Answers to questions in Lab 1: Filtering operations

Name: Rakin Ali Program: CINTE19, TCSM 22

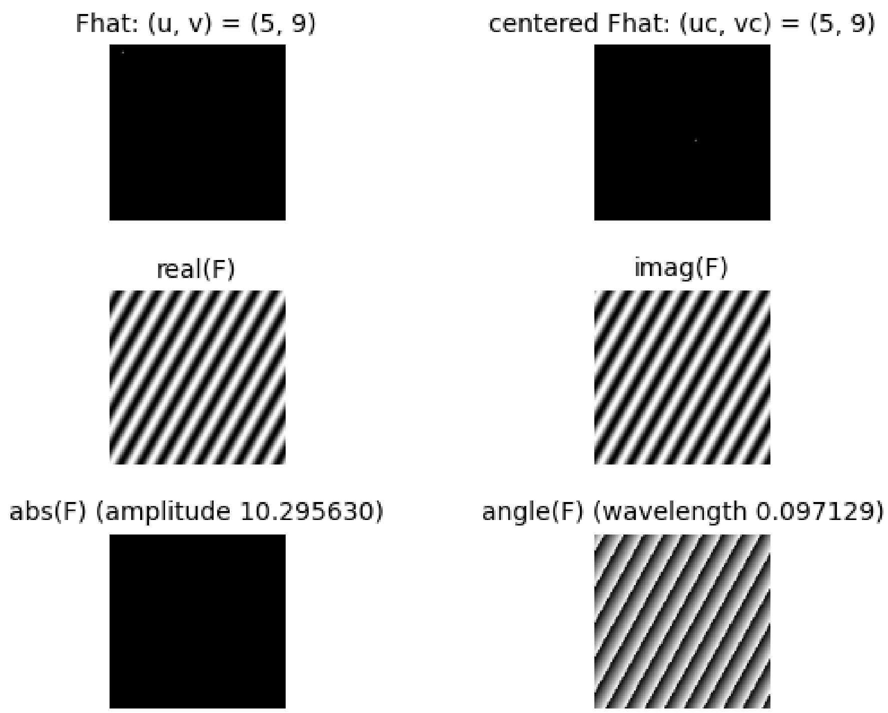
Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

- Where the dot lies determines the wave's direction.
- Origin seems to lie on the top left corner. Distance to origin determines the frequency. The closer you are to it, the lower the frequency.
- Combination of p(y axis) and q (x axis) gives you diagonal sine wave



Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

One point in the frequency domain represents a sine wave in the spatial/wave domain. The position in the frequency domain represents the frequency and orientation of the sinus waves. Below is a 2D figure of how it will look like:

Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers: Below is the image taken from my Jupyter Notebook as Microsoft Word was not cooperating with me.

- The amplitute of the signal is given by

$$Amplitute = \sqrt{\mathbb{R}e^2[\hat{f}(u,v)] + Im^2[\hat{f}(u,v)]}$$

The below equation represents the inverse Fourier transform in the discrete domain.
$F(x) = \frac{1}{N} \sum_{u \in [0:N-1]^2} \hat{(F)} \exp(\frac{2(\pi)iu^T x}{N})$
Applying the inverse Fourier transform at the point (p,q), we get,
$F(x) = \frac{1}{N} \exp(\frac{2(\pi)i(p,q)^T x}{N})$
By applying the Euler Identity to the inverse Fourier transform, we get,
$F(x) = \frac{1}{N} cos(\frac{2\pi(p,q)^T x}{N}) + i\frac{1}{N} sin(\frac{2\pi(p,q)^T x}{N})$
The amplitude of the inverse Fourier Transform is, $Amplitude =$
$\frac{1}{N}\sqrt{cos^2(\frac{2\pi(p,q)^T x}{N}) + sin^2(\frac{2\pi(p,q)^T x}{N})}$
$Amplitude = \frac{1}{N}\sqrt{1}$
$Amplitude = \frac{1}{N}$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

These are images taken from my Jupyter Notebook

- The angle of the signal is given by

$$\theta = tan^{-1}\frac{u}{v}$$

- where

$$(p = u) \vee (q = v)$$

- and the wavelength of the signal is given by:

$$\lambda = \frac{1}{\sqrt{u^2 + v^2}}$$

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

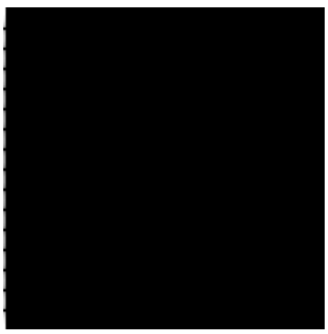
Answers:
Assuming the function is periodic where the zero frequency is centered, if p or q is greater than half the image size, the and p coordinates becomes p-N or q-N respectively. This should create the exact the image.

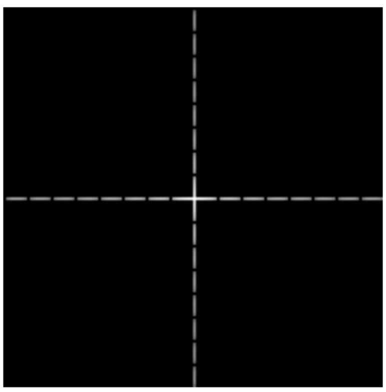
Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:
It gives the coordinates for the x and y frequency after centering the zero frequency on the graph.

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

		
Fhat	Ghat	Hhat



FFT shifted Hhat

H hat seems to be a linear combination of F and G hat respectively. If we do not use fftshift then origin is at the top left corner. Since it’s a rectangular, the frequencies of H hat just seem to move in strictly X and Y direction and not diagonal. This is why there is “some whiteness” on the borders of the H hat By taking the FFTshift of it, the spectra is relocated and moves from the border to the center and becomes cross which is seen in the Hhat picture.

Question 8: Why is the logarithm function applied?

Answers:
The logarithm function is used to make details more visible by compress the dynamic range of the image. This allows lower ranges to become more visible.

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

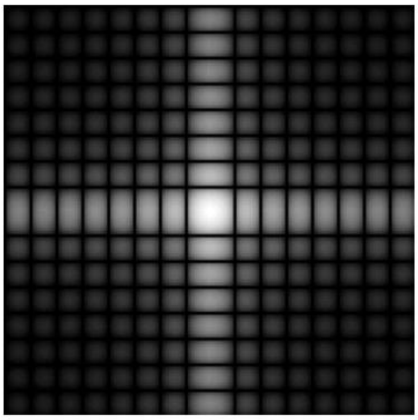
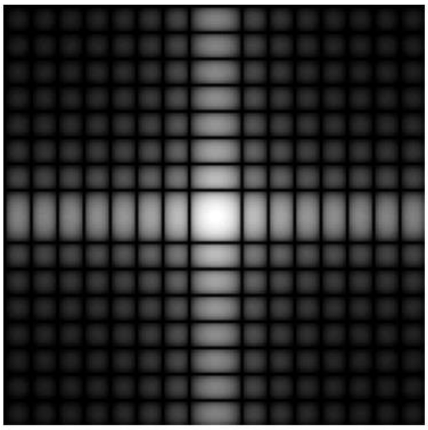
Answers:

$$\mathcal{F}\left(f\left(t\right)+g\left(t\right)\right)=F\left(\omega\right)+G\left(\omega\right)$$

The linearity of the signal is preserved. This is why H hat looks like a linear combination of F and G in the fourier domain but also in the spatial.

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:
Make use of convolution theorem. It simply states that a convolution in the spatial domain is the same as multiplication in the Fourier domain. Also multiplication in the spatial domain is the same as convolution in the Fourier domain.

	
With Fourier	With Convolve 2D and normaised

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

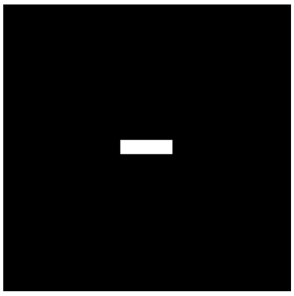
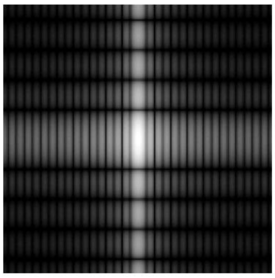
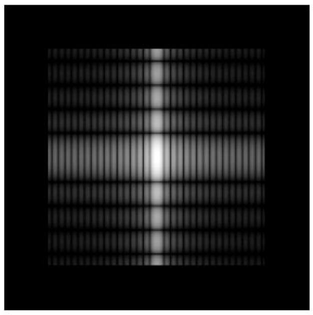
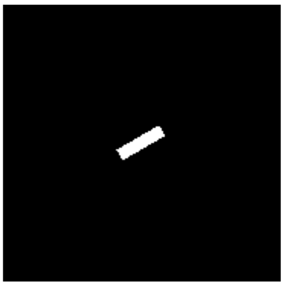
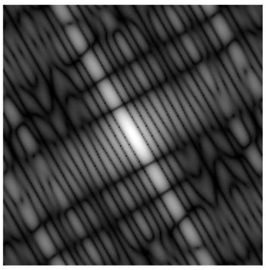
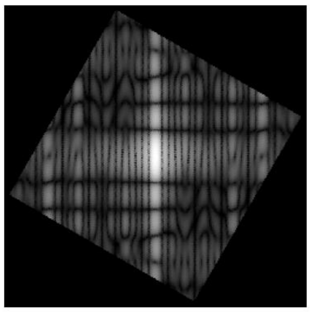
Answers:

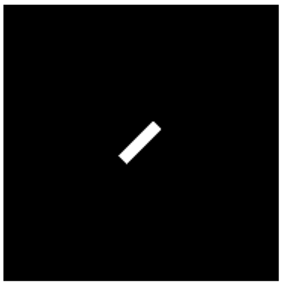
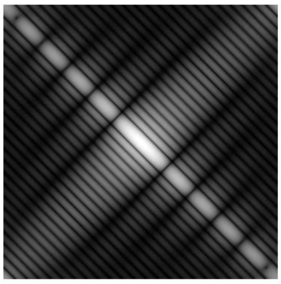
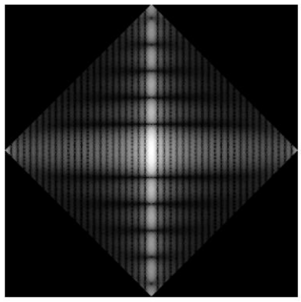
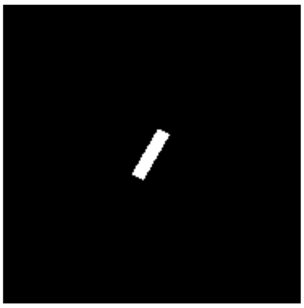
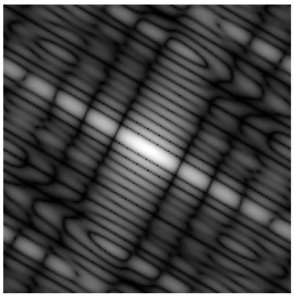
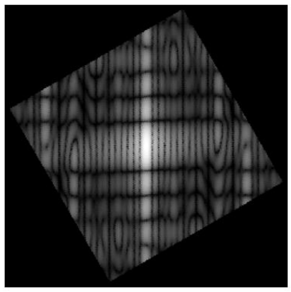
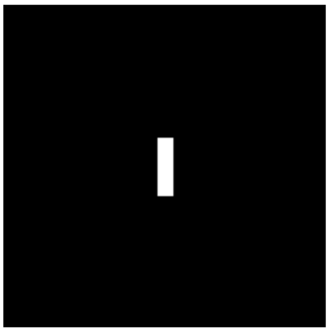
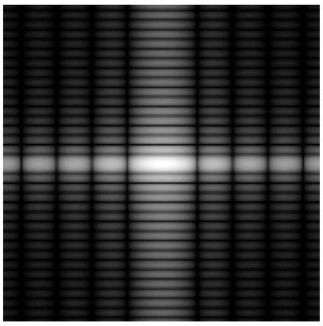
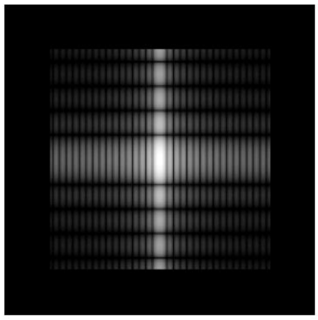
- Scaling down in the spatial domain is equivalent to scaling up in the frequency domain and vice versa.

- Scaling up in spatial domain is equivalent to scaling down in the frequency domain and vice versa.

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

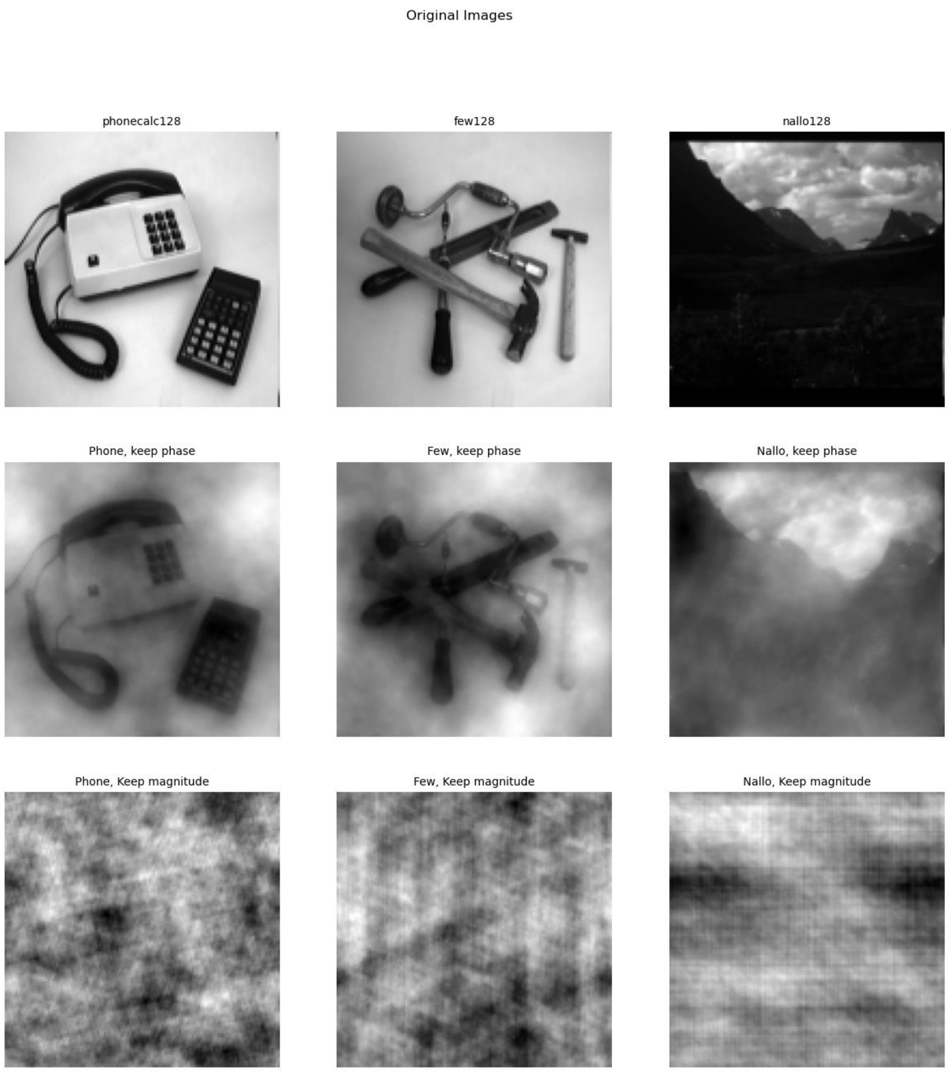
Answers:
When taking analyzing the frequency domain of a rotated image in the spatial plane, it seems that the frequency domain also rotates by the same amount of angles counterclockwise. In the fourier domain, the distance to the origin seems to be the same thus the frequencies will have the same values The noticeable difference is that the direction of the wave changes. Some information becomes lost when angle is 30 and 60 degrees but not 45 or 90 degrees as by looking at “rotated back” pictures of it and the waves in the Fourier looks distorted.

	Spatial	Fourier	Rotated back
0			
30			

45			
60			
90			

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

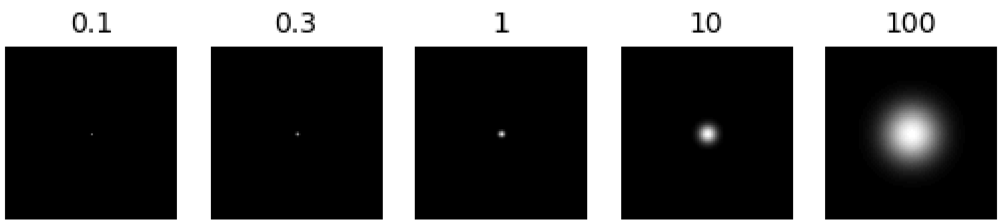
Answers:



Extremely short answer: Phases tells you were “everything” is in the image and basically where everything lies. Magnitude tells how much stuff like brightness, contrast et. This is why Phase is much more important than magnitude.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:



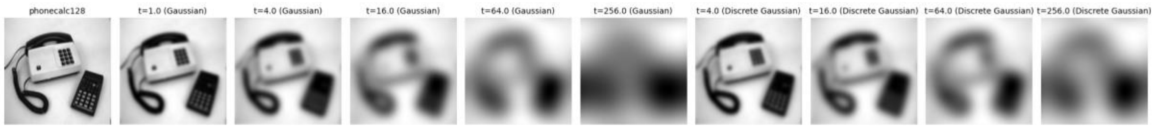
Discrete Gausian kernel					
Sigma	0.1	0.3	1.0	10	100
Variance	0.013	0.3	0.999	9.999	99.99

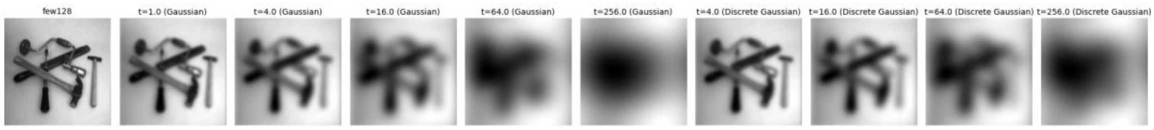
Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

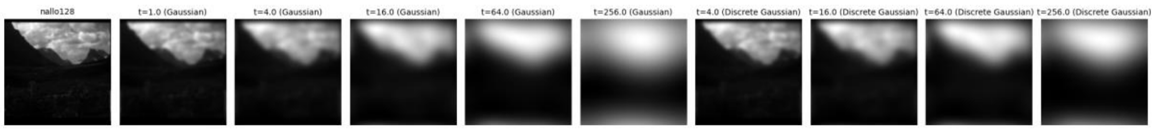
Answers:
The variances match the expected values for larger variance but not for smaller ones. Probably due to incorrect sampling.

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:







The larger the variance, the more the recreated image becomes blurrier and blurrier. The Gaussian filter removes the higher frequency components from the image. The higher frequency corresponds to the edges, contras and contours within an image

---

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

<u>Gaussian Filter:</u> It smoothens the image so that it looks less blocky. It also seems to handel gaussian noise the best. The larger the t values, the more of the edges are lost and the more blurred the image becomes. Does that seem to handle salt & pepper that well as the noise is still there just less.

<u>Median filter:</u> The most effective and removes all salt and pepper noise. Edges seems preserved in both images for small window sizes. Looks blocky as contours looked a bit messed. The window size describes "how blocky" it'll look

<u>Low pass filtering:</u> More effective for Gaussian noise than salt and pepper noise. High frequencies were removed hence why edges look bad. When increased the cut-off value, edges are somewhat preserved. Overall, the worst filter.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

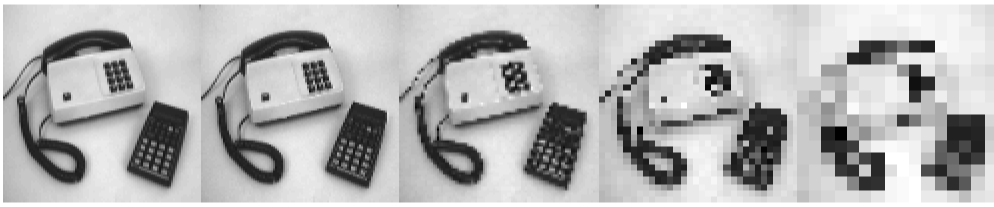
Answers:
Gaussian filter is the most effective when applied to images where there's only Gaussian noise. Median Filtering is most effective when applied to the to Salt and pepper. Low filter is just bad and impractical.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

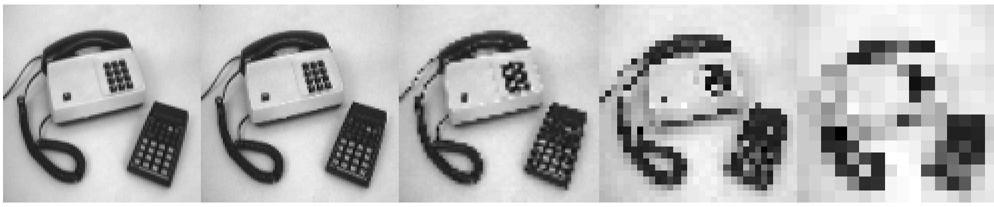
Answers:
When the sampling frequency is less than twice of the image bandwidth (bandwith being the highest frequency in the sigal) then allias effect will happens as it's below the Nyquist rate. This is why the images become more and more alias effected for each iteration

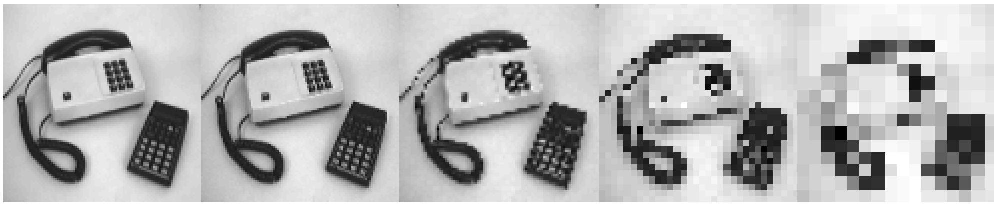
Gaussian filter smoothens the image and removes the highest frequencies thus the "subsampling" rate needed to accurately represent the image decrease. Low filtering does not smoothen the images that well thus at iteration four compared to the gaussian, there's a noticeable difference.

(top one is just subsampling, below is subsampling + Gaussian



(Top one is just subsampled, the below is ideal filtering)





Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
Smoothing first before subsampling is done to avoid the aliasing effect. Smoothing removes the high frequency components of the image. Thereafter subsampling it reduces the overall frequency of the image and avoid artifacts due to mirroring of high frequencies being present in the image. Of course information will be lost however less will be loss if smoothen then subsampled compared to just sampling it.