

$$\left(\begin{array}{cccc|c} 5 & 7 & 6 & 5 & 23 \\ 7 & 10 & 8 & 7 & 32 \\ 6 & 8 & 10 & 9 & 33 \\ 5 & 7 & 9 & 10 & 31 \end{array} \right)$$

$$i=1, a_{11}=5, k=2, \tilde{a}_{21} = \frac{a_{21}}{a_{11}} = \frac{7}{5}$$

$$\tilde{a}_{21} = a_{21} - \tilde{a}_{21} \cdot a_{11} = 7 - \frac{7}{5} \cdot 5 = 0$$

$$\tilde{a}_{22} = a_{22} - \tilde{a}_{21} \cdot a_{12} = 10 - \frac{7}{5} \cdot 7 = \frac{1}{5}$$

$$\tilde{a}_{23} = a_{23} - \tilde{a}_{21} \cdot a_{13} = 8 - \frac{7}{5} \cdot 6 = -\frac{2}{5}$$

$$\tilde{a}_{24} = a_{24} - \tilde{a}_{21} \cdot a_{14} = 7 - \frac{7}{5} \cdot 5 = 0$$

$$\tilde{a}_{25} = a_{25} - \tilde{a}_{21} \cdot a_{15} = 32 - \frac{7}{5} \cdot 23 = -\frac{1}{5}$$

$$k=3, \tilde{a}_{31} = a_{31} - \tilde{a}_{31} \cdot a_{11} = 6 - \frac{6}{5} \cdot 5 = 0$$

$$\tilde{a}_{32} = a_{32} - \tilde{a}_{31} \cdot a_{12} = 8 - \frac{6}{5} \cdot 7 = -\frac{2}{5}$$

$$\tilde{a}_{33} = a_{33} - \tilde{a}_{31} \cdot a_{13} = 10 - \frac{6}{5} \cdot 6 = \frac{14}{5}$$

$$\tilde{a}_{34} = a_{34} - \tilde{a}_{31} \cdot a_{14} = 9 - \frac{6}{5} \cdot 5 = 3$$

$$\tilde{a}_{35} = a_{35} - \tilde{a}_{31} \cdot a_{15} = 33 - \frac{6}{5} \cdot 23 = \frac{24}{5}$$

$$k=4, \tilde{a}_{41} = \frac{a_{41}}{a_{11}} = \frac{5}{5} = 1$$

$$\tilde{a}_{41} = a_{41} - \tilde{a}_{41} \cdot a_{11} = 5 - 5 = 0$$

$$\tilde{a}_{42} = a_{42} - \tilde{a}_{41} \cdot a_{12} = 7 - 7 = 0$$

$$\tilde{a}_{43} = a_{43} - \tilde{a}_{41} \cdot a_{13} = 9 - 6 = 3$$

$$\tilde{a}_{44} = a_{44} - \tilde{a}_{41} \cdot a_{14} = 10 - 5 = 5$$

$$\tilde{a}_{45} = a_{45} - \tilde{a}_{41} \cdot a_{15} = 31 - 23 = 8$$

$$\left(\begin{array}{cccc|c} 5 & 7 & 6 & 5 & 23 \\ 0 & \frac{1}{5} & -\frac{2}{5} & 0 & -\frac{1}{5} \\ 0 & -\frac{2}{5} & \frac{14}{5} & 3 & \frac{24}{5} \\ 0 & 0 & 3 & 5 & 8 \end{array} \right)$$

$$i=2, a_{22} = \frac{1}{5}, k=3, \tilde{a}_{32} = \frac{a_{32}}{a_{22}} = -\frac{2}{5} \cdot 5 = -2$$

$$\tilde{a}_{32} = a_{32} - \tilde{a}_{32} \cdot a_{22} = -\frac{2}{5} - (-2) \cdot \frac{1}{5} = 0$$

$$\tilde{a}_{33} = a_{33} - \tilde{a}_{32} \cdot a_{23} = \frac{14}{5} - (-2) \cdot \left(-\frac{2}{5}\right) = 2$$

$$\tilde{a}_{34} = a_{34} - \tilde{a}_{32} \cdot a_{24} = 3 + 2 \cdot 0 = 3$$

$$\tilde{a}_{35} = a_{35} - \tilde{a}_{32} \cdot a_{25} = \frac{24}{5} + 2 \cdot \left(-\frac{1}{5}\right) = 5$$

$$k=4, \tilde{a}_{42} = \frac{a_{42}}{a_{22}} = 0$$

$$\tilde{a}_{42} = a_{42} - \tilde{a}_{42} \cdot a_{22} = 0$$

$$\tilde{a}_{43} = a_{43} - \tilde{a}_{42} \cdot a_{23} = 3$$

$$\tilde{a}_{44} = a_{44} - \tilde{a}_{42} \cdot a_{24} = 5$$

$$\tilde{a}_{45} = a_{45} - \tilde{a}_{42} \cdot a_{25} = 8$$

$$\left(\begin{array}{cccc|c} 5 & 7 & 6 & 5 & 23 \\ 0 & \frac{1}{5} & -\frac{2}{5} & 0 & -\frac{1}{5} \\ 0 & 0 & 2 & 3 & 5 \\ 0 & 0 & 3 & 5 & 8 \end{array} \right)$$

$$i=3, a_{33}=2, k=4, \tilde{a}_{43} = \frac{a_{43}}{a_{33}} = \frac{3}{2}$$

$$\tilde{a}_{43} = a_{43} - \tilde{a}_{43} \cdot a_{33} = 3 - \frac{3}{2} \cdot 2 = 0$$

$$\tilde{a}_{44} = a_{44} - \tilde{a}_{43} \cdot a_{34} = 5 - \frac{3}{2} \cdot 3 = \frac{1}{2}$$

$$\tilde{a}_{45} = a_{45} - \tilde{a}_{43} \cdot a_{35} = 8 - \frac{3}{2} \cdot 5 = \frac{1}{2}$$

$$\left(\begin{array}{cccc|c} 5 & 7 & 6 & 5 & 23 \\ 0 & \frac{1}{5} & -\frac{2}{5} & 0 & -\frac{1}{5} \\ 0 & 0 & 2 & 3 & 5 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{array} \right)$$

Обратный ход:

$$x_4 = \frac{\tilde{a}_{45}}{\tilde{a}_{44}} = \frac{\frac{1}{2}}{\frac{1}{5}} \cdot 2 = 1$$

$$x_3 = \frac{\tilde{a}_{35} - \sum_{j=4}^4 \tilde{a}_{3j} \cdot x_j}{\tilde{a}_{33}} = \frac{5 - 3 \cdot 1}{2} = 1$$

$$x_2 = \frac{\tilde{a}_{25} - \sum_{j=3}^3 \tilde{a}_{2j} \cdot x_j}{\tilde{a}_{22}} = \frac{-\frac{1}{5} - (-\frac{2}{5} \cdot 1 + 0 \cdot 1)}{\frac{1}{5}} = \frac{-\frac{1}{5} + \frac{2}{5}}{\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{1}{5}} \cdot 5 = 1$$

$$x_1 = \frac{\tilde{a}_{15} - \sum_{j=2}^4 \tilde{a}_{1j} \cdot x_j}{\tilde{a}_{11}} = \frac{23 - (7 \cdot 1 + 6 \cdot 1 + 5 \cdot 1)}{5} = \frac{23 - 18}{5} = 1$$

Ответ: (1; 1; 1; 1)