Моисенко Павел, ИКНИТО, ИВТ, 1.2.

Обратная матрица. Матричн. ур-ия

1.4.37. $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$ $\Delta A = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{vmatrix} = 1 \neq 0$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & 2 \\ 0{,}5 & 0 \end{vmatrix} = -1$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 0{,}5 \end{vmatrix} = 0$ $A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$ $A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = 0$ $A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 0 \\ 0{,}5 & 0 \end{vmatrix} = 0$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0{,}5 \end{vmatrix} = 0{,}5$ $A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = 2$

$\tilde{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0{,}5 \\ 0 & 2 & 0 \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$ $A^{-1} = \frac{1}{1} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$

1.4.38 $\begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{vmatrix} = 3 - 4 = -1$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 3 & -6 \\ -1 & 3 \end{vmatrix} = 3$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 8 & -6 \\ -4 & 3 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 8 & 3 \\ -4 & -1 \end{vmatrix} = 4$ $A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = -1$ $A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -3$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} = -5$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -2$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -3$ $A_{22} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & -1 \\ 8 & -6 \end{vmatrix} = -2$

$\tilde{A} = \begin{pmatrix} 3 & 0 & 4 \\ -2 & -1 & -3 \\ -3 & -2 & -5 \end{pmatrix}^T = \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix}$ $A^{-1} = \frac{1}{-1} \cdot \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ -4 & 3 & 5 \end{pmatrix}$

1.4.39. $\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix} = -6 + 12 = 6$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -6$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6$ $A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = -4$ $A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 4$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -2$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = 3$ $A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2$ $A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$

$\tilde{A} = \begin{pmatrix} -6 & 0 & 6 \\ -2 & -4 & 3 \\ 4 & 2 & -3 \end{pmatrix}^T = \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix}$ $A^{-1} = \frac{1}{6} \cdot \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ 1 & 0{,}5 & -0{,}5 \end{pmatrix}$

1.4.40. $\begin{pmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{vmatrix} = 3 \cdot 11 - 4 \cdot 5 + 2 \cdot 14 = 41$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -4 & -3 \\ 5 & 1 \end{vmatrix} = 11$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 14$ $A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1$ $A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 4 & 2 \\ -4 & -3 \end{vmatrix} = -4$ $A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} = -20$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} = 6$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -11$ $A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = 13$

$\tilde{A} = \begin{pmatrix} 11 & -5 & 14 \\ 6 & 1 & -11 \\ -4 & 13 & -20 \end{pmatrix}^T = \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix}$ $A^{-1} = \frac{1}{41} \cdot \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix} = \begin{pmatrix} 11/41 & 6/41 & -4/41 \\ -5/41 & 1/41 & 13/41 \\ 14/41 & -11/41 & -20/41 \end{pmatrix}$

1.4.41. $\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 3 \cdot (-9) + 1(-3) + 2 \cdot 15 = 0 \Rightarrow$ вырожд. матрица $\Rightarrow$ обратн. матрица не существует.

1.4.42. $\begin{pmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 5 \cdot (-13) - 8 \cdot 4 - 1 \cdot 7 = -104$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix} = -13$ $A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = -4$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 7$ $A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 8 & -1 \\ 2 & 3 \end{vmatrix} = -26$ $A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16$ $A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 5 & 8 \\ 1 & 2 \end{vmatrix} = -2$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 8 & -1 \\ -3 & 2 \end{vmatrix} = 13 \quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 5 & -1 \\ 2 & 2 \end{vmatrix} = -12 \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 5 & 8 \\ 2 & -3 \end{vmatrix} = -31$

$\tilde{A} = \begin{pmatrix} -13 & -4 & 7 \\ -26 & 16 & -2 \\ 13 & -12 & -31 \end{pmatrix}^T = \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix} \quad A^{-1} = \frac{1}{-104} \cdot \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix} = \begin{pmatrix} 1/8 & 1/4 & -1/8 \\ 1/26 & -2/13 & 3/26 \\ -7/104 & 1/52 & 31/104 \end{pmatrix}$

1.4.43. $\begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \quad \Delta A = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 = 1 \quad \Gamma = \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \text{II}+\text{I} \\ \text{III}+\text{I} \end{matrix} \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \begin{matrix} \\ \\ \text{I}+\text{II}+\text{III} \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \quad A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

1.4.44. $\begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} \quad \Delta A = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{vmatrix} = 2 \cdot 7 - 7 \cdot 5 + 3 \cdot 6 = -3 \quad \Gamma = \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 2 & 7 & 3 & 1 & 0 & 0 \end{array}\right) \begin{matrix} \\ \text{II}-3\text{I} \\ \text{III}-2\text{I} \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -6 & -5 & 0 & 1 & -3 \\ 0 & -3 & -3 & 1 & 0 & -2 \end{array}\right) \begin{matrix} \\ \\ -2\text{III}+\text{II} \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -6 & -5 & 0 & 1 & -3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{matrix} \text{I}-3\text{III} \\ \text{II}+5\text{III} \\ \\ \end{matrix} \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 0 & 6 & -3 & -2 \\ 0 & -6 & 0 & -10 & 6 & 2 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{array}\right) \begin{matrix} \\ \text{II}:(-6) \\ \\ \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 0 & 6 & -3 & -2 \\ 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{matrix} \text{I}-5\text{II} \\ \\ \\ \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/3 & 2 & -1/3 \\ 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \quad A^{-1} = \begin{pmatrix} -7/3 & 2 & -1/3 \\ 5/3 & -1 & -1/3 \\ -2 & 1 & 1 \end{pmatrix}$

1.4.45. $\begin{pmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{pmatrix} \quad \Delta A = 1 \cdot \begin{vmatrix} 6 & 1 & 0 \\ 0 & 1 & 2 \\ 4 & 5 & -4 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & -2 & 4 \\ 0 & 1 & 2 \\ 4 & 5 & -4 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & -2 & 4 \\ 6 & 1 & 0 \\ 4 & 5 & -4 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & -2 & 4 \\ 6 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} =$

$= (6 \cdot (-14) + 8) - 2 \cdot (-28 - 32) + 3 \cdot (4 \cdot 26 - 4 \cdot 14) + (4 + 48) = -76 + 120 + 144 + 52 = 240$

$\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \\ \text{IV}+\text{I} \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \\ \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \\ \text{III}+\frac{\text{II}}{3} \\ \text{IV}+\text{II} \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 0 & 23/3 & -34/3 & -3 & 1 & 1/3 & 0 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \end{array}\right) \begin{matrix} \\ \\ \\ \text{IV}-\frac{11}{15}\text{III} \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \\ 0 & 0 & 23/3 & -34/3 & -3 & 1 & 1/3 & 0 \end{array}\right) \begin{matrix} \\ \\ \\ \text{IV}:(-4) \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \begin{matrix} \text{I}-4\text{IV} \\ \text{II}+10\text{IV} \\ \text{III}+10\text{IV} \\ \\ \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 0 & -8/15 & 1 & -2/5 & -11/15 \\ 0 & -6 & 7 & 0 & 5/6 & -5/2 & 2 & 11/6 \\ 0 & 0 & 10 & 0 & 11/6 & -5/2 & 2 & 17/6 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \begin{matrix} \\ \\ \text{III}:10 \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 0 & -8/15 & 1 & -2/5 & -11/15 \\ 0 & -6 & 7 & 0 & 5/6 & -5/2 & 2 & 11/6 \\ 0 & 0 & 1 & 0 & 11/60 & -1/4 & 1/5 & 17/60 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \begin{matrix} \text{I}+2\text{III} \\ \text{II}-7\text{III} \\ \\ \\ \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & -1/6 & 1/2 & 0 & -1/6 \\ 0 & -6 & 0 & 0 & -9/20 & -3/4 & 3/5 & -3/20 \\ 0 & 0 & 1 & 0 & 11/60 & -1/4 & 1/5 & 17/60 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \begin{matrix} \\ \text{II}:(-6) \\ \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & -1/6 & 1/2 & 0 & -1/6 \\ 0 & 1 & 0 & 0 & 3/40 & 1/8 & -1/10 & 1/40 \\ 0 & 0 & 1 & 0 & 11/60 & -1/4 & 1/5 & 17/60 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \begin{matrix} \text{I}-2\text{II} \\ \\ \\ \\ \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -13/60 & 1/4 & 1/5 & -13/60 \\ 0 & 1 & 0 & 0 & 3/40 & 1/8 & -1/10 & 1/40 \\ 0 & 0 & 1 & 0 & 11/60 & -1/4 & 1/5 & 17/60 \\ 0 & 0 & 0 & 1 & 23/60 & -1/4 & 1/10 & 11/60 \end{array}\right) \quad A^{-1} = \begin{pmatrix} -13/60 & 1/4 & 1/5 & -13/60 \\ 3/40 & 1/8 & -1/10 & 1/40 \\ 11/60 & -1/4 & 1/5 & 17/60 \\ 23/60 & -1/4 & 1/10 & 11/60 \end{pmatrix}$

1.4.50. $X \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad XA = B \Rightarrow X = B \cdot A^{-1} \quad \Delta A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad A^{-1} = \frac{1}{-2} \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -3/2 & 1/2 \end{pmatrix}$

$X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 \\ -3/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

1.4.51. $X \cdot \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad XA = B \Rightarrow X = B \cdot A^{-1} \quad \Delta A = \begin{vmatrix} 4 & 3 \\ -5 & -4 \end{vmatrix} = -1 \quad A^{-1} = \frac{1}{-1} \cdot \begin{pmatrix} 4 & -3 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 5 & -4 \end{pmatrix}$

$X = \begin{pmatrix}1&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}4&3\\-5&-4\end{pmatrix} = \begin{pmatrix}4&3\\-5&-4\end{pmatrix}$

1.4.52. $\begin{pmatrix}1&1\\1&1\end{pmatrix}\cdot X = \begin{pmatrix}2\\3\end{pmatrix}$ $AX=B \Rightarrow X = A^{-1}\cdot B$ $\Delta A = \begin{vmatrix}1&1\\1&1\end{vmatrix} = 0 \Rightarrow A^{-1}$ не сущ. $\Rightarrow X$ – не сущ.

1.4.53. $\begin{pmatrix}1&1\\1&1\end{pmatrix}\cdot X = \begin{pmatrix}2\\2\end{pmatrix}$ — то же, что и в 1.4.52 — $X$ – не сущ.

1.4.54. $\begin{pmatrix}1&-1\\2&3\end{pmatrix}\cdot X\cdot\begin{pmatrix}-5&6\\-4&5\end{pmatrix} = \begin{pmatrix}1&-1\\2&3\end{pmatrix}$ $AXC=B \Rightarrow X = A^{-1}BC^{-1}$

$\Delta A = \begin{vmatrix}1&-1\\2&3\end{vmatrix} = 5$ $\Delta C = \begin{vmatrix}-5&6\\-4&5\end{vmatrix} = -1$

$A^{-1} = \frac{1}{5}\cdot\begin{pmatrix}3&1\\-2&1\end{pmatrix} = \begin{pmatrix}3/5&1/5\\-2/5&1/5\end{pmatrix}$ $C^{-1} = -1\begin{pmatrix}5&-6\\4&-5\end{pmatrix} = \begin{pmatrix}-5&6\\-4&5\end{pmatrix}$ $A^{-1}B = \begin{pmatrix}3/5&1/5\\-2/5&1/5\end{pmatrix}\cdot\begin{pmatrix}1&-1\\2&3\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$X = (A^{-1}\cdot B)\cdot C^{-1} = \begin{pmatrix}1&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}-5&6\\-4&5\end{pmatrix} = \begin{pmatrix}-5&6\\-4&5\end{pmatrix}$

1.4.55. $\begin{pmatrix}1&-1\\2&3\end{pmatrix}\cdot X\cdot\begin{pmatrix}2&-2\\-4&5\end{pmatrix} = \begin{pmatrix}1&-1\\2&3\end{pmatrix}$ $AXC=B \Rightarrow X = A^{-1}BC^{-1}$

$\Delta A = \begin{vmatrix}1&-1\\2&3\end{vmatrix} = 5$ $\Delta C = \begin{vmatrix}2&-2\\-4&5\end{vmatrix} = 2$

$A^{-1} = \frac{1}{5}\cdot\begin{pmatrix}3&1\\-2&1\end{pmatrix} = \begin{pmatrix}3/5&1/5\\-2/5&1/5\end{pmatrix}$ $C^{-1} = \frac{1}{2}\begin{pmatrix}5&2\\4&2\end{pmatrix} = \begin{pmatrix}2{,}5&1\\2&1\end{pmatrix}$ $A^{-1}\cdot B = \begin{pmatrix}3/5&1/5\\-2/5&1/5\end{pmatrix}\cdot\begin{pmatrix}1&-1\\2&3\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$X = (A^{-1}\cdot B)\cdot C^{-1} = \begin{pmatrix}1&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}2{,}5&1\\2&1\end{pmatrix} = \begin{pmatrix}2{,}5&1\\2&1\end{pmatrix}$

1.4.56. $X\cdot\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix} = \begin{pmatrix}0&0&1\\0&2&0\\3&0&0\end{pmatrix}$ $XA=B \Rightarrow X = B\cdot A^{-1}$ $\Delta A = \begin{vmatrix}1&0&0\\0&2&0\\0&0&3\end{vmatrix} = 6$

$A_{11} = (-1)^{1+1}\begin{vmatrix}2&0\\0&3\end{vmatrix} = 6$ $A_{12} = (-1)^{1+2}\cdot\begin{vmatrix}0&0\\0&3\end{vmatrix} = 0$

$A_{13} = (-1)^{1+3}\cdot\begin{vmatrix}0&2\\0&0\end{vmatrix} = 0$ $A_{22} = (-1)^{2+2}\cdot\begin{vmatrix}1&0\\0&3\end{vmatrix} = 3$ $A_{31} = (-1)^{3+1}\cdot\begin{vmatrix}0&0\\2&0\end{vmatrix} = 0$ $A_{32} = (-1)^{3+2}\cdot\begin{vmatrix}1&0\\0&2\end{vmatrix} = 2$

$A_{21} = (-1)^{2+1}\cdot\begin{vmatrix}0&0\\0&3\end{vmatrix} = 0$ $A_{23} = (-1)^{2+3}\cdot\begin{vmatrix}1&0\\0&0\end{vmatrix} = 0$ $A_{33} = (-1)^{3+3}\cdot\begin{vmatrix}1&0\\0&0\end{vmatrix} = 0$

$A^{-1} = \frac{1}{6}\begin{pmatrix}6&0&0\\0&3&0\\0&0&2\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1/2&0\\0&0&1/3\end{pmatrix}$ $X = \begin{pmatrix}0&0&1\\0&2&0\\3&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&1/2&0\\0&0&1/3\end{pmatrix} = \begin{pmatrix}0&0&1/3\\0&1&0\\3&0&0\end{pmatrix}$

1.4.57. $\begin{pmatrix}1&-2&3\\2&3&-1\\0&-2&1\end{pmatrix}\cdot X = \begin{pmatrix}2\\-1\\3\end{pmatrix}$ $AX=B \Rightarrow X = A^{-1}\cdot B$ $\Delta A = \begin{vmatrix}1&-2&3\\2&3&-1\\0&-2&1\end{vmatrix} = 1-2\cdot 4 = -7$

$A_{11} = (-1)^{1+1}\cdot\begin{vmatrix}3&-1\\-2&1\end{vmatrix} = -1$ $A_{12} = (-1)^{1+2}\cdot\begin{vmatrix}-2&-2\\3&1\end{vmatrix} = -4$

$A_{13} = (-1)^{1+3}\cdot\begin{vmatrix}-2&3\\3&-1\end{vmatrix} = -7$ $A_{22} = (-1)^{2+2}\cdot\begin{vmatrix}1&0\\3&1\end{vmatrix} = 1$ $A_{31} = (-1)^{3+1}\cdot\begin{vmatrix}2&0\\3&-2\end{vmatrix} = -4$

$A_{21} = (-1)^{2+1}\cdot\begin{vmatrix}2&0\\-1&1\end{vmatrix} = -2$ $A_{23} = (-1)^{2+3}\cdot\begin{vmatrix}1&2\\3&-1\end{vmatrix} = 7$ $A_{32} = (-1)^{3+2}\cdot\begin{vmatrix}1&0\\2&-2\end{vmatrix} = 2$ $A_{33} = (-1)^{3+3}\cdot\begin{vmatrix}1&2\\-2&3\end{vmatrix} = 7$

$A^{-1} = \frac{1}{-7}\begin{pmatrix}1&-4&-7\\-2&1&7\\-4&2&7\end{pmatrix} = \begin{pmatrix}-1/7&4/7&1\\2/7&-1/7&-1\\4/7&-2/7&-1\end{pmatrix}$ $X = \begin{pmatrix}-1/7&4/7&1\\2/7&-1/7&-1\\4/7&-2/7&-1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}15/7\\-16/7\\-11/7\end{pmatrix}$

1.4.58. $\begin{pmatrix}1&-2&3\\2&3&-1\\0&-2&1\end{pmatrix}\cdot X\cdot\begin{pmatrix}1&2&3\\4&5&6\\7&8&0\end{pmatrix} = \begin{pmatrix}1&2&3\\4&5&6\\7&8&0\end{pmatrix}$ $AXC=B \Rightarrow X = A^{-1}\cdot B\cdot C^{-1}$

$A^{-1} = \begin{pmatrix}-1/7&4/7&1\\2/7&-1/7&-1\\4/7&-2/7&-1\end{pmatrix}$ – не нужно

$\Delta C = \begin{vmatrix}1&2&3\\4&5&6\\7&8&0\end{vmatrix} = 27$ $C_{11} = \begin{vmatrix}5&3\\8&0\end{vmatrix} = -48$ $C_{12} = -1\begin{vmatrix}2&8\\3&0\end{vmatrix} = 24$ $C_{13} = \begin{vmatrix}2&5\\3&8\end{vmatrix} = -3$ $C_{21} = -1\begin{vmatrix}4&6\\7&0\end{vmatrix} = 42$ $C_{22} = \begin{vmatrix}1&3\\7&0\end{vmatrix} = -21$

$C_{23} = -1\begin{vmatrix}1&2\\7&8\end{vmatrix} = 6$ $C_{31} = \begin{vmatrix}4&6\\5&8\end{vmatrix} = -3$ $C_{32} = -1\begin{vmatrix}1&3\\4&8\end{vmatrix} = 6$ $C_{33} = \begin{vmatrix}1&2\\4&5\end{vmatrix} = -3$

$C^{-1} = \frac{1}{27}\cdot\begin{pmatrix}-48&24&-3\\42&-21&6\\-3&6&-3\end{pmatrix} = \begin{pmatrix}-48/27&24/27&-3/27\\42/27&-21/27&6/27\\-3/27&6/27&-3/27\end{pmatrix}$ $A^{-1}\cdot B = \begin{pmatrix}-1/7&4/7&1\\2/7&-1/7&-1\\4/7&-2/7&-1\end{pmatrix}\cdot\begin{pmatrix}1&2&3\\4&5&6\\7&8&0\end{pmatrix} = \begin{pmatrix}64/7&74/7&3\\-51/7&-57/7&0\\-53/7&-58/7&0\end{pmatrix}$

$X = (A^{-1}\cdot B)\cdot C^{-1} = \begin{pmatrix}64/7&74/7&3\\-51/7&-57/7&0\\-53/7&-58/7&0\end{pmatrix}\cdot\begin{pmatrix}-48/27&24/27&-1/9\\42/27&-21/27&2/9\\-1/9&2/9&-1/9\end{pmatrix} = \begin{pmatrix}-1/7&4/7&1\\2/7&-1/7&-1\\4/7&-2/7&-1\end{pmatrix}$