Моисеенко Павел, ИВТ, 2 курс

Производные, часть 5

11.4.42. $z = \operatorname{arctg}\frac{y}{x}$, $x = e^{2t}+1$, $y = e^{2t}-1$, $\frac{dz}{dt}$ - ?; $z'_x = \left(\operatorname{arctg}\frac{y}{x}\right)'_x = \frac{1}{1+\frac{y^2}{x^2}}\cdot\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2}\cdot\frac{x^2}{x^2+y^2} = -\frac{y}{x^2+y^2}$

$z'_y = \left(\operatorname{arctg}\frac{y}{x}\right)'_y = \frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x} = \frac{x^2}{x^2+y^2}\cdot\frac{1}{x} = \frac{x}{x^2+y^2}$; $x'_t = (e^{2t}+1)'_t = 2e^{2t}$; $y'_t = (e^{2t}-1)'_t = 2e^{2t}$;

$\frac{dz}{dt} = -\frac{y\cdot 2e^{2t}}{x^2+y^2} + \frac{x\cdot 2e^{2t}}{x^2+y^2} = \frac{2e^{2t}(x-y)}{x^2+y^2}$

11.4.43. $z = x^4+y^4-4x^2y^2$, $x = e^{2t}$, $y = e^{2t}$, $\frac{dz}{dt}$ - ?; $z'_x = 4x^3 - 4\cdot 2xy^2 = 4x^3 - 8xy^2$;
$z'_y = 4y^3 - 4x^2 2y = 4y^3 - 8x^2y$; $x'_t = 2e^{2t}$; $y'_t = 2e^{2t}$; $\frac{dz}{dt} = (4x^3-8xy^2)\cdot 2e^{2t} + (4y^3-8x^2y)\cdot 2e^{2t} =$
$= 8e^{2t}(x^3-2xy^2+y^3-2x^2y)$

11.4.44. $z = xy + \frac{x}{y}$, $x = \operatorname{tg} t$, $y = \ln t$, $\frac{dz}{dt}$ - ?; $z'_x = y + \frac{1}{y}$; $z'_y = x - \frac{x}{y^2}$; $x'_t = \frac{1}{\cos^2 t}$; $y'_t = \frac{1}{t}$;

$\frac{dz}{dt} = \left(y+\frac{1}{y}\right)\cdot\frac{1}{\cos^2 t} + \left(x - \frac{x}{y^2}\right)\cdot\frac{1}{t}$

11.4.45. $z = \frac{x}{y^2}$, $x = \operatorname{arctg} 2t$, $y = \arcsin t$, $\frac{dz}{dt}$ - ?; $z'_x = \frac{1}{y^2}$; $z'_y = -\frac{2x}{y^3}$; $x'_t = \frac{1}{1+(2t)^2}\cdot 2 = \frac{2}{1+4t^2}$;

$y'_t = \frac{1}{\sqrt{1-t^2}}$; $\frac{dz}{dt} = \frac{2}{y^2(1+4t^2)} - \frac{2x}{y^3\sqrt{1-t^2}}$

11.4.46. $z = \frac{x}{\sqrt{x^2+y^2}}$, $x = 5^{t^2}$, $y = \arccos 2t$, $\frac{dz}{dt}$ - ?; $z'_x = \frac{1\sqrt{x^2+y^2} - x\cdot\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x}{(\sqrt{x^2+y^2})^2} = \frac{\sqrt{x^2+y^2} - x^2/\sqrt{x^2+y^2}}{x^2+y^2} =$

$= \frac{\frac{x^2+y^2-x^2}{\sqrt{x^2+y^2}}}{x^2+y^2} = \frac{y^2}{(x^2+y^2)\sqrt{x^2+y^2}}$; $z'_y = x\cdot\left(-\frac{1}{2(\sqrt{x^2+y^2})^3}\right)\cdot 2y = \frac{-xy}{(x^2+y^2)\sqrt{x^2+y^2}}$; $x'_t = 5^{t^2}\cdot\ln 5\cdot 2t$; $y'_t =$

$= -\frac{1}{\sqrt{1-(2t)^2}}\cdot 2$; $\frac{dz}{dt} = \frac{y^2}{(x^2+y^2)\sqrt{x^2+y^2}}\cdot(5^{t^2}\cdot\ln 5\cdot 2t) + \frac{xy}{(x^2+y^2)\sqrt{x^2+y^2}}\cdot\frac{2}{\sqrt{1-4t^2}} =$

$= \frac{y}{(x^2+y^2)\sqrt{x^2+y^2}}\left(2ty\cdot 5^{t^2}\ln 5 + \frac{2x}{\sqrt{1-4t^2}}\right)$

11.4.47. $z = x\cdot\sin(x+y)$, $x = 1/t^3$, $y = (t-1)^2$, $\frac{dz}{dt}$ - ?; $z'_x = 1\cdot\sin(x+y) + x\cdot\cos(x+y)\cdot 1 = \sin(x+y) +$
$x\cdot\cos(x+y)$; $z'_y = x\cdot\cos(x+y)\cdot 1 = x\cdot\cos(x+y)$; $x'_t = -3/t^4$; $y'_t = 2(t-1)\cdot 1 = 2(t-1)$; $\frac{dz}{dt} = (-3(\sin(x+y) +$
$+ x\cdot\cos(x+y)))/t^4 + 2x(t-1)\cdot\cos(x+y)$

11.4.48. $z = \frac{\cos x^2}{y}$, $x = \ln(t+2)$, $y = \operatorname{tg} t$, $z'_x = \frac{2x\cdot(-\sin x^2)}{y} = -\frac{2x\sin x}{y}$; $z'_y = -\frac{\cos x^2}{y^2}$; $x'_t =$

$= \frac{1}{t+2}$; $y'_t = \frac{1}{\cos^2 t}$; $\frac{dz}{dt} = -\frac{2x\sin x^2}{y(t+2)} - \frac{\cos x^2}{y^2\cdot\cos^2 t}$

11.4.49. $z = \operatorname{tg}\frac{x^2}{y}$, $x = \cos^2 t$, $y = \sin 2t$; $z'_x = \frac{1}{\cos^2\frac{x^2}{y}}\cdot\frac{2x}{y}$; $z'_y = \frac{1}{\cos^2\frac{x^2}{y}}\cdot\left(-\frac{x^2}{y^2}\right)$; $x'_t = 2\cos t\cdot(-\sin t) =$

$= -\sin 2t$; $y'_t = 2\cos 2t$; $\frac{dz}{dt} = \frac{2x}{y}\cdot\frac{(-\sin 2t)}{\cos^2\frac{x^2}{y}} - \frac{x^2}{y^2}\cdot\frac{2\cos 2t}{\cos^2\frac{x^2}{y}}$

11.4.62. $y^4 - 6x^2y^2 + \operatorname{arctg} 2x = 0$; $y^4 - 6x^2y^2 + (3x^2)^2 - (3x^2)^2 + \operatorname{arctg} 2x = 0$; $(y^2-3x^2)^2 =$
$= 9x^4 - \operatorname{arctg} 2x$; $y^2 - 3x^2 = \pm\sqrt{9x^4 - \operatorname{arctg} 2x}$; $y^2 = 3x^2 \pm \sqrt{9x^4 - \operatorname{arctg} 2x}$; $y = \pm\sqrt{3x^2 \pm \sqrt{9x^4 - \operatorname{arctg} 2x}}$

11.4.63. $e^{-x+y^3} - 20x - 18x^3 - 1 = 0$; $e^{-x+y^3} = 20x + 18x^3 + 1$; $\ln e^{-x+y^3} = \ln(20x+18x^3+1)$; $-x+y^3 =$
$= \ln(18x^3+20x+1)$; $y^3 = \ln(18x^3+20x+1) + x$; $y = \sqrt[3]{\ln(18x^3+20x+1)+x}$

11.4.64. $\operatorname{tg}(x^2+y^4) - 3x^2 - 17 = 0$; $\operatorname{tg}(x^2+y^4) = 3x^2 + 17$; $x^2 + y^4 = \operatorname{arctg}(3x^2+17)$;
$y^4 = \operatorname{arctg}(3x^2+17) - x^2$; $y = \pm\sqrt[4]{\operatorname{arctg}(3x^2+17) - x^2}$

11.4.67. $z = u^2\ln v$, $u = y/x$, $v = x^2+y^2$, z'_x - ?, z'_y - ?; $z'_u = 2u\ln v$; $z'_v = u^2/v$; $u'_x =$

$=-\frac{y}{x^2}$; $u'_y=\frac{1}{x}$; $v'_x=2x$; $v'_y=2y$; $z'_x=2u\ln v\cdot\left(-\frac{y}{x^2}\right)+\frac{u^2}{v}\cdot 2x=2u\left(\frac{ux}{v}-\frac{y\ln v}{x^2}\right)$;

$z'_y=2u\ln v\cdot\frac{1}{x}+\frac{u^2}{v}\cdot 2y=2u\left(\frac{uy}{v}+\frac{\ln v}{x}\right)$

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11.4.68. $z=f(u;v)$, $u=\frac{2y}{x+y}$, $v=x^2-3y$, $dz-?$; $z'_u=f'_u(u;v)$; $z'_v=f'_v(u;v)$;
$u'_x=-\frac{2y}{(x+y)^2}$; $u'_y=\frac{2(x+y)-2y}{(x+y)^2}=\frac{2x+2y-2y}{(x+y)^2}=\frac{2x}{(x+y)^2}$; $v'_x=2x$; $v'_y=-3$; $dz=f'_u(u;v)\cdot\left(-\frac{2y}{(x+y)^2}dx+\right.$
$\left.+\frac{2x}{(x+y)^2}dv\right)+f'_v(u;v)(2xdx-3dy)$

11.4.69. $z=f(u;v)$, $u=\ln(x^2-y^2)$, $v=xy^2$, $z'_x-?$, $z'_y-?$; $z'_u=f'_u(u;v)$, $z'_v=f'_v(u;v)$;
$u'_x=\frac{2x}{x^2-y^2}$; $u'_y=-\frac{2y}{x^2-y^2}$; $v'_x=y^2$; $v'_y=2xy$; $z'_x=f'_u(u;v)\cdot\frac{2x}{x^2-y^2}+f'_v(u;v)\cdot y^2$; $z'_y=f'_u(u;v)\left(\frac{-2y}{x^2-y^2}\right)+f'_v(u;v)\cdot 2xy$

11.4.70. $z=u^2v$, $u=x\sin y$, $v=y\cos x$; $dz-?$; $z'_u=2uv$; $z'_v=u^2$; $u'_x=\sin y$; $u'_y=x\cos y$;
$v'_x=-y\sin x$; $v'_y=\cos x$; $dz=2uv(\sin y\,dx+x\cos y\,dy)+u^2(\cos x\,dy-y\sin x\,dx)$

11.4.71. $z=f(u;v)$, $u=\cos(x;y)$, $v=x^5-7y$; $dz-?$; $z'_u=f'_u(u;v)$, $z'_v=f'_v(u;v)$; $u'_x=-y\sin(xy)$;
$u'_y=-x\sin(xy)$; $v'_x=5x^4$; $v'_y=-7$; $dz=f'_u(u;v)(-y\sin(xy)dx-x\sin(xy)dy)+f'_v(u;v)(5x^4dx-7dy)$

11.4.72. $z=f(u;v)$; $u=\sin\frac{x}{y}$; $v=\sqrt{\frac{x}{y}}$, $dz-?$; $z'_u=f'_u(u;v)$; $z'_v=f'_v(u;v)$; $u'_x=\frac{1}{y}\cos\frac{x}{y}$;
$u'_y=-\frac{x}{y^2}\cos\frac{x}{y}$; $v'_x=\frac{1}{y}\cdot\frac{1}{2\sqrt{x/y}}$; $v'_y=-\frac{x}{y^2}\cdot\frac{1}{2\sqrt{x/y}}$; $dz=f'_u(u;v)\left(\frac{1}{y}\cos\frac{x}{y}dx-\frac{x}{y^2}\cos\frac{x}{y}dy\right)+$
$+f'_v(u;v)\cdot\left(\frac{1}{2y\sqrt{\frac{x}{y}}}dx-\frac{x}{2y^2\sqrt{\frac{x}{y}}}dy\right)$

11.4.73. $x=\frac{u^2+v^2}{2}$, $y=\frac{u^2-v^2}{2}$; $z=uv$, $dz-?$; $2x=u^2+v^2$, $u^2=2x-v^2$; $2y=u^2-v^2$, $u^2=2y+v^2$; $2x-v^2=2y+v^2$;
$2v^2=2x-2y$, $v^2=x-y$, $v=\sqrt{x-y}$; $2x=u^2+v^2$, $v^2=2x-u^2$; $2y=u^2-v^2$, $v^2=u^2-2y$; $2x-u^2=u^2-2y$, $2u^2=2x+2y$; $u^2=x+y$, $u=\sqrt{x+y}$; $z=$
$=\sqrt{(x-y)(x+y)}=\sqrt{x^2-y^2}$; $z'_x=\frac{2x}{2\sqrt{x^2-y^2}}=\frac{x}{\sqrt{x^2-y^2}}$; $z'_y=\frac{-y}{\sqrt{x^2-y^2}}$; $dz=\frac{xdx}{\sqrt{x^2-y^2}}-\frac{ydy}{\sqrt{x^2-y^2}}=\frac{xdx-ydy}{z}$

11.4.75. $x=u+v$, $y=u-v$; $z=u^2v^2$, $dz-?$; $u=x-v$, $u=y+v$; $x-v=y+v$, $2v=x-y$, $v=\frac{x-y}{2}$; $x-u=u-y$, $2u=x+y$, $u=\frac{x+y}{2}$; $z=$
$=\left(\frac{x-y}{2}\right)^2\left(\frac{x+y}{2}\right)^2=\left(\frac{x^2-y^2}{16}\right)^2$; $z'_x=\frac{1}{16}\cdot 2(x^2-y^2)\cdot 2x=\frac{x^2-y^2}{4}\cdot x$; $z'_y=\frac{1}{16}\cdot 2(x^2-y^2)(-2y)=-y\frac{x^2-y^2}{4}$;
$dz=\frac{(x^2-y^2)xdx}{4}-\frac{(x^2-y^2)ydy}{4}=\frac{(x^2-y^2)(xdx-ydy)}{4}=\sqrt{z}(xdx-ydy)\left[\sqrt{z}=\frac{x^2-y^2}{4}\right]$