Моисеенко Павел, ИВТ, 2 курс

Интегрирование, часть 2

8.2.33. $\int \cos(6x+1)dx = [t=6x+1 \to dt=(6x+1)'dx=6dx \to dx=\frac{dt}{6}] = \frac{1}{6}\int \cos t\, dt =$
$= \sin t/6 + C = \sin(6x+1)/6 + C$

8.2.34. $\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = [t=5x-2 \to dt=(5x-2)'dx=5dx \to dx=\frac{dt}{5}] = \frac{1}{5}\int\frac{dt}{t^{4/3}} = \frac{1}{5}\int t^{-\frac{4}{3}}dt = \frac{1}{5}\frac{t^{-\frac{1}{3}}}{-\frac{1}{3}} +$
$+ C = -\frac{3}{5\sqrt[3]{t}} + C = -\frac{3}{5\sqrt[3]{5x-2}} + C$

8.2.35. $\int \frac{\sqrt{tg x}\,dx}{\cos^2 x} = [tg x = t \to dt = (tg x)'dx = \frac{1}{\cos^2 x}dx \to dx = \cos^2 x\, dt] = \int\frac{\sqrt{t}\cos^2 x\, dt}{\cos^2 x} = \int\sqrt{t}\,dt =$
$= \int t^{\frac{1}{2}}dt = \frac{t^{3/2}}{3/2} + C = \frac{2}{3}tg^{\frac{3}{2}}x + C$

8.2.36. $\int\frac{e^x dx}{e^{2x}+9} = [t=e^x \to dt=e^x dx] = \int\frac{dt}{t^2+3^2} = \frac{1}{3}arctg\frac{x}{3} + C = \frac{1}{3}arctg\frac{e^x}{3} + C$

8.2.37. $\int\frac{x^5 dx}{\sqrt{x^6+7}} = [t=x^6+7 \to dt=6x^5dx \to x^5dx=\frac{dt}{6}] = \frac{1}{6}\int t^{-\frac{1}{2}}dt = \frac{1}{6}\cdot\frac{t^{\frac{1}{2}}}{1/2} + C = \frac{\sqrt{t}}{3} + C = \frac{\sqrt{x^6+7}}{3} + C$

8.2.38. $\int\frac{dx}{\arccos x\cdot\sqrt{1-x^2}} = [t=\arccos x \to dt = \frac{-dx}{\sqrt{1-x^2}}] = -\int\frac{dt}{t} = -\ln|t| + C = -\ln|\arccos x| + C$

8.2.39. $\int\frac{(2x+3)dx}{(x^2+3x-1)^4} = [t=x^2+3x-1 \to dt=(2x+3)dx] = \int\frac{dt}{t^4} = \frac{t^{-3}}{-3} + C = -\frac{1}{3(x^2+3x-1)^3} + C$

8.2.40. $\int\cos^{11}2x\cdot\sin 2x\,dx = [\cos 2x = t \to dt = -2\sin 2x\,dx \to \sin 2x\,dx = -dt/2] =$
$= -\frac{1}{2}\int t^{11}dt = -\frac{1}{2}\cdot\frac{t^{12}}{12} + C = -\frac{t^{12}}{24} + C = -\frac{\cos^{12}2x}{24} + C$

8.2.41. $\int\frac{7^{\sqrt{x}}}{\sqrt{x}}dx = [7^{\sqrt{x}} = t \to dt = \frac{1}{2\sqrt{x}}\cdot 7^{\sqrt{x}}\ln 7\,dx \to \frac{dx}{\sqrt{x}} = \frac{2dt}{t\cdot\ln 7}] = \int\frac{t\,2dt}{\ln 7\, t} = \frac{2}{\ln 7}\int dt =$
$= \frac{2}{\ln 7}t + C = \frac{2}{\ln 7}\cdot 7^{\sqrt{x}} + C$

8.2.42. $\int\frac{e^{1/x}}{x^2}dx = [t=e^{1/x} \to dt = \frac{-1}{x^2}\cdot e^{1/x}dx \to \frac{e^{1/x}}{x^2}dx = -dt] = -\int dt = -t + C = -e^{1/x} + C$

8.2.43. $\int\frac{\ln 5x}{x}dx = [t=\ln 5x \to dt = \frac{5}{5x}dx = \frac{dx}{x}] = \int t\,dt = \frac{t^2}{2} + C = \frac{\ln^2 5x}{2} + C$

8.2.44. $\int ctg x\,dx = \int\cos x\,dx/\sin x = [t=\sin x \to dt = \cos x\,dx] = \int dt/t = \ln|t| + C = \ln|\sin x| + C$

8.2.45. $\int 4x\cdot\sqrt[3]{x^2+8}\,dx = [x^2+8=t \to dt=2xdx] = \int 2\sqrt[3]{t}\,dt = 2\int t^{1/3}dt = 2\cdot t^{4/3}/\frac{4}{3} + C = 3t\sqrt[3]{t}/2 + C =$
$= 3\sqrt[3]{(x^2+8)^4}/2 + C$

8.2.46. $\int\frac{\cos x\,dx}{\sin^2 x} = [\sin x = t \to dt = \cos x\,dx] = \int\frac{dt}{t^2} = \int t^{-2}dt = \frac{t^{-1}}{-1} + C = \frac{1}{-\sin x} + C$

8.2.47. $\int tg 2x\,dx = \int\sin 2x\,dx/\cos 2x = [t=\cos 2x \to dt = -\frac{1}{2}\sin 2x\,dx] = -\frac{1}{2}\int\frac{dt}{t} = -\frac{1}{2}\ln|t| + C = -\frac{1}{2}\ln|\cos 2x| + C$

8.2.48. $\int\frac{x\,dx}{x^4+1} = [t=x^2 \to dt=2xdx \to xdx=\frac{dt}{2}] = \frac{1}{2}\int\frac{dt}{t^2+1} = \frac{1}{2}arctg\, t + C = \frac{1}{2}arctg\, x^2 + C$

8.2.49. $\int e^{-x^3}\cdot x^2 dx = [t=e^{-x^3} \to dt = e^{-x^3}\cdot(-3x^2)dx \Rightarrow e^{-x^3}\cdot x^2 dx = \frac{dt}{-3}] = \int\frac{dt}{-3} = -\frac{1}{3}\int dt = -\frac{1}{3}t + C = -\frac{1}{3}e^{-x^3} + C$

$\int\frac{x^2dx}{\sqrt{x^6-4}}=\left[\begin{matrix}x^3=t\to\\ \to dt=3x^2dx\end{matrix}\right]=\frac{1}{3}\int\frac{dt}{\sqrt{t^2-4}}=\frac{1}{3}\ln|t+\sqrt{t^2-4}|+C=\frac{1}{3}\ln|x^3+\sqrt{x^6-4}+C$ — 8.2.50

8.2.51. $\int(8\cos\frac{x}{3}-5)^2\sin\frac{x}{3}dx=[8\cos\frac{x}{3}-5=t\to dt=-\frac{1}{8\cdot3}\sin\frac{x}{3}dx\to\sin\frac{x}{3}dx=\frac{3dt}{-8}]=-\frac{3}{8}\int t^2dx=$
$=-\frac{3}{8}\cdot\frac{t^3}{3}+C=-\frac{(8\cos\frac{x}{3}-5)^3}{8}+C$

8.2.52. $\int\frac{(3x^2-2x+7)dx}{\sqrt{x^3-x^2+7x-2}}=\left[\begin{matrix}x^3-x^2+7x-2=t\to\\ \to dt=(3x^2-2x+7)dx\end{matrix}\right]=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}+C=2\sqrt{x^3-x^2+7x-2}+C$

8.2.53. $\int x(2x+1)^{35}dx=[t=2x+1,\ x=\frac{t-1}{2}\to dt=2dx\to dx=\frac{dt}{2}]=\int\frac{t-1}{2}\cdot t^{35}\cdot\frac{dt}{2}=\frac{1}{4}\int(t^{36}-t^{35})dt=$
$=\frac{1}{4}(\int t^{36}dt-\int t^{35}dt)=\frac{1}{4}(t^{37}/37-t^{36}/36)+C=\frac{1}{4}((2x+1)^{37}/37-(2x+1)^{36}/36)+C$

8.2.54. $\int(x-2)\sqrt{x+4}\,dx=[t=x+4\to x=t-4,\to dt=dx]=\int(t-4-2)\sqrt{t}\,dt=\int(t-6)\sqrt{t}\,dt=\int t\sqrt{t}\,dt-$
$-6\int\sqrt{t}\,dt=\int t^{3/2}dt-6\int t^{1/2}dt=t^{5/2}/\tfrac{5}{2}-6\cdot\frac{t^{3/2}}{3/2}+C=2(x+4)^{5/2}/5-4(x+4)^{3/2}+C$

8.2.55. $\int\frac{3\sqrt{x}-2\cos\frac{1}{x^2}}{x^3}dx=\int\frac{3\sqrt{x}}{x^3}dx-\int\frac{2\cos\frac{1}{x^2}}{x^3}dx=[2.\ t=\frac{1}{x^2}\to dt=\frac{-2}{x^3}dx\to\frac{dx}{x^3}=-\frac{dt}{2}]=3\int x^{-\frac{5}{2}}dx+$
$+2\cdot\frac{1}{2}\int\cos t\,dt=\frac{3\cdot x^{-\frac{3}{2}}}{-3/2}+\sin t+C=-\frac{2}{x\sqrt{x}}+\sin\frac{1}{x^2}+C$

8.2.56. $\int\frac{7x+2}{\sqrt{x^2+10}}dx=\int\frac{7x}{\sqrt{x^2+10}}dx+\int\frac{2}{\sqrt{x^2+10}}dx=[1.\ t=x^2+10\to dt=2xdx]=\frac{7}{2}\int\frac{dt}{\sqrt{t}}+2\int\frac{1}{\sqrt{x^2+10}}dx=\frac{7}{2}\cdot2\sqrt{t}+$
$+2\ln|x+\sqrt{x^2+10}|+C=7\sqrt{x^2+10}+2\ln|x+\sqrt{x^2+10}|+C$

8.2.57. $\int\frac{dx}{e^x+e^{-x}}=\int\frac{dx}{e^x+1/e^x}=\int\frac{dx}{(e^{2x}+1)/e^x}=\int\frac{e^xdx}{e^{2x}+1}=\left[\begin{matrix}t=e^x\to\\ \to dt=e^xdx\end{matrix}\right]=\int\frac{dt}{t^2+1}=\mathrm{arctg}\,t+C=\mathrm{arctg}\,e^x+C$

8.2.58. $\int\frac{x+8}{x^2+3}dx=\int\frac{x}{x^2+3}dx+8\int\frac{dx}{x^2+3}=\left[\begin{matrix}1.\ t=x^2+3\to dt=2xdx\to\\ \to xdx=dt/2\end{matrix}\right]=\frac{1}{2}\int\frac{dt}{t}+8\int\frac{dx}{x^2+(\sqrt{3})^2}=\frac{1}{2}\ln|t|+$
$+8\frac{1}{\sqrt{3}}\mathrm{arctg}\frac{x}{\sqrt{3}}+C=\frac{\ln|x^2+3|}{2}+\frac{8}{\sqrt{3}}\mathrm{arctg}\frac{x}{\sqrt{3}}+C$

8.2.59. $\int\frac{x+4\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx=\int\frac{xdx}{\sqrt{1-x^2}}+4\int\frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx=\left[\begin{matrix}1.\ t=1-x^2\to dt=-2xdx\to xdx=-\frac{dt}{2}\\ 2.\ z=\arcsin x\to dz=1/\sqrt{1-x^2}dx\end{matrix}\right]=-\frac{1}{2}\int\frac{dt}{\sqrt{t}}+$
$+4\int\sqrt{z}\,dz=-\frac{1}{2}\cdot\frac{t^{1/2}}{1/2}+4\frac{z^{3/2}}{3/2}+C=-\sqrt{t}+\frac{8\cdot z^{3/2}}{3}+C=-\sqrt{1-x^2}+\frac{8\sqrt{\arcsin^3x}}{3}+C$

8.2.60. $\int\frac{1-6x}{(x+1)(x-1)}dx=\int\frac{1-6x}{x^2-1}dx=\int\frac{dx}{x^2-1}-6\int\frac{x}{x^2-1}dx=\left[\begin{matrix}2.\ t=x^2-1\to\\ \to dt=2xdx\end{matrix}\right]=\int\frac{dx}{x^2-1}-6\frac{1}{2}\int\frac{dt}{t}=$
$=\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-3\ln|t|+C=\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-3\ln|t|+C=\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-3\ln|x^2-1|+C$

8.2.61. $\int(\cos^2x-\sin^2x)\sqrt[3]{1+\sin2x}\,dx=\int\cos2x\sqrt[3]{1+\sin2x}\,dx=[t=1+\sin2x\to dt=2\cos2x\,dx]=\frac{1}{2}\int\sqrt[3]{t}\,dt=$
$=1/2\cdot t^{4/3}/\tfrac{4}{3}+C=3\cdot(1+\sin2x)^{4/3}/8+C$

8.2.66. $\int dx/(x+1)\sqrt{x}=[x=t^2\to dx=2tdt\to t=\sqrt{x}]=\int2tdt/(t^2+1)t=2\int\frac{dt}{t^2+1}=2\,\mathrm{arctg}\,t+C=2\,\mathrm{arctg}\sqrt{x}+C$

8.2.69. $\int x\ln x\,dx=\left[\begin{matrix}u=\ln x\to u'=\frac{1}{x}dx\\ v'=x\to v=\int v'dx=\frac{x^2}{2}\end{matrix}\right]=\ln x\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\frac{dx}{x}=\ln x\cdot\frac{x^2}{2}-\frac{1}{2}\int xdx=\ln x\frac{x^2}{2}-\frac{1}{2}\frac{x^2}{2}+C=\frac{x^2}{2}(\ln x-\frac{1}{2})+C$

8.2.70. $\int(2x+3)\cos x\,dx=[u=2x+3\to u'=2;\ v'=\cos x\to v=\int v'dx=\sin x]=(2x+3)\sin x-\int\sin x\cdot2dx=\sin x(2x+$
$+3)-2\int\sin x\,dx=\sin x(2x+3)+2\cos x+C$

8.2.71. $\int x\cdot\mathrm{sh}\,5x\,dx=[u=x\to u'=1,\ v'=\mathrm{sh}\,5x\to v=\int v'dx=1/5\,\mathrm{ch}\,5x]=1/5\,\mathrm{ch}\,5x\cdot x-\int1/5\,\mathrm{ch}\,5x\cdot1dx=\frac{1}{5}x\,\mathrm{ch}\,5x-$
$-\frac{1}{5}\cdot\frac{1}{5}\int\mathrm{ch}\,5x\,d(5x)=\frac{1}{5}x\,\mathrm{ch}\,5x-\frac{1}{25}\int\mathrm{sh}\,5x+C$

$$8.2.73.\ \int x^2 \ln x\,dx = \begin{bmatrix} u = \ln x \to u' = \frac{1}{x} \\ v' = x^2 \to v = \frac{x^3}{3} \end{bmatrix} = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx = \frac{x^3}{3}\ln x -$$

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$$- \frac{1}{3}\cdot\frac{x^3}{3} + C = \frac{x^3}{9}(3\ln x - 1) + C$$

$$8.2.81.\ \int e^{\sqrt{x}}dx = [x = t^2 \to dx = 2t\,dt] = \int e^t 2t\,dt = 2\int t\cdot e^t dt = [u = t \to u' = 1,\ v' = e^t \to v = e^t] =$$
$$= 2(t\cdot e^t - \int 1\cdot e^t dt) = 2t\cdot e^t - 2e^t + C = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$$

$$8.2.82.\ \int \frac{x\,dx}{\cos^2 x} = \begin{bmatrix} u = x \to u' = 1 \\ v' = \frac{1}{\cos^2 x} \to v = tg\,x \end{bmatrix} = x\cdot tg\,x - \int 1\cdot tg\,x\,dx = x\,tg\,x + \ln|\cos x| + C$$

$$8.2.89.\ \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = \begin{bmatrix} x = t^2 \to t = \sqrt{x} \\ dx = 2t\,dt \end{bmatrix} = \int \frac{\cos t}{t}\cdot 2t\,dt = 2\int \cos t\,dt = 2\sin t + C = 2\sin\sqrt{x} + C$$

$$8.2.91.\ \int \frac{dx}{\sin x} = \int \frac{\sin x\,dx}{\sin^2 x} = \begin{bmatrix} t = \cos x \\ dt = -\sin x\,dx \end{bmatrix} = -\int \frac{dt}{1 - t^2} = \int \frac{dt}{t^2 - 1} = \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C = \frac{1}{2}\left|\frac{\cos x - 1}{\cos x + 1}\right| + C =$$
$$= \frac{1}{2}\left|tg\frac{x}{2}\right| + C$$