CAPM and Time-Varying Beta: The Cross-Section of Expected Returns

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Abstract

The failure of the static-beta CAPM to explain the cross-section of returns on portfolios sorted on firm size, book-to-market ratio, momentum, and even portfolios sorted on past CAPM betas, is well documented. In this paper we show that the model's performance dramatically improves when portfolio betas are allowed to be time-varying functions of (lagged) business cycle variables. We use an approach based on Hansen and Richard (1987) to construct a candidate stochastic discount factor (SDF), using the excess return on the market portfolio as the single factor, scaled by a time-varying coefficient. The result is a model in which the conditional factor risk premium is a non-linear function of the business cycle variables. We assess the performance of our model by computing the R^2 of the cross-sectional regression of realized on model-implied expected returns, as for example in Jagannathan and Wang (1996). While this is not a formal test of the model's ability to price the assets correctly, it does provide an informative summary statistic that allows us to compare the performance of our scaled model with that of the static version, and also to compare our findings to those of other similar studies.

In the post-1980 period, where the static CAPM is known to perform particularly poorly, our scaled model explains around 60% of the cross-sectional variation in returns on beta and book-to-market portfolios, and 87% for momentum portfolios. Moreover, the model captures 70% of the value premium (the return spread between the highest and lowest book-to-market decile portfolios), and 75% of the momentum premium (the spread between the past 'winner' and 'loser' portfolios). Our results thus confirm the crucial importance of time-varying risk premiums in explaining the cross-section of average returns on these sets of portfolios. Moreover, the conditional market risk premium and hence also the betas implied by our model exhibits considerable non-linearity in the business cycle instruments.

JEL Classification: C12, G11, G12

Keywords: Capital Asset Pricing Model, Time-Varying Risk Premium

1 Introduction

In this paper we propose a conditional version of the single-factor CAPM, in which the factor risk premium is a non-linear, time-varying function of a set of (lagged) business cycle variables. We show that this model explains a substantial portion of the cross-sectional variation in expected returns, in particular for portfolios for which the traditional (static) CAPM is known to perform poorly. The performance of our model matches that of many others proposed in the literature, which often require the inclusion of additional factors[1].

The inability of the static CAPM to explain the value premium (the difference in returns between stocks with high and low book-to-market ratios), or the momentum premium (the difference in return between portfolios of stocks consisting of past 'winners' and 'losers'), is well documented. A number of papers, for example Ball and Kothari (1989), have suggested that these premiums could be due to the non-stationarity of expected returns, which in turn could be explained by time-variation in market risk premiums and/or assets' exposures to factor risk (beta). A number of recent papers have proposed extensions of the CAPM which allow for such time-variation. For example, Jagannathan and Wang (1996) propose a conditional version of the CAPM, augmented by a human-capital factor to address the Roll (1977) critique, and show that it explains a substantial fraction of the cross-sectional variation in returns on 100 portfolios sorted on size and book-to-market ratio. Lettau and Ludvigson (2001b) advocate a conditional version of the consumption (C-)CAPM, scaled by consumption-wealth ratio[2], while Lustig and Van Nieuwerburgh (2004) scale the C-CAPM by a variable that captures time-variation in housing collateral. Both these models are shown to explain a substantial portion of the cross-section of returns on the 25 portfolios sorted on size and book-to-market ratio, matching the performance of the Fama and French (1992)

[1] For example, the size and value factors in Fama and French (1992), the momentum factor in Carhart (1997), or the human-capital factor in Jagannathan and Wang (1996).

[2] As introduced in their earlier paper Lettau and Ludvigson (2001a).

three-factor model. Ang and Chen (2005) also find that a conditional version of the CAPM can explain the value premium over the 1926–2001 period.

In this paper, we restrict our attention to the single-factor CAPM, but allow the conditional factor risk premium to be time-varying as a non-linear function of a set of (lagged) business cycle variables. We assess the ability of this model to explain the cross-section of returns on three sets of portfolios where the static CAPM is known to perform poorly. These are the portfolios sorted on momentum (past realized return), book-to-market ratio, and past CAPM beta. Our analysis is more broadly based than many of those mentioned earlier, in that we use a wider range of base assets, and at the same time use only standard scaling instruments and no additional factors. Specifically, as conditioning instruments we use the one-month Treasury bill rate, the Treasury term spread (the difference in yield between the 10-year and one-year Treasury bond), and the credit spread (the difference in yield between the 10-year AAA-rated corporate and the corresponding Treasury bond). For the momentum portfolios, we also use the lagged return on the market portfolio.

To model the time-varying factor risk premium as a non-linear function of these variables, we use an approach based on the results of Hansen and Richard (1987), and related to Ferson, Siegel, and Xu (2006). This methodology allows us to construct a candidate stochastic discount factor (SDF), given as an affine transformation of the market risk factor. The coefficients of this transformation are non-linear functions of the conditioning variables, capturing the time-variation in the conditional factor risk premium. Our methodology thus differs from the more common approach[3] of adding additional factors by 'scaling' each of the original factors by each of the instruments, and then estimating constant coefficients for each of these new factors. This linear scaling procedure is not only less parsimonious, but is also theoretically sub-optimal[4]. Once the candidate SDF has been constructed, the

[3]Followed for example by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b).

[4]Empirical evidence (Ghysels 1998) shows that linearly scaled models often under-perform even their static counterparts.

model-implied expected return for a given asset or portfolio can be computed, based on the conditional covariance between the asset and the SDF, using standard asset pricing theory.

To assess the performance of our model, we follow Jagannathan and Wang (1996) and analyze the regression of realized on model-implied expected returns. While this is not a formal test of the model's ability to price the assets correctly, it does provide an informative summary statistic that allows us to compare the performance of our scaled model with that of the static version, and also to compare our findings to those of other similar studies.

We first focus on portfolios sorted on prior CAPM beta. It is well documented that, in particular after 1980, beta is not sufficient to explain the cross-section of expected returns. In fact, the pattern of expected returns on portfolios sorted on past beta exhibits a distinct 'U'-shape, which the fixed-beta CAPM is entirely unable to capture. In contrast, we find that the conditional model, scaled by business cycle variables, explains around 60% of the cross-sectional variation in returns on the beta decile portfolios over the 1980–2004 period (compared to less than 2% for the unscaled model). Moreover, optimal scaling halves the mean error between model-implied and realized returns.

For the momentum portfolios, we concentrate on the 1961–1999 period, where the momentum premium (the return spread between the top and bottom momentum decile portfolios) was around 9.8% per annum. The static CAPM explains only 20% of this premium, and only about 19% of the cross-sectional return variation. For this set of portfolios, we use two sets of scaling instruments, business cycle variables following Chordia and Shivakumar (2002), and the lagged return on the market portfolio as suggested in Cooper, Gutierrez, and Hameed (2004). We find that both sets of variables improve the performance of the model considerably, with the CAPM scaled by business cycle variables explaining 78% of the cross-sectional return variation and around 65% of the spread. The model scaled by lagged market return performs even better, explaining 87% of the cross-section and 75% of the premium.

Finally, we focus on the Fama-French book-to-market portfolios and the value premium, defined as the difference in returns between the high (value stocks) and low (growth stocks) book-to-market decile portfolios. During the 1961–1990 period, this premium was about 7.7% per annum, of which the static CAPM explains only 5%. In contrast, the scaled model (using business cycle variables) explains 70% of the value premium, and about 62% of the cross-sectional variation of returns on these portfolios (in contrast to 13% for the unscaled model).

Across all three sets of portfolios, we find that the difference in return error (realized minus model-implied return) between the scaled and unscaled model is most pronounced for the extreme portfolios. For the momentum portfolios for example, the unscaled CAPM under-estimates the return on the 'winner' portfolio by about 5.7% per annum, while over-estimating the return on the 'loser' portfolio by 2.5%. In contrast, the errors for the scaled model are only 1.7% and 0.6%, respectively. Interestingly, while the magnitude of the error is dramatically reduced, the sign remains the same in almost all cases: whenever the unscaled model under-estimates the true return, so does the scaled model, and vice versa.

Overall, our results confirm the crucial importance of time-varying risk premiums in explaining the cross-section of average returns on these sets of portfolios. The time-variation is particularly pronounced for the momentum portfolios, where the risk premium (using the lagged market return as the conditioning instrument) has a statistically significant 9% correlation with the momentum premium.

In order to facilitate comparison with other scaled models considered in the literature, we also test our model on the 5×5 and 10×10 portfolios sorted on size *and* and book-to-market ratio. For the former, our model achieves an R^2 of 56%, matching the results of Lettau and Ludvigson (2001b) and Lustig and Van Nieuwerburgh (2004). For the 10×10 portfolios, our model achieves an R^2 of 57%, identical to that achieved by Jagannathan and Wang's CAPM augmented by the human-capital factor.

Finally, as suggested by Lewellen, Nagel, and Shanken (2006), we also consider the case of adding industry portfolios to the size and book-to-market portfolios. For this set, our model achieves an R^2 of around 21%, which compares favorably to the models of Lettau and Ludvigson (2001b) and Lustig and Van Nieuwerburgh (2004), where the R^2 in this case drops to less than 5%.

The remainder of the paper is organized as follows. Section 2 outlines our methodology and describes the data, while the results are reported in Section 3. Section 4 concludes.

2 Methodology and Data

In this section, we briefly describe our methodology and the data used in our empirical applications. The theoretical basis for our approach is derived from Hansen and Richard (1987), Ferson, Siegel, and Xu (2006), and some of our own earlier work.

2.1 The Conditional CAPM

To describe the conditional ('scaled') CAPM, we embed the model within the framework of *arbitrage pricing theory (APT)*. In this context, the conditional CAPM can be written as,

$$r_t^k - r^f = \beta_{t-1}^k (r_t^M - r^f) + \varepsilon_t^k, \tag{1}$$

where r_t^k and r_t^M are the returns in period t on an individual asset k and the market portfolio M, respectively, r^f is the return on the (conditionally) risk-free asset, and ε_t^k is the residual capturing asset k's idiosyncratic risk. APT then implies that for the model to be viable, the ε_t^k must have zero mean, be serially and cross-sectionally independent, and uncorrelated with the systematic risk factor r_t^M. Note that, if the asset's factor exposure β_{t-1}^k is constant, then (1) delivers the well-known risk-return implications of the classic CAPM.

In this paper we allow the β_{t-1}^k to be time-varying functions of a set of *conditioning instruments*, i.e. variables that capture the information available to investors at the beginning of each period. It is now easy to show[5] that, equivalent to (1) is that there exist time-varying coefficients a_{t-1} and b_{t-1} so that

$$m_t = a_{t-1} + b_{t-1}(\, r_t^M - r^f \,) \tag{2}$$

is a viable *stochastic discount factor (SDF)* that prices the risky assets, i.e. $E_{t-1}(\, m_t r_t^k \,) = 1$ for all assets $k = 1 \dots n$. Note that he coefficient b_{t-1} in (2) is proportional to the conditional market price of risk associated with the market factor.

In what follows, we show how to construct *optimal* coefficients a_{t-1} and b_t in (2) so that the implied SDF has minimal pricing error. As a result of this process, we endogenously determine the time-varying risk premium as a function of the conditioning information, as well as the time-varying factor risk exposures β_{t-1}^k of the risky assets.

2.2 Conditioning Information

To capture the time-varying information available to investors at the beginning of any one period, we take as given a set of p (lagged) *conditioning instruments*[6], denoted $y_{t-1}^1 \, \dots \, y_{t-1}^p$. We postulate that both the market factor and the risky asset returns are related to these instruments via a linear predictive model of the form,

$$\begin{bmatrix} r_t^M - r^f \\ R_t - r^f e \end{bmatrix} = \begin{bmatrix} \bar{\nu} \\ \bar{\mu} \end{bmatrix} + \begin{bmatrix} \delta' \\ \Gamma' \end{bmatrix} \cdot Y_{t-1} + \begin{bmatrix} \xi_t \\ \zeta_t \end{bmatrix}. \tag{3}$$

Here, R_t is the n-vector of risky asset returns, and Y_{t-1} is the p-vector of conditioning instruments. The predictive relation between factors, assets, and conditioning instruments, is captured by the p-vector δ and the $p \times n$ matrix Γ. For notational convenience, we

[5]See, for example, Cochrane (2001).

[6]In our empirical application, we use the 1-month Treasury bill rate, the Treasury term spread, and the credit yield spread, all of which have been shown to be correlated with the business cycle and have been used in numerous other studies.

normalize Y_{t-1} to have zero mean, so that $\bar{\nu}$ and $\bar{\mu}$ are the unconditional expectations of the left-hand side variables, respectively. While we assume the residuals ξ_t and ζ_t to be jointly *iid*, we do not assume cross-sectional independence. In other words, the conditional covariance matrix is constant but not necessarily diagonal.

From (3) we obtain the conditional expectations $\nu_{t-1} = \bar{\nu} + \delta' Y_{t-1}$ and $\mu_{t-1} = \bar{\mu} + \Gamma' Y_{t-1}$. Finally, we denote by $\Lambda_{t-1} = \mu_{t-1}\mu_{t-1}' + E_{t-1}(\,\zeta_t\zeta_t'\,)$ and $Q_{t-1} = \mu_{t-1}\nu_{t-1} + E_{t-1}(\,\zeta_t\xi_{t-1}\,)$ the cross-sectional conditional second moments of assets and factors.

2.3 Candidate Stochastic Discount Factor

Because we not necessarily assume that the market portfolio is part of the investible asset universe, we need to construct a *factor-mimicking portfolio (FMP)* within the set of traded assets. The aim is to construct a dynamically managed strategy of the base assets, with portfolio weights that are time-varying functions of the conditioning information, so that the resulting portfolio is maximally correlated[7] with the market risk factor. One can show[8] that the FMP associated with the market portfolio can be written as,

$$f_t^M = r^f + (\,R_t - r^f e\,)'\theta_{t-1}, \qquad \text{with} \quad \theta_{t-1} = \Lambda_{t-1}^{-1}(\,q_{t-1} - \kappa\mu_{t-1}\,), \tag{4}$$

where κ is a constant. To construct the optimal candidate SDF associated with the given factor, we need to find the time-varying coefficients a_{t-1} and b_{t-1}, so that the managed portfolio $a_{t-1}r^f + (\,f_t^M - r^f\,)b_{t-1}$ achieves the maximal possible unconditional Sharpe ratio. In fact, Basu and Stremme (2005) show that if the Sharpe ratio of this portfolio matches that spanned by the base assets, then the corresponding SDF prices the assets correctly. More importantly, even if this is not the case, the SDF thus constructed is *optimal*, given the chosen factor, in the sense that it minimizes pricing error.

[7] An alternative approach, used for example in Ferson, Siegel, and Xu (2006), is to construct the FMP via of an orthogonal projection. However, it can be shown that these two approaches are in fact equivalent.

[8] See for example Basu and Stremme (2005).

It can be shown that the coefficient b_{t-1} of the SDF is proportional to the expression

$$\left[Y'_{t-1}\Lambda^{-1}_{t-1}\Sigma\Lambda^{-1}_{t-1}Y_{t-1}\right]^{-1} Y'_{t-1}\Lambda^{-1}_{t-1}\mu_{t-1}, \tag{5}$$

where $Y_{t-1} = Q_{t-1} - \kappa\mu'_{t-1}$. It is important to note that our candidate stochastic discount is different for different sets of base assets, as we construct the factor-mimicking portfolio in each case. The factor loadings are nonlinear functions of the predictive variables and in fact exhibit considerably non-linearities. These loadings enable the stochastic discount factor to attain the maximum factor Sharpe ratio in the presence of conditioning information. In contrast, there is no guarantee that the standard procedure of modeling risk premiums as linear functions of the predictive variables leads to this.

Finally, once the candidate SDF has been constructed, the model-implied returns can be computed using standard asset pricing theory, based on the conditional covariance between an asset's return and the SDF. To assess the performance of the model, we follow the Fama and MacBeth (1973) procedure of regressing realized on model-implied returns.

2.4 Data

We focus on three sets of test assets: first, we construct decile portfolios sorted on their CAPM beta. For each year in the period from 1980 to 2004, we first calculate the beta of each stock relative to the S&P 500, then sort stocks by beta and group them into deciles (by equal number of stocks in each group). From each decile we form a value-weighted portfolio and compute its monthly returns throughout the year following portfolio formation.

The next set of base assets are value-weighted momentum decile portfolios over the 1961–1999 period, whose construction is described in Chordia and Shivakumar (2002). We also use the decile portfolios sorted on book-to-market ratio over the 1961–1990 period, and the 25 and 100 portfolios sorted by size and book-to-market ratio over the 1963–2004 and

1963–1990 periods respectively. These data are available from Kenneth French's web site[9].

As conditioning variables we use the one-month Treasury Bill rate, the term spread (defined as the difference in yield between the 10-year and the one-year Treasury bond) Both these variables are constructed from data available from the Federal Reserve Bank of St. Louis[10]. We also use the credit spread (defined as the difference in 10-year yield between the AAA-rated corporate and the corresponding government bond), which we obtained from Datastream.

All of these variables have been shown to predict long and short term business cycles and have been employed in a number of studies. The one-month US Treasury bill rate has been shown to be a proxy for future economic activities by Fama and Schwert (1977), and more recently by Ang and Bekaert (2006), who find that it outperforms the dividend yield as a predictive variable. The term spread has been shown to be closely correlated with the short-term business (Fama and French 1988), and the credit spread tracks long-term business cycle conditions (Fama and French 1988). We also use the convexity of the yield curve (defined as twice the 5-year yield minus the sum of the 10-year and 1-year yields).

3 Empirical Analysis

The business cycle instruments that we consider for the beta, momentum, B/M portfolios and 25 portfolios sorted on size and book to market are the 1 month Treasury Bill rate, the term spread and the credit spread. For the 100 portfolios sorted on size and book-to-market in addition we consider the convexity of the yield curve. We first analyze the performance on the beta, momentum and book-to-market portfolios and then on the 25 and 100 portfolios sorted by size and book-to-market.

[9] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`.

[10] `http://research.stlouisfed.org/fred2/`.

3.1 The Beta, Momentum and Book-to-Market Portfolios

We first discuss the results for the 10 portfolios sorted on CAPM betas over the 1980–2004 period. It has been observed (e.g. Black 1990) that the static CAPM performs poorly on these portfolios over this period, while it does relatively well prior to 1980. Our findings confirm this as the static CAPM explains only 1.6% of the cross-sectional return variation (Table 1). The 'U'-shaped pattern of expected returns of these portfolios is clear from Table 1 and is contrary to what the CAPM predicts. Unsurprisingly, the expected returns implied by the static CAPM pick up very little of this variation with high positive errors for the extreme portfolios and large negative errors for the middle portfolios. In contrast, from Table 1 we see that the CAPM scaled by business cycle instruments explains 60% of the cross-sectional variation in expected returns. The model also predicts the pattern of expected returns much better, with considerably lower errors for the extreme as well as middle portfolios. The root mean squared error (RMSE) across the 10 portfolios is 0.12% per month for the unscaled CAPM, but only 0.08% per month for the scaled CAPM. The monthly spread in expected return across the 10 portfolios (highest minus lowest beta) is 0.38%, and the scaled CAPM explains around 70% of this premium (with a model-implied spread of 0.28%). The return on the top beta decile portfolio has a negative correlation of around 5% with the factor risk premium, and it is this correlation which leads to the scaled CAPM predicting a return of 0.36%, which is close to the realized return of 0.39%.

Momentum Portfolios

Next we turn next to the momentum decile portfolios over the 1961–1999 period, over which the average monthly momentum premium 0.78% (i.e. about 9.8% per annum). The inability of the static CAPM to explain this premium (or in fact the cross-section of returns on momentum portfolios) is well documented (Jegadeesh and Titman 1993). We confirm these findings as the unscaled CAPM explains only 19% of the cross-sectional return variability (Table 2). The return errors for the unscaled CAPM are the highest for the top and bottom decile portfolio (-0.21% and 0.46%, respectively), with the model-implied premium being only 0.09%. The unscaled CAPM comes quite close to explaining the average returns of the middle deciles, but not the extreme deciles which account for most of the cross-sectional variation in expected returns.

For the scaled CAPM, we consider two sets of scaling variables. The first are the business cycle variables we used for the beta portfolios, a subset of those considered in Chordia and Shivakumar (2002) who find that abnormal returns to momentum strategies disappeared once stock returns were adjusted for their predictability based on these variables. Second, we consider the lagged return on the market portfolio, which Cooper, Gutierrez, and Hameed (2004) find to work better than business cycle variables in explaining momentum profits. From Table 2 we see that scaling by business cycle variables does improve performance considerably, with the cross-sectional R^2 rising to 78% (using business cycle variables). The scaled model comes much closer to explaining the average returns of the extreme deciles, with errors of -0.09% and 0.19% for the bottom and top decile (compared to -0.21% and 0.46% for the unscaled model). The RMSE is 0.09%, about half that implied by the unscaled model. The momentum premium implied by the scaled model is 0.51%, almost 65% of the realized premium.

Turning to the CAPM scaled by the lagged market return, we find from Table 2 that the cross-sectional R^2 increases to 87%. This model predicts a momentum premium of 0.61% around 75% of the realized premium. The model model has the lowest return errors overall, with an RMSE of 0.07% per month.

The time-varying risk premiums, normalized to have unit second moment, for the two sets of scaling variables are shown in Panels A1 and A2 of Figure 5, while the time series of the momentum premium is shown in Panel A of Figure 6. The time series of the risk premium based on the lagged market more closely resemble that of the momentum premium, and indeed the unconditional correlation between the risk premium and the momentum premium is 9.4% for the lagged market, while it is 5.7% for the business cycle variables. If we regress the momentum premium on the lagged risk premiums we find that the slope coefficient is significant at the 5% level for the lagged market but not for the business cycle variables.

BOOK-TO-MARKET PORTFOLIOS

Finally, we consider the 10 portfolios sorted by book-to-market ratio and the value premium, defined as the difference in expected return between the top ('value' stocks) and the bottom decile ('growth' stocks). We consider these portfolios over the 1961–1990 period, over which the value premium was 0.63% per month[11]. The inability of the static CAPM to explain the cross-sectional variation in these portfolios, or in fact the value premium itself, is well known and we again confirm these findings with a cross-sectional R^2 of 13% (Table 3). We see that the unscaled CAPM over-estimates the return on the low book-to-market portfolio and under-estimates that of the high book-to-market portfolio with errors of -0.19% and 0.40%, respectively, leading to a model-implied value premium of 0.03%. The CAPM scaled by business cycle variables achieves a cross-sectional R^2 of 62%, with the errors for the bottom and top deciles reduced to 0.06% and 0.25%, respectively. The scaled model predicts a value premium of 0.44%, almost 70% of the realized premium. The time series of the normalized risk premium is shown in Panel B1 of Figure 5, while the time series of the value premium is shown in Panel B of Figure 6. Both premiums exhibit considerable time-variation and the unconditional correlation between the them is -4.5%.

3.2 The 25 and 100 Portfolios Sorted by Size and B/M

We now focus on the 25 and 100 portfolios sorted by size *and* book-to-market ratio to facilitate comparison with other studies, particularly Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b). We first consider the 25 portfolios over the 1963–2004 period. The unscaled CAPM explains only 16% of the cross-section of expected returns, while the scaled CAPM achieves an R^2 of 52% (Figure 4), which is almost identical to that for the Consumption CAPM of Lettau and Ludvigson (2001b) and Lustig and Van Nieuwerburgh (2004), although their analysis is at a quarterly frequency. We also consider the 'small'

[11]The value premium declines subsequently to 0.49% over the 1961–1999 period.

value premium (the difference in expected return between the small value and small growth portfolios), which is 0.92% per month. The unscaled CAPM has a model-implied premium of only 0.13% while for the scaled CAPM this increases to 0.66%, just over 70% of the realized premium.

In a recent paper, Lewellen, Nagel, and Shanken (2006) point out that these portfolios have a tight factor structure, with the Fama-French three factor model explaining over 75% of the cross-sectional variation in these returns. They point out that factor models that are weakly correlated with the Fama-French factors but not with the residuals of these portfolios with respect to the factors are likely to achieve high cross-sectional R^2s[12]. They propose adding the 30 industry portfolios to the 25 size and book-to-market portfolios, and find that the R^2's for the scaled Consumption CAPM of Lettau and Ludvigson (2001b) and Lustig and Van Nieuwerburgh (2004) drop to less than 5%. When we augment the set of conditioning variables by the convexity of the yield curve, our scaled CAPM achieves an R^2 about 21%. While this is lower than for the other sets of portfolios, it still compares favorably to the 34% achieved by the Fama-French model. These 55 portfolios clearly span a higher-dimensional space than the 25 size and book-to-market portfolios alone, and our model does better than most of those considered in Lewellen, Nagel, and Shanken (2006).

We next consider the 100 portfolios sorted by size and book-to-market ratio, which are used in Jagannathan and Wang (1996). Over the July 1963–December 1990 period considered in that paper, our scaled model achieves an R^2 of 57%, identical to that achieved by Jagannathan and Wang (1996), and considerably higher than the 21% achieved by the Fama-French model.

[12]Daniel and Titman (2005) also make a similar point about the low-dimensionality of the space spanned by these portfolios.

3.3 Nonlinearity in the Factor Risk Premiums

We construct our candidate stochastic discount factor as the combination of factors which has minimum second moment in the presence of conditioning information, following Hansen and Richard (1987). This procedure has several advantages, the first of which is that the factor risk premiums are pre-specified functions of the predictive variables and do not need to be estimated via GMM for example. With N assets and L instruments this would require the estimation of an $NL \times NL$ covariance matrix, while our procedure requires an $(N + L) \times (N + L)$ covariance matrix. Secondly, our factor risk premiums are nonlinear functions of predictive variables. This is in contrast to the standard procedure of estimating factor risk premiums as linear functions of predictive variables. Our candidate stochastic discount factor attains the maximum Sharpe ratio in the factor space in the presence of conditioning information, while there is no guarantee that the standard linear-beta specifications do, which could lead to suboptimal performance[13]. Finally, these risk-premiums exhibit a conservative response to extreme values of the conditioning variables, due to the conservative response of the portfolio weights underlying their construction as noted by Ferson and Siegel (2001).

Figure 7 shows the factor risk premium for the momentum portfolios as a function of each of the predictive variables. We note that there are significant non-linearities for the short rate and the credit spread. These findings complement the evidence of nonlinearities in factor sensitivities and the term structure found by Boudoukh, Richardson, and Whitelaw (1997), and suggest that capturing these nonlinearities is important for conditional asset pricing.

[13]Ghysels (1998), or Brandt and Chapman (2005).

4 Conclusions

In this paper, we focus on the performance of the CAPM scaled by business cycle and market variables in explaining the cross-section of returns on three sets of assets where the CAPM performs poorly. These are the beta portfolios, the momentum portfolios and the value portfolios. We use the methodology in Hansen and Richard (1987) and Ferson and Siegel (2001) to construct the minimum second moment stochastic discount factor in the presence of conditioning information, which leads to a model with time-varying risk premiums and analyze its performance. We judge the performance of the scaled CAPM by using the cross-sectional R^2 introduced in Jagannathan and Wang (1996).

We find that the CAPM scaled by business cycle instruments explains around 60% of the cross-sectional variation in the beta portfolios over the 1980–2004 period, in contrast to 1.5% for the unscaled CAPM. The scaled CAPM explains 87% of the cross-sectional variation in the returns of the momentum portfolios, and 75% of the spread in average return between ‘winners’ and ’losers’. The CAPM scaled by business cycle variables explains 60% of the cross-sectional return variation of the value portfolios and around 70% of the value premium. Our results thus confirm the crucial role of time-varying risk premiums in explaining the cross-section of average returns on these sets of portfolios. Our risk-premiums exhibit considerable time-variation which are correlated with the various spreads.

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Table 1: Realized and Model-Implied Returns (Beta Decile Portfolios)

portfolio	**average (monthly) return**				
beta		model-implied (error in brackets)			
decile	realized	unscaled CAPM		scaled CAPM (business cycle)	
1 (lowest)	0.30%	0.17%	(+0.13%)	0.38%	(−0.08%)
2	0.26%	0.17%	(+0.09%)	0.17%	(+0.09%)
3	0.28%	0.18%	(+0.10%)	0.20%	(+0.08%)
4	0.01%	0.18%	(−0.17%)	0.07%	(−0.06%)
5	0.08%	0.19%	(−0.11%)	0.12%	(−0.04%)
6	0.23%	0.19%	(+0.04%)	0.18%	(+0.05%)
7	0.01%	0.18%	(−0.17%)	0.12%	(−0.11%)
8	0.24%	0.20%	(+0.04%)	0.13%	(+0.11%)
9	0.09%	0.21%	(−0.12%)	0.16%	(−0.07%)
10 (highest)	0.39%	0.22%	(+0.17%)	0.36%	(+0.03%)
R^2 (RMSE)		1.6%	(0.123%)	60.4%	(0.076%)

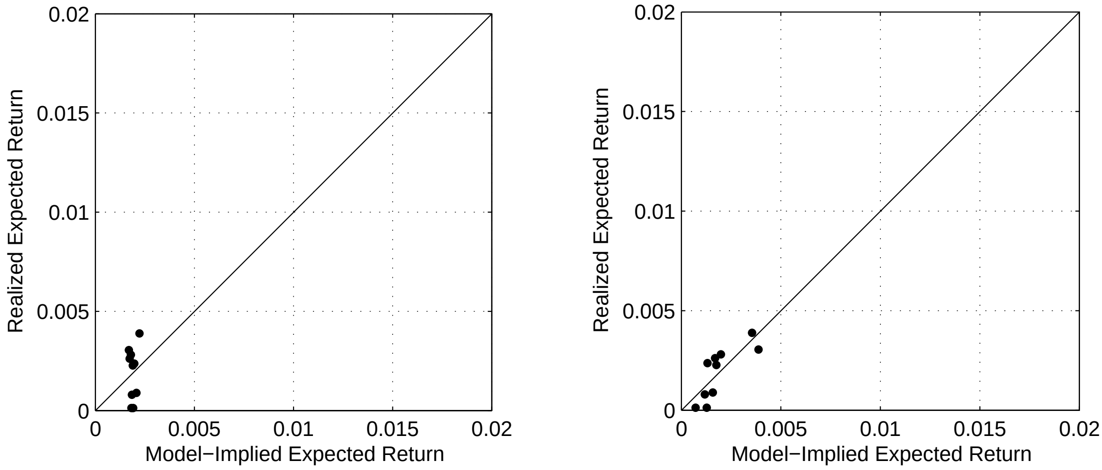
This table reports the realized and model-implied average monthly returns for the decile portfolios sorted on CAPM beta, for the period from 1980 to 2004. Figures in parentheses are the errors (for each portfolio) between realized and model-implied return. The scaled model uses the business cycle variables (short rate, term spread, and credit spread) as conditioning instruments.

Figure 1: Realized and Model-Implied Return (Beta Decile Portfolios)

panel A: unscaled CAPM

panel B: scaled CAPM

(business cycle variables)



($R^2 = 1.6\%$) ($R^2 = 60.4\%$)

These figures plot the realized average returns on the beta decile portfolios against the expected returns implied by the unscaled (panel A) and scaled (panel B) CAPM. For the scaled model, the conditioning instruments are the business cycle variables (short rate, term spread, and credit spread).

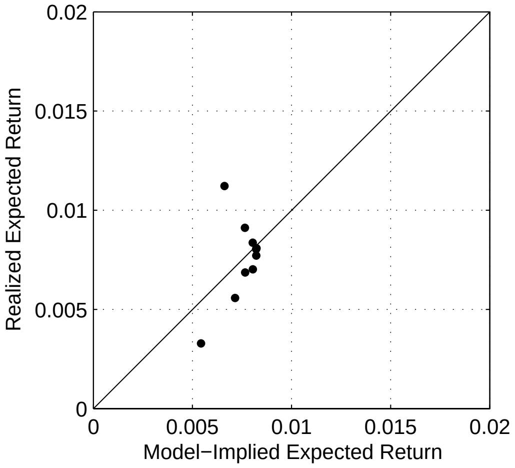
Table 2: Realized and Model-Implied Returns (Momentum Decile Portfolios)

portfolio	**average (monthly) return**						
momentum		model-implied (error in brackets)					
decile	realized	unscaled CAPM		scaled CAPM (business cycle)		scaled CAPM (lagged market)	
1 (lowest)	0.33%	0.54%	(−0.21%)	0.42%	(−0.09%)	0.38%	(−0.05%)
2	0.56%	0.72%	(−0.16%)	0.60%	(−0.04%)	0.59%	(−0.03%)
3	0.69%	0.77%	(−0.08%)	0.59%	(+0.10%)	0.65%	(+0.04%)
4	0.70%	0.80%	(−0.10%)	0.69%	(+0.01%)	0.66%	(+0.04%)
5	0.77%	0.82%	(−0.05%)	0.72%	(+0.05%)	0.69%	(+0.08%)
6	0.81%	0.83%	(−0.02%)	0.77%	(+0.04%)	0.80%	(+0.01%)
7	0.80%	0.82%	(−0.02%)	0.86%	(−0.06%)	0.86%	(−0.06%)
8	0.84%	0.81%	(+0.03%)	0.97%	(−0.13%)	0.91%	(−0.07%)
9	0.91%	0.76%	(+0.15%)	0.98%	(−0.07%)	1.01%	(−0.10%)
10 (highest)	1.12%	0.66%	(+0.46%)	0.93%	(+0.19%)	0.98%	(+0.14%)
R^2 (RMSE)		19.2%	(0.180%)	77.8%	(0.092%)	87.2%	(0.072%)

This table reports the realized and model-implied average monthly returns for the decile portfolios sorted on momentum (prior return), for the period from 1961 to 2000. Figures in parentheses are the errors (for each portfolio) between realized and model-implied return. The scaled model uses the business cycle variables (short rate, term spread, and credit spread), or the lagged market return, as conditioning instruments.

Figure 2: Realized and Model-Implied Return (Momentum Decile Portfolios)

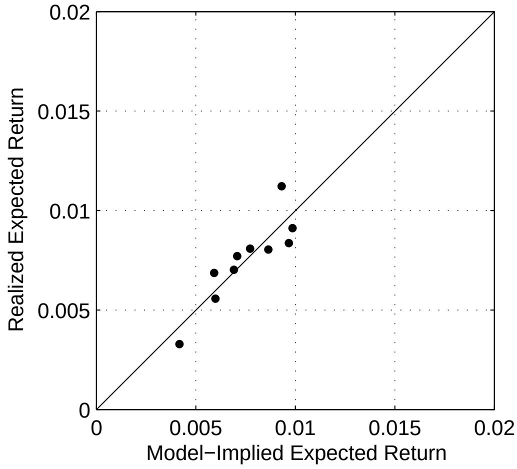
panel A: unscaled CAPM



($R^2 = 19.2\%$)

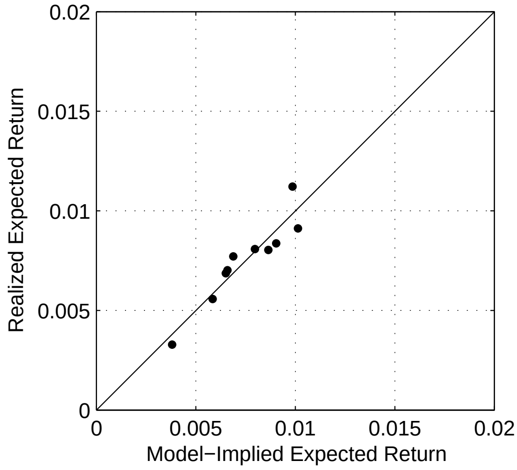
panel B: scaled CAPM

B1: business cycle variables



($R^2 = 77.8\%$)

B2: lagged market return



($R^2 = 87.2\%$)

These figures plot the realized average returns on the momentum decile portfolios against the expected returns implied by the unscaled (panel A) and scaled (panel B) CAPM. For the scaled model, the conditioning instruments are the business cycle variables (short rate, term spread, and credit spread) in panel B1, and the lagged market return in panel B2.

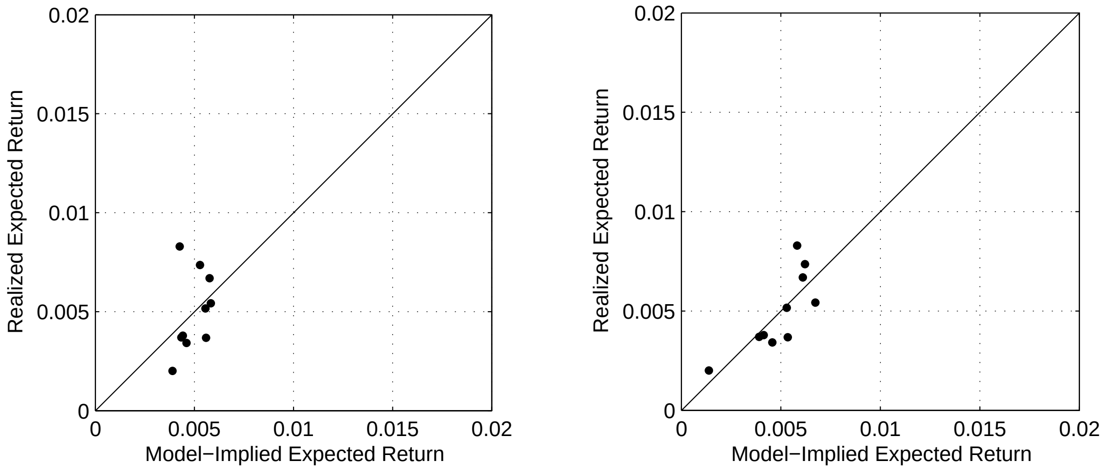
Table 3: Realized and Model-Implied Returns (Book-to-Market Decile Portfolios)

portfolio	**average (monthly) return**				
book/market		model-implied (error in brackets)			
decile	realized	unscaled CAPM		scaled CAPM (business cycle)	
1 (lowest)	0.20%	0.39%	(−0.19%)	0.13%	(+0.07%)
2	0.37%	0.43%	(−0.06%)	0.39%	(−0.02%)
3	0.38%	0.44%	(−0.06%)	0.42%	(−0.04%)
4	0.34%	0.46%	(−0.12%)	0.46%	(−0.12%)
5	0.37%	0.56%	(−0.19%)	0.54%	(−0.17%)
6	0.52%	0.56%	(−0.04%)	0.53%	(−0.01%)
7	0.54%	0.58%	(−0.04%)	0.67%	(−0.13%)
8	0.67%	0.58%	(+0.09%)	0.61%	(+0.06%)
9	0.74%	0.53%	(+0.21%)	0.63%	(+0.11%)
10 (highest)	0.83%	0.43%	(+0.40%)	0.58%	(+0.25%)
R^2 (RMSE)		13.3%	(0.176%)	61.9%	(0.120%)

This table reports the realized and model-implied average monthly returns for the decile portfolios sorted on book-to-market ratio, for the period from 1961 to 1990. Figures in parentheses are the errors (for each portfolio) between realized and model-implied return. The scaled model uses the business cycle variables (short rate, term spread, and credit spread) as conditioning instruments.

Figure 3: Realized and Model-Implied Return (Book-to-Market Decile Portfolios)

panel A: unscaled CAPM | **panel B: scaled CAPM** (business cycle variables)



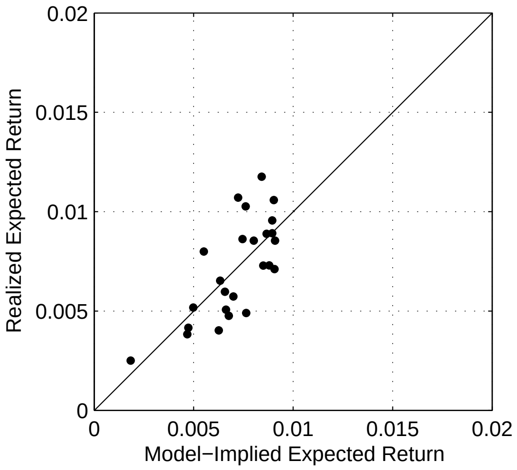
($R^2 = 13.3\%$) | ($R^2 = 61.9\%$)

These figures plot the realized average returns on the book-to-market decile portfolios against the expected returns implied by the unscaled (panel A) and scaled (panel B) CAPM. For the scaled model, the conditioning instruments are the business cycle variables (short rate, term spread, and credit spread).

Figure 4: Mixed Size, Book-to-Market, and Industry Portfolios

panel A: size/BM portfolios

A1: 5×5 size/BM

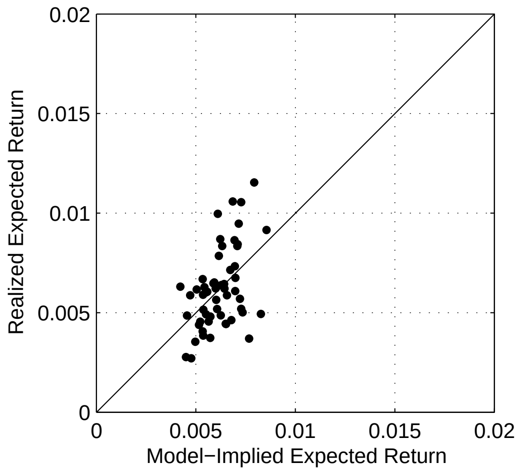


Rsq=51.98%

($R^2 = 52.0\%$)

panel B: + industry portfolios

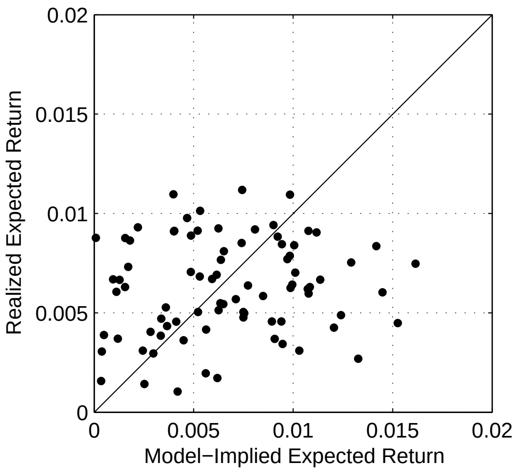
B1: 5×5 size/BM + 30 industry



Rsq=23.54%

($R^2 = 23.5\%$)

A2: 10×10 size/BM



Rsq=57.34%

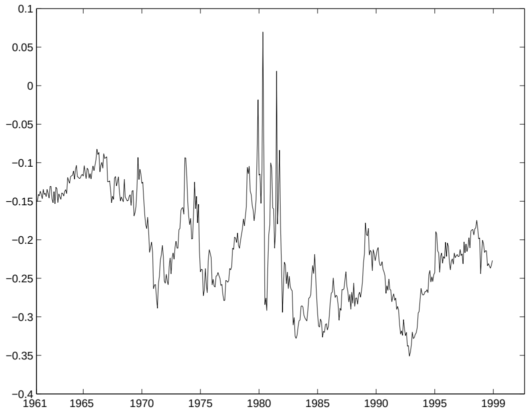
($R^2 = 57.3\%$)

These figures plot the realized average returns on the momentum decile portfolios against the expected returns implied by the unscaled (panel A) and scaled (panel B) CAPM. For the scaled model, the conditioning instruments are the business cycle variables (short rate, term spread, and credit spread) in panel B1, and the lagged market return in panel B2.

Figure 5: Time-Series of (Conditional) Factor Risk Premium

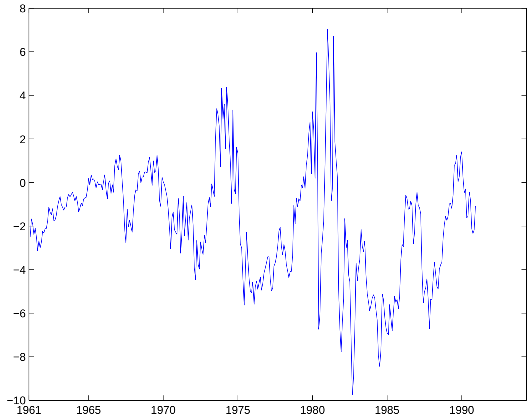
panel A: momentum portfolios

A1: business cycle variables

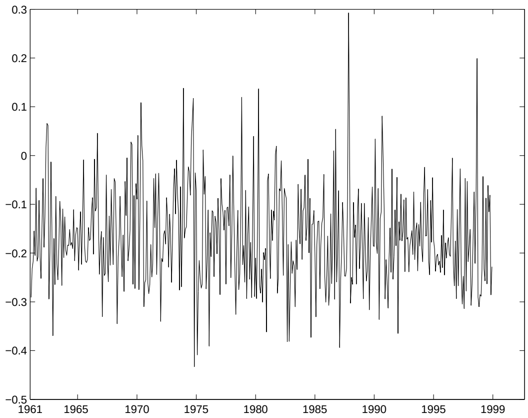


panel B: book-to-market portfolios

B1: business cycle variables



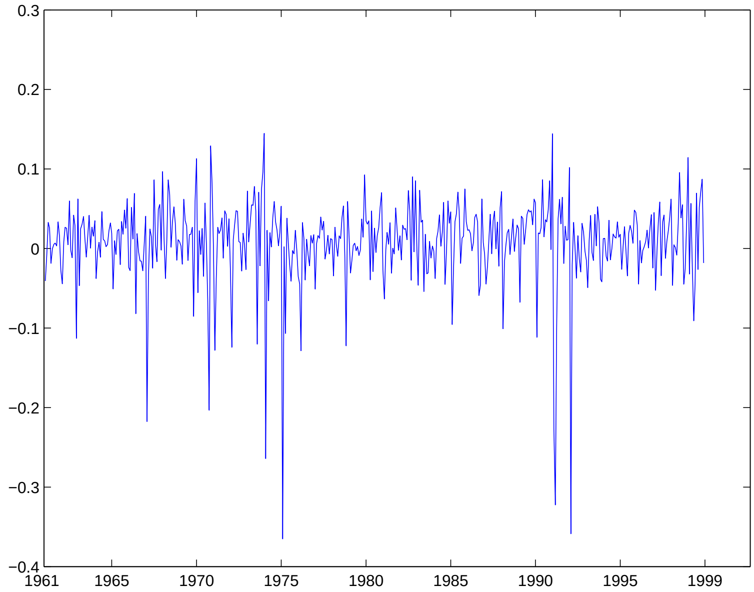
A2: lagged market return



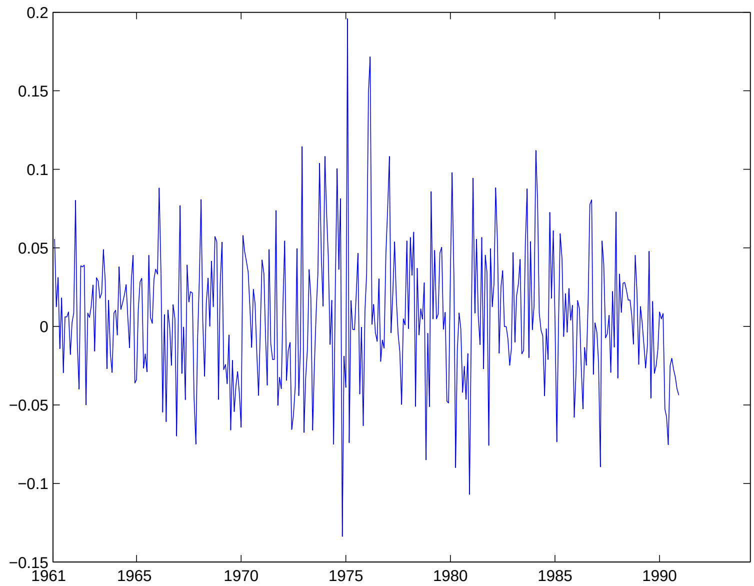
This figure shows the time series of the factor risk premium (normalized to have unit second moment) for the scaled CAPM, as described in Section 3. The base assets are the momentum decile portfolios in panel A, and the book-to-market decile portfolios in panel B. The factor risk premium is assumed to be functions of the business cycle variables (short rate, term spread, and credit spread) in A1 and B1, while the lagged market return is used as conditioning instrument in panel A2.

Figure 6: Time Series of (Conditional) Momentum and Value Premium

panel A: momentum premium

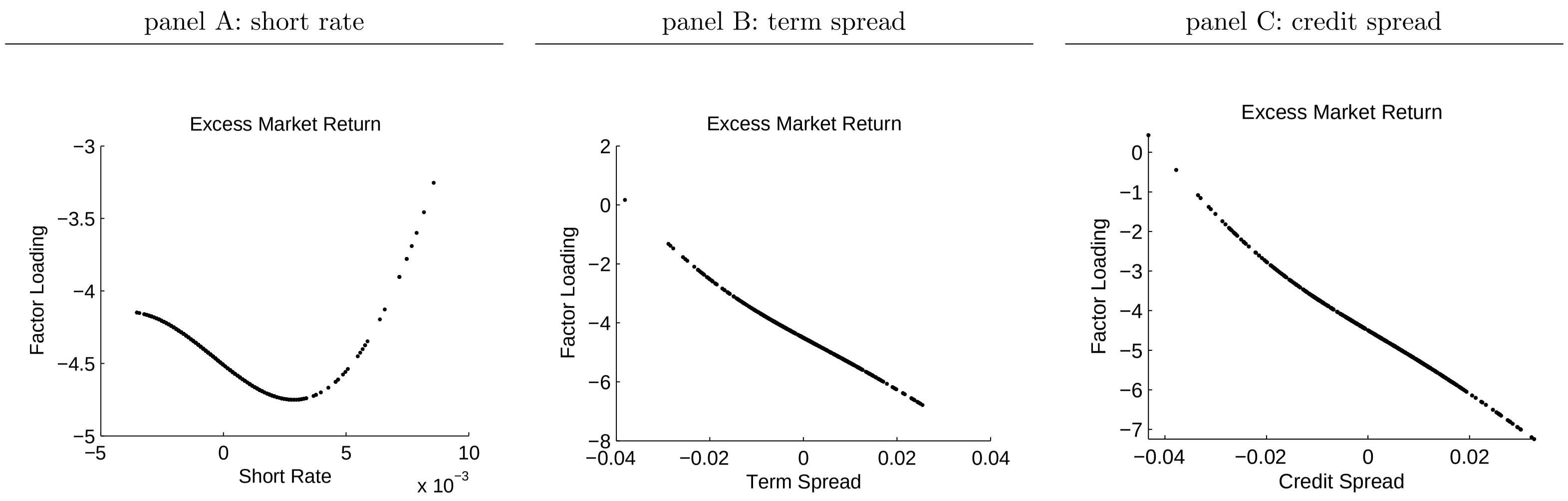


panel B: value premium



These figures show the time series of the (conditional) momentum (panel A) and value (panel B) premium, defined as the difference in return between the top and bottom decile in the respective sets of portfolios.

Figure 7: Factor Risk Premium as Function of Conditioning Variables



These figures show the factor risk premium as functions of individual conditioning variables. The base assets in this case are the momentum decile portfolios. Panel A shows the risk premium as a function of the short rate, Panel B as a function of the terms spread, and Panel C as a function of the credit spread.