Интегрирование, часть 4, практическая часть

8.4.2. $\int \frac{\sqrt[3]{x}dx}{\sqrt[3]{x^2}-\sqrt{x}} = [n=3, q=2 \to k = \text{НОК}(3,2) = 6 \to x = t^6 \to 6t^5dt] = \int \frac{\sqrt[3]{t^6}*6t^5dt}{\sqrt[3]{(t^6)^2}-\sqrt{t^6}} =$
$6\int \frac{t^2*t^5dt}{t^4-t^3} = 6\int \frac{t^7dt}{t^3(t-1)} = 6\int \frac{t^4dt}{t-1} = 6\int \frac{t^4-1+1}{t-1}dt = 6\left(\int \frac{t^4-1}{t-1}dt + \int \frac{1}{t-1}dt\right) = 6\left(\int \frac{(t^2-1)(t^2+1)}{t-1} +\right.$
$\left.\int \frac{dt}{t-1}\right) = 6\left(\int \frac{(t-1)(t+1)(t^2+1)}{t-1} + \int \frac{dt}{t-1}\right) = 6\left(\int (t+1)(t^2+1) + \int \frac{dt}{t-1}\right) = 6\left(\int (t^3+t+t^2+1)dt +\right.$
$\left.\int \frac{dt}{t-1}\right) = 6\left(\int t^3dt + \int t^2dt + \int tdt + \int dt + \int \frac{d(t-1)}{t-1}\right) = 6\left(\frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} + t + \ln|t-1|\ \right) + \frac{3}{2}t^4 +$
$2t^3 + 3t^2 + 6t + 6\ln|t-1| + C = [x = t^6 \to t = \sqrt[6]{x}] = \frac{3}{2}\left(\sqrt[6]{x}\right)^4 + 2\left(\sqrt[6]{x}\right)^3 + 3\left(\sqrt[6]{x}\right)^2 + 6\sqrt[6]{x} +$
$6\ln\left|\sqrt[6]{x}-1\right| + C = \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln\left(\sqrt[6]{x}-1\right) + C$

8.4.3. $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}} = [n=2, q=4 \to k = \text{НОК}(2;4) = 4 \to x = t^4 \to dx = 4t^3dt \to t = \sqrt[4]{x}] =$
$\int \frac{4t^3dt}{\sqrt{t^4}+\sqrt[4]{t^4}} = \int \frac{4t^3dt}{t^2+t} = 4\int \frac{t^3dt}{t(t+1)} = 4\int \frac{t^2dt}{t+1} = 4\int \frac{t^2+1-1}{t+1}dt = 4\left(\int \frac{t^2-1}{t+1}dt + \int \frac{1}{t+1}dt\right) =$
$4\left(\int \frac{(t-1)(t+1)dt}{t+1} + \int \frac{dt}{t+1}\right) = 4\left(\frac{t^2}{2} - t + \ln|t+1|\right) + C = 2t^2 - 4t + 4\ln|t+1| + C = [t = \sqrt[4]{x}] =$
$2\left(\sqrt[4]{x}\right)^2 - 4\sqrt[4]{x} + 4\ln\left|\sqrt[4]{x}+1\right| + C = 2\sqrt{x} - 4\sqrt[4]{x} + 4\ln\left(\sqrt[4]{x}+1\right) + C$

8.4.5. $\int \frac{dx}{\sqrt[3]{(2x+1)^2}-\sqrt{2x+1}} = \left[n=3, q=2 \to k = \text{НОК}(3,2) = 6 \to 2x+1 = t^6 \to 1.\ 2x = t^6 - 1 \to\right.$
$\left.x = \frac{t^6-1}{2} \to dx = 3t^5dt;\ 2.\ t = \sqrt[6]{2x+1}\right] = \int \frac{3t^5dt}{\left(\sqrt[3]{t^6}\right)^2-\sqrt{t^6}} = 3\int \frac{t^5dt}{t^4-t^3} = 3\int \frac{t^5dt}{t^3(t-1)} = 3\frac{t^2dt}{t-1} =$
$3\int \frac{t^2-1+1}{t-1}dt = 3\left(\int \frac{t^2-1}{t-1}dt + \int \frac{dt}{t-1}\right) = 3\left(\int \frac{(t-1)(t+1)}{t-1}dt + \int \frac{dt}{t-1}\right) = 3\left(\int (t+1)dt + \int \frac{dt}{t-1}\right) =$
$3\left(\int tdt + \int dt + \int \frac{d(t-1)}{t-1}\right) = 3\left(\frac{t^2}{2} + t + \ln|t-1| + C\right) = \frac{3}{2}\sqrt[6]{(2x+1)^2} + 3\sqrt[6]{2x+1} +$
$3\ln\left|\sqrt[6]{2x+1}-1\right| = \frac{3}{2}\sqrt[3]{2x+1} + 3\sqrt[6]{2x+1} + 3\ln\left|\sqrt[6]{2x+1}\right| + C$

8.4.6. $\int \frac{dx}{1+\sqrt[3]{x+1}} = [x+1 = t^3 \to x = t^3 - 1 \to dx = 3t^2dt;\ t = \sqrt[3]{x+1}] = \int \frac{3t^2dt}{1+\sqrt[3]{t^3}} = 3\int \frac{t^2dt}{t+1} =$
$3\int \frac{t^2-1+1}{t+1}dt = 3\left(\int \frac{t^2-1}{t+1}dt + \int \frac{dt}{t+1}\right) = 3\left(\int \frac{(t-1)(t+1)}{t+1}dt + \int \frac{dt}{t+1}\right) = 3\left(\int (t-1)dt + \int \frac{dt}{t+1}\right) =$
$3\left(\int tdt - \int dt + \int \frac{d(t+1)}{t+1}\right) = 3\left(\frac{t^2}{2} - t + \ln|t+1|\right) + C = \frac{3}{2}\left(\sqrt[3]{x+1}\right)^2 - 3\sqrt[3]{x+1} +$
$3\ln\left|\sqrt[3]{x+1}+1\right| + C = \frac{3}{2}\sqrt[3]{(x+1)^2} - 3\sqrt[3]{x+1} + 3\ln\left(\sqrt[3]{x+1}+1\right) + C$

8.4.8. $\int \frac{\sqrt{x}}{x^2*\sqrt{x-1}}dx = \int \frac{1}{x^2} * \sqrt{\frac{x}{x-1}}dx = \left[\frac{x}{x-1} = t^2 \to x = t^2(x-1) \to xt^2 - x = \frac{t^2}{to}x(t^2-1) = t^2 \to\right.$
$\left.x = \frac{t^2}{t^2-1};\ dx = d\left(\frac{t^2}{t^2-1}\right)' = \frac{2t(t^2-1)-t^2(2t)}{(t^2-1)^2}dt = -\frac{2t}{(t^2-1)^2}dt\right] = \int \frac{1}{\left(\frac{t^2}{(t^2-1)}\right)^2} * \sqrt{t^2} * \left(-\frac{2t}{(t^2-1)^2}\right)dt =$
$\int \frac{(t^2-1)^2}{t^4} * t * \left(-\frac{2t}{(t^2-1)^2}\right)dt = -\int \frac{2t^2}{t^4}dt = -2\int \frac{1}{t^2}dt = -2*(-1)*\frac{1}{t} + C = 2\sqrt{\frac{x-1}{x}} + C$

8.4.10. $\int \sqrt{x}\left(1+\sqrt[3]{x}\right)^4dx = \int x^{\frac{1}{2}}\left(1+x^{\frac{1}{3}}\right)^4 dx = \left[m = \frac{1}{2}, n = \frac{1}{3}, p = 4;\ \text{рассм. случай №1}(p \in\right.$
$\left.Z?);\ p = 4 \to x = t^6 \to dx = (t^6)'dt * 6t^5dt;\ t = \sqrt[6]{x}\right] = \int \sqrt{t^6}\left(1+\sqrt[3]{t^6}\right)^4 * 6t^5dt =$
$\int t^3(1+t^2)^4 * 6t^5dt = 6\int t^8(1+t^2)^4dt = [(1+t^2)^4 = (1+t^2)^2 * (1+t^2)^2 = (1+2t^2+$
$t^4)(1+2t^2+t^4) = 1+2t^2+t^4+2t^2+4t^4+2t^6+t^4+2t^6+t^8 = 1+4t^2+6t^4+4t^6+t^8] =$
$6\int t^8(1+4t^2+6t^4+4t^6+t^8)dt = 6\int t^{16}+4t^{14}+6t^{12}+4t^{10}+t^8)dt = 6\int t^{16}dt +$
$24\int t^{14}dt + 36\int t^{12}dt + 24\int t^{10}dt + 6\int t^8dt = \frac{6t^{17}}{17} + \frac{24t^{15}}{15} + \frac{36t^{13}}{13} + \frac{24t^{11}}{11} + \frac{6t^9}{9} + C = \frac{6\left(\sqrt[6]{x}\right)^{17}}{17} +$

$$\frac{24\left(\sqrt[6]{x}\right)^{15}}{15}+\frac{36\left(\sqrt[6]{x}\right)^{13}}{13}+\frac{24\left(\sqrt[6]{x}\right)^{11}}{11}+\frac{2\left(\sqrt[6]{x}\right)^{9}}{3}+C=\frac{6}{17}*x^2\sqrt[6]{x^5}+\frac{8}{5}*x^2\sqrt{x}-\frac{36}{13}*x^2\sqrt[6]{x}+\frac{24}{11}*x\sqrt[6]{x^5}+\frac{2}{3}*x*\sqrt{x}+C$$