Моисеенко Павел, ИКНиТО, ИВТ, 1.2.

1

Предел функции (4 часть)

6.4.76. $\lim\limits_{x\to2}\frac{3x-1}{2x+5}=\left[\frac{3\cdot2-1}{2\cdot2+5}\right]=\frac{5}{9}$

6.4.77. $\lim\limits_{x\to2,5}\sqrt{4x-1}=[\sqrt{4\cdot2,5-1}]=[\sqrt{9}]=3$

6.4.78. $\lim\limits_{x\to-1}\frac{\sqrt[3]{x+2}+1}{\sqrt{x+5}}=\left[\frac{\sqrt[3]{-1+2}+1}{\sqrt{-1+6}}\right]=\left[\frac{1+1}{\sqrt{4}}\right]=1$

6.4.79. $\lim\limits_{x\to\sqrt{2}}\left(x^2+\frac{1}{x^4}-5\right)=\left[\sqrt{2}^2+\frac{1}{\sqrt{2}^4}-5\right]=-\frac{3}{4}$

6.4.81. $\lim\limits_{x\to1}\frac{x^2-2x+1}{x^3-1}=\left[\frac{0}{0}\right]=\lim\limits_{x\to1}\frac{(x-1)^2}{(x-1)(x^2+x+1)}=\lim\limits_{x\to1}\frac{x-1}{x^2+x+1}=\frac{1-1}{1^2+1+1}=\frac{0}{3}=0$

6.4.83. $\lim\limits_{x\to0}\frac{x^5-x^3+x^2}{x^4+2x^2}=\left[\frac{0}{0}\right]=\lim\limits_{x\to0}\frac{x^2(x^3-x+1)}{x^2(x^2+2)}=\lim\limits_{x\to0}\frac{x^3-x+1}{x^2+2}=\frac{0-0+1}{0+2}=\frac{1}{2}$

6.4.85. $\lim\limits_{a\to0}\frac{(x+a)^3-x^3}{a}=\left[\frac{x^3-x^3}{0}\right]=\lim\limits_{a\to0}\frac{a^3+3a^2x+3ax^2+x^3-x^3}{a}=\lim\limits_{a\to0}\frac{a^3+3a^2x+3x^2a}{a}=$

$=\lim\limits_{a\to0}\frac{a^2+3ax+3x^2}{1}=\lim\limits_{a\to0}a^2+3ax+3x^2=0+0+3x^2=3x^2$

6.4.86. $\lim\limits_{x\to8}\frac{x^2-8x}{\sqrt{x+1}-3}=\left[\frac{0}{0}\right]=\lim\limits_{x\to8}\frac{(x^2-8x)(\sqrt{x+1}+3)}{(\sqrt{x+1}-3)(\sqrt{x+1}+3)}=\lim\limits_{x\to8}\frac{(x^2-8x)(\sqrt{x+1}+3)}{(x+1)-9}=\lim\limits_{x\to8}$

$\frac{x(x-8)(\sqrt{x+1}+3)}{x-8}=\lim\limits_{x\to8}x(\sqrt{x+1}+3)=8\cdot(\sqrt{9}+3)=48$

6.4.87. $\lim\limits_{x\to5}\frac{\sqrt{9-x}-2}{3-\sqrt{x+4}}=\left[\frac{0}{0}\right]=\lim\limits_{x\to5}\frac{(\sqrt{9-x}-2)(3+\sqrt{x+4})}{(3-\sqrt{x+4})(3+\sqrt{x+4})}=\lim\limits_{x\to5}\frac{(\sqrt{9-x}-2)(3+\sqrt{x+4})}{9-x-4}=$

$=\lim\limits_{x\to5}\frac{(\sqrt{9-x}-2)(3+\sqrt{x+4})}{(\sqrt{9-x}-2)(\sqrt{9-x}+2)}=\lim\limits_{x\to5}\frac{3+\sqrt{x+4}}{\sqrt{9-x}+2}=\frac{9}{4}$

6.4.88. $\lim\limits_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\left[\frac{\sqrt{x}-\sqrt{x}}{0}\right]=\lim\limits_{h\to0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}=\lim\limits_{h\to0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=$

$=\lim\limits_{h\to0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$

6.4.89. $\lim\limits_{x\to\sqrt{3}}\frac{\sqrt{x^2+1}-2}{\sqrt{x^2+6}-3}=\left[\frac{0}{0}\right]=\lim\limits_{x\to\sqrt{3}}\frac{(\sqrt{x^2+1}-2)(\sqrt{x^2+6}+3)}{(\sqrt{x^2+6}-3)(\sqrt{x^2+6}+3)}=\lim\limits_{x\to\sqrt{3}}\frac{(\sqrt{x^2+1}-2)(\sqrt{x^2+6}+3)(\sqrt{x^2+1}+2)}{(\sqrt{x^2+6}-3)(\sqrt{x^2+6}+3)(\sqrt{x^2+1}+2)}=$

$=\lim\limits_{x\to\sqrt{3}}\frac{(x^2+1-4)(\sqrt{x^2+6}+3)}{(\sqrt{x^2+1}+2)(x^2+6-9)}=\lim\limits_{x\to\sqrt{3}}\frac{\sqrt{x^2+6}+3}{\sqrt{x^2+1}+2}=\frac{\sqrt{3+6}+3}{\sqrt{3+1}+2}=1,5$

6.4.90. $\lim\limits_{t\to0}\frac{\sqrt[3]{1+t}-1}{t}=\left[\frac{0}{0}\right]=\lim\limits_{t\to0}\frac{(\sqrt[3]{1+t}-1)(\sqrt[3]{(1+t)^2}+\sqrt[3]{1+t}+1)}{t(\sqrt[3]{(1+t)^2}+\sqrt[3]{1+t}+1)}=\lim\limits_{t\to0}\frac{1+t-1}{t(\sqrt[3]{(1+t)^2}+\sqrt[3]{1+t}+1)}=$

$=\lim\limits_{t\to0}\frac{1}{\sqrt[3]{(1+t)^2}+\sqrt[3]{1+t}+1}=\frac{1}{\sqrt[3]{1^2}+\sqrt[3]{1}+1}=\frac{1}{3}$

6.4.91. $\lim\limits_{y\to1}\frac{y-1}{\sqrt[4]{y}-1}=\left[\frac{0}{0}\right]=\lim\limits_{y\to1}\frac{(\sqrt[4]{y}-1)(\sqrt[4]{y}+1)(\sqrt{y}+1)}{\sqrt[4]{y}-1}=\lim\limits_{y\to1}\frac{(\sqrt[4]{y}+1)(\sqrt{y}+1)}{1}=\frac{(1+1)\cdot(1+1)}{1}=4$

6.4.92. $\lim\limits_{x\to\infty}\frac{5x^4-2x+3}{x^2-3x^4}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to\infty}\frac{x^4\left(5-\frac{2}{x^3}+\frac{3}{x^4}\right)}{x^4\left(\frac{1}{x^2}-3\right)}=\lim\limits_{x\to\infty}\frac{5-\frac{2}{x^3}+\frac{3}{x^4}}{\frac{1}{x^2}-3}=\frac{5-0+0}{0-3}=-\frac{5}{3}$

6.4.93. $\lim\limits_{x\to\infty}\frac{7x^3-x^2+3x-1}{10x^2+x}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to\infty}\frac{x^2\left(7x-1+\frac{3}{x}-\frac{1}{x^2}\right)}{x^2\left(10+\frac{1}{x}\right)}=\frac{\infty-1+0+0}{10+0}=\infty$

6.4.94. $\lim\limits_{x\to\infty}\frac{(x^2-3)(2x+9)}{(x^2+x+1)(3x^2-4)}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to\infty}\frac{x^4\left(1-\frac{3}{x^2}\right)\left(\frac{2}{x}+\frac{9}{x^2}\right)}{x^4\left(1+\frac{1}{x}+\frac{1}{x^2}\right)\left(3-\frac{4}{x^2}\right)}=\frac{(1-0)(0+0)}{(1+0+0)(3-0)}=0$

6.4.95. $\lim\limits_{x\to-\infty}(\sqrt{x^2+4}-10x)=[\infty+\infty]=\infty$

6.4.96. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)=[\infty-\infty]=\lim\limits_{x\to\infty}\frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}=$

$=\lim\limits_{x\to\infty}\frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}}=\lim\limits_{x\to\infty}\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}=\left[\frac{2}{\infty+\infty}\right]=0$

6.4.99. $\lim\limits_{x\to\pi}\frac{\sin 3x}{\sin 2x}=\left[\frac{0}{0}\right]=\lim\limits_{x\to\pi}\frac{\sin 2x\cos x+\cos 2x\sin x}{\sin 2x}=\lim\limits_{x\to\pi}\frac{\sin 2x\cos x}{\sin 2x}+\lim\limits_{x\to\pi}\frac{\cos 2x\sin x}{2\sin x\cos x}=$

$=\lim\limits_{x\to\pi}\cos x+\lim\limits_{x\to\pi}\frac{\cos 2x}{2\cos x}=-1+\frac{1}{-2}=-\frac{3}{2}$

6.4.101. $\lim\limits_{h\to 0}\frac{h-\sin h}{h+\sin h}=\left[\frac{0}{0}\right]=\lim\limits_{h\to 0}\frac{\frac{h-\sin h}{h}}{\frac{h+\sin h}{h}}=\lim\limits_{h\to 0}\frac{1-\frac{\sin h}{h}}{1+\frac{\sin h}{h}}=\frac{\lim\limits_{h\to 0}1-\lim\limits_{h\to 0}\frac{\sin h}{h}}{\lim\limits_{h\to 0}1+\lim\limits_{h\to 0}\frac{\sin h}{h}}=\frac{1-1}{1+1}=0$

6.4.105. $\lim\limits_{x\to\infty}x\sin\left(\frac{2}{x}\right)=[\infty]=\left[\begin{matrix}ax=0\Rightarrow x=\frac{2}{a}\\ x\to\infty\Rightarrow\frac{2}{a}\to\infty\Rightarrow a\to 0\end{matrix}\right]=\lim\limits_{a\to 0}\frac{2}{a}\cdot\sin a=\lim\limits_{a\to 0}2\cdot\lim\limits_{a\to 0}\frac{\sin a}{a}=2$

6.4.107. $\lim\limits_{x\to 0}(1+\operatorname{tg}x)^{\operatorname{ctg}x}=[0^{\infty}]=\lim\limits_{x\to 0}\left(1+\frac{1}{\operatorname{ctg}x}\right)^{\operatorname{ctg}x}=\lim\limits_{x\to 0}(1+\operatorname{tg}x)^{\frac{1}{\operatorname{tg}x}}=e$

6.4.108. $\lim\limits_{x\to\infty}\left(\frac{x^2+2}{x^2-2}\right)^{x^2}=\left[\left(\frac{\infty}{\infty}\right)^{\infty}\right]=\lim\limits_{x\to\infty}\left(\frac{x^2\left(1+\frac{2}{x^2}\right)}{x^2\left(1-\frac{2}{x^2}\right)}\right)^{x^2}=\lim\limits_{x\to\infty}\left(\frac{\left(1+\frac{2}{x^2}\right)}{\left(1-\frac{2}{x^2}\right)}\right)^{x^2}=\frac{\lim\limits_{x\to\infty}\left(1+\frac{2}{x^2}\right)^{x^2}}{\lim\limits_{x\to\infty}\left(1-\frac{2}{x^2}\right)^{x^2}}=\frac{e^2}{e^{-2}}=e^4$

6.4.109. $\lim\limits_{x\to 0}\frac{e^{3x}-1}{\sin x}=\left[\frac{0}{0}\right]=\lim\limits_{x\to 0}\frac{\frac{e^{3x}-1}{x}}{\frac{\sin x}{x}}=\frac{\lim\limits_{x\to 0}\frac{e^{3x}-1}{x}}{\lim\limits_{x\to 0}\frac{\sin x}{x}}=\frac{\log_e e^3}{1}=\ln e^3=3$

6.4.112. $\lim\limits_{t\to\infty}\left(1+\frac{1}{t^2}\right)^{t}=[0^{\infty}]=\lim\limits_{t\to\infty}\left(1+\frac{\frac{1}{t}}{t}\right)^{t}=e^0=1$

6.4.118. $\lim\limits_{x\to 0}\frac{\ln(1+x^2)}{\operatorname{tg}^2 8x}=\left[\frac{0}{0}\right]=\left[\begin{matrix}\ln(1+x^2)\approx e^0\cdot x^2\ x\to 0\\ \operatorname{tg}^2 x8\approx(8x)^2\ x\to 0\end{matrix}\right]=\lim\limits_{x\to 0}\frac{e^0\cdot x^2}{(8x)^2}=\frac{e^0}{8^2}=\frac{1}{64}$

6.4.119. $\lim\limits_{x\to 0}\frac{2^{\sin x}-1}{x}=\left[\frac{0}{0}\right]=[\sin x\approx x,\ x\to 0]=\lim\limits_{x\to 0}\frac{2^x-1}{x}=\ln 2$

6.4.120. $\lim\limits_{x\to 0}\frac{x\cdot\arcsin\sqrt{x}}{\operatorname{arctg}^{\frac{3}{2}}2x}=\left[\frac{0}{0}\right]=\left[\begin{matrix}\arcsin\sqrt{x}\sim\sqrt{x}\\ \operatorname{arctg}2x\sim 2x\end{matrix}\ \text{при}\ x\to 0\right]=\lim\limits_{x\to 0}\frac{1}{2^{\frac{3}{2}}}=\frac{1}{\sqrt{8}}$

6.4.123. $\lim\limits_{x\to 0}\frac{\sqrt{1+x\cdot\sin x}-1}{x^2}=\left[\frac{0}{0}\right]=[\sin x\sim x,\ x\to 0]=\lim\limits_{x\to 0}\frac{\sqrt{1+x^2}-1}{x^2}=$

$=\lim\limits_{x\to 0}\frac{\left(\sqrt{1+x^2}-1\right)\left(\sqrt{1+x^2}+1\right)}{x^2\left(\sqrt{1+x^2}+1\right)}=\lim\limits_{x\to 0}\frac{1+x^2-1}{x^2\left(\sqrt{1+x^2}+1\right)}=\lim\limits_{x\to 0}\frac{1}{\sqrt{1+x^2}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$