Моисеенко Павел, ИВТ, 2 курс

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Интегрирование, часть 1

8.1.29. $\int \frac{dx}{x^2\sqrt{x}} = \int \frac{dx}{x^{2\frac{1}{2}}} = \int x^{-2\frac{1}{2}}dx = \frac{x^{-1\frac{1}{2}}}{-3/2} + C = -\frac{2}{3x\sqrt{x}} + C$

8.1.30. $\int \frac{dx}{x^2+3} = \int \frac{dx}{x^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + C$

8.1.31. $\int \frac{1}{5^x}dx = \int 5^{-x}dx = -\int 5^{-x}d(-x) = -\frac{5^{-x}}{\ln 5} + C = -\frac{1}{5^x \ln 5} + C$

8.1.32. $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{2^2-x^2}} = \arcsin\frac{x}{2} + C$

8.1.33. $\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{dx}{\sqrt{x^2+(-1)}} = \ln|x+\sqrt{x^2+1}| + C$

8.1.34. $\int \frac{dx}{x^2-25} = \int \frac{dx}{x^2-5^2} = \frac{1}{10}\cdot\ln\left|\frac{x-5}{x+5}\right| + C$

8.1.35. $\int (x+\frac{2}{x})^2dx = \int (x^2+4+\frac{4}{x^2})dx = \int x^2dx + \int 4dx + \int \frac{4}{x^2}dx = \int x^2dx + 4\int dx + 4\int x^{-2}dx =$
$= \frac{x^3}{3} + 4x + 4\cdot\frac{x^{-1}}{-1} + C = \frac{x^3}{3} + 4x - \frac{4}{x} + C$

8.1.36. $\int \frac{dx}{4x^2+1} = \frac{1}{4}\int \frac{dx}{x^2+\frac{1}{4}} = \frac{1}{4}\int \frac{dx}{x^2+(\frac{1}{2})^2} = \frac{1}{4}\cdot 2\operatorname{arctg} 2x + C = \frac{1}{2}\operatorname{arctg} 2x + C$

8.1.37. $\int (7^x - 8/x + 4\cos x)dx = \int 7^x dx - \int 8/x\, dx + \int 4\cos x\, dx = \int 7^x dx - 8\int x^{-1}dx + 4\int \cos x dx =$
$= 7^x/\ln 7 - 8\ln|x| + 4\sin x + C$

8.1.38. $\int (\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4})dx = \int \frac{\sqrt{3}}{\cos^2 x}dx - \int \sqrt[3]{x}dx - \int \frac{2}{x^4}dx = \sqrt{3}\int \frac{dx}{\cos^2 x} - \int x^{\frac{1}{3}}dx - 2\int x^{-4}dx =$
$= \sqrt{3}\operatorname{tg} x - \frac{x^{\frac{4}{3}}}{4/3} - 2\cdot\frac{x^{-3}}{-3} + C = \sqrt{3}\operatorname{tg} x - \frac{3x\sqrt[3]{x}}{4} + \frac{2}{3x^3} + C$

8.1.39. $\int \frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}}dx = \int x^{\frac{1}{2}-\frac{1}{4}}dx - \int 3\cdot x^{\frac{2}{5}-\frac{1}{4}}dx + \int x^{-\frac{1}{4}}dx = \int x^{\frac{1}{4}}dx - 3\int x^{\frac{3}{20}}dx + \int x^{-\frac{1}{4}}dx = \frac{x^{\frac{5}{4}}}{5/4} -$
$- 3\frac{x^{\frac{23}{20}}}{23/20} + \frac{x^{3/4}}{3/4} + C = \frac{4x\sqrt[4]{x}}{5} - \frac{60x\sqrt[20]{x^3}}{23} + \frac{4\sqrt[4]{x^3}}{3} + C$

8.1.40. $\int (0{,}7\cdot x^{-0{,}1} + 0{,}2\cdot(0{,}5)^x)dx = \int 0{,}7x^{-0{,}1}dx + \int 0{,}2\cdot(0{,}5)^x dx = 0{,}7\int x^{-0{,}1}dx + 0{,}2\int 0{,}5^x dx =$
$= 0{,}7\cdot\frac{x^{0{,}9}}{0{,}9} + 0{,}2\cdot\frac{(0{,}5)^x}{\ln(0{,}5)} + C = \frac{7x^{0{,}9}}{9} + 0{,}2\cdot\frac{(0{,}5)^x}{\ln(0{,}5)} + C$

8.1.41. $\int (5\operatorname{sh} x - 7\operatorname{ch} x + 1)dx = 5\int \operatorname{sh} x dx - 7\int \operatorname{ch} x + \int dx = 5\operatorname{ch} x - 7\operatorname{sh} x + x + C$

8.1.42. $\int (x^2-1)(\sqrt{x}+4)dx = \int (x^2\sqrt{x} + 4x^2 - \sqrt{x} - 4)dx = \int x^{2\frac{1}{2}}dx + 4\int x^2dx - \int x^{\frac{1}{2}}dx - 4\int dx =$
$= \frac{x^{3\frac{1}{2}}}{7/2} + 4\frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{3/2} - 4x + C = \frac{2x^3\sqrt{x}}{7} + \frac{4x^3}{3} - \frac{2x\sqrt{x}}{3} - 4x + C$

8.1.43. $\int \frac{7 - \sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}dx = \int \frac{7dx}{\sqrt{x^2+\pi}} - \int dx = 7\int \frac{dx}{\sqrt{x^2+\pi}} - \int dx = 7\ln|x + \sqrt{x^2+\pi}| - x + C$

8.1.44. $\int \left(\frac{\sqrt{x}-5}{x}\right)^3dx = \int \frac{x^{\frac{3}{2}} - 3\cdot 5x + 3\cdot 25\sqrt{x} - 125}{x^3}dx = \int (x^{-\frac{3}{2}} - \frac{15}{x^2} + 75x^{-\frac{5}{2}} - 125x^{-3})dx =$

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$= \int x^{-\frac{3}{2}}dx - 15\int x^{-2}dx + 75\int x^{-\frac{5}{2}} - 125\int x^{-3}dx = \frac{x^{-\frac{1}{2}}}{-1/2} - 15\frac{x^{-1}}{-1} + \frac{75x^{-\frac{3}{2}}}{-3/2} - 125\frac{x^{-2}}{-2} + C =$

$= -\frac{2}{\sqrt{x}} + \frac{15}{x} - \frac{50}{x\sqrt{x}} + \frac{125}{2x^2} + C$

8.1.45. $\int \sin 7x\,dx = 1/7\int \sin 7x\,d(7x) = 1/7\cdot(-\cos(7x)) + C = -\cos 7x/7 + C$

8.1.46. $\int \sqrt[5]{2x-8}\,dx = \frac{1}{2}\int \sqrt[5]{2x-8}\,d(2x-8) = \frac{1}{2}\cdot\frac{(2x-8)^{\frac{6}{5}}}{6/5} + C = \frac{5(2x-8)^{\frac{6}{5}}}{12} + C$

8.1.47. $\int (1-4x)^{2001}dx = -\frac{1}{4}\int (1-4x)^{2001}d(1-4x) = -\frac{1}{4}\,\frac{(1-4x)^{2002}}{2002} + C = -\frac{(1-4x)^{2002}}{8008} + C$

8.1.48. $\int \frac{dx}{9x+7} = \frac{1}{9}\int \frac{d(9x+7)}{(9x+7)} = \frac{1}{9}\ln|9x+7| + C$

8.1.49. $\int \frac{dx}{(6x+11)^4} = \frac{1}{6}\int (6x+11)^{-4}d(6x+11) = \frac{1}{6}\cdot\frac{(6x+11)^{-3}}{(-3)} + C = -\frac{(6x+11)^{-3}}{18} + C$

8.1.50. $\int \frac{dx}{25x^2+1} = \frac{1}{25}\int \frac{dx}{x^2+(1/5)^2} = \frac{1}{25}\cdot 5\,\mathrm{arctg}\,5x + C = \frac{1}{5}\mathrm{arctg}\,5x + C$

8.1.51. $\int 3^{2-11x}dx = \frac{1}{11}\int 3^{2-11x}d(2-11x) = -\frac{1}{11}\cdot\frac{3^{2-11x}}{\ln 3} + C$

8.1.52. $\int \frac{dx}{\sqrt{4x^2-1}} = \frac{1}{2}\int \frac{dx}{\sqrt{x^2-\frac{1}{4}}} = \frac{1}{2}\ln\left|x+\sqrt{x^2-\frac{1}{4}}\right| + C$

8.1.53. $\int \sin^2 3x\,dx = \int \frac{1-\cos 6x}{2}dx = \int \frac{1}{2}dx - \int \frac{\cos 6x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\cdot\frac{1}{6}\int \cos(6x)d(6x) = \frac{1}{2}x -$
$- \frac{1}{12}\sin 6x + C$

8.1.54. $\int \cos^2 8x\,dx = \int \frac{1+\cos 16x}{2}dx = \int \frac{1}{2}dx + \int \frac{\cos 16x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\cdot\frac{1}{16}\cos 16x\,d(16x) =$
$= \frac{1}{2}x + \frac{\sin 16x}{32} + C$

8.1.55. $\int \mathrm{tg}^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int\left(\frac{1}{\cos^2 x} - 1\right)dx = \int \frac{dx}{\cos^2 x} - \int dx = \mathrm{tg}\,x - x + C$

8.1.56. $\int \frac{4x+1}{x-5}dx = 4\int \frac{x+\frac{1}{4}}{x-5}dx = 4\int \frac{x-5+21/4}{x-5}dx = 4\int\left(1+\frac{21}{4(x-5)}\right)dx = 4\left(\int dx + \frac{21}{4}\int \frac{1}{x-5}dx\right) =$
$= 4\left(x+\frac{21}{4}\ln|x-5|\right) + C = 4x + 21\ln|x-5| + C$

8.1.59. $\int \frac{\cos 2x\,dx}{\sin^2 x\cos^2 x} = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x\cos^2 x}dx = \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} = -\mathrm{ctg}\,x - \mathrm{tg}\,x + C$

8.1.60. $\int \frac{\sin 2x}{\cos x}dx = \int \frac{2\sin x\cos x}{\cos x}dx = \int 2\sin x\,dx = 2\int \sin x\,dx = -2\cos x + C$