Интегрирование, часть 2, практическая часть

8.2.11. $\int \frac{4x+3}{\sqrt{x^2-5}} = \int \frac{4xdx}{\sqrt{x^2-5}} + \int \frac{3dx}{\sqrt{x^2-5}} = \left[t = x^2 - 5 \rightarrow dt = 2xdx; \int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x + \sqrt{x^2+a}\right| + C, a = -5\right] = \int \frac{2dt}{\sqrt{t}} + 3\int \frac{dx}{\sqrt{x^2+(-5)}} = 2 * 2\sqrt{t} + 3\ln\left|x + \sqrt{x^2-5}\right| + C = 4\sqrt{x^2-5} + 3\ln\left|x + \sqrt{x^2-5}\right| + C$

8.2.12. $\int e^{\sin^2 x} * \sin 2x\, dx = [t = \sin^2 x \rightarrow dt = 2\sin x \cos x\, dx] = \int e^t dt = e^t + C = e^{\sin^2 x} + C$

8.2.13. $\int \frac{1-2\sin x}{\cos^2 x} dx = \int \frac{dx}{\cos^2 x} - \int \frac{2\sin x dx}{\cos^2 x} = \left[\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C, t = \cos x \rightarrow dt = -\sin x\, dx\right] = \int \frac{dx}{\cos^2 x} - \int \frac{-2dt}{t^2} = \int \frac{dx}{\cos^2 x} + 2\int \frac{dt}{t^2} = \operatorname{tg} x - \frac{2}{t} + C = \operatorname{tg} x - \frac{2}{\cos x} + C$

8.2.14. $\int \frac{3x-4}{x^2-4} dx = \int \frac{3xdx}{x^2-4} - \int \frac{4}{x^2-4} dx = \left[t = x^2 - 4 \rightarrow dt = 2xdx \rightarrow xdx = \frac{dt}{2}; \int \frac{dx}{\sqrt{x^2+a}} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C, a = 2\right] = 3\int \frac{\frac{1}{2}dt}{t} - 4\int \frac{dx}{x^2-4} = 3 * \frac{1}{2}\ln|t| - 4 * \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C = \frac{3}{2}\ln|x^2-4| - \ln\left|\frac{x-2}{x+2}\right| + C$

8.2.16. $\int \sqrt{9-x^2}dx = [x = 3\sin t \rightarrow x^2 = 9\sin^2 t\,; dx = (3\sin t)'dt = 3\cos t\, dt] = \int \sqrt{9 - 9\sin^2 t} * 3\cos t\, dt = \int \sqrt{9(1-\sin^2 t)} * 3\cos t\, dt = 3\int \sqrt{9\cos^2 t} * \cos t\, dt = 9\int \cos t * \cos t\, dt = \left[\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C\right] = \left[\cos^2 x = \frac{1+\cos 2x}{2}\right] = 9\int \frac{1+\cos 2t}{2} dt = 9\int \frac{1}{2} dt + \frac{9}{2}\int \cos 2t\, dt = \frac{9}{2}\int dt + \frac{9}{2}\int \cos 2t\, dt = \frac{9}{2}\left(\int dt + \frac{1}{2} * 2 * \int \cos 2t\, dt\right) = \frac{9}{2}\left(\int dt + \frac{1}{2}\int \cos 2t\, d(2t)\right) = \frac{9}{2}\left(t + \frac{1}{2}\sin 2t\right) + C = \left[x = 3\sin t \rightarrow \sin t = \frac{x}{3}, t = \arcsin\frac{x}{3}\right] = \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{1}{2}\sin\left(2\arcsin\frac{x}{3}\right)\right) + C = \left[\sin\left(2\arcsin\frac{x}{3}\right) = 2\sin\left(\arcsin\frac{x}{3}\right) * \cos\left(\arcsin\frac{x}{3}\right) = 2 * \frac{x}{3} * \sqrt{1 - \left(\frac{x}{3}\right)^2} = \frac{2x}{3} * \frac{\sqrt{9-x^2}}{3} = \frac{2x\sqrt{9-x^2}}{9}\right] = \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{1}{2} * \frac{2x\sqrt{9-x^2}}{9}\right) + C = \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{x\sqrt{9-x^2}}{9}\right) + C$