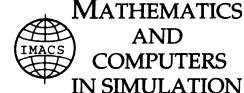




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# Time-varying estimates of CAPM betas

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## Abstract

It is well known that the CAPM beta is not stable over time. We investigate the nature of the time-variation in betas using monthly Australian data from 1979 to 1994 for 23 sectors. We discuss beta estimates for sub-periods and tests of the statistical adequacy of the market model used to estimate the betas. We estimate time-varying betas using recursive regressions, rolling regressions and using the Kalman Filter. We find considerable time-variation in the estimated betas and find that many are non-stationary. We estimate a simple model which explains the variation in each of the betas in terms of a time trend, allowing for a break both in level and in trend at October 1987. The model explains a large proportion of the variation in the betas over the sample period for most of the sectors. © 1999 IMACS/Elsevier Science B.V. All rights reserved.

**Keywords:** Time-varying estimate; CAPM beta; Recursive beta

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## 1. Introduction

The CAPM beta ( $\beta$ ) is a parameter which plays a central role in modern finance as a measure of an asset's risk. It is, however, unobservable and consequently a great deal of energy has been devoted to its estimation.

The unobservability of  $\beta$  can be resolved by simply regressing an asset's return on the return to the market portfolio (the "market model") using time-series data as long as the asset returns are stationary so that their distribution has time-invariant parameters (including the CAPM  $\beta$ ). The assumption of time-invariant parameters is implicit in tests of the CAPM where the mean return to an asset is measured by the sample mean over some period of time and the asset's  $\beta$  is estimated as the (constant) slope parameter in the market model. The time-invariance assumption is also implied in the practice of using a  $\beta$  estimated over a given period to make inferences (about asset value, say) in some different (normally future) time period.

However, it has long been recognised that asset returns may not be stationary in practice, resulting in  $\beta$  instability over time.<sup>1</sup> This recognition is reflected in the common practice of estimating  $\beta$ s over only

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<sup>1</sup>See, e.g., [2] for the US and [3–5] for evidence using Australian data.

relatively short periods (usually five years) and the finding that CAPM performs better over short periods than long periods.<sup>2</sup>

The instability of  $\beta$ s over time leads to important practical problems. Apart from those posed by the interpretation of  $\beta$ s which change over time, there are problems of estimation both for practical use and for use in testing the CAPM. Various methods of dealing with the estimation problem have been applied. The most natural one is to estimate a market model which makes explicit allowance for time-varying parameters. Examples of this approach are [2] for the US, [1] for the UK, [7] using Swedish data and [5] for Australia.

We contribute to the literature on time-variation in  $\beta$ s in various ways. We use data for monthly returns to 23 Australian industry portfolios for the period 1979–1994 and proceed in a number of stages. We start by a brief discussion of the results of estimating the market model over the full sample on the basis of a constant- $\beta$  assumption and testing the adequacy of this specification in a number of ways on the presumption that variability of the  $\beta$ s will show up in mis-specification of a model which assumes the opposite. We also compare the full-sample  $\beta$ s with those estimated over various shorter sub-samples to gauge the variability of  $\beta$ s.

After this preliminary investigation of the issue, we proceed to estimate the  $\beta$ s allowing explicitly for time-variation. We employ three alternative methods in order to assess the sensitivity of the  $\beta$ s to the method used. The first is recursive estimation, the second involves the use of rolling regressions and the third is based on the Kalman Filter. We compare the estimated  $\beta$ s with each other and then assess the time-series properties of the  $\beta$ s by testing them for stationarity and then regressing them against a time trend and structural breaks.

We find evidence of widespread  $\beta$  instability for a set of 23 Australian industry portfolios. However, an important finding is that the nature of the time-variation depends on the method of estimation used. Almost all the  $\beta$ s estimated by the recursive or rolling regression methods were found to be non-stationary, while approximately half the industries were found to have non-stationary  $\beta$ s when they were estimated by the Kalman Filter procedure. Much of the variability of the  $\beta$ s over the sample is explained by a time trend and there was strong evidence of a widespread break in the relationship at October 1987. The break was found to exist in both the level and the trend of the  $\beta$ s.

## 2. The data and preliminary evidence<sup>3</sup>

The results are based on monthly returns calculated from 23 industry share-price indexes obtained from the Australian Stock Exchange for the period December 1979 to February 1994. The return to the market portfolio was based on the All Ordinaries Index. Summary statistics for the 23 industry return series indicate widespread departure from normality, although these departures are less marked when the observation for October 1987 is omitted. These features are not unusual for financial data.

Preliminary investigation of beta stability was carried out in various ways. The first was to estimate the betas over sub-periods and compare the estimated betas for these shorter samples with each other and with the full-sample betas. Considerable variability of betas over time was found.

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<sup>2</sup>See [6] for recent Australian evidence.

<sup>3</sup>The preliminary evidence is reported in detail in [6].

Secondly, more formal diagnostic tests of the market model were carried out on the basis of the argument that if the betas are time-varying, a model which assumes them to be constant will show evidence of mis-specification.

The Durbin–Watson statistic, tests based on the recursive residuals (Harvey’s  $t$  and  $F$  tests) and the RESET test of functional form all provide little evidence of mis-specification – at most five of the 23 sectors appear to have problems. A more direct test of parameter stability is the Chow test which, however, requires a prior specification of the break-point. If October 1987 is chosen as a break-point, stability of beta is rejected for five of the 23 sectors. If a break-point is permitted at the end of 1984 and again at the end of 1989 to match the five-year sub-samples used earlier, a dummy variable interacting with beta is significant in 13 of the 23 equations suggesting considerable instability in the betas over shorter periods.

### 3. Time-varying parameter estimates

Having established in the previous section that there is evidence of  $\beta$  instability, as had Brooks and Faff et al. in earlier work, we go on in this section to the main purpose of our paper: the estimation of time-varying  $\beta$ s, an analysis of the nature of their time-variation and a comparison of alternative methods of estimating time-variation. We present  $\beta$ s based on three different methods of estimation: recursive regression, rolling regression and the Kalman Filter.

A selection of the estimated  $\beta$ s for various of the 23 sectors analysed is pictured in Fig. 1.

There is clearly considerable variability in the recursive and rolling  $\beta$ s, although less in the Kalman Filter  $\beta$ s. In many cases there is a distinct break at the date of the October 1987 crash (observation 40), usually in level but often also in trend. The way in which the recursive and rolling  $\beta$ s vary over the sample period parallels the sub-period OLS results discussed in the previous section; the correspondence is less clear for the Kalman Filter  $\beta$ s. Generally, the Kalman Filter  $\beta$ s show the least variation especially where, as we shall see, the optimal autoregressive parameter in the  $\beta$  transition equation is close to zero. The recursive  $\beta$ s show greater variation early in the sample but less as time goes on. This is not surprising given that the estimation procedure means that each successive observation carries less weight. For the same reason the rolling  $\beta$ s show the greatest variation over the sample.

Having presented the estimated  $\beta$  series, we now go on to an analysis of their time-series properties. We can consider this as a formalisation of the conclusions drawn informally from an inspection of Fig. 1.

#### 3.1. Recursive betas

We begin by examining the time-series properties of each of the recursively estimated  $\beta$ s. In each case we test for stationarity using two tests – an augmented Dickey–Fuller (ADF) test and a Phillips–Perron (PP) test. Both are tests of difference-stationarity against an alternative hypothesis of trend-stationarity, i.e. the “ADF regression” has the form:

$$\beta_{it} = \alpha_0 + \alpha_1 t + \alpha_2 \beta_{it-1} + \sum_{j=1}^J \gamma_j \Delta \beta_{it-j} + \epsilon_{it}$$

and the null hypothesis is  $H_0 = \alpha_1 = \alpha_2 = 0$ .

We go on to regress each  $\beta$  against a time trend, allowing for a break in trend and a shift in the intercept at October 1987 suggested by the graphs in Fig. 1. The results are reported in Table 1.

The  $\beta$ s for all sectors are clearly non-stationary – for all sectors both ADF and PP statistics are clearly less than the critical value of 5.34. Thus the  $\beta$ s not only change over time but have no tendency to vary about a fixed mean. This finding is consistent with (although it does not necessarily imply) the  $\beta$ s themselves being generated by a random-walk process so that the current  $\beta$  is the best forecast of next period's  $\beta$  thus seeming to support the practice of using historical  $\beta$ s as though they were relevant for the future. The remainder of the table shows that most of the time variation in the  $\beta$ s is explained by a trend term and a break both in level and trend, with the break occurring at October 1987. The regression results coincide with the graphs presented in Fig. 1. The striking exception to the general features of the results is the other metals sector, the  $\beta$  for which is only weakly related to the explanatory variables. The graph for this sector in Fig. 1 indicates very little variation in the recursive

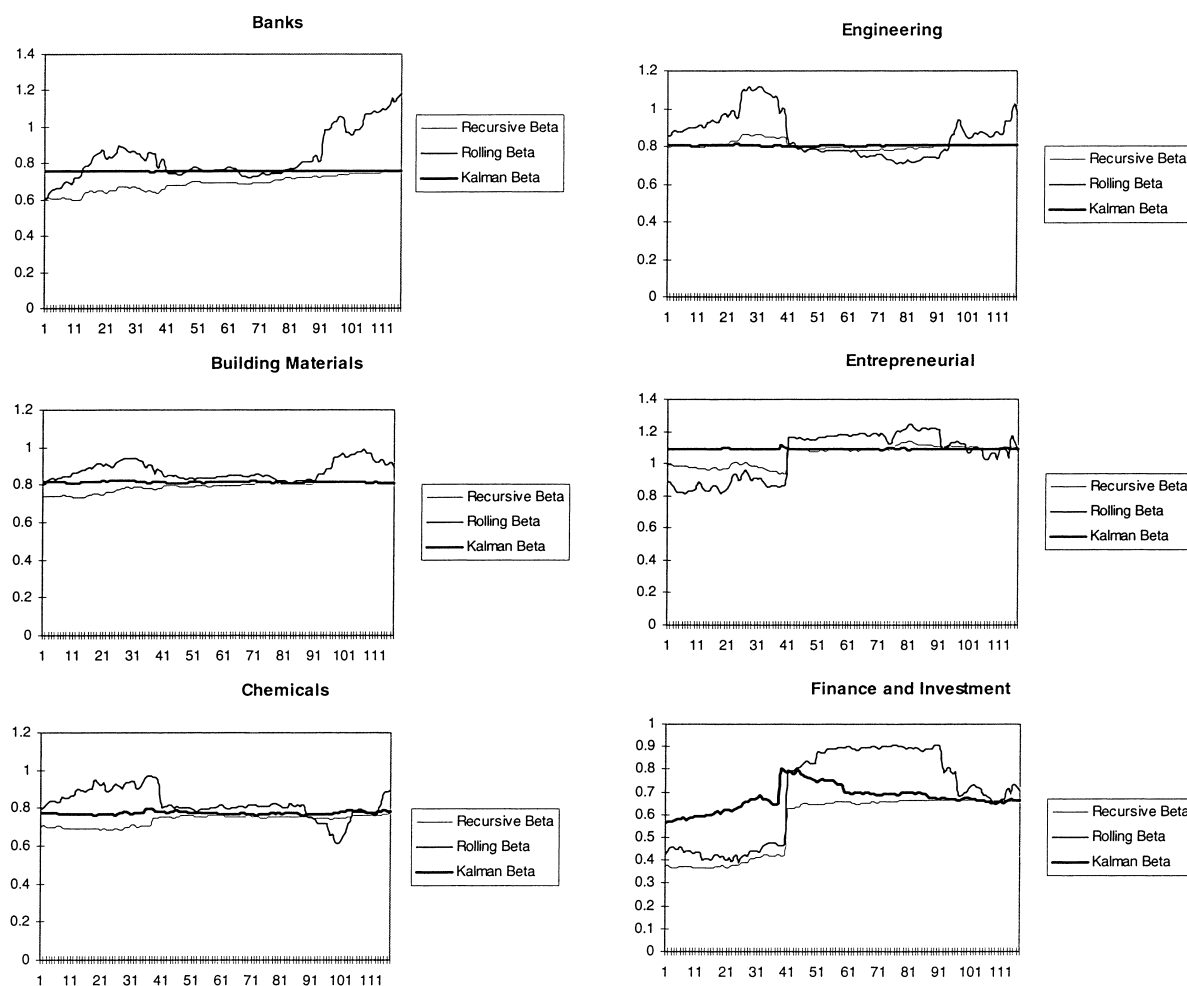


Fig. 1. Selected estimated time-varying betas.

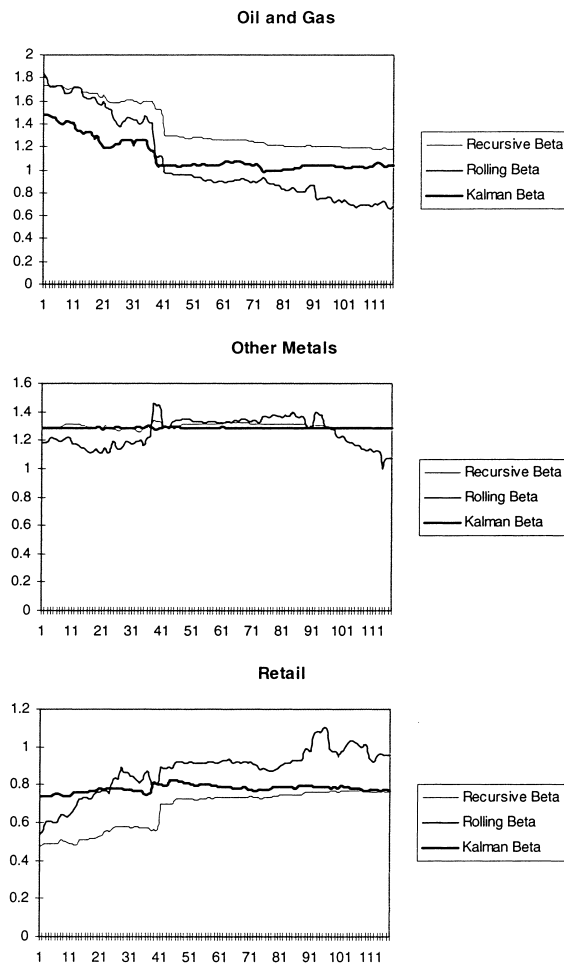


Fig. 1. (Continued)

$\beta$ . The Durbin–Watson statistic is always very low which, together with the high  $R^2$ , indicates that the non-stationarity is not removed by regressing against trend and two breaks.

### 3.2. Rolling betas

The second approach to the estimation of time-varying  $\beta$ s is based on rolling regressions. A window of 50 months was used, chosen so as to correspond to the 4–5-year period over which  $\beta$ s are often estimated in practice. Again, we start with an analysis of how the resulting  $\beta$ s change over time by testing for stationarity and regressing them on a trend with a break at October 1987. The results are reported in Table 2.

The results of the tests for stationarity show that only two sectors have stationary  $\beta$ s – Diversified Industrials and Property Trusts – although the first of these is not significant at the 5% level. The degree of explanatory power is again high, although  $R^2$ s are not as high as they were for the recursively

Table 1  
Time-series properties of recursive betas

Sector	Stationarity		$\hat{\beta}_t = \gamma_0 + \gamma_1 t + \gamma_2 D87 + \gamma_3 t D87 + \epsilon_t$				
	ADF	PP	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$R^2$	DW
Alcohol and tobacco	1.64	1.62	0.0020 (18.79)	0.2633 (27.31)	−0.0018 (15.56)	0.9884	0.57
Banks	4.36	4.49	0.0016 (10.90)	0.0567 (4.28)	−0.0005 (3.52)	0.9424	0.33
Building materials	2.39	2.08	0.0016 (19.17)	0.1111 (15.02)	−0.0012 (13.56)	0.9481	0.24
Chemicals	1.51	1.39	0.0006 (4.57)	0.0969 (7.96)	−0.0006 (4.04)	0.8856	0.48
Developers and contractors	1.12	1.12	0.0011 (8.20)	0.2377 (19.45)	−0.0010 (7.05)	0.9855	0.23
Diversified industrials	1.30	1.97	0.0013 (11.42)	0.1353 (13.36)	−0.0005 (4.45)	0.9863	0.36
Diversified resources	1.40	1.36	−0.0024 (17.37)	−0.3359 (27.20)	0.0023 (16.01)	0.9852	0.34
Engineering	1.61	1.69	0.0020 (15.80)	0.0807 (7.02)	−0.0017 (12.85)	0.8468	0.36
Entrepreneurial	2.93	3.06	−0.0007 (3.52)	0.0230 (1.24)	0.0011 (4.81)	0.9393	0.22
Finance and investment services	1.10	1.10	0.0016 (15.38)	0.3510 (36.16)	−0.0013 (11.71)	0.9964	0.27
Food and household goods	1.60	1.60	0.0025 (17.26)	0.2651 (20.41)	−0.0018 (12.03)	0.9847	0.17
Gold	2.92	4.23	−0.0019 (6.28)	−0.1458 (5.25)	0.0005 (1.47)	0.9509	0.36
Insurance	1.67	1.61	0.0065 (40.14)	0.7542 (51.22)	−0.0056 (32.23)	0.9963	0.42
Media	1.87	1.67	0.0058 (16.98)	0.3964 (12.81)	−0.0036 (9.79)	0.9686	0.16
Misc industrials	1.44	1.53	0.0013 (10.70)	0.2748 (24.40)	−0.0019 (14.09)	0.9705	0.32
Misc services	1.85	1.84	−0.0008 (7.58)	−0.1009 (9.98)	0.0007 (6.28)	0.9064	0.71
Oil and gas	1.91	1.36	−0.0055 (28.61)	−0.6047 (34.88)	0.0038 (18.78)	0.9954	0.54
Other metals	1.93	4.12	−0.0003 (1.74)	0.0322 (1.97)	−0.0000 (0.01)	0.3189	0.62
Paper and packaging	2.19	2.06	0.0002 (0.25)	−0.1125 (14.03)	0.0010 (10.62)	0.9181	0.56
Property trusts	3.17	4.45	−0.0010 (14.51)	−0.0249 (3.89)	0.0009 (12.36)	0.9389	0.50
Retail	1.70	1.71	0.0029 (21.71)	0.3186 (25.85)	−0.0021 (14.57)	0.9911	0.31
Solid fuels	1.21	1.21	−0.0043 (20.77)	−0.4668 (24.63)	0.0036 (15.98)	0.9850	0.19
Transport	1.98	1.91	0.0018 (14.15)	0.2070 (17.90)	−0.0017 (12.48)	0.9566	0.42

Notes: 10% critical values for ADF and PP: 5.34. *t*-ratios in parentheses.

estimated  $\beta$ s. There is again pervasive evidence of a break both in level and in trend at October 1987. The low Durbin–Watson statistic in conjunction with high  $R^2$ s indicate stationarity problems reflecting the widespread nonstationarity of the  $\beta$ s.

### 3.3. Kalman Filter betas

We turn finally to the  $\beta$ s estimated by the Kalman Filter. Consider the linear regression model with time-varying coefficients:

$$y_t = x_t \beta_t + \epsilon_t \quad (1)$$

where  $x_t$  and  $\beta_t$  are  $k$ -component vectors and  $\epsilon_t$  is a random error term with  $E(\epsilon_t) = 0$  and  $E(\epsilon_t^2) = n_t$ . In the Kalman Filter model Eq. (1) is the measurement equation. The evolution of the time-varying parameter vector,  $\beta_t$ , is given by the state equation which in the model used in this paper has the AR(1) form:

$$\beta_t = \rho \beta_{t-1} + (1 - \rho) \bar{\beta} + \eta_t \quad (2)$$

Table 2  
Time-series properties of rolling betas

Sector	Stationarity		$\beta_t = \gamma_0 + \gamma_1 t + \gamma_2 D87 + \gamma_3 D87t + \epsilon_t$			$R^2$	$DW$
	ADF	PP	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$		
Alcohol and tobacco	3.53	3.55	0.0023 (4.19)	0.4731 (9.65)	−0.0035 (6.14)	0.7630	0.36
Banks	0.88	0.65	0.0058 (6.25)	−0.2411 (2.88)	−0.0002 (0.18)	0.7460	0.22
Building materials	2.23	1.86	0.0025 (5.43)	−0.0377 (0.90)	−0.0009 (1.92)	0.5100	0.21
Chemicals	1.63	1.92	0.0036 (6.06)	0.2446 (4.55)	−0.0043 (6.74)	0.6669	0.18
Developers and contractors	1.28	1.96	−0.0005 (0.76)	0.5619 (9.58)	−0.0025 (3.63)	0.8474	0.31
Diversified industrials	5.92	6.28	0.0014 (2.94)	−0.0964 (2.39)	0.0013 (2.66)	0.8792	0.35
Diversified resources	2.13	2.20	−0.0048 (7.97)	−0.9849 (17.95)	0.0079 (12.16)	0.8896	0.37
Engineering	1.43	0.83	0.0062 (8.50)	0.0482 (0.73)	−0.0044 (5.68)	0.7832	0.30
Entrepreneurial	1.79	1.77	0.0013 (2.29)	0.5533 (10.47)	−0.0026 (4.20)	0.9143	0.43
Finance and investment services	1.05	1.04	0.0007 (0.88)	0.7742 (11.15)	−0.0032 (3.92)	0.9223	0.19
Food and household goods	5.10	5.10	0.0062 (10.76)	0.4612 (8.87)	−0.0050 (8.19)	0.8186	0.24
Gold	2.39	3.06	−0.0067 (3.89)	0.2098 (1.34)	−0.0010 (0.54)	0.7403	0.23
Insurance	2.73	3.22	0.0129 (18.87)	1.6586 (26.82)	−0.0147 (20.23)	0.9581	0.25
Media	1.84	1.67	0.0145 (8.26)	−0.0284 (0.18)	−0.0031 (1.65)	0.8497	0.18
Misc industrials	2.58	1.58	0.0066 (6.16)	1.2527 (13.75)	−0.0121 (11.26)	0.6958	0.23
Misc services	4.10	6.80	0.0024 (4.39)	−0.3719 (7.44)	0.0034 (5.80)	0.4691	0.68
Oil and gas	2.15	2.39	−0.0141 (22.6)	−1.2024 (21.20)	0.0100 (14.89)	0.9845	0.68
Other metals	1.57	1.47	0.0027 (2.72)	0.6835 (7.65)	−0.0056 (5.31)	0.5121	0.38
Paper and packaging	1.06	1.06	0.0022 (2.30)	−0.6331 (7.30)	0.0046 (4.52)	0.7932	0.20
Property trusts	6.10	9.24	−0.0030 (6.46)	−0.1267 (2.98)	0.0021 (4.12)	0.4472	0.49
Retail	5.06	5.07	0.0078 (13.85)	0.5956 (11.60)	−0.0064 (10.59)	0.8851	0.33
Solid fuels	2.50	2.49	−0.0107 (15.48)	−1.0956 (17.56)	0.0105 (14.23)	0.9226	0.34
Transport	4.32	4.31	0.0048 (9.93)	0.5028 (11.37)	−0.0058 (11.10)	0.5379	0.46

Note: 10% critical value for ADF and PP is 5.34.  $t$ -statistics in parentheses.

where  $\rho$  is the AR(1) parameter,  $\bar{\beta}$  is a constant and  $\eta_t$  is a vector of random variables each of which is uncorrelated with  $\epsilon_t$ ,  $E(\eta_t) = 0$  and  $E(\eta_t \eta_t') = M_t$ .

The Kalman Filter estimates  $\beta_t$  conditional on  $y_t$ ,  $x_t$ ,  $\rho$ ,  $\bar{\beta}$ ,  $\hat{\beta}_{t-1}$ , and  $\hat{\Sigma}_{t-1}$ , the estimated covariance matrix of  $\hat{\beta}_{t-1}$ . The estimation procedure assumes that  $\rho$ ,  $\bar{\beta}$ ,  $n_t$ , and  $M_t$  are known. Hence to apply the Kalman Filter to the estimation of  $\beta_t$  in (1) we need starting values for  $\hat{\beta}$  and  $\hat{\Sigma}$ , values for  $\rho$  and  $\bar{\beta}$  and for the entire time series for  $n_t$  and  $M_t$ . The starting values for  $\hat{\beta}$  and  $\hat{\Sigma}$  were obtained from a constant parameter OLS regression for the first  $k$  periods and the value of  $\bar{\beta}$  was set as the OLS  $\beta$  for the entire sample. Clearly, insufficient information is available to estimate  $n_t$  and  $M_t$  and it was assumed that both are constant over time. Recall that  $n$  is the variance of the error term in the measurement equation (the market model) and an estimate was obtained in the process of estimating starting values for  $\hat{\beta}$  and  $\hat{\Sigma}$ . The matrix  $M$  was further simplified by assuming that only the slope parameter is time-varying so that only one element of  $M$  needs to be obtained for the two-parameter market model, viz. the variance of the innovation in the  $\beta$  equation. Various values were experimented with and it was found that the resulting  $\beta$  estimates are quite sensitive to choice of  $M$ . It was decided, therefore, to estimate  $M$  as the variance of the error term in an AR(1) equation estimated using the recursive  $\beta$ s. The obvious alternative is to use the rolling  $\beta$ s to estimate the AR(1) model. This was also done but made little difference to the resulting Kalman  $\beta$ s, a rather surprising result given the quite different behaviour of

Table 3  
Time-series properties of the Kalman betas

Sector	$\hat{\rho}$	Stationarity		$\beta_t = \gamma_0 + \gamma_1 t + \gamma_2 D87 + \gamma_3 D87t + \epsilon_t$			$R^2$	$DW$
		ADF	PP	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$		
Alcohol and tobacco	0.81	7.52	7.62	0.0001 (2.14)	0.0203 (4.81)	−0.0002 (3.86)	0.2283	0.51
Banks	−0.00	4.28	65.95	−0.0000 (0.52)	−0.0006 (0.86)	0.0000 (0.88)	0.0244	2.17
Building materials	0.90	2.70	3.20	0.0001 (3.22)	0.0048 (1.24)	−0.0001 (2.47)	0.1802	0.31
Chemicals	0.86	5.21	4.79	0.0002 (1.53)	0.0116 (1.26)	−0.0001 (1.32)	0.0312	0.32
Developers and contractors	0.94	2.52	2.51	0.0010 (5.60)	0.2147 (12.82)	−0.0019 (9.51)	0.7098	0.17
Diversified industrials	0.92	1.56	5.89	0.0002 (3.51)	0.0528 (9.64)	−0.0004 (6.62)	0.6382	0.61
Diversified resources	0.00	11.53	56.15	−0.0000 (1.34)	−0.0049 (2.02)	0.0001 (1.89)	0.0458	2.08
Engineering	0.82	2.57	4.68	0.0000 (0.13)	−0.0113 (5.08)	0.0001 (2.70)	0.4410	0.40
Entrepreneurial	0.35	11.71	31.86	−0.0000 (1.06)	0.0025 (0.58)	0.0000 (0.15)	0.0628	1.40
Finance and investment services	0.95	2.50	2.49	0.0030 (13.56)	0.5330 (26.91)	−0.0047 (19.99)	0.9247	0.32
Food and household goods	0.95	3.48	6.51	0.0008 (16.37)	0.1018 (22.34)	−0.0010 (18.79)	0.8837	0.28
Gold	0.00	8.79	59.49	0.0001 (1.00)	−0.0001 (0.18)	−0.0000 (0.42)	0.0387	2.04
Insurance	0.97	4.42	6.88	0.0048 (25.23)	0.5505 (31.65)	−0.0056 (27.38)	0.9407	0.23
Media	0.94	2.67	5.16	0.0012 (13.15)	0.0908 (10.94)	−0.0011 (11.07)	0.7634	0.19
Misc industrials	0.00	6.72	51.83	−0.0000 (0.21)	0.0010 (0.76)	−0.0000 (0.33)	0.0239	1.88
Misc services	−0.74	13.80	685.53	−0.0001 (0.83)	−0.0077 (0.60)	0.0001 (0.75)	0.0068	3.66
Oil and gas	0.98	2.82	3.28	−0.0080 (19.43)	−0.8459 (22.67)	0.0078 (17.72)	0.9589	0.32
Other metals	0.39	1.88	35.38	0.0001 (1.82)	0.0070 (2.26)	−0.0001 (2.27)	0.0541	1.59
Paper and packaging	−0.00	7.40	65.17	−0.0000 (1.60)	−0.0017 (2.06)	0.0000 (2.14)	0.0709	2.24
Property trusts	0.00	14.18	52.39	−0.0000 (0.12)	0.0004 (0.61)	−0.0000 (0.24)	0.0172	1.89
Retail	0.95	3.62	3.71	0.0010 (6.63)	0.1506 (11.68)	−0.0013 (8.68)	0.7281	0.27
Solid fuels	0.96	4.35	4.45	−0.0029 (28.23)	−0.3178 (33.75)	0.0033 (29.42)	0.9529	0.30
Transport	0.73	4.62	8.76	0.0000 (1.66)	0.0026 (2.39)	−0.0000 (2.41)	0.1008	0.60

Notes: 10% critical value for ADF and PP is 5.34. Figures in parentheses are *t*-ratios.

the recursive and rolling  $\beta$ s described in Sections 3.1 and 3.2. Finally, the value of  $\rho$  was chosen so as to maximise the quasi-likelihood function which RATS evaluates for each iteration of the Kalman Filter. The optimal value of  $\rho$  was found to be sensitive to the starting value in most sectors so that a variety of starting values (in the range 0.1–1.0) was experimented with. For each sector the starting value was chosen to lead to the optimal  $\rho$  with the highest log-likelihood function. It turned out that only starting values of 0.1 and 1.0 were needed. The optimal value of  $\rho$  for each sector is reported in Table 3 together with the time-series characteristics of the Kalman  $\beta$ s.

The first column of figures in the table gives the optimal value for  $\rho$ , the AR(1) parameter in the transition equations for the  $\beta$ s. Approximately half the sectors have  $\beta$ s based on an optimal  $\rho$  in the range 0.8–1.0 and the next two columns show that in most cases the two unit-root tests used are unable to reject a (false) null hypothesis of non-stationarity. There is one sector with a value of  $\rho$  in the 0.7–0.8 range and here the tests provide mixed evidence of a unit root – the ADF test indicated non-stationarity while the PP test rejects the non-stationary null hypothesis. Even the value of  $\rho$  of 0.39 for the other metals sector produces an ADF statistic which points to non-stationarity. The six sectors which have optimal  $\rho$  values of approximately zero are all found to have stationary  $\beta$ s with the exception of the banks sector which fails to reject non-stationarity with the ADF test. Finally, there is one sector with a value of  $\rho$  which is substantially negative. The Kalman Filter  $\beta$ s are, therefore, in some contrast to those



obtained from the other two estimation procedures. The remainder of Table 3 reports the results of regressing the  $\beta$ s on a trend, a dummy variable for October 1987 and an interaction term formed from these two variables. There is clearly a great deal of variation in the extent to which  $\beta$  could be explained, judging from the  $R^2$ s.

The results suggest that the sectors may be roughly divided into two groups; the first has high estimated  $\rho$ , non-stationary  $\beta$ , high  $R^2$ , low Durbin–Watson statistic and significant breaks both in level and trend at October 1987. The other group has generally stationary  $\beta$ , low  $\rho$ , low  $R^2$  and often insignificant regressors in the equations explaining the estimated  $\beta$ s. An inspection of the graphs of the estimated betas suggests that the first group corresponds to those sectors which have relatively volatile Kalman  $\beta$ s, while the second group has relatively constant Kalman  $\beta$ s.

#### 4. Conclusions

The single-factor market model is commonly estimated using time-series data to obtain CAPM  $\beta$ s on the assumption that the  $\beta$ s are stable over time. In practice, however, they have been found to be unstable and are generally estimated over only short periods – five-year periods are common.

We have used monthly data on 23 Australian industry share-price indexes for the period 1979(12)–1994(2) to test the specification of the constant- $\beta$  market model and to estimate and examine time-varying  $\beta$ s. Standard tests of the market model fail to detect widespread specification problems but significant structural breaks were found when dummy variables were used at approximately five-year intervals for over one half of the sectors in the sample.

The time-varying model was estimated using three different approaches – recursive regression, rolling regression and the Kalman Filter. The nature of the time-variation in the estimated  $\beta$ s differed considerably depending on the method of estimation. In most cases the  $\beta$ s were found to be non-stationary but generally well explained by a time trend and a dummy variable at October 1987 both alone and in interaction with the trend.

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