Интегрирование, часть 5, практическая часть

8.5.12. $\int \sin^2 x * \cos^2 x\, dx = [\text{сл. } 4] = \int \frac{1-\cos 2x}{2} * \frac{1+\cos 2x}{2} dx = \int \frac{(1-\cos^2 2x)dx}{4} = \frac{1}{4}\int (1 - \cos^2 2x)dx = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x\, dx = \frac{1}{4}\int dx - \frac{1}{4}\int \frac{1+\cos 4x}{2} dx = \frac{1}{4}\int dx - \frac{1}{4} * \frac{1}{2}\int (1 + \cos 4x)dx = \frac{1}{4}\int dx - \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x\, dx = \frac{x}{4} - \frac{x}{8} - \frac{1}{32}\sin 4x + C = \frac{x}{8} - \frac{1}{32}\sin 4x + C$

8.5.14. $\int \cos 2x * \sin 4x\, dx = \int \sin 4x * \cos 2x\, dx = \left[\text{сл. } 5.2; \sin\alpha * \cos\beta = \frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta))\right] = \int \frac{1}{2}(\sin 2x + \sin 6x)dx = \frac{1}{2}\int \sin 2x\, dx + \frac{1}{2}\int \sin 6x\, dx = -\frac{1}{2} * \frac{1}{2}\cos 2x - \frac{1}{2} * \frac{1}{6}\cos 6x + C = -\frac{1}{4}\cos 2x - \frac{1}{12}\cos 6x + C$

8.5.15. $\int \sin\frac{x}{2}\sin\frac{3x}{2} dx = \left[\text{сл. } 5.2; \sin\alpha * \cos\beta = \frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta))\right] = \int \frac{1}{2}\left(\cos\left(\frac{x}{2} - \frac{3x}{2}\right) - \cos\left(\frac{x}{2} + \frac{3x}{2}\right)\right) dx = \frac{1}{2}\int (\cos x - \cos 2x)dx = \frac{1}{2}\int \cos x\, dx - \frac{1}{2}\int \cos 2x\, dx = \frac{1}{2}\sin x - \frac{1}{4}\sin 2x + C$

8.5.17. $\int \operatorname{ctg}^3 x\, dx = [\text{сл. } 6] = \int \operatorname{ctg} x * \left(\frac{1}{\sin^2 x} - 1\right) dx = \int \left(\frac{\cos x}{\sin^3 x} - \operatorname{ctg} x\right) dx = \int \frac{\cos x}{\sin^3 x} - \int \operatorname{ctg} x = \left[t = \sin x \to dt = \cos x\, dx; dx = \frac{dt}{\cos x}\right] = \int \frac{\cos x\, dt}{t^3 \cos x} - \int \operatorname{ctg} x\, dx = \frac{t^{-3+1}}{-3+1} - \ln|\sin x| + C = -\frac{1}{2\sin^2 x} - \ln|\sin x| + C$

8.5.18. $\int \operatorname{tg}^2 x\, dx = [\text{сл. } 6] = \int \frac{dx}{\cos^2 x} - 1 = \int \frac{dx}{\cos^2 x} - \int dx = \operatorname{tg} x - x + C$