Интегрирование, часть 1, практическая часть

8.1.2. $\int x^{10}dx = \frac{x^{11}}{11} + C$

8.1.3. $\int \frac{dx}{x^7} = \int x^{-7}dx = -\frac{1}{6x^6} + C$

8.1.4. $\int \sqrt[4]{x}dx = \int x^{\frac{1}{4}}dx = \frac{4}{5}\sqrt[4]{x^5} + C$

8.1.5. $\int \frac{dx}{x^2+9} = \int \frac{dx}{x^2+3^2} = \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$

8.1.6. $\int \frac{dx}{x^2-\frac{1}{2}} = \int \frac{dx}{x^2-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2*\frac{1}{\sqrt{2}}} * \ln\left|\frac{x-\frac{1}{\sqrt{2}}}{x+\frac{1}{\sqrt{2}}}\right| + C = \frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}\right| + C$

8.1.7. $\int \frac{dx}{\sqrt{x^2+3}} = \ln\left|x + \sqrt{x^2+3}\right| + C$

8.1.9. $\int \frac{x^4+x^2-6x}{x^3}dx = \int \frac{x^3+x-6}{x^2}dx = \int \frac{x^3}{x^2}dx + \int \frac{x}{x^2}dx - \int \frac{6}{x^2}dx = \int xdx + \int x^{-1}dx - 6\int x^{-2}dx = \frac{x^2}{2} + \ln|x| - 6*\left(-\frac{1}{x}\right) + C = \frac{x^2}{2} + \frac{6}{x} + \ln|x| + C$

8.1.10. $\int \left(\frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7}\right)dx = 5\int \frac{dx}{x} - 10\int x^{-\frac{3}{4}}dx - 3\int \frac{1}{x^2+(\sqrt{7})^2}dx = 5\ln|x| - 10*4\sqrt[4]{x} - 3*\frac{1}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C = 5\ln|x| - 40\sqrt[4]{x} - \frac{3}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C$

8.1.11. $\int \sqrt{x}(x^2+1)dx = \int \left(x^{\frac{5}{2}} + \sqrt{x}\right)dx = \int x^{\frac{5}{2}}dx + \int \sqrt{x}dx = \frac{2}{7}*x^{\frac{7}{2}} + \frac{2}{3}*x^{\frac{3}{2}} + C$

8.1.12. $\int \frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx = 3\int \frac{1}{\sqrt{4-x^2}} + \int dx = 3\arcsin\frac{x}{2} + x + C$

8.1.13. $\int \frac{(x^3+2)^2}{\sqrt{x}}dx = \int \frac{x^6}{\sqrt{x}}dx + \int \frac{4x^3}{\sqrt{x}}dx + \int \frac{4}{\sqrt{x}}dx = \int x^{\frac{11}{2}}dx + 4\int x^{\frac{5}{2}}dx + 4\int \frac{1}{\sqrt{x}}dx = \frac{2}{13}x^{\frac{13}{2}} + 4*\frac{2}{7}x^{\frac{7}{2}} + 4*2\sqrt{x} + C = \frac{2}{13}x^{\frac{13}{2}} + \frac{8}{7}x^{\frac{7}{2}} + 8\sqrt{x} + C$

8.1.14. $\int \left(4\sin x + 8x^3 - \frac{11}{\cos^2 x}\right)dx = 4\int \sin x\,dx + 8\int x^3dx - 11\int \frac{dx}{\cos^2 x} = -4\cos x + \frac{8x^4}{4} - 11\operatorname{tg}x + C = -4\cos x + 2x^4 - 11\operatorname{tg}x + C$

8.1.16. $\int \cos 2x\,dx = \int \frac{\cos(2x)d(2x)}{2} = \frac{\sin 2x}{2} + C$

8.1.17. $\int (9x+2)^{17}dx = \left[t = 9x+2, x = \frac{t-2}{9} \rightarrow dx = \left(\frac{t-2}{9}\right)'dt = \frac{1}{9}dt\right] = \int t^{17}*\frac{1}{9}dt = \frac{t^{18}}{18*9} + C = \frac{(9x+2)^{18}}{162} + C$

8.1.18. $\int \frac{dx}{8x-1} = \left[t = 8x-1, x = \frac{t+1}{8} \rightarrow dx = \frac{1}{8}dt\right] = \int \frac{dt}{8t} = \frac{1}{8}\ln|t| + C = \frac{1}{8}\ln|8x-1| + C$

8.1.19. $\int 4^{3-5x}dx = \left[t = 3-5x, x = \frac{3-t}{5} \rightarrow dx = -\frac{1}{5}dt\right] = \int 4^t*\left(-\frac{1}{5}\right)dt = \frac{4^t}{\ln 4}*\left(-\frac{1}{5}\right) + C = -\frac{4^{3-5x}}{5\ln 4} + C$

8.1.20. $\int \sqrt{3x + 4}\,dx = \left[t = 3x + 4, x = \frac{t-4}{3} \to dx = \frac{1}{3}dt\right] = \int \sqrt{t} * \frac{1}{3}dt = \frac{1*2t^{\frac{3}{2}}}{9} + C = \frac{2t^{\frac{3}{2}}}{9} + C =$

$\frac{2}{9}\sqrt{(3x + 4)^3} + C|$

8.1.21. $\int \frac{dx}{3x^2 - 25} = \left[t = \sqrt{3x^2} = \sqrt{3}x, x = \frac{t}{\sqrt{3}} \to dx = \frac{1}{\sqrt{3}}dt\right] = \int \frac{1}{\sqrt{3}} * \frac{dt}{t^2 - 25} = \frac{1}{\sqrt{3}} * \frac{1}{10} * \ln\left|\frac{t-5}{t+5}\right| + C =$

$\frac{1}{10\sqrt{3}} \ln\left|\frac{\sqrt{3}x-5}{\sqrt{3}x+5}\right| + C$

8.1.23. $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}(\int 1dx + \int \cos 2x\,dx) = \frac{1}{2}\left(\int 1dx + \int \frac{\cos(2x)d(2x)}{2}\right) =$

$\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C = \frac{x}{2} + \frac{\sin 2x}{4} + C$

8.1.24. $\int \frac{x-2}{x+3}\,dx = \int \frac{x+3-5}{x+3}\,dx = \int 1dx - 5\int \frac{1dx}{x+3} = [x + 3 = t, x = t - 3 \to dx = 1dt] = \int dx -$

$5\int \frac{1}{t}dt = x - 5\ln|t| + C = x - 5\ln|x + 3| + C$

8.1.25. $\int \frac{x^2 dx}{x^2 - 9} = \int \frac{x^2 - 9 + 9}{x^2 - 9}\,dx = \int 1dx + \int \frac{9}{x^2 - 9}\,dx = x + 9 * \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + C = x + \frac{3}{2}\ln\left|\frac{x-3}{x+3}\right| + C$

8.1.26. $\int \frac{5+\sin^3 x}{\sin^2 x}\,dx = 5\int \frac{1dx}{\sin^2 x} + \int \sin x\,dx = -5\operatorname{ctg} x - \cos x + C$