Моисеенко Павел, ИКНиТО, ИВТ, 1.2.

Предел последовательности (3 часть)

6.3.29 $\lim\limits_{n\to\infty}\frac{(n+2)^3}{5n^3}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{n\to\infty}\frac{n^3+6n^2+12n+8}{5n^3}=\lim\limits_{n\to\infty}\frac{n^3(1+6/n+12/n^2+8/n^3)}{5n^3}=\lim\limits_{n\to\infty}\frac{1+\frac{6}{n}+\frac{12}{n^2}+\frac{8}{n^3}}{5}=$
$=[(1+0+0+0)/5]=1/5$

6.3.30 $\lim\limits_{n\to\infty}\left(\frac{3}{n+2}+\frac{5}{2n+1}\right)=[0-0]=\lim\limits_{n\to\infty}\frac{3}{n+2}-\lim\limits_{n\to\infty}\frac{5}{2n+1}=\lim\limits_{n\to\infty}\frac{n\frac{3}{n}}{n\cdot(1+\frac{2}{n})}-\lim\limits_{n\to\infty}\frac{n\frac{5}{n}}{n(2+\frac{1}{n})}=$
$=\left[\frac{0}{1+0}-\frac{0}{2+0}\right]=0$

6.3.33. $\lim\limits_{n\to\infty}\frac{5^n-1}{5^n+1}=\left[\frac{\infty}{\infty}\right]=[5^n=y\Rightarrow y\to\infty]=\lim\limits_{n\to\infty}\frac{y-1}{y+1}=\lim\limits_{n\to\infty}\frac{y(1-\frac{1}{y})}{y(1+\frac{1}{y})}=\lim\limits_{n\to\infty}\frac{1-\frac{1}{y}}{1+\frac{1}{y}}=\frac{1-0}{1+0}=1$

6.3.34. $\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2+1}=\left[\frac{\infty}{\infty}\right]=\left[\sum\limits_{i=0}^{n}i=\frac{n(n+1)}{2}\right]=\lim\limits_{n\to\infty}\frac{n(n+1)}{2\cdot(n^2+1)}=\lim\limits_{n\to\infty}\frac{n^2(1+\frac{1}{n^2})}{2n^2(1+\frac{1}{n^2})}=\lim\limits_{n\to\infty}\frac{1}{2}=\frac{1}{2}$

6.3.35. $\lim\limits_{n\to\infty}\frac{1-q^n}{1-q}=\left[\frac{1-\infty}{1-q}\right]=\infty$

6.3.36. $\lim\limits_{n\to\infty}\frac{1+\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{3^n}}{1+\frac{1}{4}+\frac{1}{16}+\ldots+\frac{1}{4^n}}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{n\to\infty}\frac{\sum\limits_{k=0}^{n}(\frac{1}{3})^k}{\sum\limits_{k=0}^{n}(\frac{1}{4})^k}=\left[\sum\limits_{k=0}^{n}a^k=\frac{a^{n+1}-1}{a-1}\Rightarrow\sum\limits_{k=0}^{n}(\frac{1}{3})^k=\frac{(\frac{1}{3})^{n+1}-1}{-\frac{2}{3}};\sum\limits_{k=0}^{n}(\frac{1}{4})^k=\right.$
$\left.=\frac{(\frac{1}{4})^{n+1}-1}{-3/4}\right]=\lim\limits_{n\to\infty}\frac{((\frac{1}{3})^{n+1}-1)(-\frac{3}{4})}{((\frac{1}{4})^{n+1}-1)(-\frac{2}{3})}=\lim\limits_{n\to\infty}\frac{((\frac{1}{3})^{n+1}-1)\cdot 9}{((\frac{1}{4})^{n+1}-1)\cdot 8}=\lim\limits_{n\to\infty}\frac{-(1-(\frac{1}{3})^{n+1})\cdot 9}{-(1-(\frac{1}{4})^{n+1})\cdot 8}=\frac{9}{8}$

6.3.37. $\lim\limits_{n\to\infty}\frac{x_n+2}{x_n^2+4}$, если $\lim\limits_{n\to\infty}x_n=-1$, $\lim\limits_{n\to\infty}\frac{x_n+2}{x_n^2+4}=\frac{\lim\limits_{n\to\infty}x_n+\lim\limits_{n\to\infty}2}{\lim\limits_{n\to\infty}x_n^2+\lim\limits_{n\to\infty}4}=\left[\frac{-1+2}{-1(-1)+4}\right]=\frac{1}{5}$

6.3.38 $\lim\limits_{n\to\infty}\frac{\sqrt{n^3+n^2-4}-\sqrt[5]{n^6}}{\sqrt[3]{n^5+2n}+\sqrt[4]{n^6+3n^4+2}}=\left[\frac{\infty-\infty}{\infty-\infty}\right]=\lim\limits_{n\to\infty}\frac{\sqrt{n^2(n+1-4/n^2)}-\sqrt[5]{n^5n}}{\sqrt[3]{n^3(n^2+2/n^2)}+\sqrt[4]{(n^2+3+2/n^4)n^4}}=$
$=\lim\limits_{n\to\infty}\frac{n(\sqrt{n+1-4/n^2}-\sqrt[5]{n})}{n(\sqrt[3]{n^2+2/n^2}+\sqrt[4]{n^2+3+2/n^4})}=\lim\limits_{n\to\infty}\frac{n\cdot\sqrt{\frac{1}{n}+\frac{1}{n^2}-\frac{4}{n}}-\sqrt[5]{\frac{1}{n^4}}\cdot n}{n\cdot\sqrt[3]{\frac{1}{n}+\frac{2}{n^5}}+n\sqrt[4]{\frac{1}{n^2}+\frac{3}{n^4}+\frac{2}{n^6}}}=\lim\limits_{n\to\infty}\frac{n\left(\sqrt{\frac{1}{n}+\frac{1}{n^2}-\frac{4}{n^4}}-\sqrt[5]{1/n^4}\right)}{n\left(\sqrt[3]{\frac{1}{n}+\frac{2}{n^5}}+\sqrt[4]{\frac{1}{n^2}+\frac{3}{n^4}+\frac{2}{n^6}}\right)}=\left[\frac{0}{0}\right]$