

Advantages of digital signal over analog

- More immune to noise
- More secure and reliable
- High data rate
- Long distance transfer reliable
- Storage
- Cheaper than analog

Numbers

- i. Binary \rightarrow It's base is 2
- ii. Octal \rightarrow It's base is 8 from (0-7)
- iii. Decimal \rightarrow It's base is 10 from (0-9)
- iv. Hexa-Decimal \rightarrow It's base is 16 from (0-9, A, B, C, D, E, F)

* $(26)_{10} = (?)_2$

$$\begin{array}{r}
 2 | 26 \rightarrow 0 \\
 2 | 13 - 1 \\
 2 | 6 - 0 \\
 2 | 3 - 1
 \end{array}$$

$$(26)_{10} = 11010_2$$

$$*(26.6725)_{10} = (?)_2$$

$$\begin{array}{r} 2 \mid 26 - 0 \\ 2 \mid 13 - 1 \\ 2 \mid 6 - 0 \\ 2 \mid 3 - 1 \\ \quad | \end{array}$$

$$\begin{aligned} 0.6725 \times 2 &= 1.349 \\ 0.349 \times 2 &= 0.68 \\ 0.68 \times 2 &= 0.96 \\ 0.96 \times 2 &= 1.872 \\ 0.872 \times 2 &= 1.744 \end{aligned}$$

$$*(26.6725)_{10} = 11010.110011$$

$$*(65.369)_8 = (?)_{10}$$

$$\begin{aligned} 6 \times 8^1 + 5 \times 8^0 + 3 \times 8^{-1} + 6 \times 8^{-2} + 9 \times 8^{-3} \\ = 48 + 5 + 0.375 \\ = (53.375)_{10} \end{aligned}$$

$$*(65.369)_8 = (?)_2$$

$$(11010.011110100)_2$$

$$*(65.369)_8 = (?)_{16}$$

$$\begin{array}{r} 11010.011110100 \\ = 0011\ 0101 \cdot 0111\ 1010 \quad 0000 \\ = 3\ 5 \cdot 7\ A \\ = (35.7A)_{16} \end{array}$$

find the value of n from number.

$$*(239)_x = (695)_8$$

$$2n^2 + 3n + 9 = 6 \times 8^2 + 9 \times 8^1 + 5 \times 8^0$$

$$\text{or, } 2n^2 + 3n + 9 = 6 \times 64 + 32 + 5$$

$$\text{or, } 2n^2 + 3n + 9 = 427$$

$$\text{or, } 2n^2 + 3n - 172 = 0$$

$$* (271)_n = (152)_8$$

$$\text{or, } 2n^2 + n + 1 = 1 \times 8^2 + 5 \times 8^1 + 2 \times 8^0$$

$$\text{or, } 2n^2 + n + 1 = 64 + 32 + 2$$

$$\text{or, } 2n^2 + n + 1 = 106$$

$$\text{or, } 2n^2 + n - 105 = 0$$

$$\text{or, } 2n^2 - 14n + 15n - 105 = 0$$

$$\text{or, } 2n(n-7) + 15(n-7) = 0$$

$$\text{or, } (n-7)(2n+15) = 0$$

$$\text{Hence, } n-7=0 \quad 2n+15=0$$

$$\therefore n=7 \quad 2n=-15$$

$$n = -\frac{15}{2}$$

Binary addition

$$* (26)_{10} = (11010)_2$$

$$(12)_{10} = + \underline{(1100)}_2$$

$$\begin{array}{r} 2 \\ | \\ 12 - 0 \end{array}$$

$$\begin{array}{r} 2 \\ | \\ 6 - 0 \end{array}$$

$$\begin{array}{r} 2 \\ | \\ 3 - 1 \end{array}$$

Binary subtraction

$$(26)_{10} = (11010)_2$$

$$(18)_{10} = + \underline{(1100)}_2$$

Binary Multiplication

$$\begin{array}{r}
 11010 \\
 \times 1100 \\
 \hline
 00000 \\
 00000x \\
 11010xx \\
 + 11010xxx \\
 \hline
 (100111000)_2
 \end{array}$$

Representing of BCD, Gray code, Excess-3 code, ASCIIBinary to Gray-code

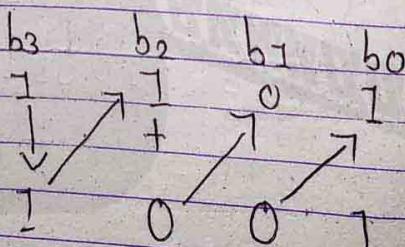
$$\begin{array}{l}
 b_3 + b_2 + b_1 + b_0 \\
 1000 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \\
 \downarrow \\
 1 \ 1 \ 0 \ 0 \\
 G_3 = b_3 \quad G_2 = b_3b_2 \quad G_1 = b_1b_2 \quad G_0 = b_0 + b_1
 \end{array}$$

Gray code to binary

$$1000 \rightarrow 1100$$

$$1001 \rightarrow 1101$$

$$(1101)_{\text{gray}} = (1001)_2$$



$$(1100)_{\text{Gray}} = (1000)_2$$

$$\begin{array}{cccc}
 1 & 1 & 0 & 0 \\
 \downarrow & \nearrow + & \nearrow + & \nearrow \\
 1 & 0 & 0 & 0
 \end{array}$$

Excess-3 code

$$0 \rightarrow +3 \rightarrow 3$$

$$1 \rightarrow +3 \rightarrow 4$$

$$2 \rightarrow +3 \rightarrow 5$$

$$3 \rightarrow +3 \rightarrow 6$$

$$4 \rightarrow +3 \rightarrow 7$$

$$5 \rightarrow +3 \rightarrow 8$$

$$6 \rightarrow +3 \rightarrow 9$$

7

8

9

$$10 + 3$$

$$\begin{array}{c}
 10 \\
 \downarrow \\
 33 \\
 \downarrow \\
 43
 \end{array}$$

$$0100001$$

Alpha numeric code

→ ASCII (American standard code for information interchange)

→ EBCDIC (Extended Binary Coded Decimal Interchange Code)

EBCD = Binary coded decimal

Addition

$$(115)_{10} = (110011)_2$$

$$(27)_{10} = + (11011)_2$$

$$(10001110)_2$$

$$\begin{array}{r} 2 \mid 115 - 1 \\ 2 \mid 57 - 1 \\ 2 \mid 28 - 0 \\ 2 \mid 14 - 0 \\ 2 \mid 7 - 1 \end{array}$$

$$\begin{array}{r} 2 \mid 27 - 1 \\ 2 \mid 13 - 1 \\ 2 \mid 6 - 0 \\ 2 \mid 3 - 1 \\ 1 \end{array}$$

$$11011 \text{ (1's complement is } 00100) \quad 2 \underline{\mid} \begin{matrix} 3 \\ 1 \end{matrix} - 1$$

Subtraction

$$\begin{array}{r} 1110011 \\ + 00100 \\ \hline 1110111 \\ 1+1 \end{array}$$

$$1111000 \rightarrow \text{(2's complement)}$$

Binary subtraction using 1's complement

$$(26)_{10} - (10)_{10}$$

↓ ↓
 Binary equivalent 1's complement
 ↓
 Binary equivalent ↓
 ↓
 1's complement ↓
 + ↓

$$(115)_{10} - (27)_{10}$$

$$\begin{array}{r} 1110011 \\ + 0011011 \rightarrow 1's \text{ complement} \\ \hline 1100100 \end{array}$$

$$\begin{array}{r} 2 \mid 115 - 1 \\ 2 \mid 57 - 1 \\ 2 \mid 28 - 0 \\ 2 \mid 14 - 1 \\ 2 \mid 3 - 1 \\ 1 \end{array}$$

$$\begin{array}{r} 1110011 \\ - 1100100 \\ \hline 011011 \\ +1 \\ \hline 1011000 \end{array}$$

$$\begin{array}{r} 2 \mid 27 - 1 \\ 2 \mid 13 - 1 \\ 2 \mid 6 - 0 \\ 2 \mid 3 - 1 \\ 1 \end{array}$$

$$(534)_8 \rightarrow BC20$$

$$5 \times 8^2 + 3 \times 8^1 + 4 \times 8^0$$

$$320 + 24 + 4$$

$$(348)_{10}$$

$$(001101001000)_{BCD}$$

Error detection and correction code:

examples {

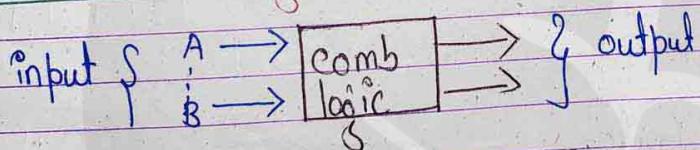
→ Parity ^{odd} even

→ check sum

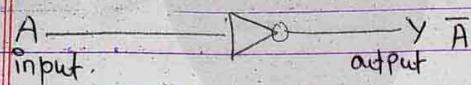
→ Cyclic Redundancy check.

{ Block codes ←
Convolution codes

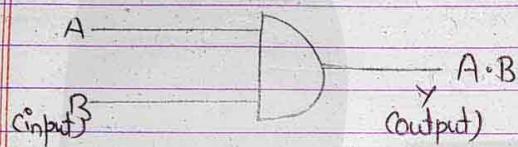
Combinational logic :-



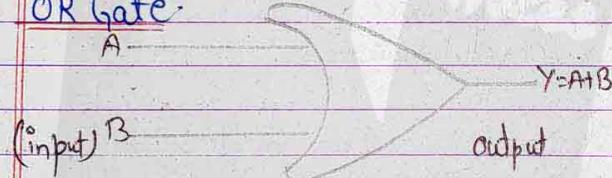
logic gates → Basic gates [eg: Not, AND, OR]
 Universal gates [eg: NAND, NOR]
 Special type gates [x-OR, x-NOR]

1 Not-Gate

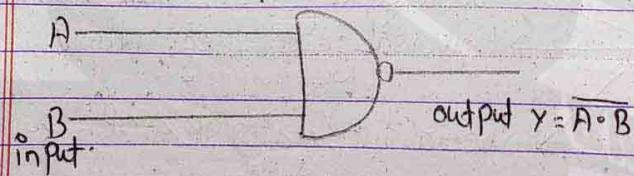
| Truth table | |
|-------------|------------|
| Input (A) | Output (Y) |
| 0 | 1 |
| 1 | 0 |

2 AND-Gate

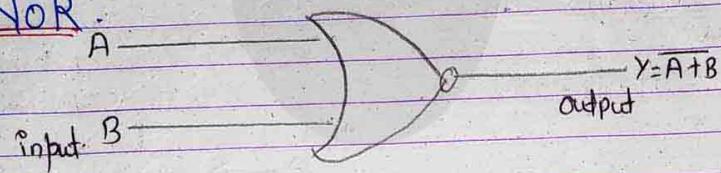
| Truth table | | |
|-------------|---------|------------|
| Input A | Input B | Output (Y) |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

3 OR Gate

| Truth table | | |
|-------------|---------|------------|
| Input A | Input B | Output (Y) |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

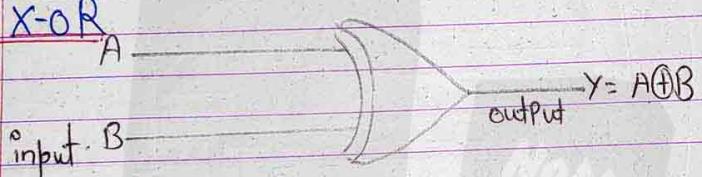
4. NAND Gate

| Truth table | | |
|-------------|---------|------------|
| Input A | Input B | Output Pwt |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

5. NOR

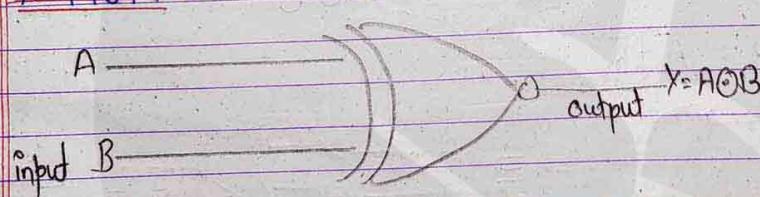
Truth table.

| Input | A | B | Output(Y) |
|-------|---|---|-----------|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |

6. X-OR

Truth table.

| Input | A | B | Output(Y) |
|-------|---|---|-----------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

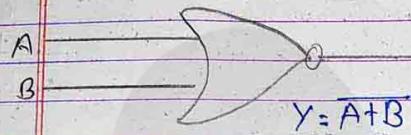
7. X-NOR

Truth table.

| Input | A | B | Output(Y) |
|-------|---|---|-----------|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

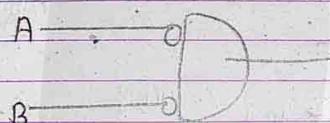
De-Morgan's Law

i. $A+B = \bar{A} \cdot \bar{B}$



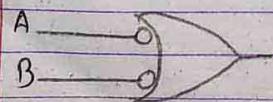
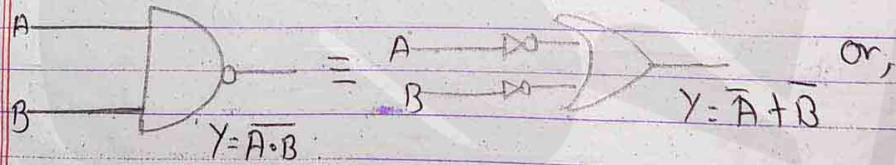
$$\begin{array}{c} \overline{A} \\ \overline{B} \end{array} \rightarrow D \quad \text{or.}$$

$$Y = \bar{A} \cdot \bar{B}$$



| A | B | $A+B$ | $\bar{A} \cdot \bar{B}$ | $\bar{A} \cdot \bar{B}$ |
|---|---|-------|-------------------------|-------------------------|
| 0 | 0 | 1 | 11 | 1 |
| 0 | 1 | 0 | 10 | 0 |
| 1 | 0 | 0 | 01 | 0 |
| 1 | 1 | 0 | 00 | 0 |

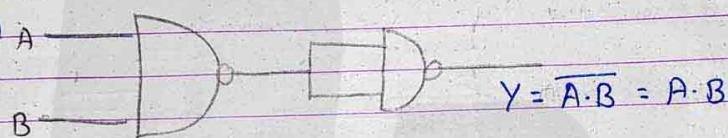
ii. $\bar{A} \cdot \bar{B} = \bar{A} + \bar{B}$



| A | B | $\bar{A}B$ | \bar{A} | \bar{B} | $\bar{A} + \bar{B}$ |
|---|---|------------|-----------|-----------|---------------------|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |

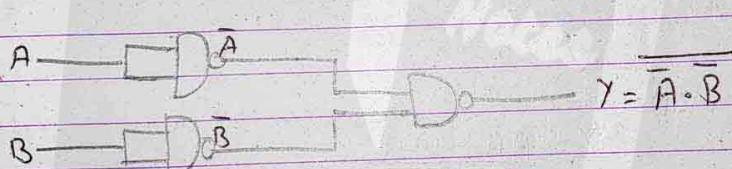
Construction of all logic gates using NAND only

(ii) AND

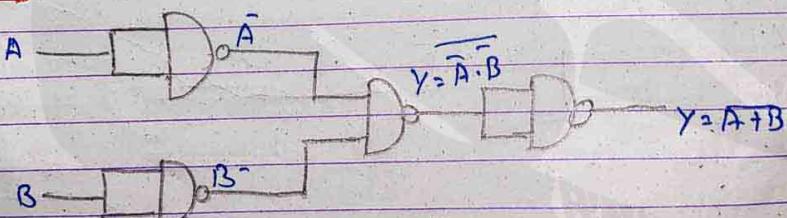


OR

(iii)

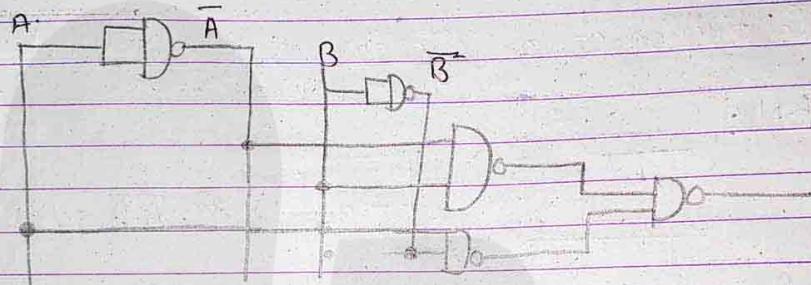


iv. NOR

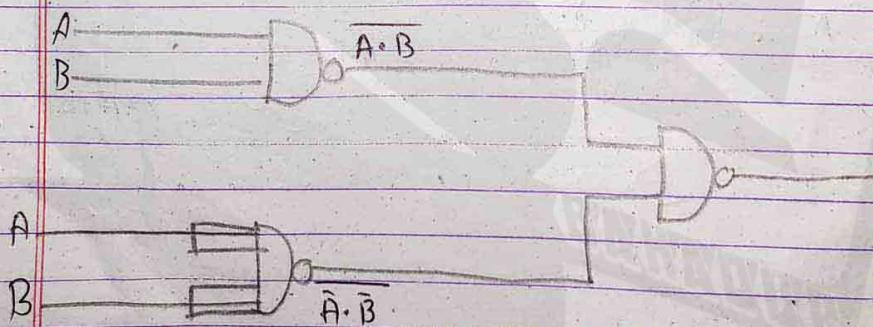


v. ~~X-OR~~

$$\begin{aligned} Y &= \overline{AB + A\bar{B}} \\ &= \overline{\bar{A}B + A\bar{B}} \\ &= (\bar{A}\bar{B})(A\bar{B}) \end{aligned}$$

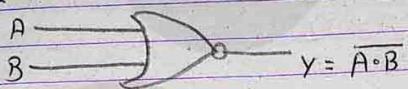
vi. ~~X-NOR~~

$$\begin{aligned} Y &= AB + \bar{A}\bar{B} \\ &= AB + \bar{A}B \\ &= (\bar{A}\cdot B)(\bar{A}\cdot \bar{B}) \end{aligned}$$

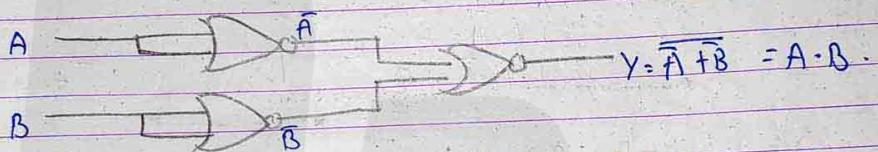


Construction of all logic gates using NOR only :-

i. NOR :-



ii. AND



iii. OR



Boolean expression or laws:

1. Associative law:

2. Commutative law:

$$(i) (A+B)+C = A+(B+C)$$

$$(ii) A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

2. Commutative law:

$$(i) A \cdot B = B \cdot A$$

$$(ii) A+B = B+A$$

3. OR Law :-

- i) $A+0 = A$
- ii) $A+1 = 1$
- iii) $A+A = A$
- iv) $A+\bar{A} = 1$

4. AND Law :-

- i) $A \cdot 0 = 0$
- ii) $A \cdot 1 = A$
- iii) $A \cdot A = A$
- iv) $A \cdot \bar{A} = 0$

$$5. A + AB = A$$

$$6. A + \bar{A}B = A + B$$

$$7. A + BC = (A+B)(A+C)$$

$$= A \cdot A + A \cdot C + B \cdot A + B \cdot C$$

$$= A + BC + BA + BC$$

$$= A(1+C) + BA + BC$$

$$= A + BA + BC$$

$$= A(A+B) + BC$$

$$= A + BC,$$

$$8. xy + yz (wz + w\bar{z})$$

$$= xy + n \{ w(z + \bar{z}) \}$$

$$= xy + nw$$

$$= n(xy + w),$$

$$\begin{aligned}
 & 10. \quad ny + ny' + n'y \\
 & = n(y + y') + n'y \\
 & = n \cdot 1 + n'y \\
 & = n + n'y \\
 & = ny
 \end{aligned}$$

Questions.

$$\begin{aligned}
 i. \quad & \bar{x}\bar{y}\bar{z} + \bar{n}y\bar{z} + \bar{n}\bar{y}\bar{z} + ny\bar{z} \\
 & = \bar{z}(\bar{s}n\bar{y} + ny + ny^2) \\
 & = \bar{z}(\bar{s}n\bar{y} + ny + ny^2) \\
 & = \bar{z}s(\bar{n} + n) + ny(\bar{n} + n) \\
 & = \bar{z}s(\bar{1} + 1) + ny(\bar{1} + 1) \\
 & = \bar{z}(1) \\
 & = \bar{z},
 \end{aligned}$$

$$\begin{aligned}
 ii. \quad & ny + \bar{n}z + yz \\
 & = ny + \bar{n}z + yz(n + \bar{n}) \\
 & = ny + \bar{n}z + nyz + \bar{n}yz \\
 & = ny(1 + z) + \bar{n}z(1 + y) \\
 & = ny(1 + z) + \bar{n}z(1 + y) \\
 & = ny + \bar{n}z,
 \end{aligned}$$

$$\begin{aligned}
 iii. \quad & AB + A(CB + C) + BC(CB + C) \\
 & = AB + A \cdot B + AC + B \cdot B + BC \\
 & = AB + AC + BC(CB + C) \\
 & = B + AB + AC + BC \\
 & = BC(1 + A) + AC + BC \\
 & = BC + AC + AC \\
 & = BC(1 + C) + AC \\
 & = BC + AC,
 \end{aligned}$$

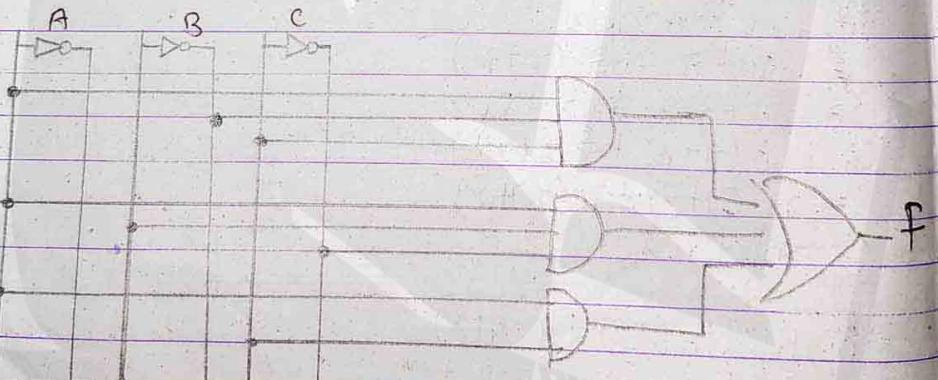
$$\begin{aligned}
 & i. \quad A(\bar{A}+C)(\bar{A}B+C)(\bar{A}BC+\bar{C}) \\
 & = (\bar{A}+AC)(A\bar{A}+AC)(\bar{A}ABC+A\bar{C}) \\
 & = (AC)(AC)(\bar{A}C) \\
 & = AC \cdot A\bar{C} \\
 & = 0,
 \end{aligned}$$

Boolean function

$$f(A, B, C) = \underbrace{\bar{A}BC}_{\substack{\text{literal} \\ \text{Boolean function}}} + \underbrace{A\bar{B}C}_{\substack{\text{Boolean function}}} + \underbrace{AB\bar{C}}_{\substack{\text{Boolean function}}} + \underbrace{ABC}_{\substack{\text{term}}}$$

Sum of Product (SoP) \Rightarrow

$$f(A, B, C) = A\bar{B}C + A\bar{B}\bar{C} + AC$$



$$\begin{aligned}
 \text{Standard Canonical form} &= A\bar{B}C + A\bar{B}\bar{C} + AC(CB + \bar{B}) \\
 &= A\bar{B}C + A\bar{B}\bar{C} + ABC + A\bar{B}C \\
 &= \underbrace{A\bar{B}C + A\bar{B}\bar{C}}_{\text{Minterm}} + ABC
 \end{aligned}$$

| Input | | | | minterm(m_i) | F_1 |
|-------|---|---|-------------------------|------------------|-------|
| A | B | C | | | |
| 0 | 0 | 0 | $\bar{A}\bar{B}\bar{C}$ | m_0 | 0 |
| 0 | 0 | 1 | $\bar{A}\bar{B}C$ | m_1 | 1 |
| 0 | 1 | 0 | $\bar{A}B\bar{C}$ | m_2 | 0 |
| 0 | 1 | 1 | $\bar{A}BC$ | m_3 | 0 |
| 1 | 0 | 0 | $A\bar{B}\bar{C}$ | m_4 | 1 |
| 1 | 0 | 1 | $A\bar{B}C$ | m_5 | 1 |
| 1 | 1 | 0 | $AB\bar{C}$ | m_6 | 0 |
| 1 | 1 | 1 | ABC | m_7 | 1 |

$$f_1 = m_1 + m_4 + m_5 + m_7$$

$$\begin{aligned}
 f &= AB + A + \bar{B}C \\
 &= ABC(C + \bar{C}) + A(CB + \bar{B})(C + \bar{C}) + \bar{B}C(A + \bar{A}) \\
 &= ABC + A\bar{B}C + (AB + A\bar{B})(C + \bar{C}) + A\bar{B}C + \bar{A}\bar{B}C \\
 &= ABC + A\bar{B}C + ABC + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}\bar{B}C \\
 &= \underbrace{ABC}_{m_7} + \underbrace{A\bar{B}C}_{m_6} + \underbrace{\bar{A}BC}_{m_5} + \underbrace{\bar{A}\bar{B}C}_{m_4} + \underbrace{A\bar{B}C}_{m_9} + \underbrace{\bar{A}\bar{B}C}_{m_1}
 \end{aligned}$$

$$f = M_1, M_4, M_5, M_6, M_7$$

$$f = \sum_m (1, 4, 5, 6, 7)$$

Product of sum (Pos) \Rightarrow

$$f = ((A+B+\bar{C})) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C})$$

↓ term. ↑ literal.

| Input | | | minterm | f |
|-------|---|---|---------------------------------------|---|
| A | B | C | | |
| 0 | 0 | 0 | (A+B+C) | 0 |
| 0 | 0 | 1 | (A+B+ \bar{C}) | 1 |
| 0 | 1 | 0 | (A+ \bar{B} +C) | 0 |
| 0 | 1 | 1 | (A+ \bar{B} + \bar{C}) | 1 |
| 1 | 0 | 0 | (\bar{A} +B+C) | 1 |
| 1 | 0 | 1 | (\bar{A} +B+ \bar{C}) | 0 |
| 1 | 1 | 0 | (\bar{A} + \bar{B} +C) | 0 |
| 1 | 1 | 1 | (\bar{A} + \bar{B} + \bar{C}) | 1 |

$$f = m_0 + m_2 + m_5 + m_6$$

$$f = ABC + A\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$$

$$= 111 + 101 + 110 + 000$$

$m_7 \quad m_5 \quad m_6 \quad m_0$

(0, 5, 6, 7)

1, 2, 3, 4

$$= A+B+\bar{C} + A+\bar{B}+C + A+\bar{B}+\bar{C} + \bar{A}+B+C$$

P T C

$$\begin{aligned}
 f &= (A+B)(A+\bar{C})(B) \\
 &= (A+B+C \cdot \bar{C})(A+\bar{C}+B \cdot \bar{B})(B+A \cdot \bar{A}+C \cdot \bar{C}) \\
 &= (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(B+A \cdot \bar{A}+C)(B+A \cdot \bar{A}+\bar{C}) \\
 &= (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})
 \end{aligned}$$

$$\begin{aligned}
 f &= \prod_{m=0,2,4,5} (A+B+\bar{C}) \\
 &= (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+B+\bar{C}) \\
 &= (1,3,6,7) \\
 f &= \sum_{m=1,3,6,7} \\
 &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC
 \end{aligned}$$

Difference between SoP and Pos

| SoP | Pos |
|---------------------------------|--|
| $f = (\sim) + \sim + \sim$ | $f = (\sim) \cdot (\sim) \cdot (\sim)$ |
| $0 \rightarrow \bar{A}$ | $0 \rightarrow A$ |
| $1 \rightarrow A$ | $1 \rightarrow \bar{A}$ |
| Minterm (m_i) | Maxterm (M_j) |
| term(output) - 1 | term(output) - 0 |
| $A\bar{B}C \rightarrow 111$ | $(\bar{A}+\bar{B}+\bar{C}) = 111$ |
| canonical form $(A+\bar{A})$ | canonical form $(A+B+C) = (A+B)(A+C)$ |

K-Map (Karnaugh Map)

| | | A'BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | BC | $B\bar{C}$ |
|--|--|-----------|------------------|------------|----|---------------|
| | | 0 | 1 | 1 | 1 | 0 |
| | | \bar{A} | 0 | 1 | 1 | 1 |
| | | A | 1 | 1 | 1 | 1 |
| | | | | | | $\bar{A} + C$ |

Questions

1. $f(x,y,z) = \Sigma(2,3,4,5)$

| | | $\bar{x}yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|--|--|-------------|------------------|------------|------|------------|
| | | 0 | 1 | 1 | 1 | 1 |
| | | \bar{x} | 0 | 1 | 1 | 1 |
| | | \bar{x} | 1 | 1 | 1 | 1 |

$$\begin{aligned} & \bar{y}yz + \bar{y}x\bar{z} \\ &= \bar{y}x(z + \bar{z}) \end{aligned}$$

2. $f(x,y,z) = \Sigma(3,4,6,7)$

| | | $\bar{x}yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|--|--|-------------|------------------|------------|------|------------|
| | | 0 | 1 | 1 | 1 | 1 |
| | | x | 1 | 1 | 1 | 1 |
| | | x | 1 | 1 | 1 | 1 |

$$= yz + x\bar{z},$$

3. $f(n, y, z) = (0, 2, 4, 5, 6)$

| \bar{x} | xz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-----------|------|------------------|------------|------|------------|
| \bar{x} | | 1 | | | 1 |
| x | | 1 | 1 | | 1 |

$$= \bar{x}y + \bar{z},$$

4. $f(n, y, z) = (1, 2, 3, 5, 7)$

| \bar{x} | xz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-----------|------|------------------|------------|------|------------|
| \bar{x} | | 1 | 1 | 1 | |
| x | | 1 | 1 | | |

$$= \bar{x}y + z,,$$

5. $f(\omega, x, y, z) = (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

| ωx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-----------------------|------|------------------|------------|------|------------|
| $\bar{\omega}\bar{x}$ | | 1 | 1 | | 1 |
| $\bar{\omega}x$ | | 1 | 1 | | 1 |
| ωx | | 1 | 1 | | 1 |
| $\omega\bar{x}$ | | 1 | 1 | | |

$$= \bar{y} + \bar{\omega}\bar{z} + x\bar{z}$$

6. $Af(A, B, C, \bar{D}) = \Sigma(0, 1, 2, 5, 8, 9, 10)$

| $AB\bar{C}\bar{D}$ | $\bar{C}\bar{D}$ | $\bar{C}D$ | $C\bar{D}$ | CD |
|--------------------|------------------|------------|------------|------|
| $\bar{A}\bar{B}$ | 1 | 1 | 1 | |
| $\bar{A}B$ | | 1 | | |
| $A\bar{B}$ | 1 | 1 | | |
| AB | | | 1 | |

$$= \bar{B}\bar{C} + \bar{A}\bar{C}\bar{D} + \bar{B}\bar{D}$$

7. $f(x, y, z) = \pi(0, 2, 5, 7)$

| $x\bar{y}z$ | $\bar{y}z$ | $y\bar{z}$ | $\bar{y}\bar{z}$ | $\bar{y}z$ |
|-------------|------------|------------|------------------|------------|
| \bar{x} | 0 | 0 | | |
| x | | 0 0 | | |

$$= \bar{x}z + (\bar{x}+z) \cdot (x+\bar{z})$$

8. $f(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$
 $d(w, x, y, z) = \Sigma(0, 2, 5)$

| $w\bar{x}y\bar{z}$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | $y\bar{z}$ | yz |
|--------------------|------------------|------------|------------|------|
| $\bar{w}\bar{x}$ | x 1 1 x | | | |
| $\bar{w}x$ | x 1 | | | |
| $w\bar{x}$ | | 1 | | |
| wx | | 1 | | |

$$= \bar{w}\bar{x} + yz$$

$$g. \quad f(n, y, z) = \sum (0, 1, 2, 4, 5)$$

$$d(n, y, z) = \sum (3, 6, 7)$$

| \bar{x} | y^2 | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-----------|-------|------------------|------------|------|------------|
| x | | (1 1 X 1) | | | |
| x | | (1 1 X X) | | | |

= 1.,

$$10. \quad f(A, B, C, \omega) = \sum (0, 6, 8, 13, 14)$$

$$f(A, B, C, \omega) = \sum (2, 9, 10)$$

| AB | CO | $\bar{C}\bar{A}$ | $\bar{C}A$ | $C\bar{A}$ | CA |
|------------------|----|------------------|------------|------------|------|
| $\bar{A}\bar{B}$ | D | | X | | |
| $\bar{A}B$ | X | | 1. | | |
| $A\bar{B}$ | 1 | | 1 | | |
| AB | 1 | | X | | |

$$\bar{B}\bar{D} + AB\bar{C}D + CD$$

Simplify the following Boolean functions, using K-maps

$$a. \quad f(w, x, y, z) = \sum (0, 1, 4, 5, 8, 12)$$

| wx | y^2 | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | | (1 1) | | | |
| $\bar{w}x$ | | (1 1) | | | |
| $w\bar{x}$ | | (1 1) | | | |
| wx | | (1) | | | |

★ $\bar{y}\bar{z} + \bar{w}\bar{y}$

b. $f(w,x,y,z) = \Sigma(9,6,9,11,12,14)$

| $w\bar{x}$ | $\bar{y}z$ | $\bar{x}z$ | yz | $x\bar{z}$ |
|------------------|------------|------------|------|------------|
| $\bar{w}\bar{x}$ | | | | |
| $\bar{w}x$ | 1 | | | 1 |
| $w\bar{x}$ | 1 | | | 1 |
| wx | 1 | 1 | | |

$$= \bar{y}\bar{z} + z \neq y\bar{z}$$

$$= x\bar{y}\bar{z} + z + x\bar{z}$$

c. $f(w,x,y,z) = \Sigma(1,3,5,6,7,9,11)$

| wx | xz | $\bar{y}z$ | $\bar{y}z$ | yz | $x\bar{z}$ |
|------------------|------|------------|------------|------|------------|
| $\bar{w}\bar{x}$ | | | | | |
| $\bar{w}x$ | | 1 | x | | |
| $w\bar{x}$ | | | | 1 | 1 |
| wx | 1 | | | | |

$$\bar{w}z + \bar{w}xy + \bar{x}z$$

$$= \bar{w}z + \bar{w}xy + xz$$

d. $f(w,x,y,z) = \Sigma(2,6,7,10,14,15)$

| wx | $y\bar{z}$ | $\bar{y}z$ | $\bar{y}z$ | xz | yz |
|------------------|------------|------------|------------|------|------|
| $\bar{w}\bar{x}$ | | | | | |
| $\bar{w}x$ | | | | 1 | 1 |
| $w\bar{x}$ | | | | 1 | 1 |
| wx | | | | 1 | 1 |

$$xy + x\bar{z}$$

Implement the following Boolean function f , together with the don't care condition, using not more than two NOR Gates.

$$f(A, B, C, D) = \Sigma(2, 4, 6, 10, 12)$$

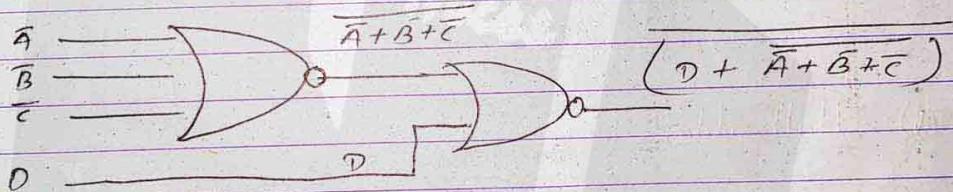
$$d(A, B, C, D) = \Sigma(0, 8, 9, 13) \quad (A')'$$

| | | | | | |
|------------|------------|------------------|------------|------------|------|
| | $\bar{A}B$ | $\bar{C}\bar{D}$ | $\bar{C}D$ | $C\bar{D}$ | CD |
| $\bar{A}B$ | X | | | | 1 |
| $\bar{A}B$ | | 1 | | | 1 |
| $A\bar{B}$ | | | 1 | X | |
| AB | | | | X | |
| $A\bar{B}$ | X | X | | | 1 |

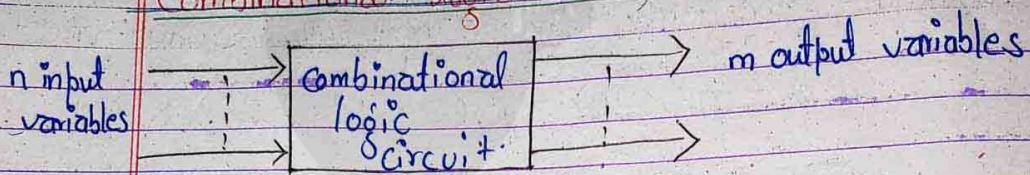
$$\begin{aligned} &= \overline{\bar{D}} (\overline{A} + \overline{B} + \overline{C}) \\ &= \{ \overline{\bar{D}} \cdot (\overline{A} + \overline{B} + \overline{C}) \}^1 \\ &= \overline{(D + \overline{A} + \overline{B} + \overline{C})} \end{aligned}$$

$(A+B) = \overline{\bar{A} \cdot \bar{B}}$

$(A \cdot B) = (\overline{\bar{A} + \bar{B}}) = \overline{\bar{C}\bar{D}} + \overline{\bar{C}D} + \overline{C\bar{D}}$



Combinational logic circuit

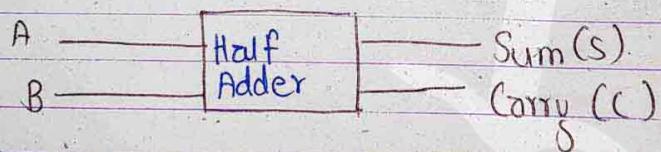


Block diagram of combinational logic circuit.

Adder
 ↗ Half adder
 ↘ full adder

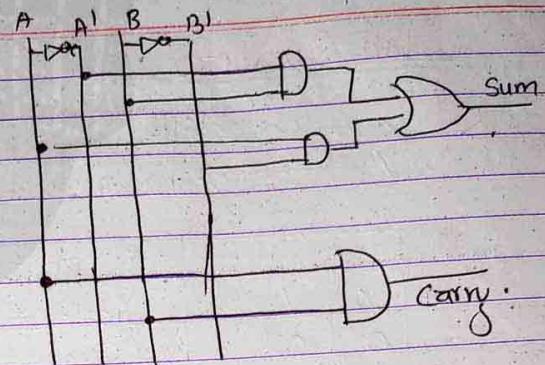
Subtractor
 ↗ Half subtractor
 ↘ full subtractor

(a) Half adder

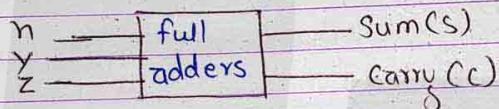


| Input | | Output | |
|-------|---|--------|---|
| A | B | S | C |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$$\begin{aligned} \text{Sum}(S) &= \bar{A}B + A\bar{B} \\ &= A \oplus B \\ \text{Carry } C &= AB \end{aligned}$$



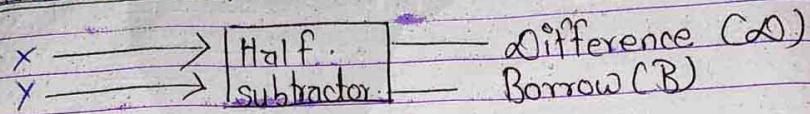
b. full adder:



| Input | | |
|-------|---|---|
| x | y | z |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

| Output | |
|--------|----------|
| Sum(S) | Carry(C) |
| 0 | 0 |
| 1 | 0 |
| 1 | 0 |
| 0 | 1 |
| 1 | 0 |
| 0 | 1 |
| 0 | 1 |
| 1 | 1 |

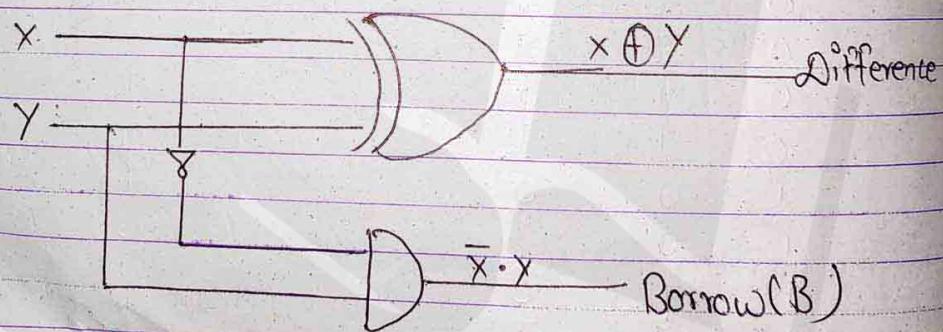
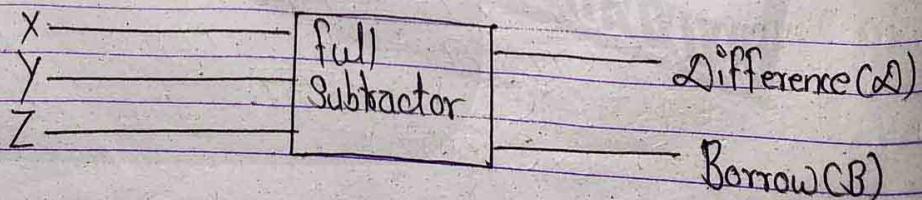
$$\begin{aligned} \text{Sum}(S) &\Rightarrow \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} \\ &\Rightarrow \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + y\bar{z}) \\ &\Rightarrow \bar{x}(y \oplus z) + x(y \oplus z) \\ &\Rightarrow (x \oplus y) \oplus z \end{aligned}$$

Half Subtractor

| Input | | Output | |
|-------|-----|--------|--------|
| X | Y | Diff | Borrow |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

$$\text{Difference } (D) = \bar{X}Y + X\bar{Y}$$

$$\text{Borrow } (B) = \bar{X}Y$$

Full Subtractor

| Input | | | Output | |
|-------|---|---|--------|--------|
| x | y | z | Diff | Borrow |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

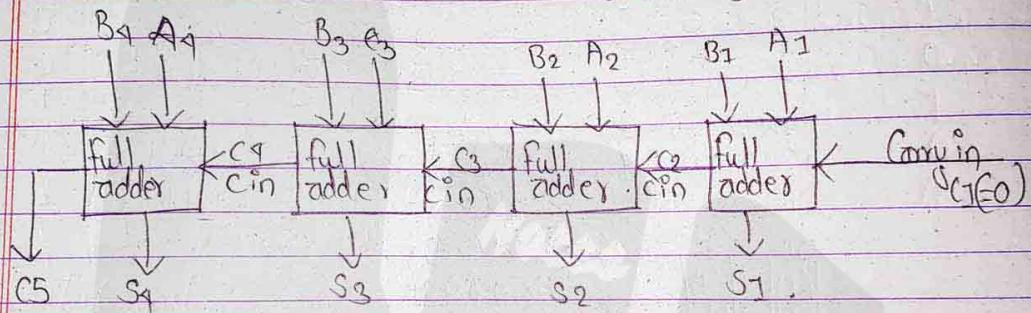
Difference (Δ) $\Rightarrow x'y'z + x'y'z' + xy'z' + xyz$
 $\Rightarrow z'(x'y + xy) + z(x'y' + xy)$
 $\Rightarrow z(x \oplus y) + z(x \oplus y)$
 $\Rightarrow x \oplus y \oplus z$

Borrow (B) $\Rightarrow \bar{x}yz + \bar{x}y\bar{z} + \bar{x}y\bar{z} + xyz$
 $\Rightarrow \bar{x}z(\bar{y} + y) + y(\bar{x}\bar{z} + xz)$
 $\Rightarrow \bar{x}z + y(x \oplus z)$
 $\Rightarrow y(x \oplus z) + \bar{x}z$

- i) Binary Parallel adder
- ii) Binary Parallel subtractor
- iii) Binary adder and subtractor

i. 4-bit binary parallel adder \rightarrow

$$\begin{aligned} A &\Rightarrow A_4 A_3 A_2 A_1 \\ B &\Rightarrow B_4 B_3 B_2 B_1 \end{aligned}$$



$$\therefore c_{in} = \text{Carry in}$$

Binary Parallel subtractor

$$A = 9 \quad B = 1$$

$$1001$$

$$0100$$

$$1011 \text{ (1's complement)}$$

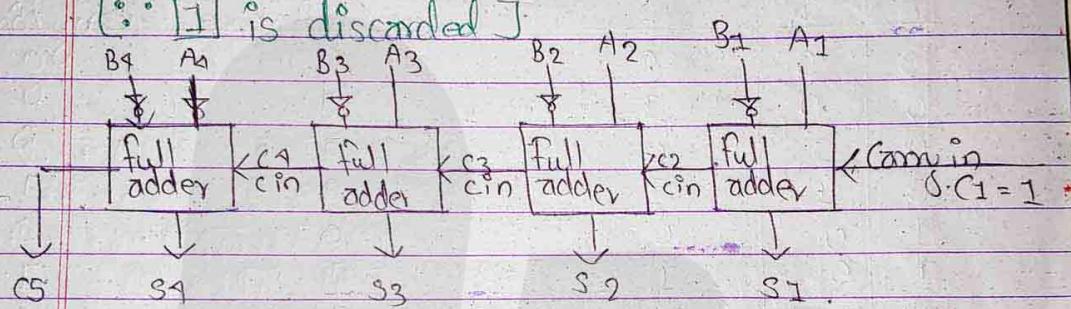
$$+1 \text{ (2's complement)}.$$

$$1100$$

\therefore The inverted form of 0100 is 1011 which is 1's complement & the addition of 1 in Lsp is 2's complement)

$$\begin{array}{r}
 101 \\
 - 1100 \\
 \hline
 001
 \end{array}
 \quad
 \begin{array}{r}
 100 \\
 + 1100 \\
 \hline
 10101
 \end{array}$$

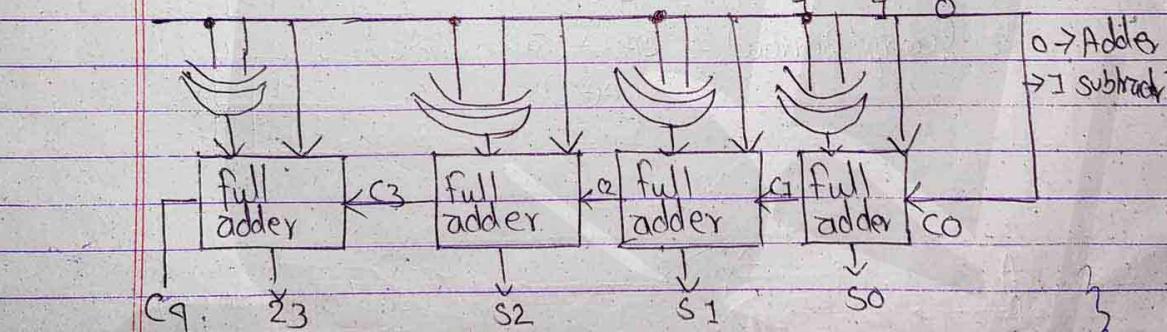
$\therefore [1]$ is discarded



iii. Binary adder and subtractor.

$$\begin{aligned}
 S & B \oplus 0 = B \\
 & B \oplus 1 = \bar{B}
 \end{aligned}$$

| X | 0 | R |
|---|---|---|
| A | 0 | 0 |
| | 0 | 1 |
| | 1 | 0 |



P.T.O

Adder subtractor \Rightarrow

$C=0$, addition

$C=1$, subtraction

| C | A | B | sum/difference | Carry/Borrow (S) |
|---|---|---|----------------|---------------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$$Y_1(\text{sum/difference}) = \bar{A}B + A\bar{B}$$

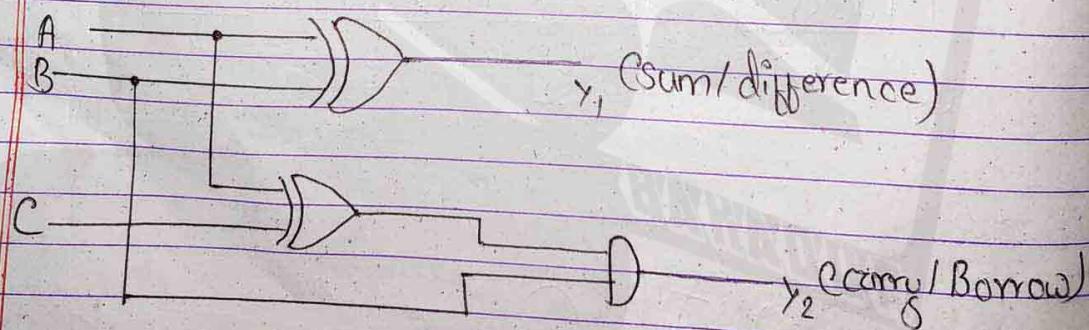
$$= A \oplus B$$

| C | A | B | 00 | 01 | 11 | 10 |
|---|---|---|----|----|----|----|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 |

$$Y_2(\text{Carry/Borrow}) = \bar{C}AB + C\bar{A}B$$

$$= B(C \oplus A)$$

| C | A | B | 00 | 01 | 11 | 10 |
|---|---|---|----|----|----|----|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 |

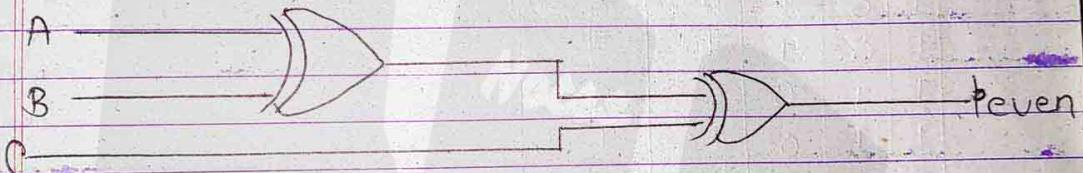


Parity Generator and checker.

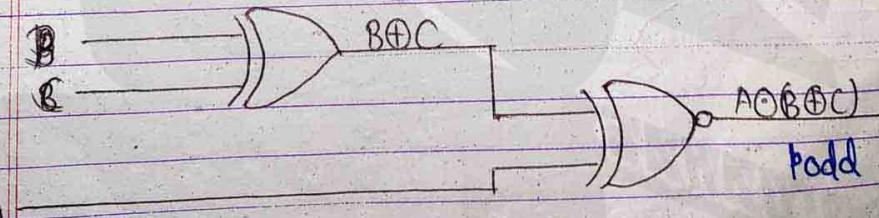
Information

| A | B | C | P _{odd} | P _{even} |
|---|---|---|------------------|-------------------|
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$$\begin{aligned}
 P_{\text{even}} &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\
 &= \bar{A}(C\bar{B} + B\bar{C}) + A(\bar{C}\bar{B} + BC) \\
 &= \bar{A}(CB \oplus C) + A(CB \oplus C) \\
 &= A \oplus B \oplus C
 \end{aligned}$$



$$\begin{aligned}
 P_{\text{odd}} &\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C} \\
 &= \bar{A}C\bar{B}\bar{C} + BC + A\bar{C}\bar{B}C + B\bar{C} \\
 &= \bar{A}(C\bar{B} \oplus C) + A(C\bar{B} \oplus C) \\
 &= A \odot (C\bar{B} \oplus C)
 \end{aligned}$$

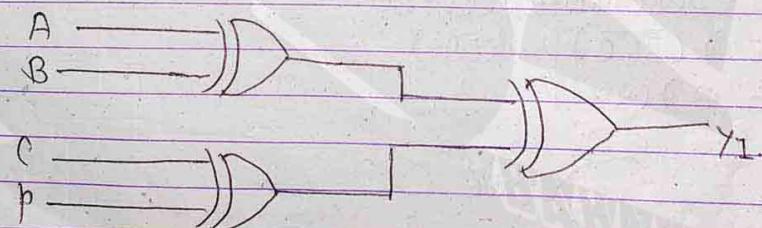


Parity checker (even):

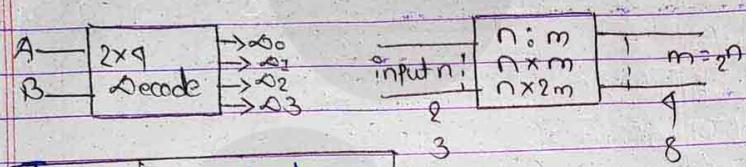
| A | B | C | P | P Check |
|---|---|---|---|---------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

| AB | CP | 00 | 01 | 11 | 10 |
|----|----|----|----|----|----|
| 00 | 0 | 0 | 1 | 0 | 1 |
| 01 | 1 | 1 | 0 | 1 | 0 |
| 11 | 0 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 1 | 1 | 0 |

$$\begin{aligned}
 Y_1 &= \bar{A}\bar{B}\bar{C}P + \bar{A}\bar{B}C\bar{P} + A\bar{B}\bar{C}\bar{P} + AB\bar{C}P + \\
 &\quad A\bar{B}CP + ABC\bar{P} + A\bar{B}\bar{C}\bar{P} + A\bar{B}CP \\
 &\hat{=} \bar{A}\bar{B}C(C\oplus P) + \bar{A}B(C\oplus P) + A\bar{B}(C\oplus P) \\
 &= (C\oplus P)(A\oplus B) + (C\oplus P)(A\oplus B) \\
 &\hat{=} (A\oplus B\oplus C\oplus P),
 \end{aligned}$$

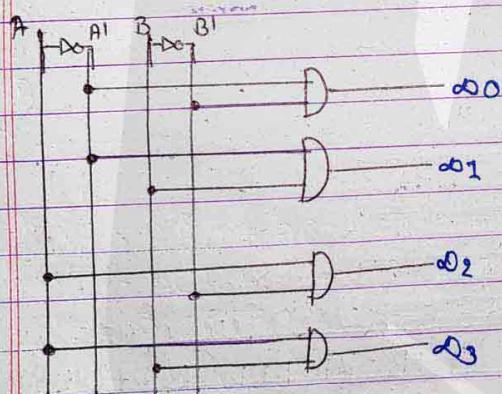


Encode & Decode :-

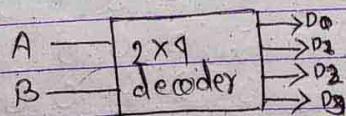


| Input | Output |
|-------|-------------|
| A B | D0 D1 D2 D3 |
| 0 0 | 1 0 0 0 |
| 0 1 | 0 1 0 0 |
| 1 0 | 0 0 1 0 |
| 1 1 | 0 0 0 1 |

$$D_0 = \bar{A}\bar{B}, D_1 = \bar{A}B, D_2 = A\bar{B}, D_3 = AB$$

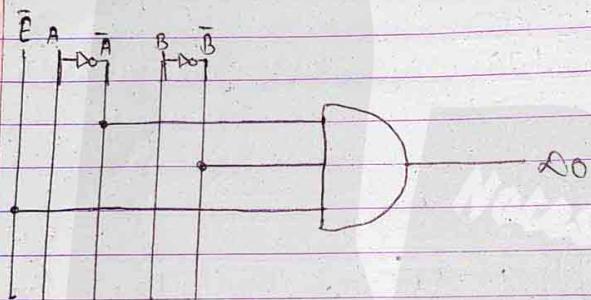


Decoder with enable pin :-



| Input | | | Output | | | |
|-------|---|---|----------------|----------------|----------------|----------------|
| F | A | B | D ₀ | D ₁ | D ₂ | D ₃ |
| 1 | x | x | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |

$$D_0 = A'B'E', D_1 = A'B'E, D_2 = AB'E', D_3 = AB'E$$



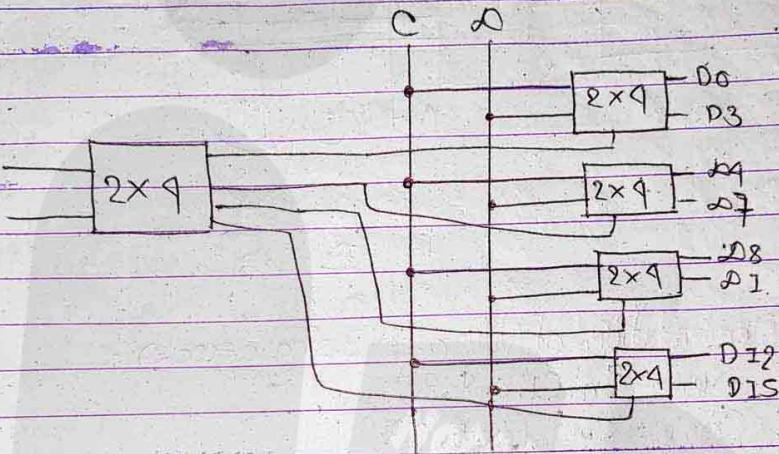
Design a 8x8 decoder using 2x4 decoder.

Let F be the MSB for input.



\therefore Note :- MSB \Rightarrow Most significant Bits

Design a 4×4 demultiplexer using 2×4 decoder.



Implementation of Boolean function using decoder :-

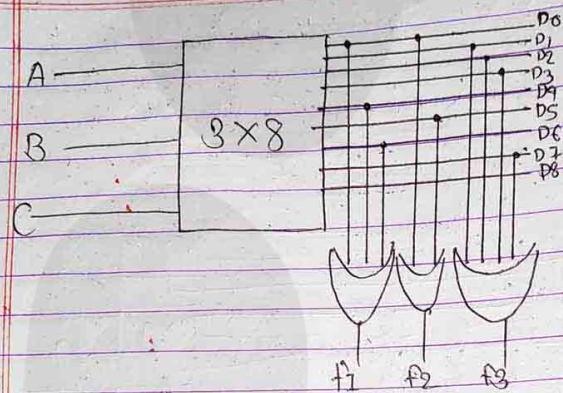
$$f_1 = \Sigma m(0, 4, 6), f_2 = \Sigma m(0, 5) f_3 = \Sigma m(1, 2, 5, 7)$$

$$f_1 = \Sigma m(0, 4, 6) \\ = \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC \quad (i)$$

$$f_2 = \Sigma m(0, 5) \\ = \bar{A}\bar{B}\bar{C} + A\bar{B}C \quad (ii)$$

$$f_3 = \Sigma m(1, 2, 5, 7) \\ = \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + ABC \quad (iii)$$

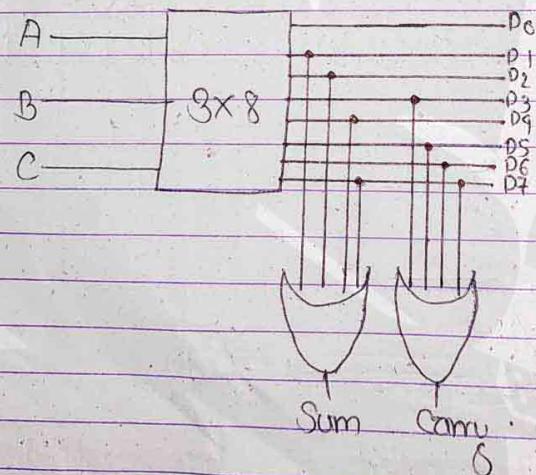
(P.T.O)



Implementation of full adder using decoder

$$\text{Sum} = \Sigma m' (1, 2, 4, 7)$$

$$\text{Carry} = \Sigma m' (3, 5, 6, 7)$$



(P + Q)

| A | B | C | Sum | Carry |
|---|---|---|-----|-------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Sum = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$
Carry = $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

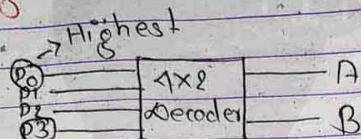
* Encoder

| Input | | | | Output | |
|-------|-------|-------|-------|--------|---|
| D_0 | D_1 | D_2 | D_3 | A | B |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |

$A = D_2 \oplus D_3$
 $B = D_1 + D_3$

$D_0 \quad D_1 \quad D_2 \quad D_3$

Briarity Encoder :-

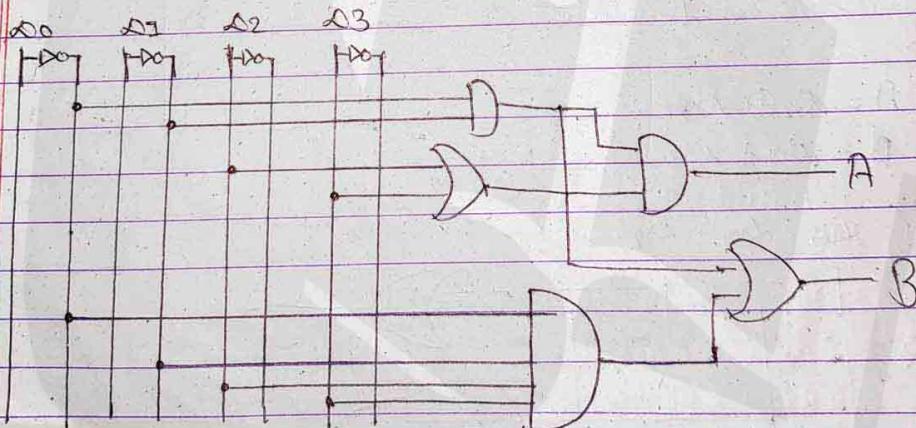


| input | | | | output | |
|-------|----|----|----|--------|---|
| D0 | D1 | D2 | D3 | A | B |
| 1 | X | X | X | 0 | 0 |
| 0 | 1 | X | X | 0 | 1 |
| 0 | 0 | 1 | X | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |

$$A = \overline{D_0} \overline{D_1} D_2 + \overline{D_0} D_1 \overline{D_2} D_3$$

$$= \overline{D_0} \overline{D_1} (D_2 + D_3)$$

$$B = \overline{D_0} D_1 + \overline{D_0} D_1 \overline{D_2} D_3$$

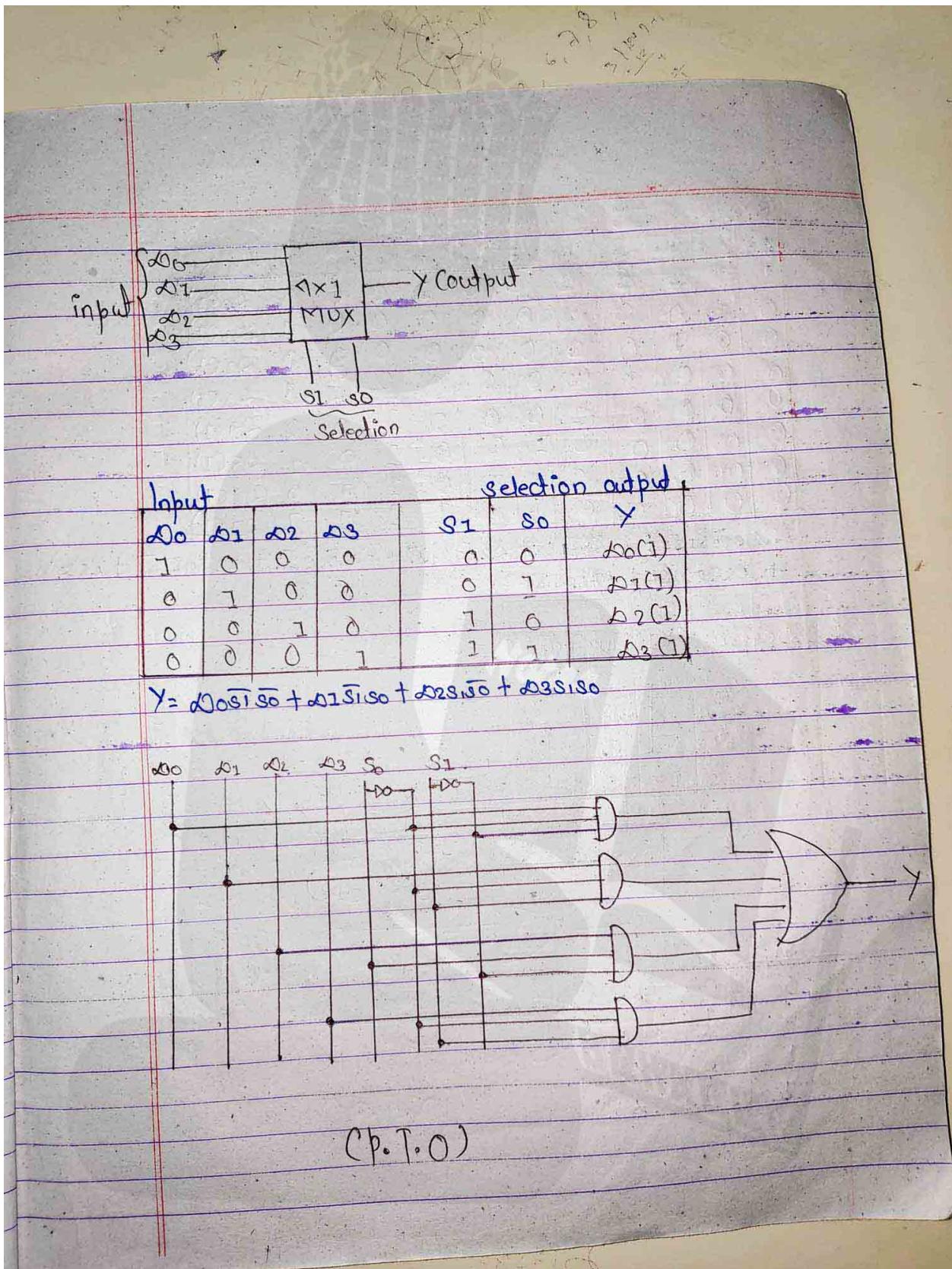


Multiplexer and Demultiplexer (Mux & Demux)

Multiplexer :

* Many to one

(4x1 Multiplexer)

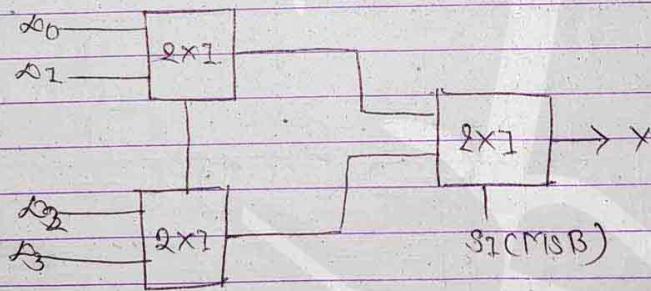


(8x1)

| s_0 | s_1 | s_2 | s_3 | s_4 | s_5 | s_6 | s_7 | s_0 | s_1 | s_2 | y |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------------|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $s_0(s_1)$ |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | $s_1(s_1)$ |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $s_2(s_1)$ |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $s_3(s_1)$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | $s_4(s_1)$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | $s_5(s_1)$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | $s_6(s_1)$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | $s_7(s_1)$ |

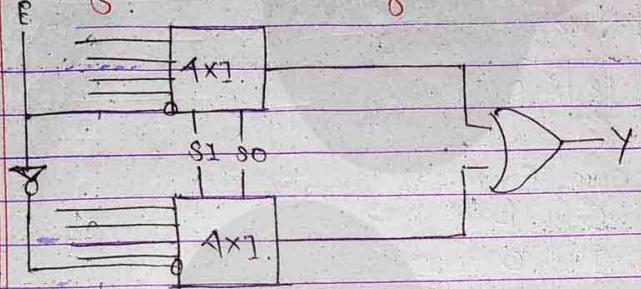
$$Y = s_0s_1s_2s_3 + s_1s_2s_3s_4 + s_2s_3s_4s_5 + s_3s_4s_5s_6 + s_4s_5s_6s_7 + s_5s_6s_7s_0 \\ + s_6s_7s_0s_1 + s_7s_0s_1s_2$$

* Design 4x1 Mux using 2x1

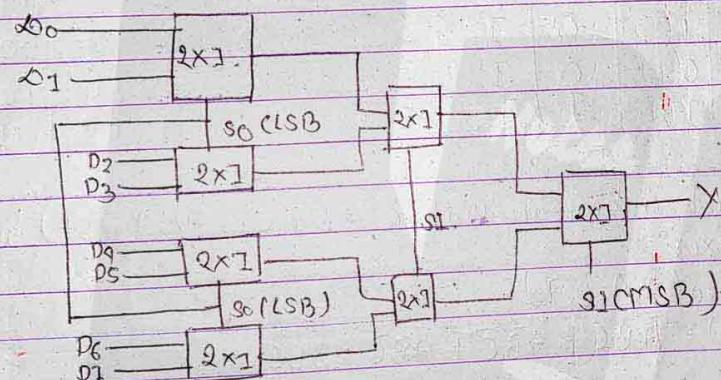


| s_1 | s_0 | Input |
|-------|-------|-------|
| 0 | 0 | d0 |
| 0 | 1 | d1 |
| 1 | 0 | d2 |
| 1 | 1 | d3 |

* Design 8x1 Mux using 4x1.



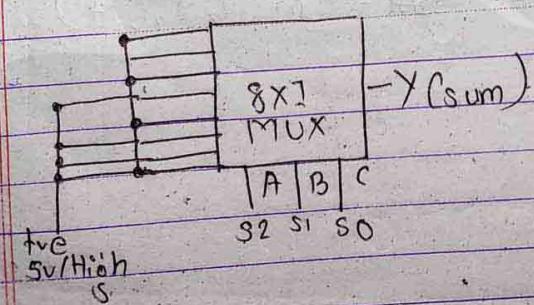
using ΔX_1

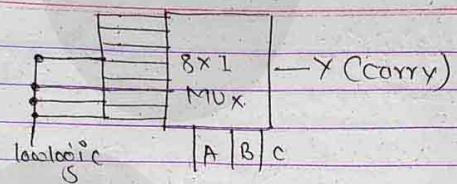


Design full adder using multiplexer

$$fsum = (1, 2, 1, 7)$$

$$f_{\text{corr}} = \{3, 5, 6, 7\}$$



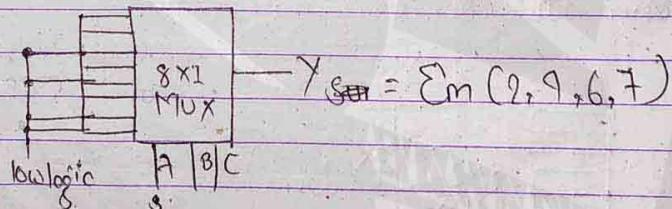


| A | B | C | Sum | Carry |
|---|---|---|-----|-------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Implement a boolean function $f = \sum m(2, 9, 6, 7)$ by using Multiplexor

$$f = \sum m(2, 9, 6, 7)$$

$$f(ABC) = \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$$



| A | B | C | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

(de-Multiplexer (deMux))

→ One to Many

$\xrightarrow{1 \times 2^n}$ $\leftarrow 2^n$
de-MUX

n = no of selection

Input (2) ————— $\xrightarrow{1 \times 4}$ Output
S1 S0 y_0 y_1 y_2 y_3

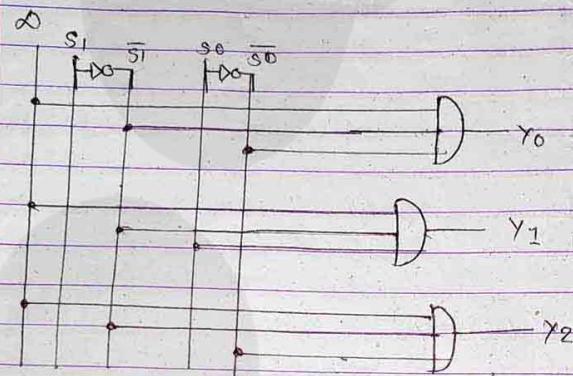
| Input | Selection | Output: |
|-------|-----------|---------------|
| 0 | 0 0 | y_0 0 0 0 0 |
| 0 | 0 1 | 0 y_2 0 0 |
| 0 | 1 0 | 0 0 y_3 0 |
| 0 | 1 1 | 0 0 0 y_1 |

$$Y_0 = A \bar{S}_1 \bar{S}_0$$

$$Y_1 = A \bar{S}_1 S_0$$

$$Y_2 = A S_1 \bar{S}_0$$

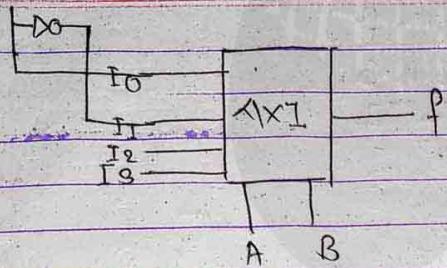
$$Y_3 = A S_1 S_0$$



1. Implement $f(A, B, C) = \sum m(1, 2, 4, 5)$ using Mux (using 1x1 Mux)

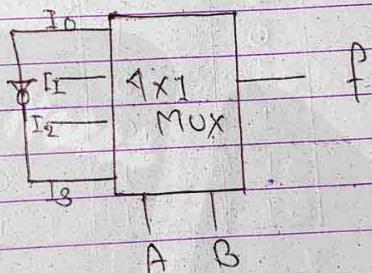
$$f(A, B, C) = \sum m(1, 2, 4, 5)$$

| | A | B | C | f |
|-----|---|---|---|---|
| 101 | 0 | 0 | 0 | 0 |
| 101 | 0 | 0 | 1 | 1 |
| 111 | 0 | 1 | 0 | 1 |
| 111 | 0 | 1 | 1 | 0 |
| 111 | 1 | 0 | 0 | 1 |
| 111 | 1 | 0 | 1 | 1 |
| 111 | 1 | 1 | 0 | 0 |
| 111 | 1 | 1 | 1 | 0 |

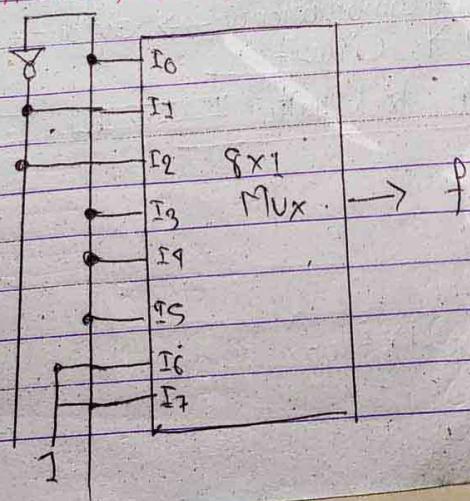


2. Implement $f(A, B, C) = \sum m(1, 3, 5, 6)$ using MUX using 1x1.

| A | B | C | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

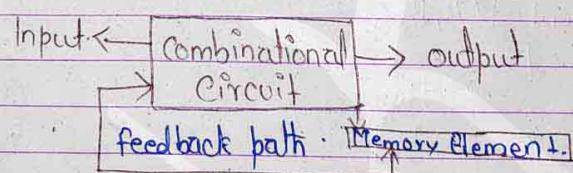


3. Implement $f(A, B, C, D) = \sum m(1, 2, 4, 7, 11, 12, 13, 14, 15)$



| A | B | C | D | f |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Sequential circuit :-



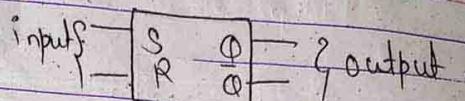
1. S-R (Set-Reset)
2. Delay (D)
3. J-k.
4. Toggle flip flop (T-flip flop)
5. Master slave flip flop

I. S-R flip flop \Rightarrow

S-R latch

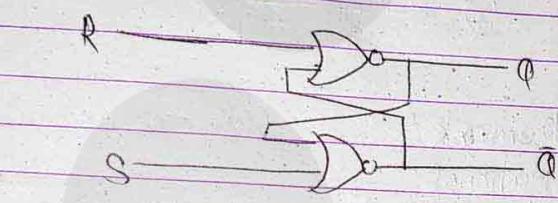
• NOR based

• NAND based



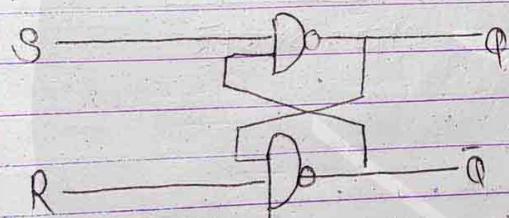
NOR

| A | B | X |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

S R Q \bar{Q} Remarks

| | | | | |
|---|---|---|---|------------|
| 0 | 0 | 0 | 1 | Not Change |
| 0 | 1 | 0 | 1 | Reset |
| 1 | 0 | 1 | 0 | Set |
| 1 | 1 | ? | ? | Invalid |

NAND based



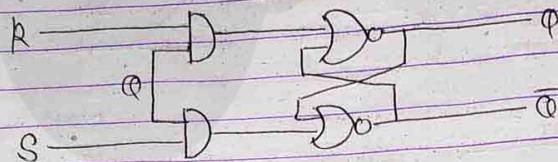
NAND

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

S R Q \bar{Q} Remarks

| | | | | |
|---|---|---|---|------------|
| 0 | 0 | 0 | 1 | Invalid |
| 0 | 1 | 1 | 0 | Set |
| 1 | 0 | 0 | 1 | Reset |
| 1 | 1 | 0 | 1 | Not Change |

S-R flip flop \Rightarrow



Truth table

| S | R | $Q + \bar{Q}$ | $\bar{Q} + \bar{Q}$ | Remark |
|---|---|---------------|---------------------|-----------|
| 0 | 0 | 0+1 | 0+1 | No change |
| 0 | 1 | 0 | 1 | Reset |
| 1 | 0 | 1 | 0 | Set |
| 1 | 1 | ? | ? | Invalid |

Characteristic table

Q_t S R Q_{t+1}

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | X |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | X |

| Q_t | SR | | | |
|-------|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 0 | | | X | 1 |
| 1 | | | 1 | X |

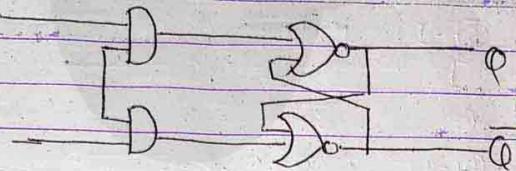
$$Q_{t+1} = S + Q_t \bar{R}$$

Execution table

Q_t Q_{t+1} S R

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |

D flip flop :-



Truth table

$D \quad Q_t$

0 0

1 1

Characteristics table

$Q_t \quad D \quad Q_{t+1}$

0 0 0

0 1 1

1 0 0

1 1 1

Execution table

$Q_t \quad Q_{t+1} \quad D$

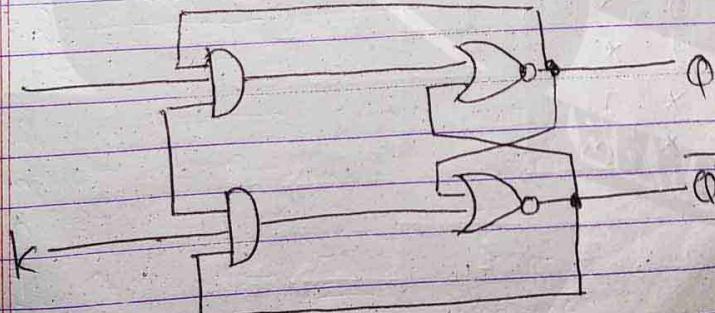
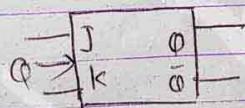
0 0 0

0 1 1

1 0 0

1 1 1

Jk flip-flop



| Truth table : | | | | Remarks |
|---------------|---|-------|-------------|--------------|
| J | k | Q_t | \bar{Q}_t | |
| 0 | 0 | 0 | 1 | No change |
| 0 | 1 | 0 | 1 | Reset |
| 1 | 0 | 1 | 0 | Set |
| 1 | 1 | 1 | 0 | Toggle SG |

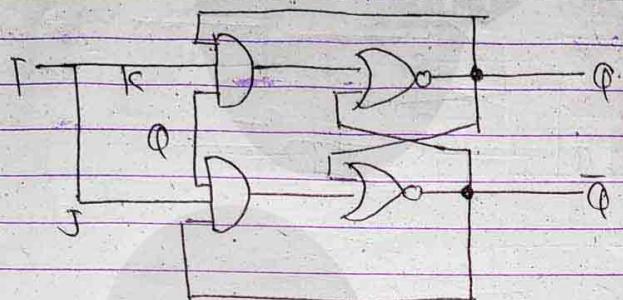
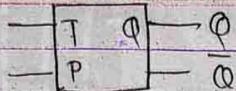
Characteristics table

| Q_t | J | k | Q_{t+1} | \bar{Q}_{t+1} | J_k |
|-------|---|---|-----------|-----------------|-------|
| 0 | 0 | 0 | 0 | 1 | - |
| 0 | 0 | 1 | 0 | 1 | - |
| 0 | 1 | 0 | 1 | 0 | - |
| 0 | 1 | 1 | 1 | 0 | - |
| 1 | 0 | 0 | 1 | 0 | - |
| 1 | 0 | 1 | 0 | 1 | - |
| 1 | 1 | 0 | 1 | 0 | - |
| 1 | 1 | 1 | 0 | 1 | - |

Execution table

| Q_t | Q_{t+1} | J | k |
|-------|-----------|---|---|
| 0 | 0 | 0 | x |
| 0 | 1 | 1 | x |
| 1 | 0 | x | 1 |
| 1 | 1 | x | 0 |

T-flipflop (Logicle) \Rightarrow



Truth table

$T \quad Q+$

0 0

1 1

Characteristic table

$Q_t \quad T \quad Q_{t+1}$

0 0 0 No change

0 1 1 Toggle

1 0 1 No change

1 1 0 Toggle

SS

Execution table

$Q+ \quad Q_{t+1} \quad T$

0 0 0

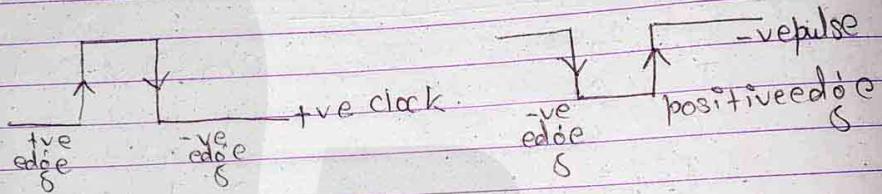
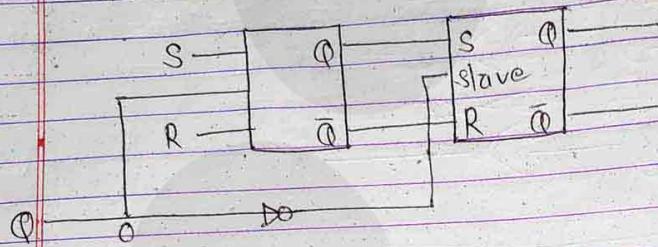
0 1 1

1 0 1

1 1 0

($\neg Q \cdot T \cdot Q$)

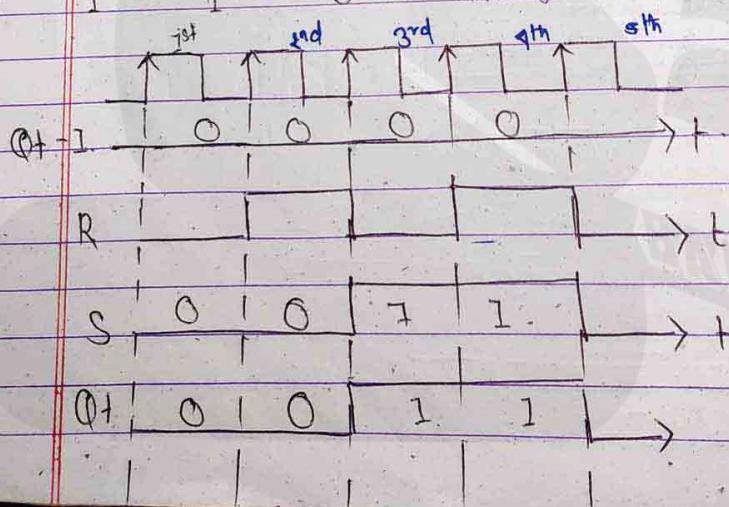
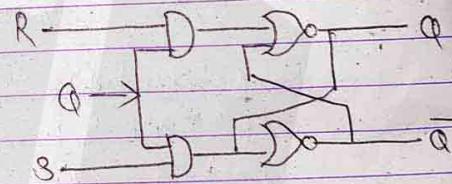
Master-slave flip-flop.



Timing diagram of flip-flop:

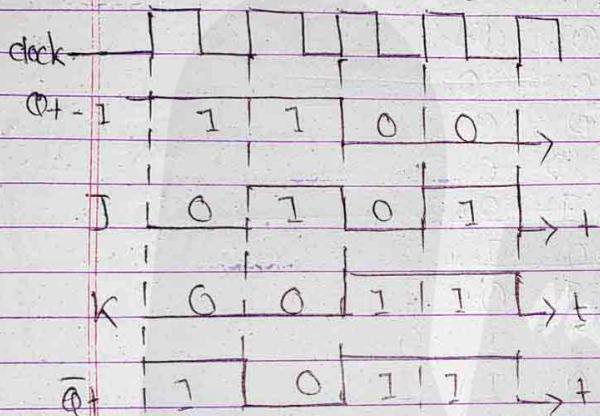
S-R-S

| S | R | Q at | Remarks |
|---|---|------|---------------|
| 0 | 0 | 0 | t-1 No change |
| 0 | 1 | 0 | Reset |
| 1 | 0 | 1 | Set |
| 1 | 1 | 0 | Invalid |



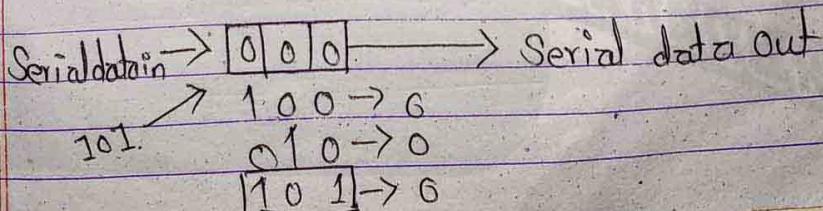
J-k

| J | k | Q_t | Remarks |
|---|---|-----------|--------------|
| 0 | 0 | Q_{t-1} | No change |
| 0 | 1 | 0 | Reset |
| 1 | 0 | 1 | Set |
| 1 | 1 | Q_{t+1} | Toggle SS |

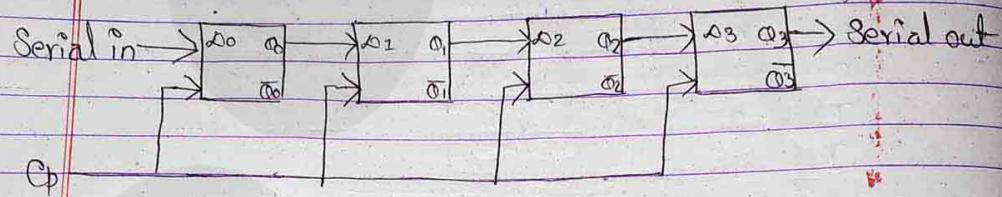


Shift - register

- i) Serial In Serial Out (SISO)
- ii) Serial In Parallel out (SIPO)
- iii) Parallel In Parallel out (PIPO)
- iv) Parallel In Serial out (PISO)

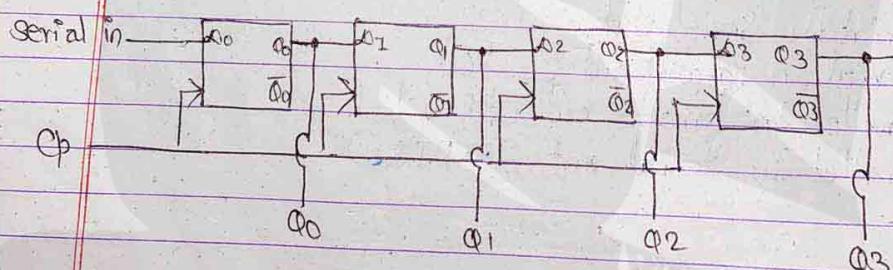


1. Serial In Serial Out



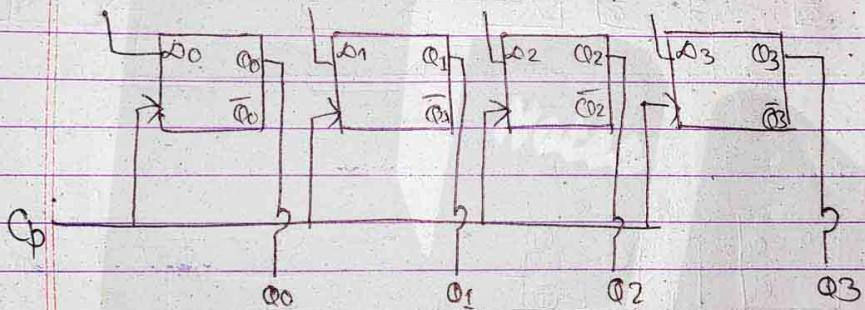
| Clock | Q ₀ | Q ₁ | Q ₂ | Q ₃ |
|-----------------|----------------|----------------|----------------|----------------|
| Initial | 0 | 0 | 0 | 0 |
| 1 st | 1 | 0 | 0 | 0 |
| 2 nd | 0 | 1 | 0 | 0 |
| 3 rd | 1 | 0 | 1 | 0 |
| 4 th | 1 | 1 | 0 | 1 |
| 5 th | 0 | 1 | 1 | 0 |
| 6 th | 0 | 0 | 1 | 1 |
| 7 th | 0 | 0 | 0 | 1 |
| 8 th | 0 | 0 | 0 | 1 |

2. Serial In Parallel Out :-



| Clock | Q_0 | Q_1 | Q_2 | Q_3 | (0011) |
|---------|-------|-------|-------|-------|-------------------------|
| Initial | 0 | 0 | 0 | 0 | |
| 1st | 1 | 0 | 0 | 0 | |
| 2nd | 1 | 1 | 0 | 0 | |
| 3rd | 0 | 1 | 1 | 0 | |
| 4th | 0 | 0 | 1 | 1 | |
| | ↓ | ↓ | ↓ | ↓ | $Q_0 \ Q_1 \ Q_2 \ Q_3$ |

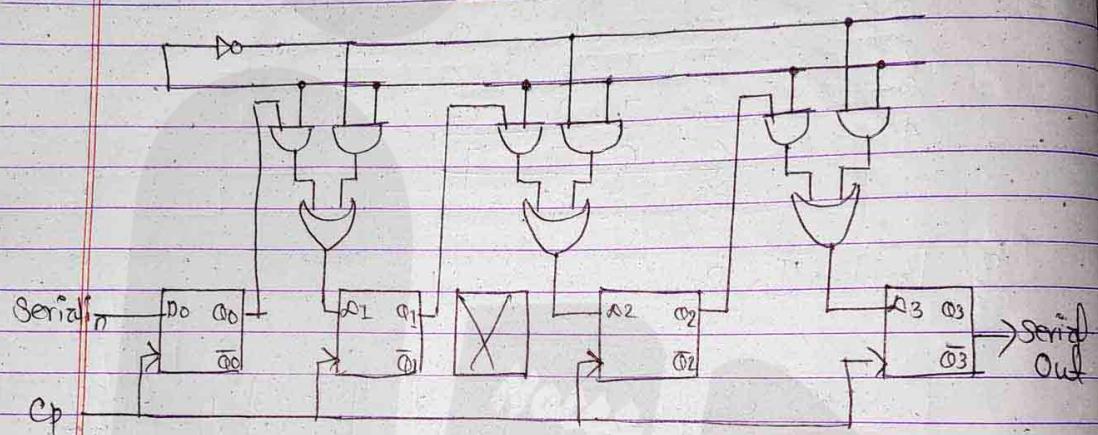
3. Parallel In Parallel Out



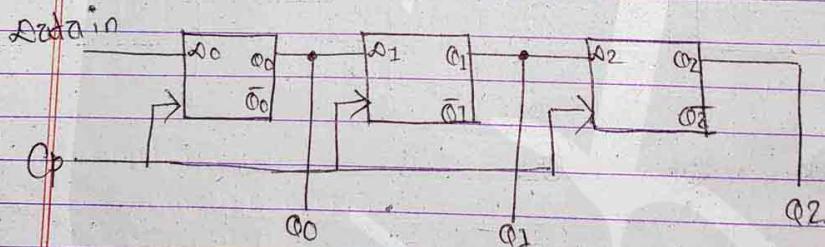
| Clock | Q_0 | Q_1 | Q_2 | Q_3 | |
|---------|-------|-------|-------|-------|-------------------------|
| Initial | 0 | 0 | 0 | 0 | |
| 1st | 0 | 0 | 1 | 1 | |
| | ↓ | ↓ | ↓ | ↓ | $Q_0 \ Q_1 \ Q_2 \ Q_3$ |

(P. T. O)

1. Parallel in Serial out :-



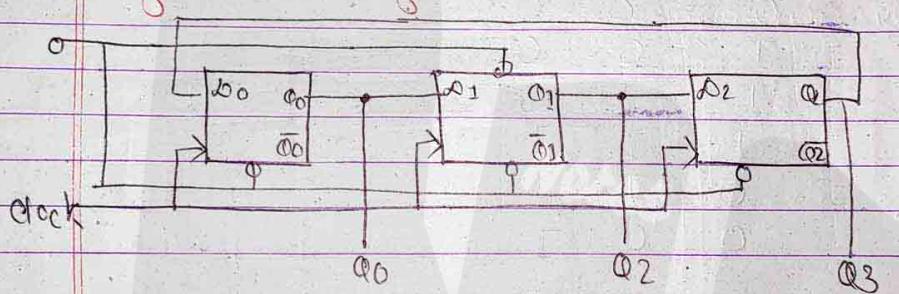
STPO :-



| clock | Q0 | Q1 | Q2 |
|-----------------|----|----|----|
| Initial | 0 | 0 | 0 |
| 1 st | 1 | 0 | 0 |
| 2 nd | 0 | 1 | 0 |
| 3 rd | 1 | 0 | 1 |

| | 1 st | 2 nd | 3 rd | 4 th |
|----------------|-----------------|-----------------|-----------------|-----------------|
| Q ₀ | 1 | 0 | 1 | |
| Q ₁ | | | 0 | |
| Q ₂ | | | 1 | |

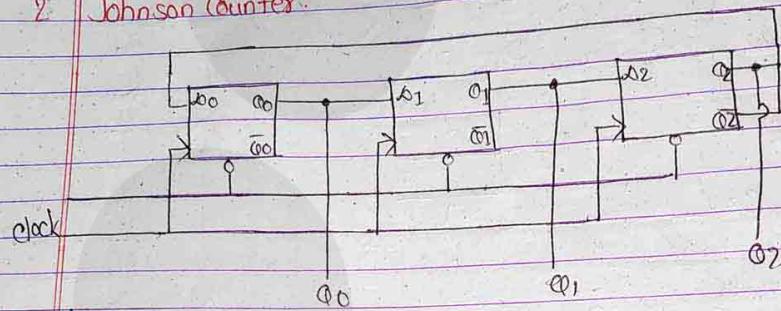
(1) Ring Counter (Straight Counter) (Q₀Q₁Q₂Q₃)



| clock | Q ₀ | Q ₁ | Q ₂ |
|-----------------|----------------|----------------|----------------|
| Initial | 0 | 1 | 0 |
| 1 st | 0 | 0 | 1 |
| 2 nd | 1 | 0 | 0 |
| 3 rd | 0 | 1 | 0 |

| | Q ₀ | Q ₁ | Q ₂ |
|----------------|----------------|----------------|----------------|
| Q ₀ | 0 | 0 | 1 |
| Q ₁ | 1 | 0 | 0 |
| Q ₂ | 0 | 1 | 0 |

2 Johnson Counter

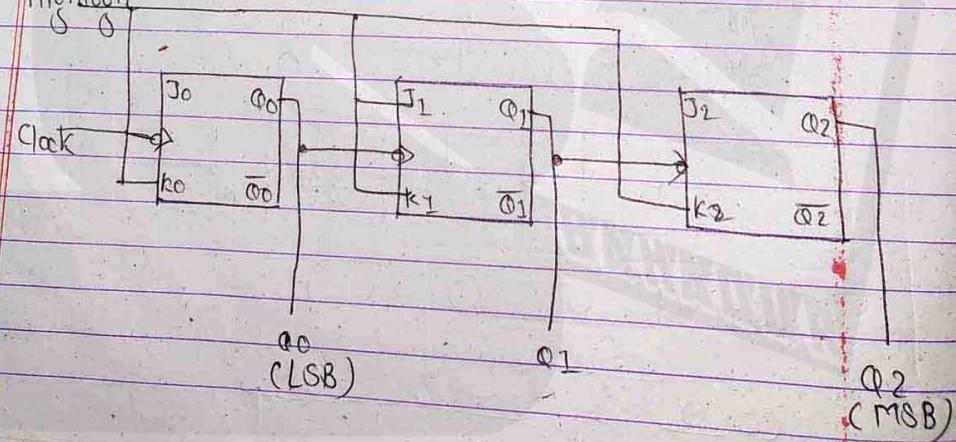


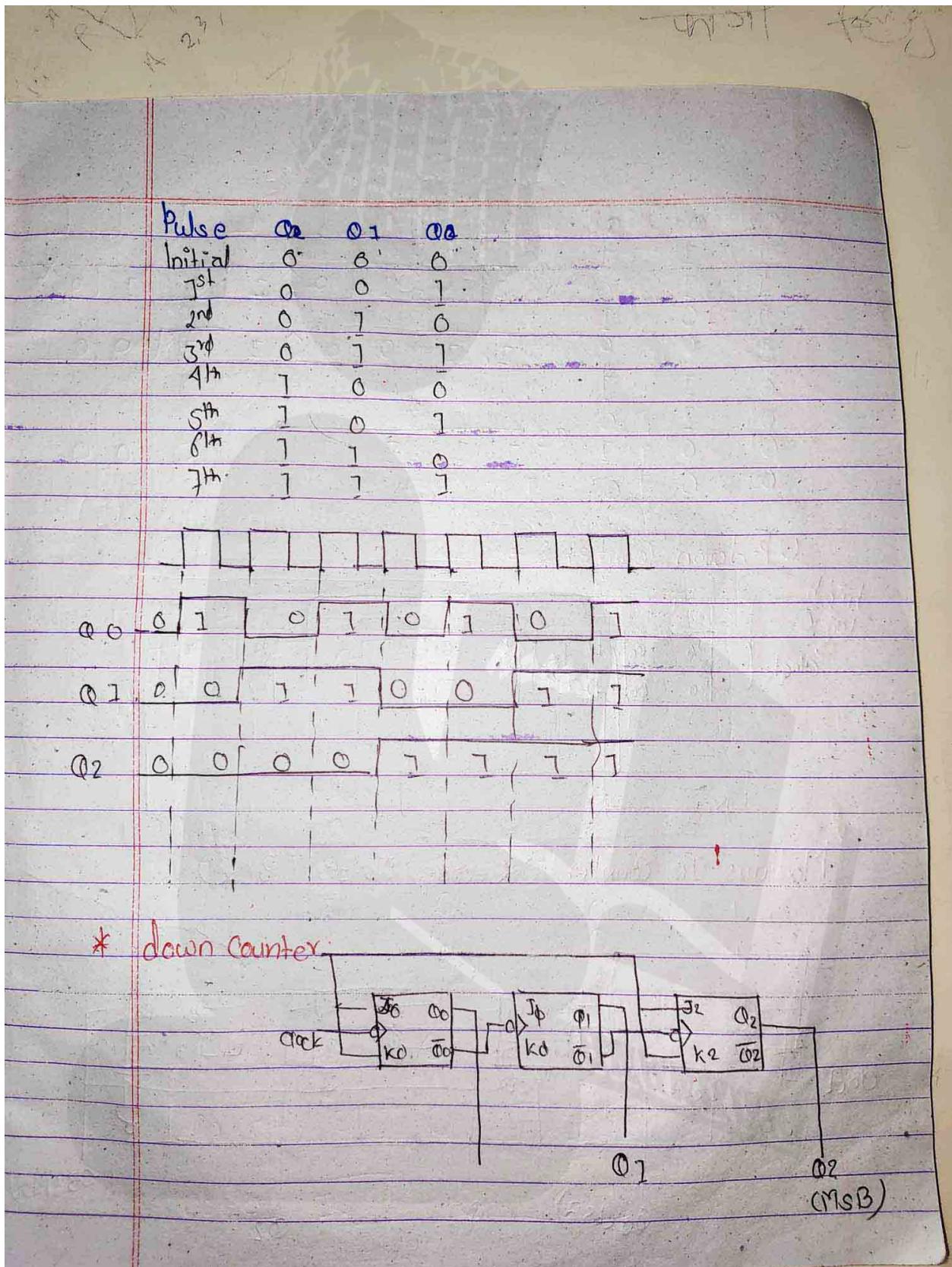
| Pulse | Q_0 | Q_1 | Q_2 |
|-----------------|-------|-------|-------|
| Initial | 0 | 0 | 0 |
| 1 st | 1 | 0 | 0 |
| 2 nd | 1 | 1 | 0 |
| 3 rd | 1 | 1 | 1 |
| 4 th | 0 | 1 | 1 |
| 5 th | 0 | 0 | 1 |

Counter :-

I Synchronous Counter :-

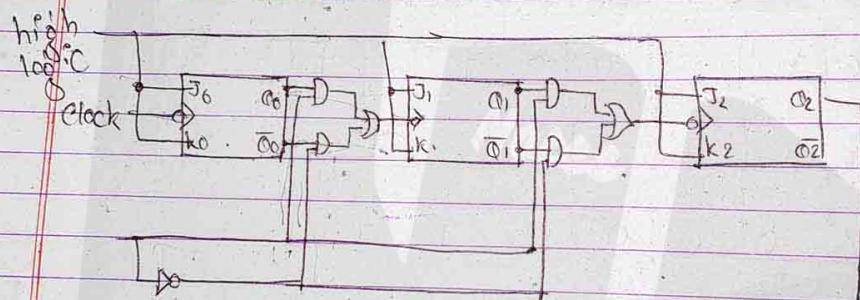
high logic 1



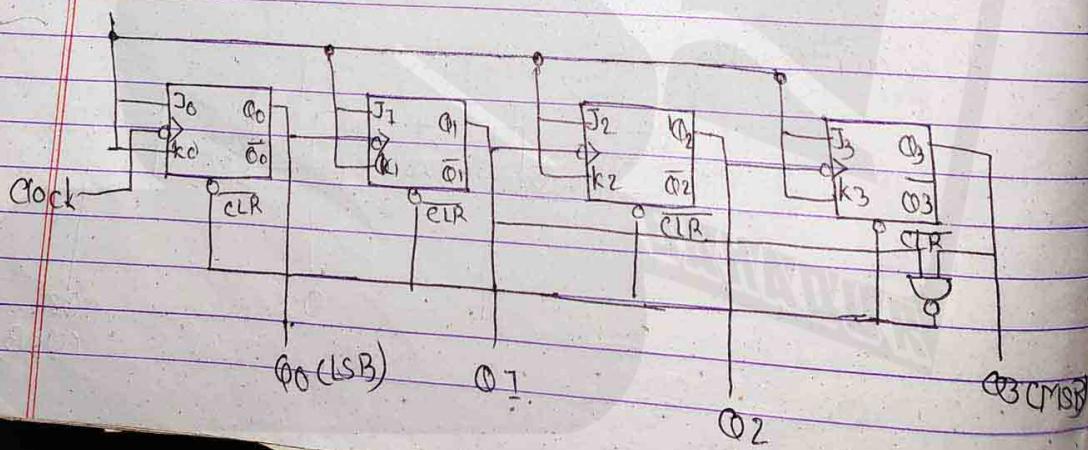


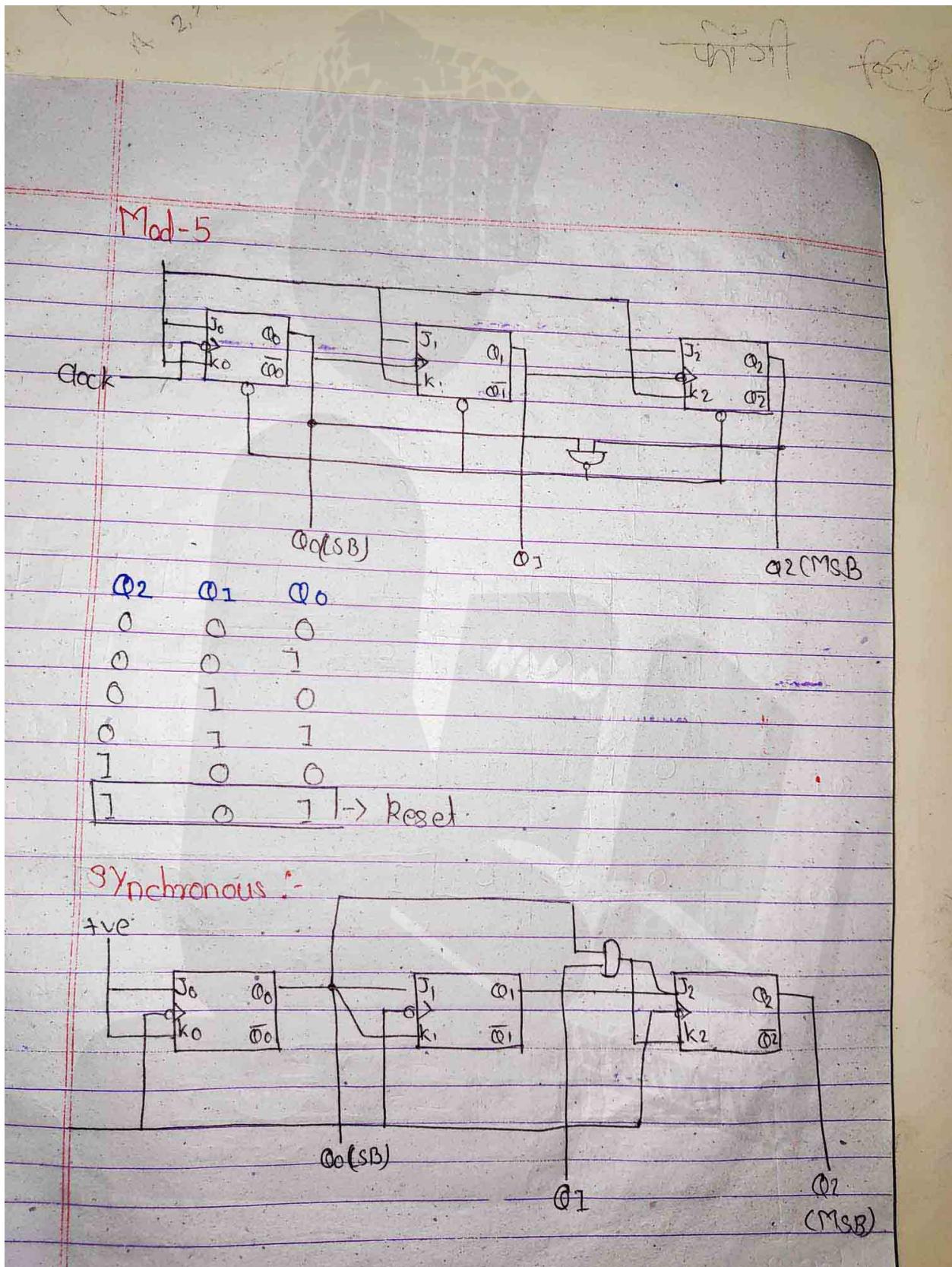
| Q_2 | Q_1 | Q_0 | | | | | | | | | | |
|-------|-------|-------|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Up-down Counter :-

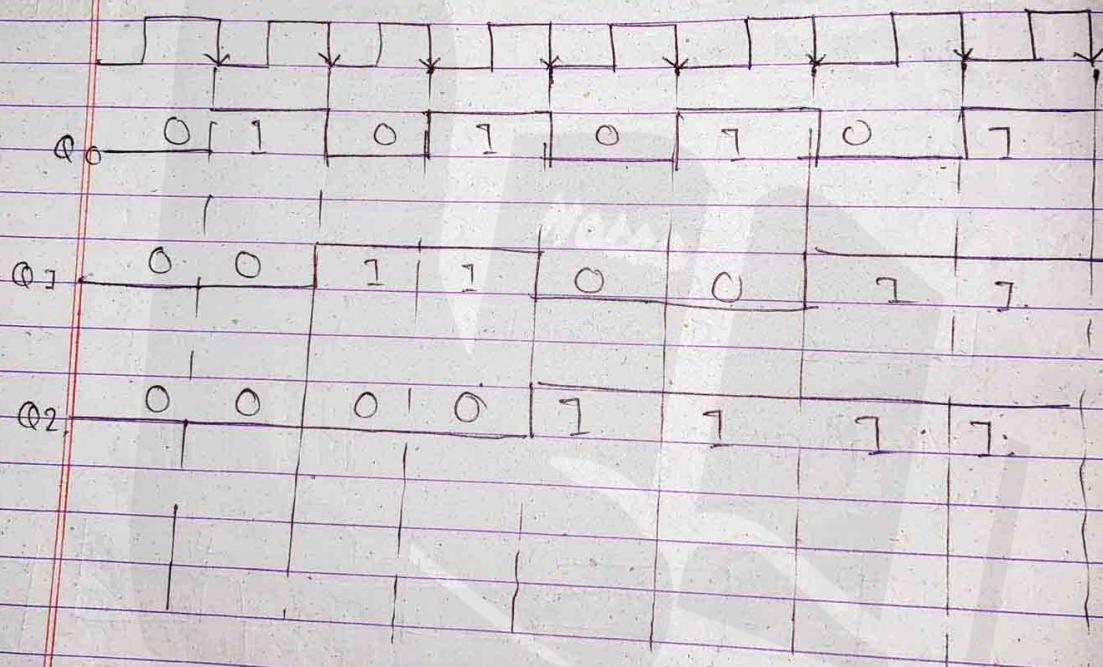


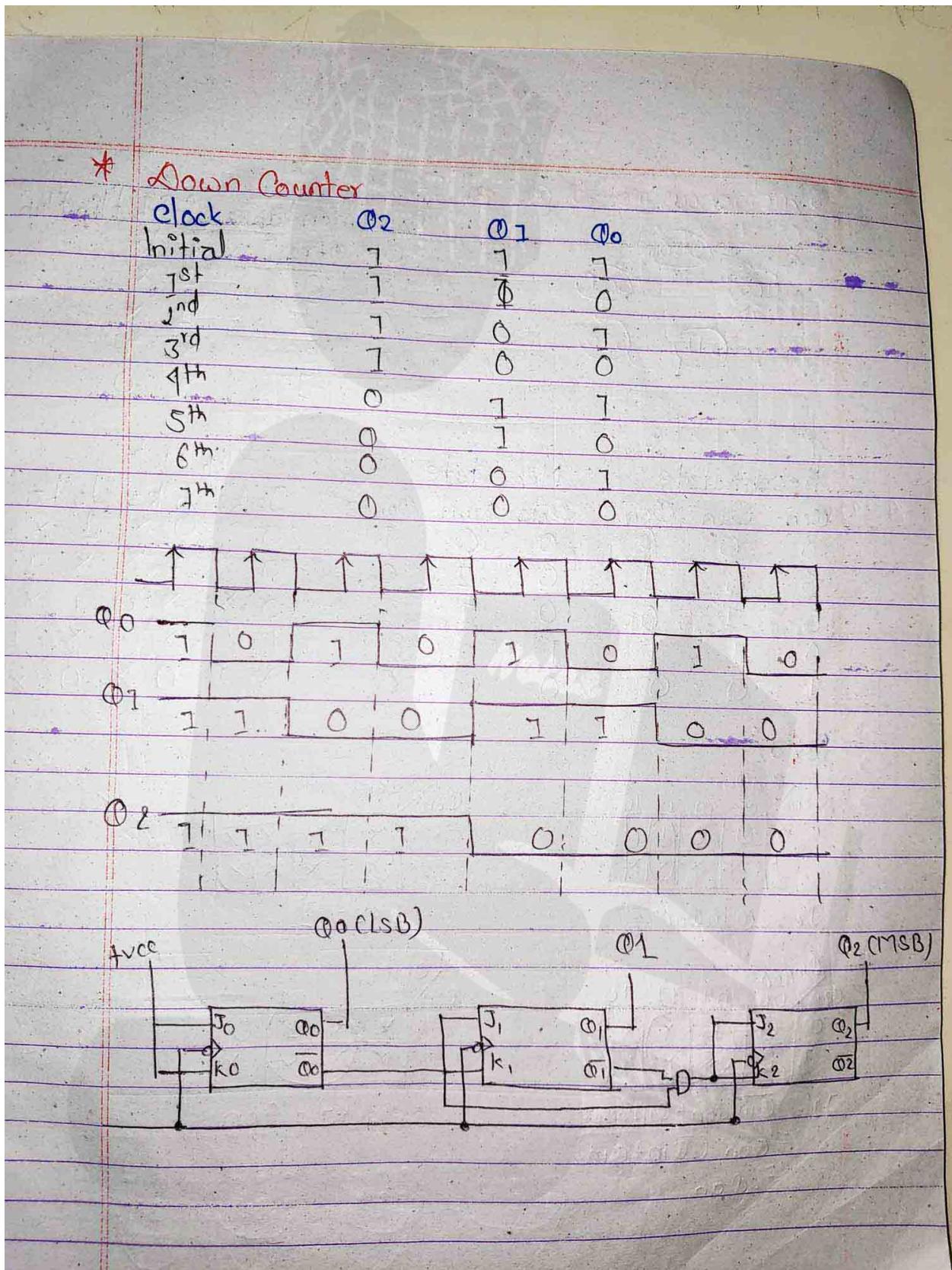
Modulus 10 Counter (Decade / Mo-10 / BCD)



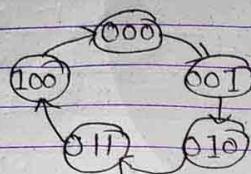


| Clock | Q_2 | Q_1 | Q_0 |
|-----------------|-------|-------|-------|
| Initial | 0 | 0 | 0 |
| 1 st | 0 | 0 | 1 |
| 2 nd | 0 | 1 | 0 |
| 3 rd | 0 | 1 | 1 |
| 4 th | 1 | 0 | 0 |
| 5 th | 1 | 0 | 1 |
| 6 th | 1 | 1 | 0 |
| 7 th | 1 | 1 | 1 |





Synchronous mod-5 Counter :-



Execution table of J-k flip flop

| Q_t | Q_{t+1} | J | K |
|-------|-----------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

| Present state Q_{2n} \bar{Q}_{1n} Q_{0n} | Next state Q_{2n+1} \bar{Q}_{1n+1} Q_{0n+1} | | | flip flop | | | |
|---|--|-------|------------|------------|---|-------|-------|
| | J_2 | k_2 | J_1, k_1 | J_0, k_0 | | | |
| 0 0 0 | 0 | 0 | 1 | 0 | X | 0 | 1 X |
| 0 0 1 | 0 | 1 | 0 | 0 | X | 1 X | X 1 |
| 0 1 0 | 0 | 1 | 1 | 0 | X | X 0 | 1 X |
| 0 1 1 | 1 | 0 | 0 | 0 | X | X 1 X | 1 |
| 1 0 0 | 0 | 0 | 0 | 0 | X | 1 0 | X 0 X |

for J_2

| Q_{2n} | \bar{Q}_{1n} | Q_{0n} | Q_{2n+1} | \bar{Q}_{1n+1} | Q_{0n+1} |
|----------|----------------|----------|------------|------------------|------------|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | X | X | X | X | X |

$$J_2 = \bar{Q}_{1n} Q_{0n}$$

| Q_{2n} | \bar{Q}_{1n} | Q_{0n} | Q_{2n+1} | \bar{Q}_{1n+1} | Q_{0n+1} |
|----------|----------------|----------|------------|------------------|------------|
| X | X | X | X | X | X |
| 1 | X | X | X | X | X |

$$k_2 = 1$$

| Q_{2n} | \bar{Q}_{1n} | Q_{0n} | Q_{2n+1} | \bar{Q}_{1n+1} | Q_{0n+1} |
|----------|----------------|----------|------------|------------------|------------|
| 0 | 0 | 1 | X | X | X |
| 1 | 0 | X | X | X | X |

$$\begin{aligned} J_2 &= \bar{Q}_{1n} Q_{0n} + \bar{Q}_{1n} \bar{Q}_{0n} \\ &= Q_{0n} (\bar{Q}_{1n} + \bar{Q}_{0n}) \\ &= Q_{0n} \end{aligned}$$

| Q_{2n} | \bar{Q}_{1n} | Q_{0n} | Q_{2n+1} | \bar{Q}_{1n+1} | Q_{0n+1} |
|----------|----------------|----------|------------|------------------|------------|
| X | X | 1 | X | X | 0 |
| X | X | X | X | X | X |

$$k_1 = Q_{0n}$$

| Q_{2n} | Q_{2n-1} | Q_{2n-2} | Q_{2n-3} |
|----------|------------|------------|------------|
| 1 | x | x | 1 |
| 0 | x | x | x |

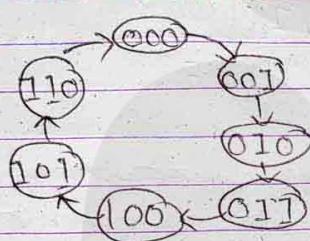
$$J_0 = \overline{Q_{2n}}\overline{Q_{2n-1}} + Q_{2n}\overline{Q_{2n-2}}$$

$$= \overline{Q_{2n}}$$

| Q_{2n} | Q_{2n-1} | Q_{2n-2} | Q_{2n-3} |
|----------|------------|------------|------------|
| x | 1 | 1 | x |
| x | x | x | x |

$$k_0 = 1$$

Synchronous mod-7 counter



Excitation table of J-k flip flop

| $Q +$ | Q_{t+1} | J | K |
|-------|-----------|---|---|
| 0 | 0 | 0 | x |
| 0 | 1 | 1 | x |
| 1 | 0 | x | 1 |
| 1 | 1 | x | 0 |

| Present state $Q_{2n} Q_{2n-1} Q_{2n-2}$ | Next state $Q_{2n+1} Q_{2n+1} Q_{2n+2}$ | flip flop $J_1 \ k_2 \ J_1 \ k_2 \ J_0 \ k_0$ |
|---|--|--|
| 0 0 0 | 0 0 1 | 0 x 0 x 1 x |
| 0 0 1 | 0 1 0 | 0 x 1 x x 1 |
| 0 1 0 | 0 1 1 | 0 x x 0 1 x |
| 0 1 1 | 1 0 0 | 1 x x 1 x x 1 |
| 1 0 0 | 1 0 1 | x 0 0 x 1 x |
| 1 0 1 | 1 1 0 | x 0 1 x x 1 |
| 1 1 0 | 0 0 0 | x 1 x 1 0 x |

for J_2

| Q_{2n} | Q_{2n-1} | Q_{2n-2} | Q_{2n-3} |
|----------|------------|------------|------------|
| 0 | 0 | 1 | 0 |
| x | x | x | x |

| Q_{2n} | Q_{2n-1} | Q_{2n-2} | Q_{2n-3} |
|----------|------------|------------|------------|
| x | x | x | x |
| 1 | x | x | x |

$$K_2 = 1$$

$$J_2 = Q_{2n} Q_{2n-1}$$

| Q _{1n} Q _{0n} | | | |
|---------------------------------|---|---|---|
| Q _{2n} | 0 | 1 | X |
| Q _{1n} | 0 | 1 | X |
| Q _{0n} | X | X | X |

J₁ =

| Q _{1n} Q _{0n} | | | |
|---------------------------------|---|---|---|
| Q _{2n} | X | X | 1 |
| Q _{1n} | X | X | X |
| Q _{0n} | X | X | 1 |

k₁ =

| Q _{1n} Q _{0n} | | | |
|---------------------------------|---|---|---|
| Q _{2n} | 1 | X | X |
| Q _{1n} | 1 | X | X |
| Q _{0n} | X | X | 0 |

J₀ =

| Q _{1n} Q _{0n} | | | |
|---------------------------------|---|---|---|
| Q _{2n} | X | 1 | 1 |
| Q _{1n} | X | 1 | 0 |
| Q _{0n} | X | 1 | X |

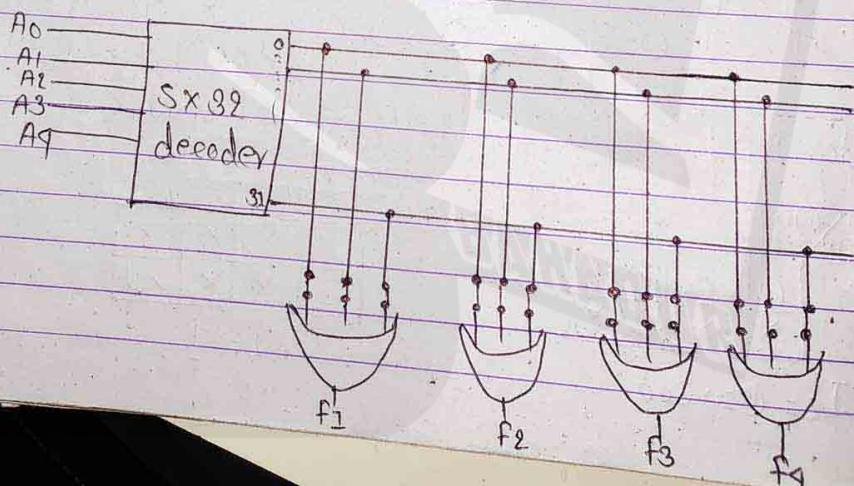
k₀ =

chapter - 3

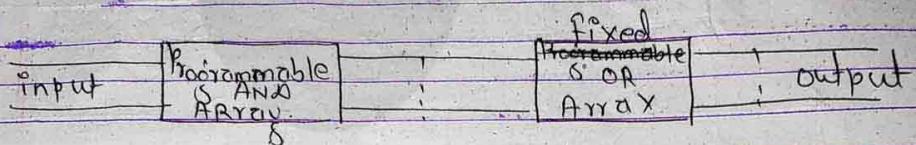
Basic concept of programmable logic :-

- PROM
- EEPROM
- PAL
- PLA

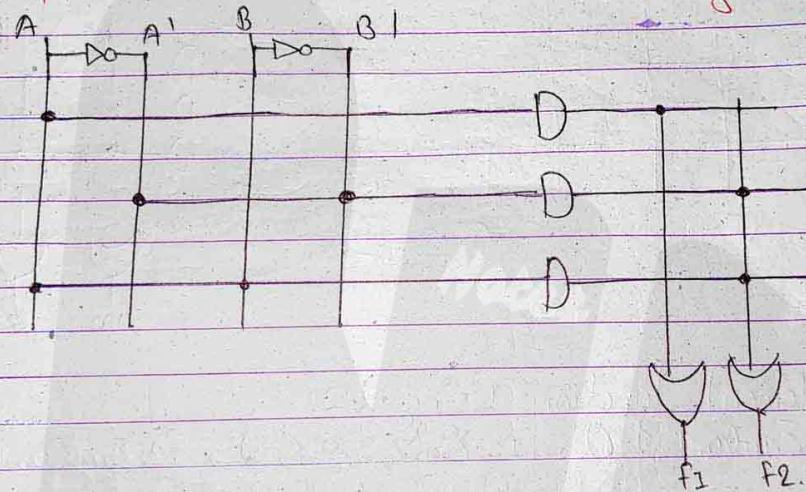
32 x 8 ROM



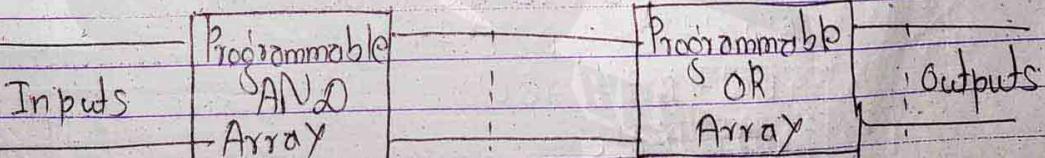
PLA (Programmable Logic Array)

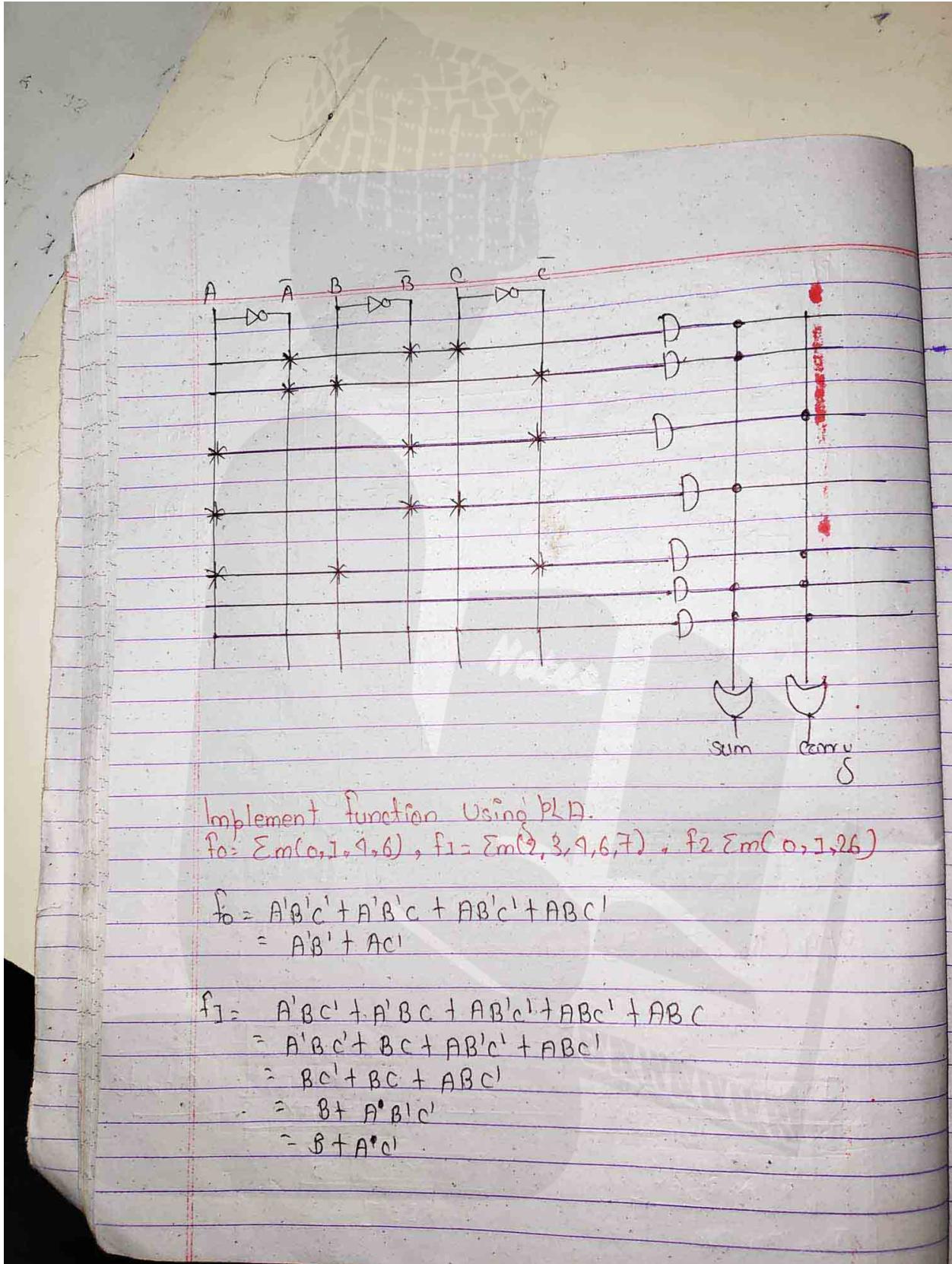


Implement function $f_1 = A_2$ $f_2 = \bar{A}\bar{B} + AB$ by using PLA



PLA (Programmable Logic Array)





Implement function Using PLN.

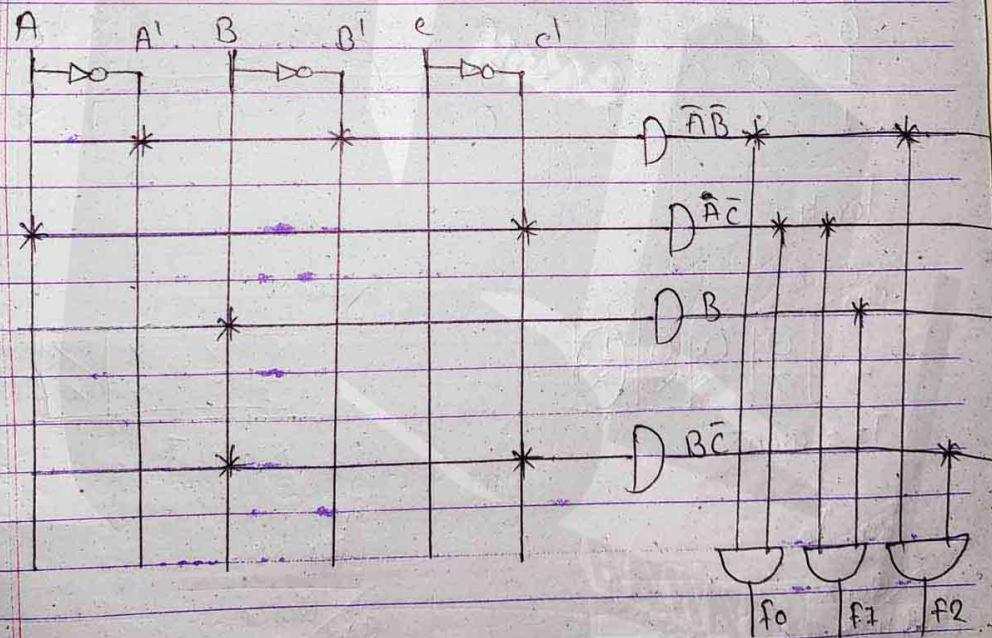
$$f_0 = \Sigma m(0, 1, 4, 6), f_1 = \Sigma m(2, 3, 7, 6, 7), f_2 = \Sigma m(0, 1, 2, 6)$$

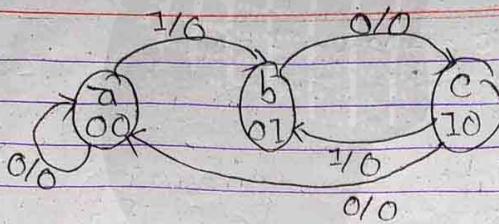
$$\begin{aligned} f_0 &= A'B'C' + A'B'C + AB'C' + ABC' \\ &= A'B' + AC' \end{aligned}$$

$$\begin{aligned} f_1 &= A'B'C' + A'B'C + AB'C' + ABC' + ABC \\ &= A'B'C' + BC + AB'C' + ABC \\ &= BC' + BC + ABC \\ &= B + A'B'C' \\ &= B + A'C' \end{aligned}$$

$$\begin{aligned}f_2 &= A'B'C' + A'B'C + A'BC' + ABC \\&= A'B' + BC'\end{aligned}$$

| Product | Inputs A B C | Outputs f ₀ f ₁ f ₂ |
|------------------|-----------------|---|
| $\bar{A}\bar{B}$ | 0 0 - | 1 0 1 |
| $A\bar{C}$ | 1 - 0 | 1 1 0 |
| B | - 1 - | 0 1 0 |
| $B\bar{C}$ | - 1 0 | 0 0 1 |



J₀₀

| Present state | Input | Nextstate | Output | J ₀ | k ₀ | J ₁ | k ₁ |
|-----------------|------------------|-----------|-------------------------------------|----------------|----------------|----------------|----------------|
| Q _{in} | Q _{out} | X | Q _{int1} Q _{int2} | Y | Q | X | |
| 0 | 0 | 0 | 0 0 | 0 | 0 | 0 | X |
| 0 | 0 | 1 | 0 1 | 0 | 1 | X | 0 |
| 0 | 1 | 0 | 1 0 | 0 | X | 1 | 1 |
| 0 | 1 | 1 | 0 1 | 0 | X | 0 | 0 |
| 1 | 0 | 0 | 0 0 | 1 | 0 | X | X |
| 1 | 0 | 1 | 0 1 | 0 | 1 | X | X |

for J₁

| Q _{in} | Q _{out} x | 0 | 0 | 0 | 1 |
|-----------------|--------------------|---|---|---|---|
| Q _{in} | Q _{out} x | X | X | X | X |
| 0 | 0 | 0 | 0 | 0 | 1 |
| X | X | X | X | X | X |

$$J_1 = Q_{outx}$$

for k₁

| Q _{in} | Q _{out} x | X | X | X | X |
|-----------------|--------------------|---|---|---|---|
| Q _{in} | Q _{out} x | 1 | 1 | X | X |
| X | X | 1 | 1 | X | X |
| X | X | X | X | X | X |

$$k_1 = 1$$

for J₀

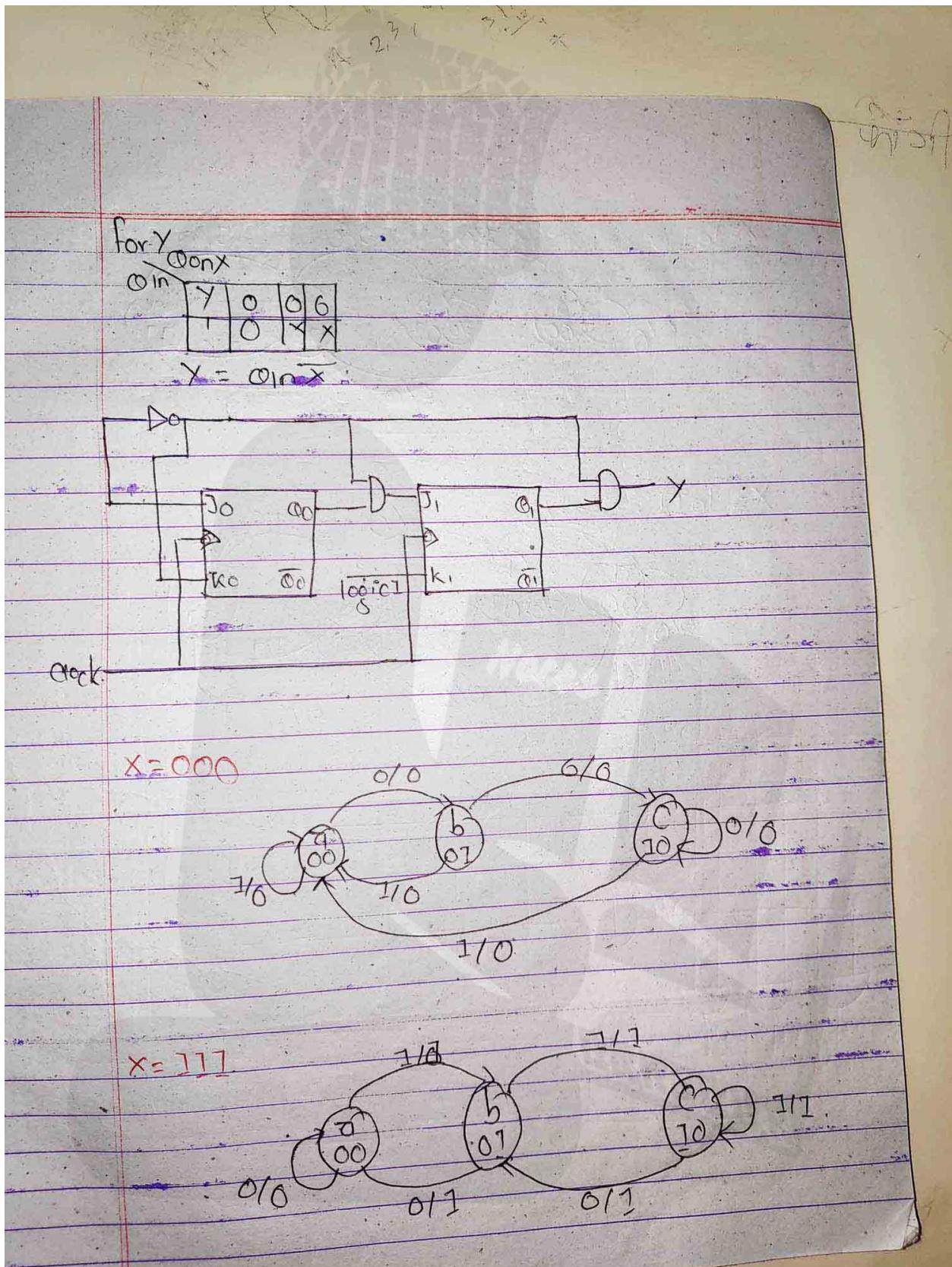
| Q _{in} | Q _{out} x | 0 | 1 | X | X |
|-----------------|--------------------|---|---|---|---|
| Q _{in} | Q _{out} x | 0 | 1 | X | X |
| 0 | 0 | 0 | 1 | X | X |
| 0 | 1 | 1 | X | X | X |

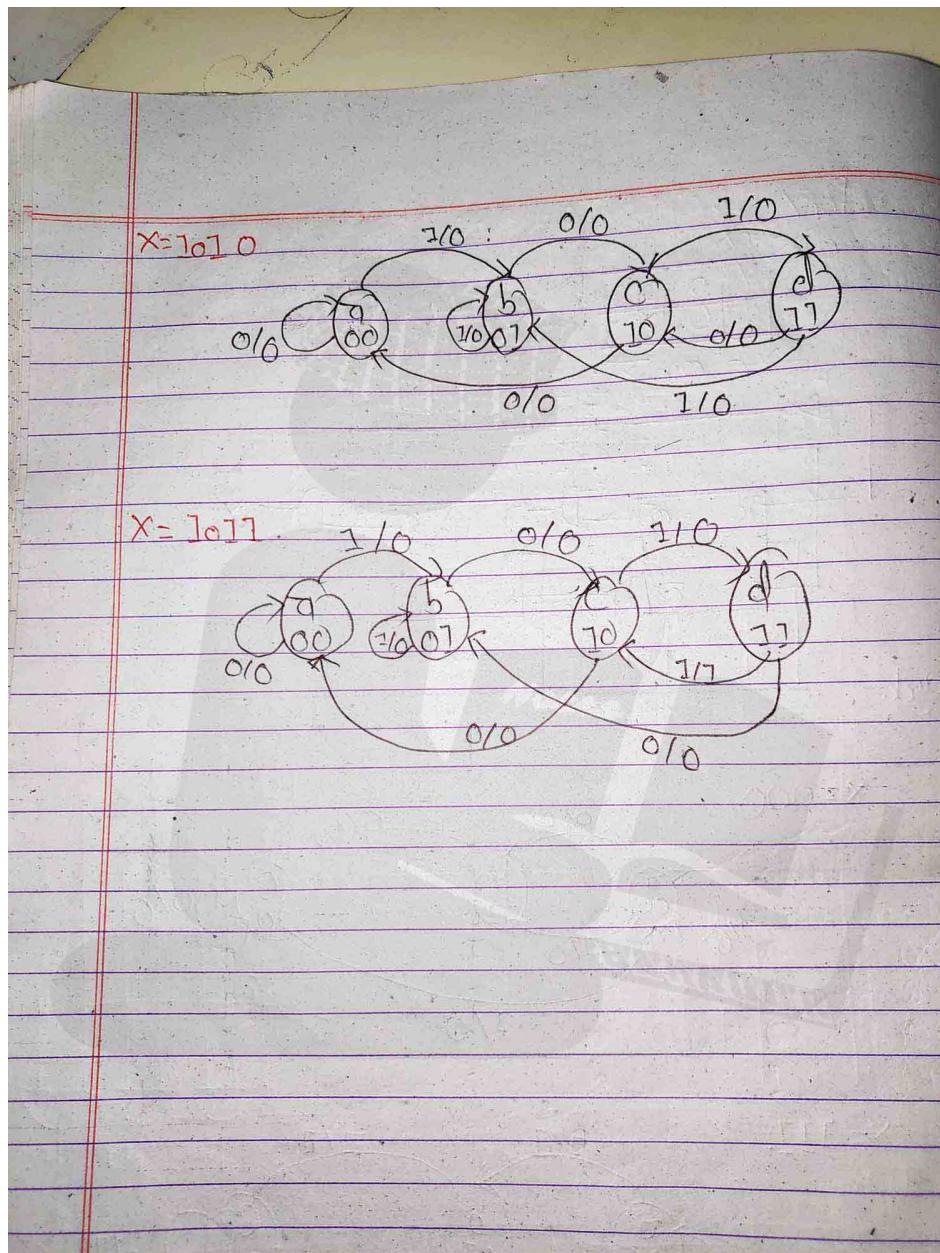
$$J_0 = X$$

for k₀

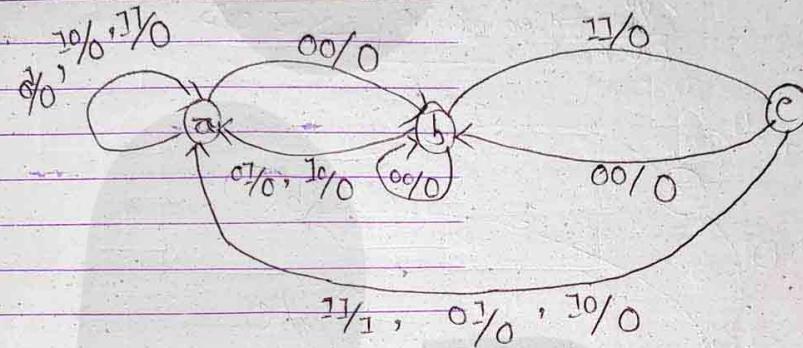
| Q _{in} | Q _{out} x | X | X | 0 | 1 |
|-----------------|--------------------|---|---|---|---|
| Q _{in} | Q _{out} x | X | X | X | X |
| X | X | X | X | X | X |
| X | X | X | X | X | X |

$$k_0 = X$$





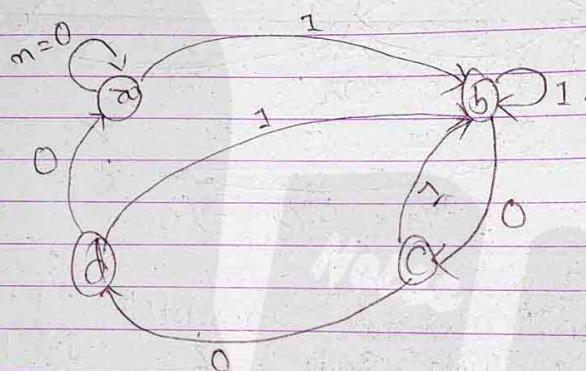
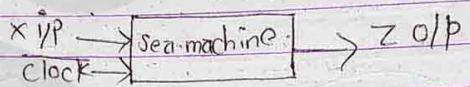
Design a machine has two several input n_1 & n_2 and one output 'Z'. The machine is required to give an output $Z=1$ when both n_1 & n_2 contain the message '011'.



Let $a = 00$, $b = 01$ and $c = 10$.

| Present state | Q _{0n} | Present IP | n ₁ | n ₂ | Next state | Q _{0nt1} | Q _{1nt1} | Present O/P FF excitation. | | | | | |
|---------------|-----------------|------------|----------------|----------------|------------|-------------------|-------------------|----------------------------|----------------|----------------|----------------|----------------|--|
| | | | | | | | | Z | J ₁ | k ₁ | J ₀ | k ₀ | |
| 0110 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | X | 1 | X | |
| | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | |
| | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | |
| | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | X | 0 | X | |
| | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | X | X | 0 | |
| | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | X | 1 | |
| | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | X | X | 1 | |
| | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | X | X | 1 | |
| | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | X | 1 | X | |
| | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | X | 1 | 1 | X | |
| | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | X | 1 | 0 | X | |
| | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | X | 1 | 0 | X | |
| | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | X | 1 | 0 | X | |

Q) Synchronous machine has one bit serial input 'n' the output Z of a machine is to be set high when live input contains the message '100'. Draw the state diagram, derive the transition table or state table and design the circuit diagram.



Transition table -

Previous state

Next state

Output

 $n=0$ $n=1$ $n=0$ $n=1$

| | | | |
|---|---|---|---|
| a | b | 0 | 0 |
| b | d | 0 | 0 |
| c | d | 0 | 0 |
| d | a | 1 | 1 |

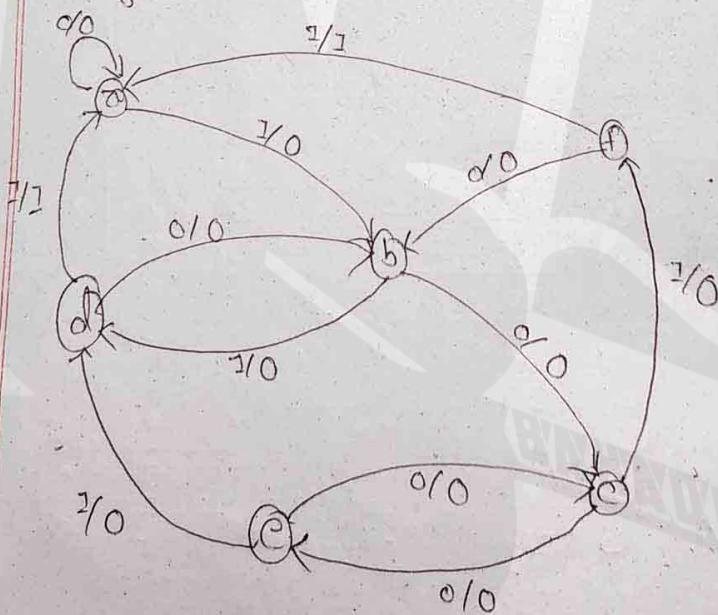
(P.T.O)

| Present state | | | Present I/P | | Nextstate output | | | FF excitation. | | |
|---------------|-----|----|-------------|-------|------------------|----|----|----------------|----|--|
| Q1n | Q0n | n1 | Q1nt1 | Q0nt1 | Z | J1 | K1 | J0 | K0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | x | 0 | x | |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | x | 1 | x | |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | x | x | 1 | |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | x | x | 0 | |
| 1 | 0 | 0 | 1 | 1 | 0 | x | 0 | 1 | x | |
| 1 | 0 | 1 | 0 | 1 | 0 | x | 1 | 1 | x | |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | x | 1 | x | |
| 1 | 1 | 1 | 0 | 1 | 1 | x | 1 | x | 0 | |

- State reduction technique :-
- Raw elimination methods
 - Implication table
 - Partitioning the state diagram

| Present state | Nextstate | | Present O/P | |
|---------------|-----------|-------|-------------|-------|
| | $n=0$ | $n=1$ | $n=0$ | $n=1$ |
| a | a | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | e | f | 0 | 0 |
| d | b | a | 0 | 1 |
| e | c | f | 0 | 0 |
| f | b | a | 0 | 1 |

(Original table)



B and E are same, so 'e' law is eliminated and substitute e=b

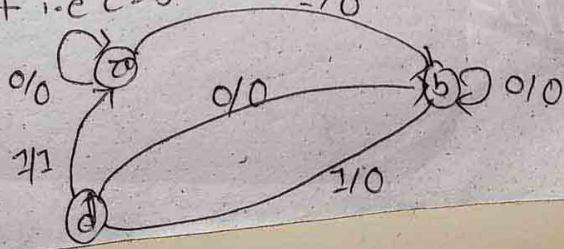
| Present state | Next state | | Present O/P. | |
|---------------|------------|-----|--------------|-----|
| | n=0 | n=1 | n=0 | n=1 |
| a | b | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | d | e | 0 | 0 |
| d | b | b | 0 | 1 |
| e | b | d | 0 | 1 |

Here for $n=0$, $b \rightarrow c$ and $c \rightarrow b$. These two are equivalent if their next state are same. Here next state are same so they are equivalent i.e $c=b$.

| Present state | Next state | | Present O/P. | |
|---------------|------------|-----|--------------|-----|
| | n=0 | n=1 | n=0 | n=1 |
| a | b | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | d | e | 0 | 0 |
| d | b | b | 0 | 1 |

| Present state | Next state | | Present O/P. | |
|---------------|------------|-----|--------------|-----|
| | n=0 | n=1 | n=0 | n=1 |
| a | b | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | d | e | 0 | 1 |

Here for $n=0$, $b \rightarrow c$ and $c \rightarrow b$. These two are equivalent if their next state are same. Here next state are same so they are equivalent i.e $c=b$.



State reduction by Implication table method

| Present state | Nextstate | | present output |
|---------------|-----------|-------|----------------|
| | $n=0$ | $n=1$ | |
| a | e | e | 1 |
| b | c | e | 1 |
| c | i | h | 0 |
| d | h | a | 1 |
| e | i | f | 0 |
| f | e | g | 0 |
| g | h | b | 1 |
| h | c | d | 0 |
| i | f | b | 1 |



| b | c-e | | | | | | | |
|---|-----|-----|------------|------------|------------|---|------------|---|
| c | x | x | | | | | | |
| d | e-h | c-h | | x | | | | |
| e | e-a | e-a | | | | | | |
| f | x | x | i-e h-g | x | i-e f-g | | | |
| g | e-h | c-h | x | a-b | x | x | | |
| h | e-b | e-b | | | | | | |
| i | x | x | i-e h-d | x | i-d f-a | x | c-c g-d | x |
| j | e-f | c-f | x | h-f a-b | x | | h-f | x |
| a | e-b | e-b | | | | | | |
| b | | | | c | d | c | f | g |
| c | | | | | | | | |
| d | | | | | | | | |
| f | | | | | | | | |

$c \equiv e$
 $h \equiv f$
 $a \equiv b$
 $d \equiv g$

| Present state | Next state $n=0 n=1$ | Present O/P |
|---------------|---------------------------|-------------|
| a | c f | 1 |
| c | d f | 0 |
| d | f g | 1 |
| f | c d | 0 |