Моисеенко Павел, ИВТ, 2 курс

Производные, часть 6

11.5.41. $z=\cos(ax+e^y)$, $\frac{\partial^3 z}{\partial x\partial y^2}$ -?; $z'_x=(\cos(ax+e^y))'_x=-a\cdot\sin(ax+e^y)$; $z''_{xy}=(-a\cdot\sin(ax+e^y))'_y=-a\cdot e^y\cdot\cos(ax+e^y)$; $z'''_{xy^2}=(-a\cdot e^y\cdot\cos(ax+e^y))'_y=-a(e^y\cdot\cos(ax+e^y)+e^y\cdot e^y\cdot(-\sin(ax+e^y)))=a\cdot e^y(e^y\sin(ax+e^y)-\cos(ax+e^y))$

11.5.43. $u=x\cdot\ln(xy)$, $\frac{\partial^3 u}{\partial x^2\partial y}$ -?; $u'_x=1\cdot\ln(xy)+xy\cdot 1/xy=\ln(xy)+1$; $u''_{x^2}=x\cdot y\cdot 1/xy=1$; $u''_{x^2y}=0$

11.5.44. $u=x^3\sin y+y^3\sin x$, $\frac{\partial^3 u}{\partial x\partial y\partial z}$ -?; $u'_x=3x^2\sin y+y^3\cos x$; $u''_{xy}=3x^2\cos y+3y^2\cos x=3(x^2\cos y+y^2\cos x)$; $u'''_{xyz}=0$

11.5.45. $u=e^{xyz}$, $\frac{\partial^3 u}{\partial x\partial y\partial z}$ -?; $u'_x=yz\cdot e^{xyz}$; $u''_{xy}=z\cdot e^{xyz}+yzxz\,e^{xyz}=ze^{xyz}+xyz^2e^{xyz}$; $u'''_{xyz}=1\cdot e^{xyz}+2xy\cdot e^{xyz}+2xyz\,e^{xyz}+xyz^2xy\,e^{xyz}=e^{xyz}+3xyz\,e^{xyz}+x^2y^2z^2\cdot e^{xyz}=e^{xyz}(1+3xyz+x^2y^2z^2)$

11.5.46. $z=\ln\frac{1}{\sqrt{(x-u)^2+(y-v)^2}}$, $\frac{\partial^4 z}{\partial x\partial y\partial u\partial v}$ -?; $z'_x=\frac{-\sqrt{(x-u)^2+(y-v)^2}\cdot 2\cdot(x-u)\cdot 1}{2(\sqrt{(x-u)^2+(y-v)^2})^3}=-\frac{x-u}{(x-u)^2+(y-v)^2}$

$z''_{xy}=\frac{(x-u)\cdot 2(y-v)\cdot 1}{((x-u)^2+(y-v)^2)^2}=\frac{2(x-u)(y-v)}{((x-u)^2+(y-v)^2)^2}$; $z'''_{xyu}=\frac{2(y-v)\cdot(-1)\cdot((x-u)^2+(y-v)^2)^2-2(x-u)(y-v)\cdot 2((x-u)^2+(y-v)^2)\cdot(-\ldots}{((x-u)^2+(y-v)^2)^4}$

$=-\frac{2(y-v)}{((x-u)^2+(y-v)^2)^2}+\frac{4(x-u)(y-v)}{((x-u)^2+(y-v)^2)^3}$; $z^{(IV)}_{xyuv}=\frac{-2\cdot(-1)((x-u)^2+(y-v)^2)^2+2(y-v)\cdot 2((x-u)^2+(y-v)^2)\cdot(-1)}{((x-u)^2+(y-v)^2)^4}+$

$+\frac{4(x-u)\cdot(-1)((x-u)^2+(y-v)^2)^3-4(x-u)(y-v)\cdot 3((x-u)^2+(y-v)^2)^2\cdot(-1)}{((x-u)^2+(y-v)^2)^6}=\frac{2}{((x-u)^2+(y-v)^2)^2}-\frac{4(y-v)}{((x-u)^2+(y-v)^2)^3}-$

$-\frac{4(x-u)}{((x-u)^2+(y-v)^2)^3}+\frac{12(x-u)(y-v)}{((x-u)^2+(y-v)^2)^4}=\frac{2}{((x-u)^2+(y-v)^2)^2}-\frac{4(y-v+x-u)}{((x-u)^2+(y-v)^2)^3}+\frac{12(x-u)(y-v)}{((x-u)^2+(y-v)^2)^4}$

11.5.47. $u=(x-x_0)^p(y-y_0)^q$, $\frac{\partial^{p+q}u}{\partial x^p\partial y^q}$ -?; $u'_x=p(x-x_0)^{p-1}(y-y_0)^q$; $u''_{x^2}=p(p-1)(x-x_0)^{p-2}(y-y_0)^q$; $u'''_{x^3}=$ $=p(p-1)(p-2)(x-x_0)^{p-3}(y-y_0)^q$; $u^{(p)}_{x^p}=p(p-1)\cdot\ldots\cdot(p-p)\cdot(x-x_0)^0(y-y_0)^q=p!(y-y_0)^q$; $u^{p+1}_{x^py}=p!q(y-y_0)^{q-1}$; $u^{p+2}_{x^py^2}=p!q(q-1)(y-y_0)^{q-2}$; …; $u^{p+q}=p!q(q-1)\cdot\ldots\cdot(q-q)(y-y_0)^0=p!q!$

11.5.48. $u=\frac{x+y}{x-y}$, $\frac{\partial^{m+n}u}{\partial x^m\partial y^n}$ -?; $u'_x=\frac{1\cdot(x-y)-(x+y)\cdot 1}{(x-y)^2}=\frac{-2y}{(x-y)^2}$; $u''_x=\frac{-2y\cdot(-2)}{(x-y)^3}=\frac{4y}{(x-y)^3}$; $u'''_x=$

$=\frac{4y\cdot(-3)}{(x-y)^4}=\frac{-12y}{(x-y)^4}$; $u^{(m)}_{x^m}=\frac{2(-1)^m m!y}{(x-y)^m}$; $u^{(m+1)}_{x^my}=\frac{2(-1)^m m!(x-y)^m-2(-1)^m m!ym(x-y)^{m-1}}{(x-y)^{2m}}=\frac{2(-1)^m m!(x-y)^{m-1}(x-y-y)}{(x-y)^{2m}}=$

$=\frac{2(-1)^m m!(x-2y)}{(x-y)^{m+1}}$; $u^{(m+2)}_{x^my^2}=\frac{2(-1)^m m!(-2)(x-y)^{m+1}-2(-1)^m m!(x-2y)(m+1)(x-y)^m}{(x-y)^{2m+2}}=\frac{2(-1)^m m!(x-y)^m(-2x+2y-(x-2y)(m+1))}{(x-y)^{2m+2}}=$

$=\frac{2(-1)^m m!(-3x+4y-xm+2ym)}{(x-y)^{m+2}}$; $u^{(m+n)}_{x^my^n}=\frac{2(-1)^m(m+n-1)!(nx+my)}{(x-y)^{m+n+1}}$

11.5.49. $u=(x^2+y^2)e^{x+y}$, $\frac{\partial^{m+n}u}{\partial x^m\partial y^n}$ -?; $u'_x=2x\cdot e^{x+y}+(x^2+y^2)1e^{x+y}=e^{x+y}(x^2+y^2+2x)$; $u''_{x^2}=1e^{x+y}(x^2+y^2+2x)+e^{x+y}(2x+2)=e^{x+y}(x^2+y^2+4x+2)$; $u'''_{x^3}=1e^{x+y}(x^2+y^2+4x+2)+e^{x+y}(2x+4)=e^{x+y}(x^2+y^2+6x+6)$; $u^{(IV)}_{x^4}=e^{x+y}(x^2+y^2+6x+6)+e^{x+y}(2x+6)=e^{x+y}(x^2+y^2+8x+12)$; $u^{(m)}_{x^m}=e^{x+y}(x^2+y^2+2mx+m(m-1))$; $u^{(m+1)}_{x^my}=$ $=1e^{x+y}(x^2+y^2+2mx+m(m-1))+e^{x+y}\cdot 2y=e^{x+y}(x^2+y^2+2mx+m(m-1)+2y)$; $u^{(m+2)}_{x^my^2}=e^{x+y}(x^2+y^2+2mx+m(m-1)+2y)+e^{x+y}(2y+2)=e^{x+y}(x^2+y^2+2mx+m(m-1)+4y+2)$; $u^{(m+3)}_{x^my^3}=e^{x+y}(x^2+y^2+2mx+m(m-1)+4y+2)+e^{x+y}(2y+4)=$ $=e^{x+y}(x^2+y^2+2mx+m(m-1)+6y+6)$; $u^{(m+n)}_{x^my^n}=e^{x+y}(x^2+y^2+2mx+m(m-1)+2ny+n(n-1))$

11.5.50. $u=\operatorname{arctg}\frac{x+y+z-xyz}{1-xy-xz-yz}$, $\frac{\partial^3 u}{\partial x\partial y\partial z}$ -?; $u'_x=\frac{1}{1+\left(\frac{x+y+z-xyz}{1-xy-xz-yz}\right)^2}\cdot\frac{(1-yz)(1-xy-xz-yz)-(x+y+z-xyz)(-y-z)}{(1-xy-xz-yz)^2}=\frac{(1-xy-xz-yz)^2}{(1-xy-xz-yz)^2+(x+y+z-xyz)^2}\cdot\frac{(y^2+1)(z^2+1)}{(1-xy-xz-yz)^2}=\frac{(y^2+1)(z^2+1)}{(x^2+1)(y^2+1)(z^2+1)}=\frac{1}{x^2+1}$; $u''_{xy}=0$; $u'''_{xyz}=0$

11.5.51. $d^{10}u$ -?; $u=\ln(x+y)$; $u'_x=\frac{1}{x+y}$; $u''_{x^2}=-\frac{1}{(x+y)^2}$; $u'''_{x^3}=\frac{2}{(x+y)^3}$; $u^{(IV)}_{x^4}=-\frac{2\cdot 3}{(x+y)^4}$; $u^{(X)}_{x^{10}}=$ $=\frac{(-1)^{11}\cdot 9!}{(x+y)^{10}}=-\frac{9!}{(x+y)^{10}}$; $u'_y=\frac{1}{x+y}$; $u''_{y^2}=-\frac{1}{(x+y)^2}$; $u'''_{y^3}=\frac{2}{(x+y)^3}$; $u^{(IV)}_{y^4}=-\frac{2\cdot 3}{(x+y)^4}$; $u^{(X)}_{y^{10}}=\frac{(-1)^{11}\cdot 9!}{(x+y)^{10}}=$

$= -\frac{9!}{(x+y)^{10}}$; $d^{10}u = -\frac{9!}{(x+y)^{10}}(dx+dy)^{10}$

11.5.52. d^4u-?; $u=\ln(x^x y^y z^z)$; $u'_x = (\ln(x^x y^y z^z))' = 1/x^x y^y z^z \cdot x^x y^y z^z(\ln(x)+1) = \ln(x)+1$; $u''_{x^2} = \frac{1}{x}$; $u'''_{x^3} = -\frac{1}{x^2}$; $u^{(IV)}_{x^4} = \frac{2}{x^3}$; аналогично $u^{(IV)}_{y^4} = \frac{2}{y^3}$ и $u^{(IV)}_{z^4} = \frac{2}{z^3}$; $d^4u = \frac{2}{x^3}dx^4 + \frac{2}{y^3}dy^4 + 2/z^3\, dz^4 = 2(dx^4/x^3 + dy^4/y^3 + dz^4/z^3)$

11.5.53. d^nu-?; $u=e^{ax+by}$; $u'_x = a\cdot e^{ax+by}$; $u''_{x^2} = a^2e^{ax+by}$; $u'''_{x^3} = a^3e^{ax+by}$; $u'_y = be^{ax+by}$; $u''_{y^2} = b^2e^{ax+by}$; $u'''_{y^3} = b^3e^{ax+by}$; $d^nu = e^{ax+by}(adx+bdy)^n$

11.5.54. d^nu-?, $u=e^{ax+by+cz}$; $u'_x = ae^{ax+by+cz}$; $u''_{x^2} = a^2e^{ax+by+cz}$; $u^{(n)}_{x^n} = a^ne^{ax+by+cz}$; $u'_y = be^{ax+by+cz}$; $u''_{y^2} = b^2e^{ax+by+cz}$; $u^{(n)}_{y^n} = b^ne^{ax+by+cz}$; $u'_z = ce^{ax+by+cz}$; $u''_{z^2} = c^2e^{ax+by+cz}$; $u^{(n)}_{z^n} = c^ne^{ax+by+cz}$; $d^nu = e^{ax+by+cz}(adx+bdy+cdz)^n$

11.5.58. $z = \frac{x}{y}e^{xy}$; $z'_x = \frac{1}{y}e^{xy} + \frac{x}{y}ye^{xy} = \frac{1}{y}e^{xy} + xe^{xy}$; $z''_{x^2} = \frac{1}{y}ye^{xy} + 1e^{xy} + xye^{xy} = e^{xy} + e^{xy} + xye^{xy} = 2e^{xy} + xye^{xy}$; $z''_{xy} = -\frac{1}{y^2}e^{xy} + \frac{1}{y}xe^{xy} + x^2e^{xy}$; $z'_y = -\frac{x}{y^2}e^{xy} + \frac{x}{y}xe^{xy}$; $z''_{y^2} = \frac{2x}{y^3}e^{xy} - \frac{x}{y^2}xe^{xy} - \frac{x^2}{y^2}e^{xy} + \frac{x}{y}xe^{xy} = \frac{2x}{y^3}e^{xy} - \frac{2x^2}{y^2}e^{xy} + \frac{x^2}{y}e^{xy}$; $d^2z = (2e^{xy} + xye^{xy})dx^2 + \left(-\frac{1}{y^2}e^{xy} + \frac{x}{y}e^{xy} + x^2e^{xy}\right)dxdy + \left(\frac{2x}{y^3}e^{xy} - \frac{2x^2}{y^2}e^{xy} + \frac{x^2}{y}e^{xy}\right)dy^2 = e^{xy}\left((2+xy)dx^2 + \left(-\frac{1}{y^2} + \frac{x}{y} + x^2\right)dxdy + \left(\frac{2x}{y^3} - \frac{2x^2}{y^2} + \frac{x^2}{y}\right)dy^2\right)$

11.5.59. $z = x^4 + 3x^3y - 4x^2y^2 + 5xy^3 - y^4$; $z'_x = 4x^3 + 3\cdot3x^2y - 4\cdot2xy^2 + 5y = 4x^3 + 9x^2y - 8xy^2 + 5y^3$; $z'_y = 3x^3 - 4x^2\cdot2y + 5x\cdot3y^2 - 4y^3 = 3x^3 - 8x^2y + 15xy^2 - 4y^3$; $z''_{x^2} = 4\cdot3x^2 + 9\cdot2xy - 8y^2 = 12x^2 + 18xy - 8y^2$; $z''_{xy} = 9x^2 - 8x\cdot2y + 5\cdot3y^2 = 9x^2 - 16xy + 15y^2$; $z''_{y^2} = -8x^2 + 15x\cdot2y - 4\cdot3y^2 = -8x^2 + 30xy - 12y^2$; $z'''_{x^2y} = 18x - 16y$; $z'''_{xy^2} = -16x + 30y$

11.5.60. $z = e^{xy}$; $z'_x = ye^{xy}$; $z'_y = xe^{xy}$; $z''_{x^2} = y^2e^{xy}$; $z''_{xy} = 1\cdot e^{xy} + yxe^{xy} = e^{xy}(xy+1)$; $z''_{y^2} = x^2e^{xy}$

11.5.61. $u = \sin xyz$, u'''_{xyz}-?; $u'_x = yz\cdot\cos xyz$; $u''_{xy} = z\cos xyz + yzxz\sin xyz = z\cos xyz + xyz^2\sin xyz$; $u'''_{xyz} = 1\cdot\cos xyz + zxy\sin xyz + 2xyz\sin xyz + xyz^2xy\cos xyz = \cos xyz + x^2y^2z^2\cos xyz + 3xyz\sin xyz = \cos xyz(1+x^2y^2z^2) + 3xyz\sin xyz$