**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

Given:

Normal Distribution with mean=45 and std=8

Let X be the amount of time it takes to complete the repair on a customer’s car

P(X>50) =1-P(X<=50)

Z= (raw score-mean)/std

Z=(X-45)/8

P(X<=50) =P (Z<= (50-45)/8) =P(Z<=0.625) =0.734

Probability that the service manager will not meet his demand will be,

=1-0.736

=0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

Given

Normal Distribution with mean=38 and std=6

A) Probability of employee's greater than age of 44=P(X>44):-

P(X>44) = 1-P(X<=44)

Z=(X-38)/6= (44-38)/6=0.841345=84.13%

Hence, P(X>44) =1-0.841345=0.1587=15.87%

Z score for 38= (38-38)/6=0=50%

Hence, People between age 38 to 44=84.13=50=34.13%

Hence, More employees at the processing center are older than 44 than between 38 to 44.Is FALSE

B) Z score for 30= (30-38)/6=-1.33=9.15%=36 out of 400

Hence, A training program for employees under the age 30 at the center would be expected to attract about 36 employees-TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: We know that if *X1* ~ *N* (μ1, σ1^2) and *X2* ~ *N* (μ2, σ2^2) are two independent random variables then X + Y ~N (μ1 + μ2, σ1^2 +, σ2^2 ) and X - Y ~N (μ1 - μ2, , σ1^2 + σ2^2 )

Similarly, if Z=aX +bY, where X & Y are as defined above, i.e. Z is a linear combination of X and Y, then and Z ~N (aμ1 + bμ2, a^2 σ1^2 +b^2 σ2^2 )

2X1~N(2μ, 4 σ^2)

X1+X2~N (μ + μ, σ^2 + σ^2 )-N(2 μ,2 σ^2)

2X1-(X1+X2) =N (4 μ,6 σ^2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The probability of getting value between a and b should be 0.99.

So, the probability outside the a and b area is 0.01(i.e. 1-0.99).

The probability towards left from a=-0.05(i.e.,0.01/2)

The probability towards right from b=-0.05(i.e.,0.01/2)

Z=(X-μ)/σ

For probability 0.05 the Z value is -2.57(from Z table)

Z\*σ+μ=X

Z (-0.05) \*20+100=-(-2.57) \*20+100=151.4

Z (+0.05) \*2 0+100=-(-2.57) \*20+100=48.6

(48.6,151.4)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Ans: Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42)

Mean=5+7

=12

Mean profit in Rs=Mean\*45

=12\*45

=540 million

SD= sqrt (32+42)

=sqrt (9+16)

=25

=5

SD in Rs=SD\*45

=5\*45

=225 million

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company.

Ans:

5th Percentile, Formula X=μ + Zσ

from z table 5 percentile = -1.645

X= 540+(-1.645) \*(225)

=170 million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Probability of Division 1 making a loss P(X<0)

Cumulative distribution function (0,5,3)

=0.0477903522728147

Probability of Division 2 making a loss P(X<0)

(0,7,4)

=0.040059156863817086

Division 1 has a larger probability of making a loss in a given year.