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Q1

$$f(z) = \log_e (1+z)$$

and $z = x^T x \quad , x \in R^d$

let,

$$f'(z) = \frac{d}{dz} \log_e (1+z)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dx} (x^T x)$$

$$= \frac{1}{1+x^T x} \cdot 2x$$

$$= \frac{2x}{1+x^T x} \quad \text{(Ans)}$$

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Q.2

$$f(z) = e^{-z/2}$$

where, $z = g(y) = y^T S^{-1} y$

$y = h(x) = x - \mu$

$x, \mu \in R^d \; , \; S \in R^{d \times d}$.

Now,

$$f'(z) = \frac{d}{dz}\left(e^{-z/2}\right)$$

$$= e^{-z/2} \cdot \frac{d}{dz}\left(-z/2\right)$$

$$= e^{-z/2}\left(-\frac{1}{2}\right) \cdot \frac{d}{dy}\, y^T S^{-1} y \cdot \frac{d}{dx}(x-\mu)$$

Here,

$$\frac{d}{dy}\left(y^T S^{-1} y\right) = \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) \cdot S^{-1} \cdot (y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T S^{-1} y + y^T S^{-1} h + h S^{-1} y + S^{-1} h^2 - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{h\,(y^T s^{-1} + s^{-1} y + s^{-1} h)}{h}$$

$$= y^T s^{-1} + s^{-1} y + \lim_{h \to 0} s^{-1} h$$

$$= y^T s^{-1} + s^{-1} y$$

and, $\frac{d}{dx}(x - \mu) = \underline{1}$

$$\therefore f'(z) = -\frac{1}{2} e^{-z/2} (y^T s^{-1} + s^{-1} y)$$

(Ans)