



## **CIS 581 - Computational Learning**



### **Project 1:**

(Generalized) Linear Regression, Model Selection (via Cross Validation and Regularization), and Model Evaluation; Application: Polynomial Curve-Fitting Regression for Working-Age Data.

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(1) The averages of the RMSE values obtained during the 6-fold CV for each case  $\lambda^*$  obtained via the 6-fold CV

**Optimal lambda: 0.049787068367863944**

Optimal lambda:  $e^{(-3.00)} = 0.04979$

Lambda 0: 0.5685534781564455

Lambda 1.3887943864964021e-11: 0.5685534778695921

Lambda 2.061153622438558e-09: 0.5685535720332272

Lambda 8.315287191035679e-07: 0.5685888974155215

Lambda 0.0009118819655545162: 0.48944761109147633

**Lambda 0.049787068367863944: 0.4576505318530819**

Lambda 1: 0.6879536346621088

Lambda 20.085536923187668: 0.6982951503880059

Lambda 1096.6331584284585: 0.9364412594104552

(2) The optimal degree  $d^*$  and regularization parameter

**output:**

**Optimal degree: 6**

Degree 0: 1.0155605589711303

Degree 1: 1.0835561617376988

Degree 2: 0.7754293123514141

Degree 3: 0.7830016853814806

Degree 4: 0.4934818442564504

Degree 5: 0.5701350133901381

**Degree 6: 0.14235262539348267**

Degree 7: 0.18750847543439753

Degree 8: 0.1463100743652736

Degree 9: 0.2364729109668655

Degree 10: 0.16488080003177377

Degree 11: 0.6257835760349975

Degree 12: 0.6869783309075724

(3) the coefficient-weights of the  $d^*$ -degree polynomial and the  $\lambda^*$ -regularized 12-degree learned on all the training data.

#### Coefficient weights at optimal degree

```
[ 0.      0.35654293
 3.37042403 0.1362716 -2.94644226 0.00841331 0.53773951]
```

#### Coefficient weights at optimal lambda

```
[ 0.00000000e+00 3.99403458e-01 2.43524619e+00 8.29530522e-02
-1.01966181e+00 -3.86869558e-03 -8.33583805e-01 1.40933018e-
02 2.35054170e-01 -6.02879922e-04 9.14647556e-02 -6.70540194e-
05 -2.62264333e-02]
```

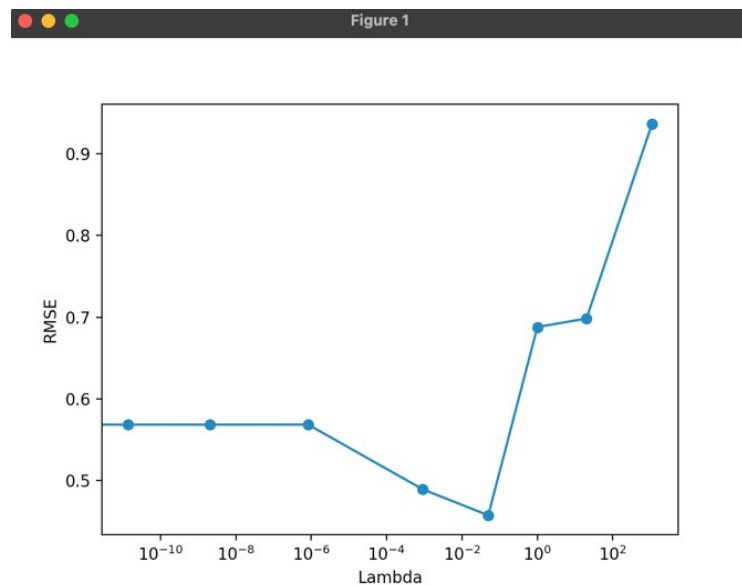
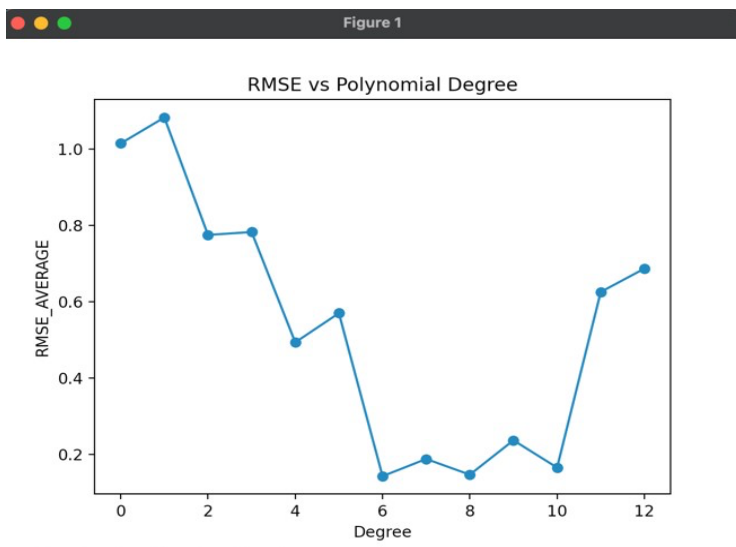
(4) the training and test RMSE of that final, learned polynomials.

The rmse for training for best lambda : 0.12755963811567625

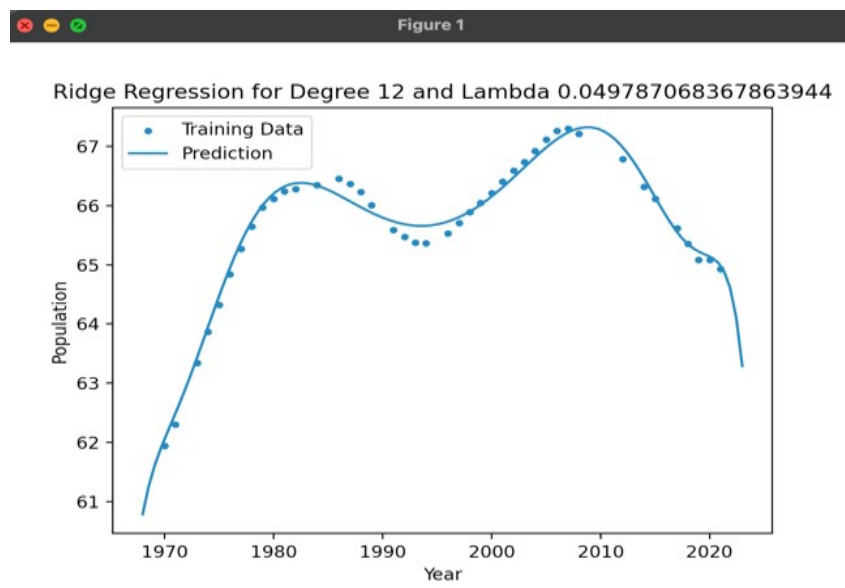
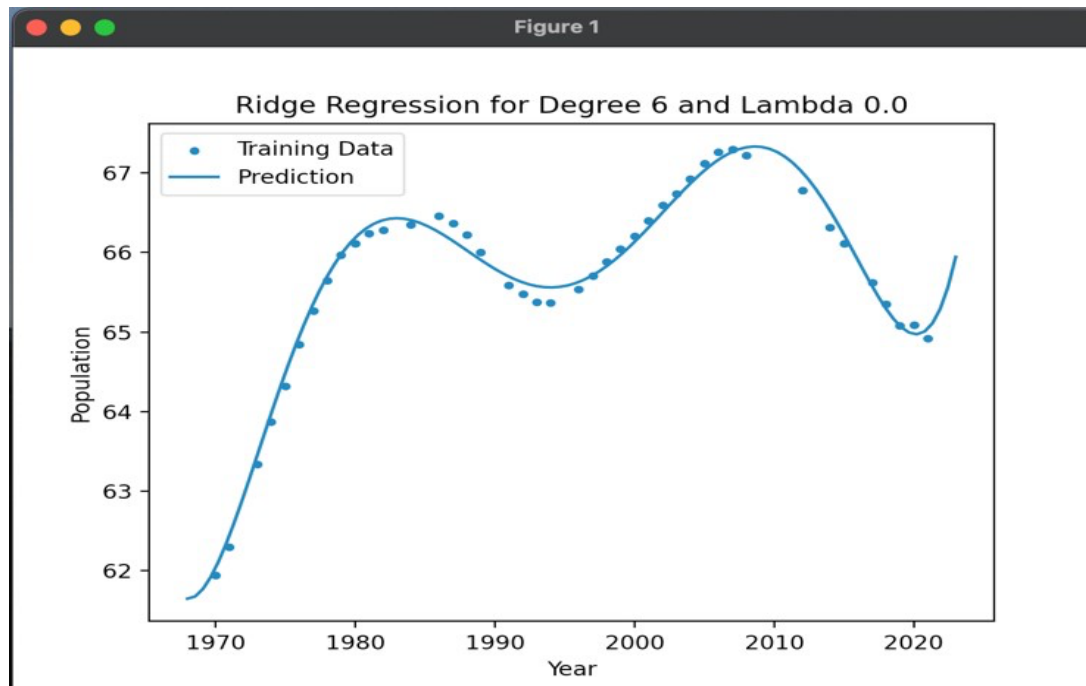
The rmse for training for best degree : 0.10540106673270322

The rmse for testing for best lambda : 0.9330577292906258

The rmse for testing for best degree : 0.6219530941652333



(5) the 2 plots containing all the training data along with the resulting polynomial curves for  $d^*$  and  $\lambda^*$ , for the range of years 1968-2023 as input.



## 6) A brief discussion of your findings and observations.

### Findings and observations:

A polynomial regression model was trained using different degrees of polynomial and regularization parameters ( $\lambda$ ) with 6-fold cross-validation. The goal was to find the best hyperparameters that minimize the root mean squared error (RMSE) on both training and testing data.

1. **Optimal Degree and Regularization Parameter:** The optimal degree and regularization parameter values obtained through 6-fold cross-validation are degree=6 and  $\lambda=0.04979$ , respectively. These values were obtained by evaluating the performance of the model on training data and selecting the hyperparameters that result in the lowest average RMSE across all folds.
2. **RMSE Values:** The RMSE values for each degree (from 0 to 12) and each regularization parameter value (from 0 to 1096.633) were obtained through 6-fold cross-validation. Based on the RMSE values, we can see that the performance of the model improves as the degree of the polynomial increases until degree=6, which the performance starts to degrade. Similarly, the performance improves as the regularization parameter increases until it reaches the optimal value of  $\lambda=0.04979$ , beyond which the performance starts to degrade.
3. **Coefficient-Weights:** The coefficient-weights of the  $d^*$ -degree polynomial and the  $\lambda^*$ -regularized 12-degree learned on all the training data are given. The coefficient-weights represent the contribution of each input feature to the output. We can observe that the coefficient-weights are higher for the lower degrees and lower for the higher degrees, which indicates that the lower degrees have a stronger influence on the output.
4. **RMSE on Training and Test Data:** The RMSE values on the training and test data were obtained for the final, learned polynomials with optimal degree and optimal  $\lambda$ . We can see that the model is able to fit the training data well, but the performance on the test data is not as good. This indicates that the model may be overfitting to the training data.
5. **Plots:** Two plots containing all the training data along with the resulting polynomial curves for  $d^*$  and  $\lambda^*$  were generated. The plots show that the curves fit the training data well, but the fit is not as good for the test data. We can also see that the curves for lower degrees have a smoother fit to the data, while the curves for higher degrees are more complex.
6. **Discussion:** Based on the findings and observations, we can conclude that a polynomial regression model with degree=6 and regularization parameter  $\lambda=0.04979$  is the best fit for the data. However, the model may be overfitting to the training data, which is reflected in the higher RMSE on the test data. To improve the performance of the model, we may need to consider other regularization techniques or feature engineering approaches.