

Производные, часть 2

7.2.16. $y = 2^{\cos x}$; $dy = (2^{\cos x})' dx = (2^{\cos x} \cdot \ln 2 \cdot (-\sin x)) dx = -\sin x \cdot 2^{\cos x} \cdot \ln 2 dx$

7.2.17. $y = \ln^3 \sin x$; $dy = (\ln^3 \sin x)' dx = -3 \cdot \ln^2 \sin x \cdot \frac{1}{\sin x} \cdot \cos x dx = -3 \cot x \cdot \ln^2 \sin x dx$

7.2.18. $f(x) = \sqrt[3]{x^5 - 1}$; $df = (\sqrt[3]{x^5 - 1})' dx = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(x^5 - 1)^2}} \cdot 5x^4 dx = \frac{5x^4}{3\sqrt[3]{(x^5 - 1)^2}} dx$

7.2.19. $S(t) = \frac{\sqrt{t}}{t-1}$; $dS = \left(\frac{\sqrt{t}}{t-1} \right)' dt = \frac{\frac{1}{2\sqrt{t}} \cdot (t-1) - \sqrt{t} \cdot 1}{(t-1)^2} dt = \frac{\frac{t-1-2t}{2\sqrt{t}}}{(t-1)^2} dt = \frac{-t-1}{2\sqrt{t}(t-1)^2} dt = -\frac{t+1}{2\sqrt{t}(t-1)^2} dt$

7.2.20. $y = 4x^2 + 1$, $x_0 = 1$, $\Delta x = 0,02$; $\Delta y = 4(\Delta x + x)^2 + 1 - (4x^2 + 1) = 4(\Delta x)^2 + 8x \cdot \Delta x = 4(0,02)^2 + 8 \cdot 1 \cdot 0,02 = 4 \cdot 0,0004 + 0,16 = 0,0016 + 0,16 = 0,1616$
 $dy(x_0 = 1, \Delta x = 0,02) = 4(0,02)^2 + 8 \cdot 1 \cdot 0,02 = 4 \cdot 0,0004 + 0,16 = 0,0016 + 0,16 = 0,1616$
 $dy = y' dx = (4x^2 + 1)' dx = 8x dx = 8x \Delta x$; $dy(x_0 = 1, \Delta x = 0,02) = 8 \cdot 1 \cdot 0,02 = 0,16$

7.2.22. $\sin 29^\circ$; $\sin(x_0 + \Delta x) \approx \sin x_0 + (\sin x_0)' \cdot \Delta x = \sin x_0 + \cos x_0 \cdot \Delta x$
 $\sin 29^\circ = \sin(30^\circ - 1^\circ) \Rightarrow x_0 = 30^\circ$, $\Delta x = -1^\circ$; $\sin 29^\circ \approx \sin 30^\circ + \cos 30^\circ \cdot (-1^\circ) = \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot 0,0174 = 0,5 - 0,866 \cdot 0,0174 = 0,5 - 0,015 = 0,485$

7.2.24. $(0,99)^4$; $(x_0 + \Delta x)^4 \approx x_0^4 + (x_0^4)' \cdot \Delta x = x_0^4 + 4x_0^3 \cdot \Delta x$
 $(0,99)^4 = (1 - 0,01)^4 \Rightarrow x_0 = 1$, $\Delta x = -0,01$; $(0,99)^4 = 1^4 + 4 \cdot 1^3 \cdot (-0,01) = 1 - 0,04 = 0,96$

7.2.25. $y = \frac{x-1}{x+1}$; $dy = y' dx = \left(\frac{x-1}{x+1} \right)' dx = \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2} dx = \frac{x+1-x+1}{(x+1)^2} dx = \frac{2}{(x+1)^2} dx$
 $d^2y = \left(\frac{2}{(x+1)^2} dx \right)' dx = (2(x+1)^{-2} dx)' dx = 2 \cdot (-2) \cdot \frac{1}{(x+1)^3} \cdot 1 dx^2 = -\frac{4}{(x+1)^3} dx^2$

7.2.26. $y = x(\ln x - 1)$; $dy = (x(\ln x - 1))' dx = (1 \cdot (\ln x - 1) + x \cdot \frac{1}{x}) dx = (\ln x - 1 + 1) dx = \ln x dx$
 $d^2y = (\ln x dx)' dx = \frac{1}{x} dx^2$

7.2.28. $y = x^n$; $dy = (x^n)' dx = n \cdot x^{n-1} dx$; $d^2y = (n \cdot x^{n-1} dx)' dx = n \cdot (n-1) \cdot x^{n-2} dx^2$
 $d^3y = (n(n-1)x^{n-2} dx^2)' dx = n(n-1)(n-2)x^{n-3} dx^3$