HOMEWORK 6

1. The goal of this homework exercise is to replicate the Black-Derman-Toy example presented in class. The data for the volatility of r is in the first sheet of the accompanying spreadsheet. The second sheet has the term structure data ($D(T)$) function. The third sheet has my BDT tree.

2. Note that exactly replicating my numbers is not required. Each program has its own particular features and quirks and it is difficult to get two numerical programs programmed to give exactly the same results.

3. The best approach may be to write a program to solve at each date for the correct drift or r^* that matches the discount bond price. This tree could also be built in Excel using the solver algorithm, but this could be a little tedious since you have to manually use the solver 20 times. It's your call on how to proceed.

4. Once the tree is built, compute the expected value as of time zero, of the value of r in .50 years, in 1.00 years, etc. out to the last date on the tree. Graph the expected r value against the horizon. Recall that the expected value of r can be viewed as a futures rate (Eurodollar futures example). Contrast this with the forward rate for the same horizon computed using the initial term structure data.

5. Using the tree, solve for the price of a five-year European call option on a two-year bond with a coupon rate of four percent and where the strike price is 98. To do this, you will need to solve for the price of the bond in five years for each of the 11 nodes in the tree in five years. The price of a two-year bond at any time and node can be computed from the values of $D(0.5)$, $D(1.0)$, $D(1.5)$, and $D(2.0)$ applicable to that time and node.

Specifically, to figure out whether the call option is in the money in five years for, say, the top node of the tree in five years, we need to compute the values of $D(0.5)$, $D(1.0)$, $D(1.5)$, and $D(2.0)$ at that node. This is done by present valuing a cash flow of \$1 received at time 5.5 back to time 5, a cash flow of \$1 received at time 6.0 back to time 5, a cash flow of \$1 received at time 6.5 back to time 5, and a cash flow of \$1 received at time 7.0 back to time 5. From these $D(T)$ values for that node, we can then compute the value of the two-year bond at the expiration date of the call option at that node and compare its value to the strike price. This process is repeated for each of the 11 nodes that are in the tree in five years. The call option can then be valued by present valuing the option payoff back through the tree all the way back to time zero.

6. Extra credit. What would be the price of the call option in the previous problem if it had an American-style exercise feature (exercisable at times 0.5, 1.0, 1.5, $\ldots$, 5.0)?