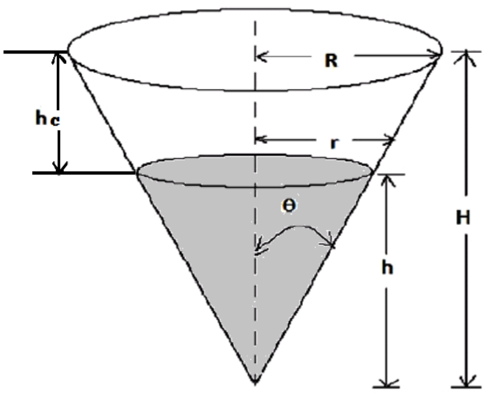

Fig. 1

Table 1.Operating Parameters

Sl. No	Parameter	Description	Value
1	R	Total radius of the cone	19.25cm
2	H	Height of the tank	73cm
3	Fin	Maximum inflow rate of the tank	400 LPH
4	Kv	Value co-efficient	55cm ^{2}/s

DERIVATION

According to mass balance equation,

A dh/dt = $F_{in} - F_{out}$, where $F_{out} = K_v \sqrt{h}$

Where A is the surface area of the fluid in the tank, $A = \pi r^2$

By using Pythagoras theorem in **Fig. 1**, $r = \frac{R\,h}{H}$

Now the mass balance equation becomes,

$$\frac{dh}{dt} = \frac{Fin - K_v\sqrt{h}}{\pi(\frac{R}{H})^2 h^2}$$

$$\alpha = \frac{1}{\pi(\frac{R}{H}^2)h^2} \qquad \beta = \alpha Kv$$

$$\frac{dh}{dt} = \alpha h^{-2} Fin - \beta h^{\frac{-3}{2}} \qquad \text{(Equation 1)}$$

$$\frac{dh_s}{dt} = \alpha F_{in.s} h_s^{-2} - \beta h_s^{\frac{-3}{2}} \qquad \text{(Equation 2)}$$

$$\alpha F_{in.s} = \beta\ h_s^{1/2} \qquad \text{(Equation 3)}$$

$$Let\ F_{in}\ be\ Q\ , \quad y = h - h_s \quad u = F_{in} - F_{in.s}$$

Equation 1 – Equation 2

$$\frac{d}{dt}(h - h_s) = [\alpha F_{in} h^{-2} - \alpha F_{in.s} h_s^{-2}] - \beta h^{\frac{-3}{2}} + \beta h_s^{\frac{-3}{2}}$$

$$\frac{dy}{dt} = [\alpha Q h^{-2} - \beta h_s^{\frac{1}{2}} . h_s^{-2} - \beta h^{\frac{-3}{2}} + \beta h_s^{\frac{-3}{2}}]$$

$$\frac{dy}{dt} = \alpha Q h^{-2} - \beta h^{-3/2} \qquad \text{(Equation 5)}$$

$$Let\ Q h^{-2} = f(h, Q)$$

$$By\ Taylor\ Series\ Expansion,\ to\,linearize\,[Qh^{-2}]\ and\ [h^{-3/2}]$$

$$f(h,Q) = f(h_s,Q_s) + \frac{\partial}{\partial h} f(h-h_s) + \frac{\partial}{\partial Q} f(Q-Q_s) + Higher\,Order$$

$$Qh^{-2} = Q_s h_s^{-2} - 2Q_s h_s^{-3}(h-h_s)\ + h_s^{-2}(Q-Q_s)$$ (Equation 6)

$$h^{-3/2} => f(h) = f(h_s) + \frac{f'(h)(h-h_s)}{1!} + Higher\,Order\,Terms$$

$$h^{-3/2} = h_s^{-3/2} - \frac{3}{2} hs^{-5/2}(h-h_s)$$ (Equation 7)

Substitute Equation 6 and Equation 7 in Equation 5; From Equation 4

$$\frac{dy}{dt} = [\alpha Q_s h_s^{-2} - 2\alpha Q_s h_s^{-3}(y) + \alpha h_s^{-2}(u)] - \beta[h_s^{-3/2} - \frac{3}{2} h_s^{-5/2} y]$$

$$\frac{dy}{dt} = -2\alpha Q_s h_s^{-3} y + \alpha h_s^{-2} u + [\frac{3}{2}\beta h_s^{-5/2} y]$$

$$\frac{dy}{dt} = -2\beta h_s^{-5/2} y + \frac{3}{2}\beta h_s^{-5/2} y + \alpha h_s^{-2} u$$

$$\frac{dy}{dt} = \beta h_s^{-5/2} y(\frac{3}{2} - 2) + \alpha h_s^{-2} u$$

$$\frac{dy}{dt} = \frac{-1}{2}\beta h_s^{-5/2} y + \alpha h_s^{-2} u$$

$$\frac{2}{\beta}\, h_s^{5/2} (\frac{dy}{dt}) + y = \frac{2\alpha}{\beta}\, h_s^{1/2}\, u$$

$$Y(s)\ [\frac{2}{\beta}\, h_s^{5/2}\ s + 1] = U(s)\ [\frac{2\alpha}{\beta}\, h_s^{1/2}\ ]$$

$$Transfer\ Function => \frac{Y(s)}{U(s)} = \frac{\frac{2\alpha}{\beta}\, h_s^{1/2}}{(\frac{2}{\beta}\, h_s^{5/2})\ s + 1}$$ (Equation 8)

$$Time\,Constant = \tau = \frac{2}{\beta}\, h_s^{5/2}$$

$$Steady\ State\ Gain = K = \frac{2}{K_v} h_s^{1/2}$$

From **Table 1**,

$$F_{in} = 400\ lph = \frac{1000}{9}\ cm^3/s \qquad K_v = 55\ cm^2\ s^{-1}$$

$$H = 73\ cm\ \ and\ \ R = 19.25\ cm \qquad \alpha = \frac{1}{\pi(\frac{R}{H})^2} = 4.57756 \qquad \beta = \alpha K_v = 251.7658$$

$$@\ Steady\ State\ (h -> h_s)$$

$$\frac{dh}{dt} = 0\ \ => \frac{F_{in} - K_v\sqrt{h_s}}{\pi(\frac{R}{H})^2 h_s^2} = 0$$

Hence, $F_{in} = K_v\sqrt{h_s}$

$$(1000/9 * 55)^2 = h_s$$

$$h_s = 4.0812\ cm$$

Substitute the steady state parameters in the transfer function,

$$TF = \frac{(\frac{2}{K_v})h_s^{1/2}}{[(\frac{2}{\beta})h_s^{5/2}]\ (s+1)}$$

$$=> \frac{0.07346}{0.26730s + 1}$$

$$TF = \frac{0.2748}{s + 3.741} \qquad \text{(Equation 9)}$$

Thus the $TF = \frac{0.2748}{s+3.741}$ is obtained by analytical method.

MATLAB SIMULATION

The differential equation is simulated in MATLAB (Simulink), with **F_{in}** = 400 lph (111.111 cm^3/s) applied as a step input, the steady state value of **h (h_s)** is obtained as 4.081 cm.

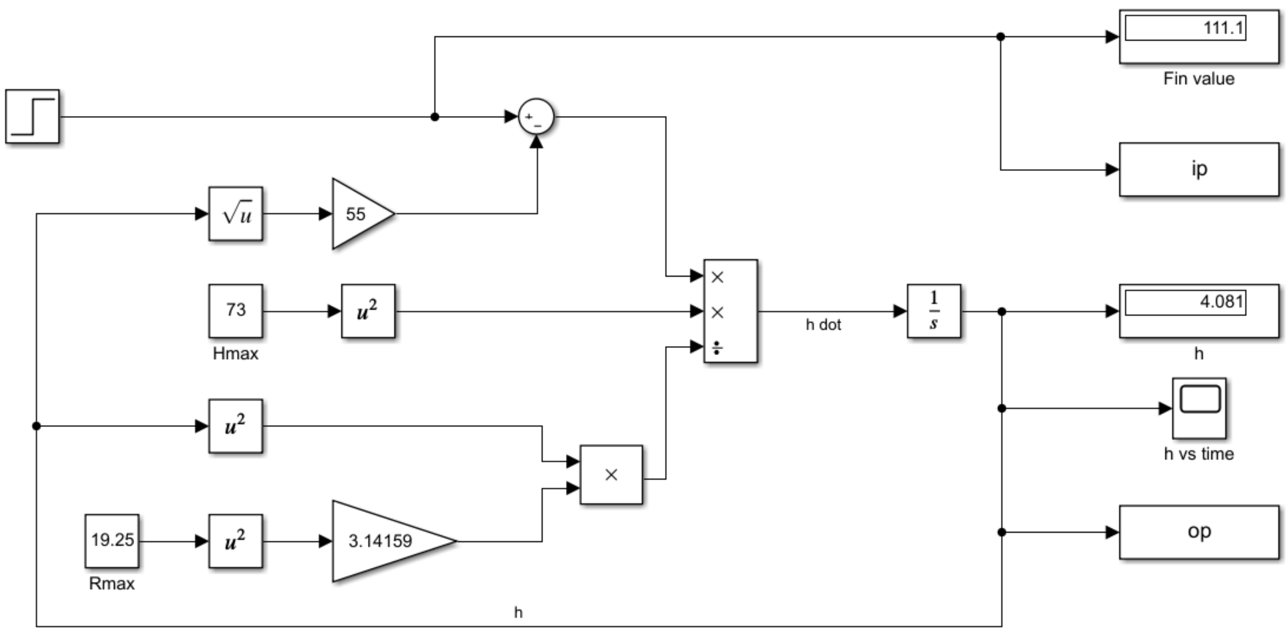

Fig. 2

GRAPH

Simulated height response, where **h** reaching a steady state of 4.0812 cm.

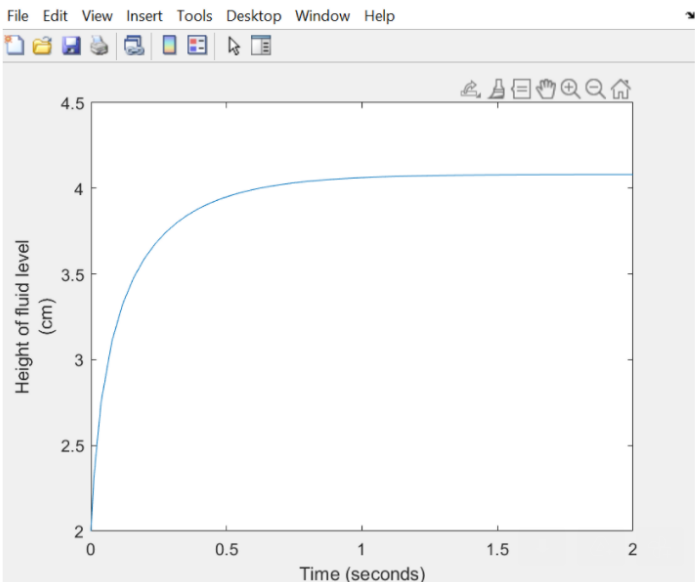

Fig. 3 *(with initial height of 2 cm)*

The transfer function of the system is obtained by using "linmod" command,

>> [numerator, denominator] = linmod ('file_name', output_value, input_value)

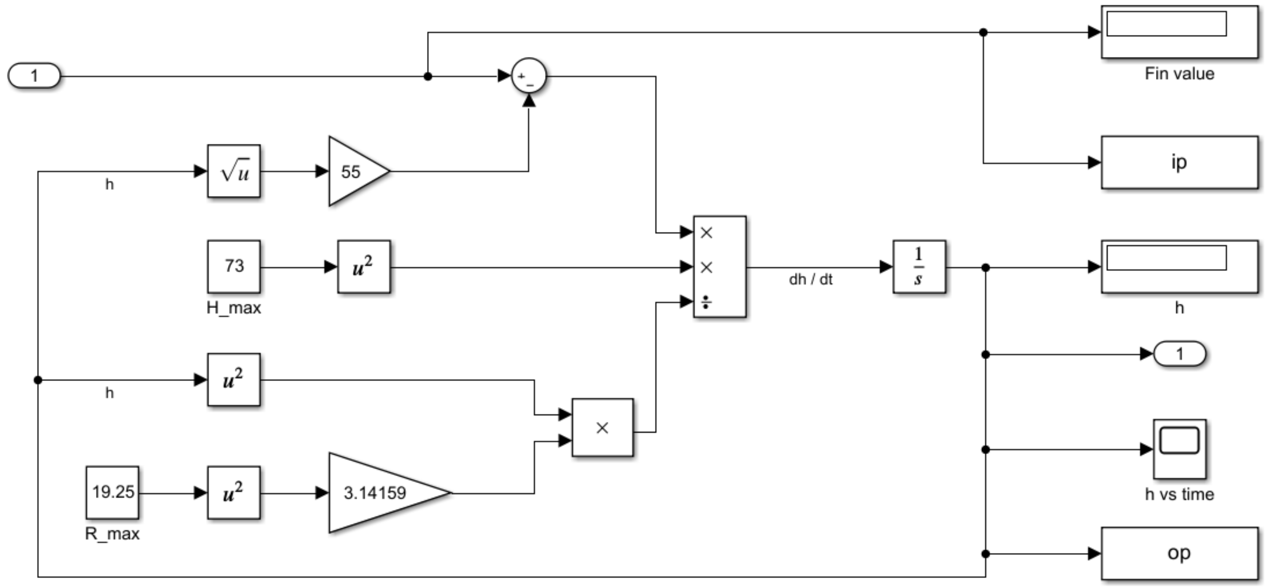

Fig. 4

TRANSFER FUNCTION BY SIMULATION

```
Command Window
>> [n, d] = linmod('conicaltank', 4.0812, 111.111)
n =

         0    0.2748


d =

    1.0000    3.7411

>> sys = tf(n,d)

sys =

   0.2748
  ---------
  s + 3.741

Continuous-time transfer function.
```

Fig. 5

VALIDATION

With 10% decrease in the input $\mathbf{F_{in}}$ from an initial value of 111.111 cm^3/s to a final value of 100 cm^3/s, the output **h** reaches a steady state of 3.306 cm.

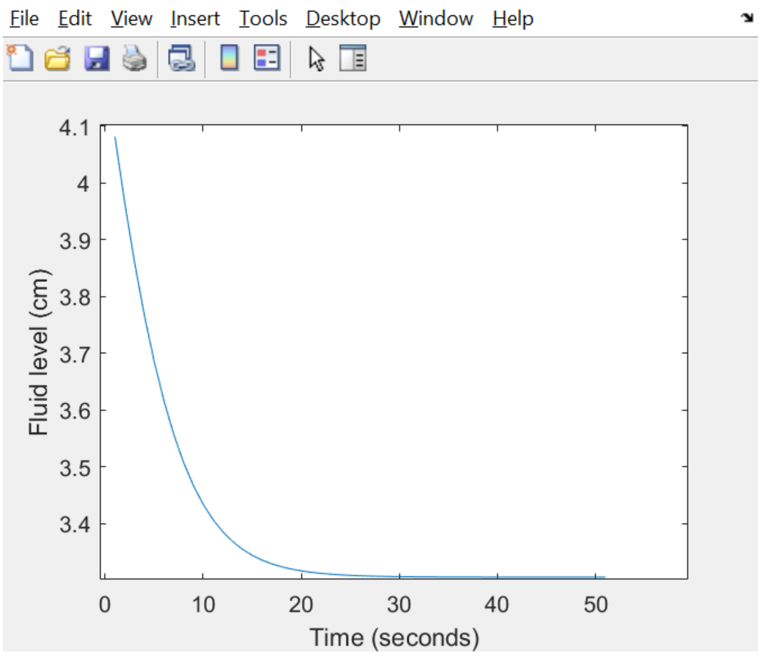

Fig. 6

With 10% increase in the input $\mathbf{F_{in}}$ from an initial value of 111.111 cm^3/s to a final value of 122.22 cm^3/s, the output **h** reaches a steady state of 4.938 cm.

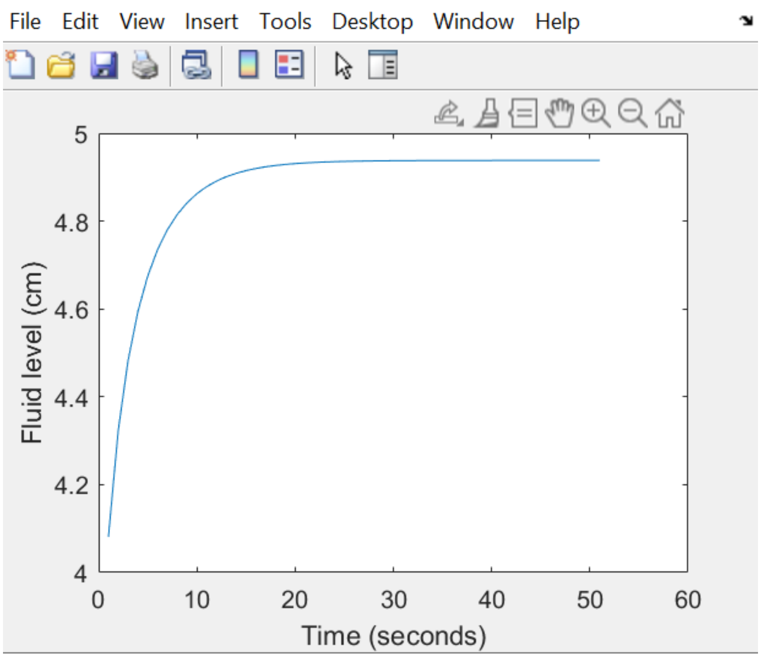

Fig. 7

STATE SPACE MODEL

Mathematical model of any physical system is given by,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where,

A = system matrix
B = input matrix
C = output matrix
D = transition matrix
x = state vector
u = input vector
y = output vector

In the conical tank system, the state space model is represented as,

$$\dot{x} = A\,x + B\,\boldsymbol{F_{in}}$$
$$\boldsymbol{h} = C\,x + D\,\boldsymbol{F_{in}}$$

which can be obtained from the transfer function, $Transfer\ Function => \frac{Y(s)}{U(s)} = \frac{\frac{2\alpha}{\beta}\,h_s^{1/2}}{(\frac{2}{\beta}\,h_s^{5/2})\ s+1}$

$$\frac{h}{Fin} = \frac{0.2748}{s+3.741}$$

and from the equation, $\frac{dy}{dt} = \frac{-1}{2}\beta h_s^{-5/2} y + \alpha h_s^{-2} u$

A = $\frac{-1}{2}\beta h_s^{-5/2}$ = - 3.7410 ; B = 1 *(form the transfer function)*

C = αh_s^{-2} = 0.2748 ; D = 0 *(form the transfer function)*

Hence, the state space model is,

$$\dot{x} = [-3.741]\,x + [1]\,\boldsymbol{F_{in}}$$
$$\boldsymbol{h} = [0.2748]\,x + [0]\,\boldsymbol{F_{in}}$$

STATE SPACE MODEL BY SIMULATION

```
Command Window

>> [A,B,C,D] = tf2ss(n,d)

A =

   -3.7411


B =

     1


C =

    0.2748


D =

     0
```

Fig. 8

RESULT

System representation	Analytical result	Simulated result
Transfer function	$\frac{0.2748}{s+3.741}$	$\frac{0.2748}{s+3.741}$
State space model *(matrices)*	A = - 3.7410 B = 1 C = 0.2748 D = 0	A = - 3.7411 B = 1 C = 0.2748 D = 0

Thus the transfer function and the state space model are obtained for conical process tank on the given operating point analytically and verified by simulating in MATLAB using simulink.