

Hopcroft-Karp Algorithm

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1 Introduction

2 Definitions

3 Algorithm

4 Time Complexity

5 Applications

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Introduction

Hopcroft-Karp Algorithm

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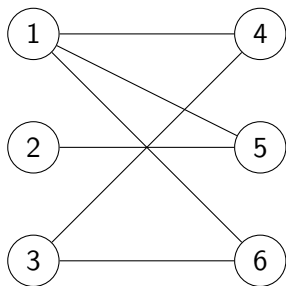


Figure: Bipartite graph

Introduction

Hopcroft-Karp Algorithm

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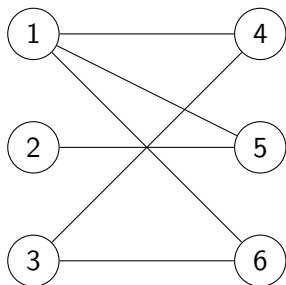


Figure: Bipartite graph

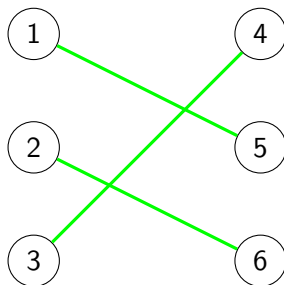


Figure: Maximum cardinality matching

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Bipartite Graph

A graph is bipartite if its vertices can be divided into two disjoint sets such that every edge connects a vertex in one set to a vertex in the other set.

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A matching in a graph is a set of edges such that no two edges share a common vertex. A matching is said to be maximum if it contains the maximum number of edges possible.

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A matching in a graph is a set of edges such that no two edges share a common vertex. A matching is said to be maximum if it contains the maximum number of edges possible.

Free vertex

A free vertex refers to a vertex in the left part of the bipartite graph that is not yet matched with any vertex in the right part of the graph.

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Hopcroft-Karp(G)

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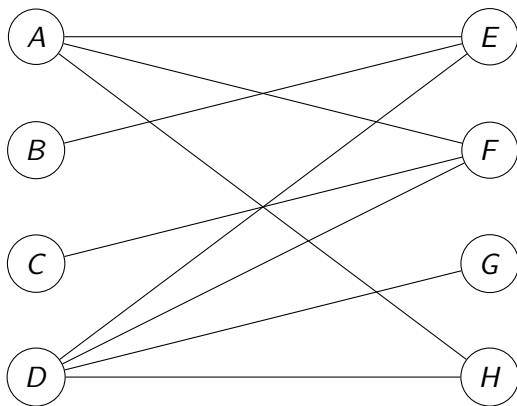
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Hopcroft-Karp(G)

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 - 2: **while** there exists an augmenting path P with respect to M **do**
 - 3: Use a breadth-first search to find an augmenting path P with respect to M .
 - 4: Use a depth-first search to augment the matching M along P .
 - 5: **end while**
 - 6: Output the matching M as the maximum cardinality matching.
-

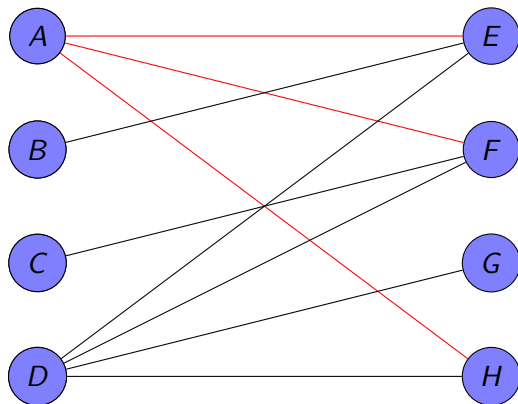
Example

A bipartite graph is given below where we will find its maximum cardinality matching:



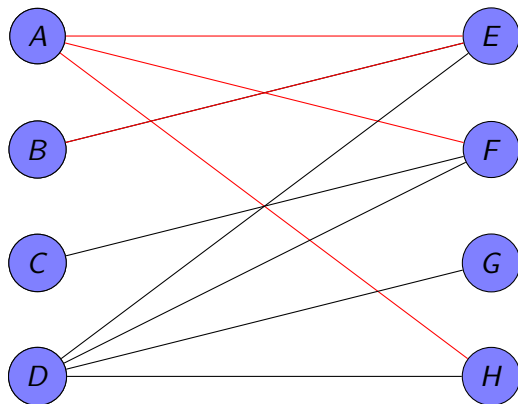
First iteration

Running BFS on the free vertices of the left side...



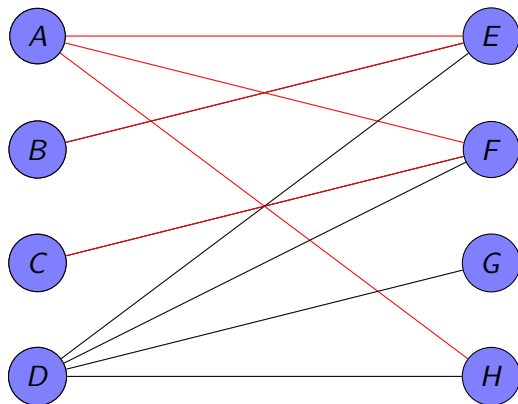
First iteration

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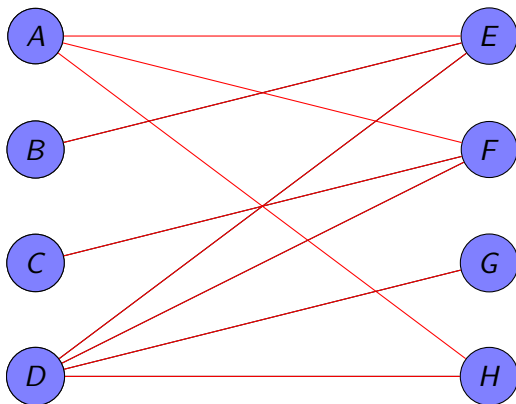
First iteration

Running BFS on the free vertices of the left side...



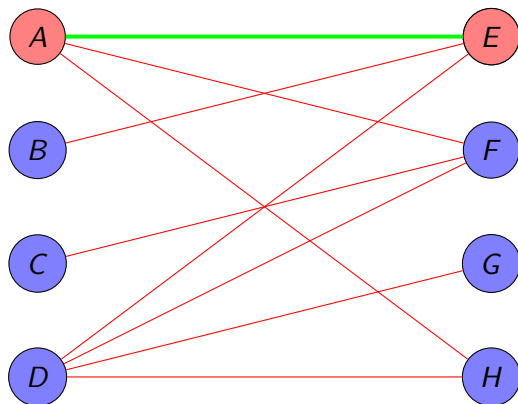
First iteration

Running BFS on the free vertices of the left side...



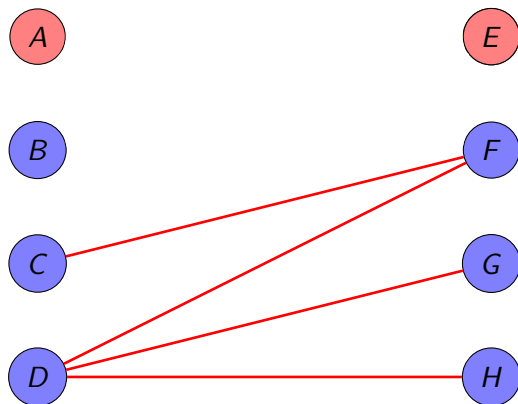
First iteration

Running DFS on the free vertices of the right side...



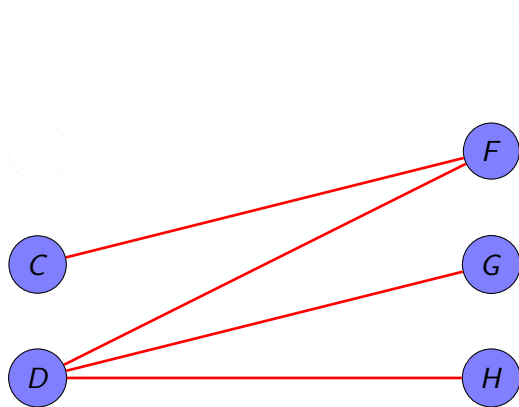
First iteration

Running DFS on the free vertices of the right side...



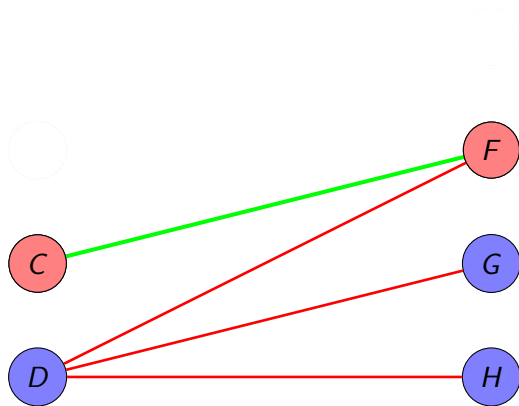
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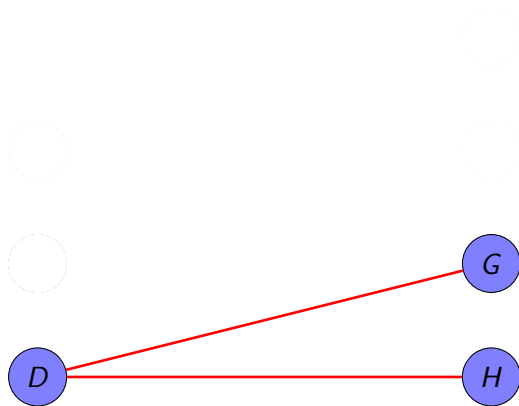
First iteration

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First iteration

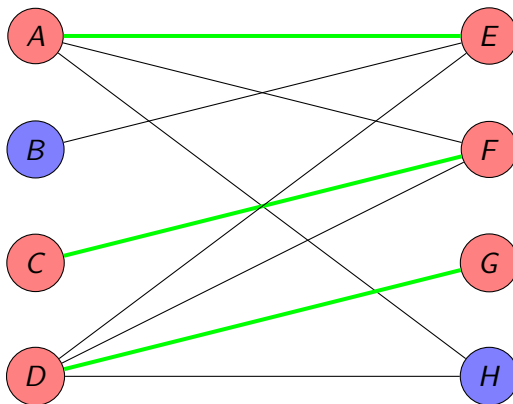
Running DFS on the free vertices of the right side...



After first iteration

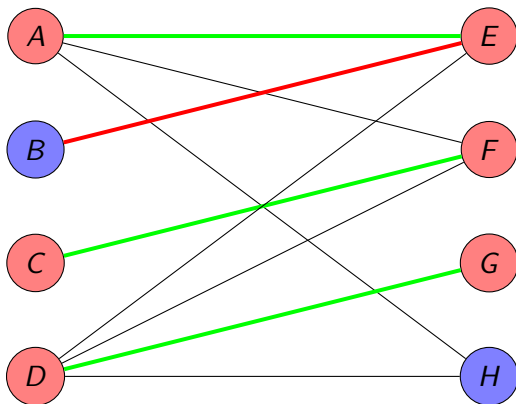
After running the DFS on the remaining part,

$$M = \{A - E, C - F, G - D\}$$



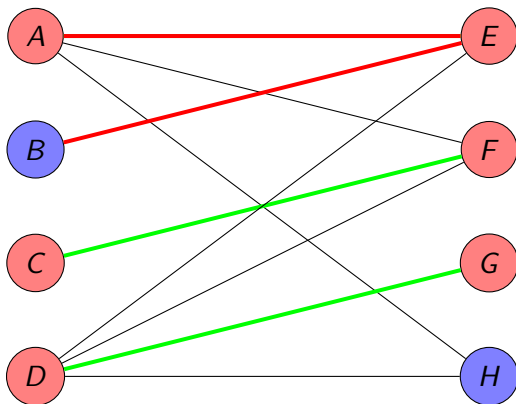
Second iteration

Running BFS on the free vertices of left side..



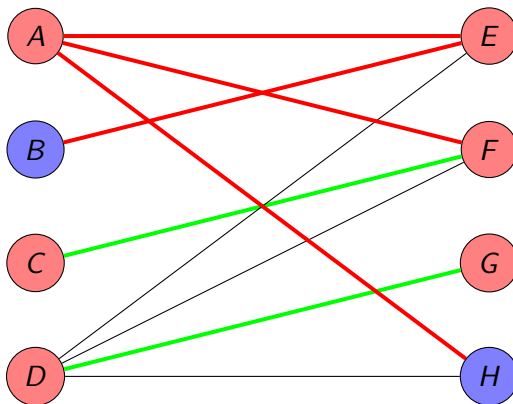
Second iteration

Running BFS on the free vertices of left side..



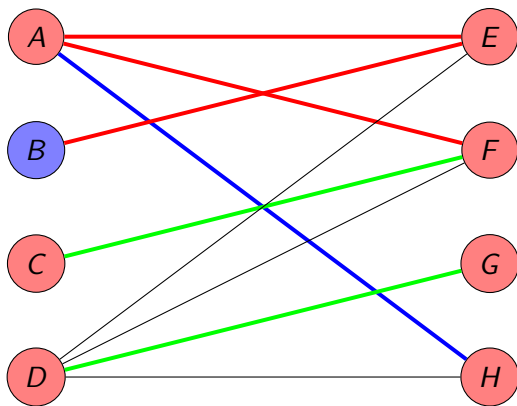
Second iteration

Running BFS on the free vertices of left side..



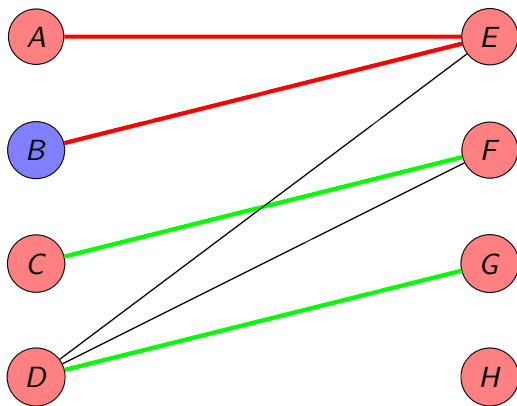
Second iteration

Running DFS on the free vertices of right side...



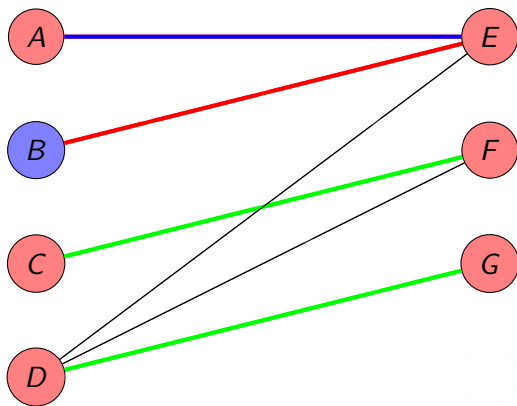
Second iteration

Running DFS on the free vertices of right side...



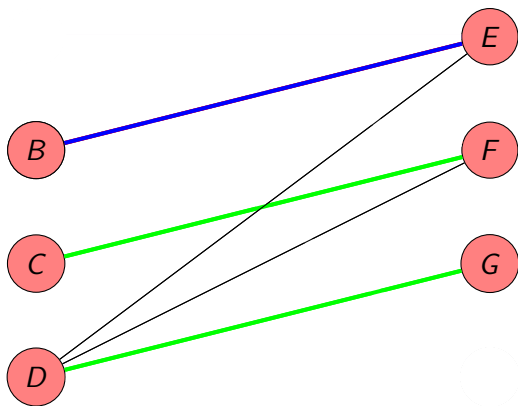
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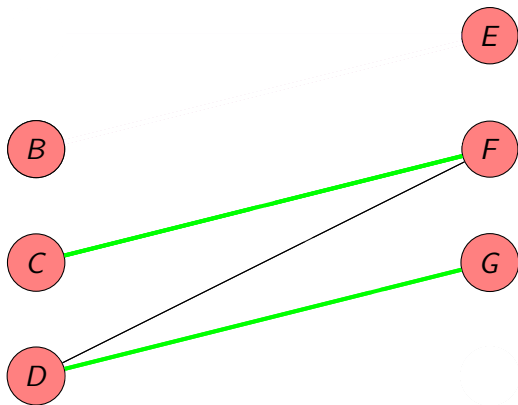
Second iteration

Running DFS on the free vertices of right side...



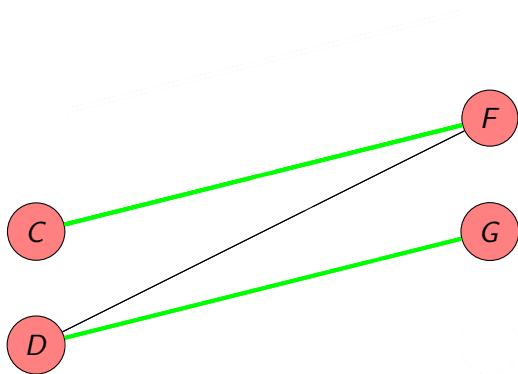
Second iteration

Running DFS on the free vertices of right side...



Second iteration

Running DFS on the free vertices of right side...



Algorithm termination..

As no more free vertex is available, the algorithm terminates and finally we get the following graph with maximum cardinality 4 where $M = \{A - H, B - E, C - F, D - G\}$

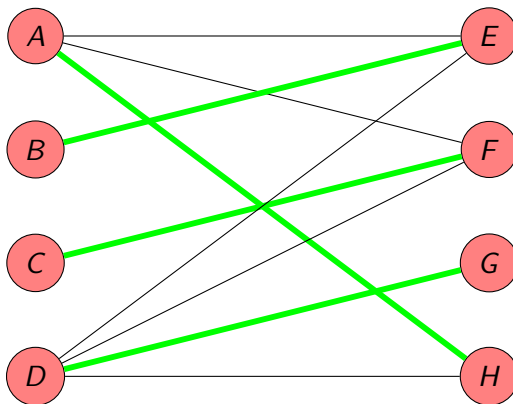


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Time Complexity

- The time complexity of the algorithm is $O(E * \sqrt{V})$, where E is the number of edges and V is the number of vertices in the graph. This means that the time required to run the algorithm increases linearly with the number of edges, but is also influenced by the square root of the number of vertices.

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- The time complexity of the algorithm is considered efficient for most bipartite graphs, but may not be the best choice for extremely large graphs.

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- **Image segmentation** - finding matches between objects in an image and a pre-defined set of object templates.

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- **Job scheduling** - matching workers with tasks based on their skills and availability.
- **Online advertising** - matching ads with potential viewers based on demographic and behavioral data.