Questions

1. The objective of this homework exercise is to implement the curve model described in class where the risk-neutral dynamics of zero-coupon bonds are given by

$$dB(t, T_i) = rB(t, T_i)dt + \sigma(T_i - t)B(t, T_i)dZ_i$$

where each bond is driven by its own Brownian motion process.

2. The accompanying spreadsheet has the data for the case. The first sheet has the 20×20 matrix of correlations among the first 20 zero-coupon bond prices (ignore the first zero-coupon bond price which is assumed to be non-stochastic). The second sheet has the Cholesky decomposition of the correlation matrix. The third sheet has the zero-coupon bond prices (the $B(T)$ function). The fourth sheet has the volatility function (the vector of σ values).

3. Following the methodology discussed in class, simulate the evolution of the vector of zero-coupon prices out to 10 years. Ideally, one would want to simulate several thousand paths of the vector of zero-coupon bond prices. We will leave it to you to decide how many paths of the evolution of the zero-coupon bond price vector to simulate.

4. Using the initial $B(T)$ function given in the spreadsheet, solve for the 2-year, 3-year, 5-year, and 10-year par rates.

5. Use the model to find the values of 2-year, 3-year, 5-year, and 10-year caps on L_t, where L_t denotes the six-month rate at time t implied by the six-month zero-coupon bond price $B(t, t + 0.50)$. Set the strike rate of each cap to the par rate identified in the previous question. For simplicity, assume that caplets are semiannual rather than quarterly. Recall that the first caplet is omitted because of the setting in advance feature.

6. Value a 5-year resettable cap with a strike of 0.07 and semiannual caplets. The cashflows from each caplet are $0.50 \max(0, L_{0.5} - L_{0.0})$, $0.50 \max(0, L_{1.0} - L_{0.50})$, etc. where L_t denotes the six-month rate at time t implied by the six-month zero-coupon bond price $B(t, t + 0.50)$.

7. Value a 5-year CMS cap with a strike of 0.05 (semiannual caplets for simplicity). The cash flow for the caplet expiring at time $t + 0.50$ is $0.50 \max(0, CMS_t - .05)$, where CMS_t denotes the 5-year par rate implied by the vector of zero-coupon bond prices at time t. Note that the first caplet is omitted because of the setting in advance feature.