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# EJERCICIOS 1.5.7

**2.** Considere que:

- $\mathbf{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} = x^i\hat{\imath}_i$
- $\mathbf{a} = \mathbf{a}(\mathbf{r}) = \mathbf{a}(x, y, z) = a^i(x, y, z)\hat{\imath}_i$ y $\mathbf{b} = \mathbf{b}(\mathbf{r}) = \mathbf{b}(x, y, z) = b^i(x, y, z)\hat{\imath}_i$
- $\phi = \phi(\mathbf{r}) = \phi(x, y, z)$ y $\psi = \psi(\mathbf{r}) = \psi(x, y, z)$

**a).** $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$.

$$\begin{aligned} \nabla(\phi\psi) &= (\partial_i\phi)\psi + \phi(\partial_i\psi) \\ &= \psi\nabla\phi + \phi\nabla\psi \\ &= \phi\nabla\psi + \psi\nabla\phi \end{aligned} \tag{1}$$

**d).** $\nabla \cdot (\nabla \times \mathbf{a})$ ¿ Qué puede dedcir de $\nabla \times (\nabla \cdot \mathbf{a})$ ?

En la ecuación $\nabla \times (\nabla \cdot \mathbf{a})$ no tiene sentido hablar del rotacional de la divergencia ya que la divergencia tiene como resultado un escalar y el rotacional es un producto cruz.

**f).** $\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2\mathbf{a}$

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{a}) &= \varepsilon^{ijk}\partial_j(\nabla \times \mathbf{a})_k \\ cc\varepsilon^{ijk}\partial_j\varepsilon_{kmn}\partial^m a^n & \\ &= \varepsilon^{kij}\varepsilon_{kmn}\partial_j\partial^m a^n \\ &= (\delta^i_m\delta^j_n - \delta^i_n\delta^j_m)\partial_j\partial^m a^n \\ &= \partial_j\partial^i a^j - \partial_j\partial^j a^i \\ &= \nabla(\nabla \cdot \mathbf{a}) - \nabla^2\mathbf{a} \end{aligned} \tag{2}$$

# EJERCICIOS 1.6.6

**2.** Demuestre:

**a).** $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha)$

**b).** $\sin(3\alpha) = 3\cos^2(\alpha)\sin(\alpha) - \sin^3(\alpha)$

$$\begin{aligned}(e^{i\alpha})^3 &= (e^{3\alpha})^i \\ (\cos(\alpha) + i\sin(\alpha))^3 &= \cos(3\alpha) + i\sin(3\alpha)\end{aligned} \tag{3}$$

$$\begin{aligned}(\cos(\alpha) + i\sin(\alpha))^3 &= \cos^3(\alpha) + 3\cos^2(\alpha)\sin(\alpha)i + 3\cos(\alpha)\sin^2(\alpha)i^2 + i^3\sin^3(\alpha) \\ &= \cos^3(\alpha) + 3\cos^2(\alpha)\sin(\alpha)i - 3\cos(\alpha)\sin^2(\alpha) - i\sin^3(\alpha) \\ \cos(3\alpha) + i\sin(3\alpha) &= \cos^3(\alpha) + 3\cos^2(\alpha)\sin(\alpha)i - 3\cos(\alpha)\sin^2(\alpha) - i\sin^3(\alpha) \\ \cos(3\alpha) &= \cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha) \\ \sin(3\alpha) &= 3\cos^2(\alpha)\sin(\alpha) - i\sin^3(\alpha)\end{aligned} \tag{4}$$

**5.** Encuentre las raices de:

**a).** $(2i)^{\frac{1}{2}}$

$$\begin{aligned}(2i)^{\frac{1}{2}} &= (2e^{i(\frac{\pi}{2}+2\pi n)})^{\frac{1}{2}} \\ &= 2^{\frac{1}{2}}e^{i(\frac{\pi}{4}+\pi n)}(n=0, n=1) \\ Z_0 &= 2^{\frac{1}{2}}e^{i(\frac{\pi}{4})}) \\ &= 2^{\frac{1}{2}}[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}] \\ &= 2^{\frac{1}{2}}[\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}] \\ Z_1 &= 2^{\frac{1}{2}}e^{i(\frac{\pi}{4}+\pi)} \\ &= 2^{\frac{1}{2}}[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}] \\ &= 2^{\frac{1}{2}}[-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}]\end{aligned} \tag{5}$$

**b).** $(1-\sqrt{3i})^{\frac{1}{2}}$

$$\begin{aligned}(1-\sqrt{3i})^{\frac{1}{2}} &= (\sqrt{1+3}e^{i\tan^{-1}\frac{-\sqrt{3}}{1}})^{\frac{1}{2}} \\ &= (2e^{i(-\frac{\pi}{8}+2\pi n)})^{\frac{1}{2}} \\ &= \sqrt{2}e^{i(-\frac{\pi}{6}+\pi n)} \\ Z_0 &= \sqrt{2}e^{i(-\frac{\pi}{6})} \\ &= \sqrt{2}[\cos-\frac{\pi}{6} + i\sin-\frac{\pi}{6}] \\ &= \sqrt{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i) \\ Z_1 &= \sqrt{2}e^{i(-\frac{\pi}{6}+\pi)} \\ &= \sqrt{2}[\cos-\frac{5\pi}{6} + i\sin-\frac{5\pi}{6}] \\ &= \sqrt{2}(-\frac{\sqrt{3}}{2} + \frac{1}{2}i)\end{aligned} \tag{6}$$

**c).** $(-1)^{\frac{1}{3}}$

$$
\begin{aligned}
(-1)^{\frac{1}{3}} &= (e^{i(\pi+2\pi n)})^{\frac{1}{3}} \\
&= e^{i(\frac{\pi}{3}+\frac{2\pi}{3}n)} \\
(n=0), Z_0 &= e^{i}(\frac{\pi}{3}) \\
&= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \\
&= \frac{1}{2} + \frac{\sqrt{3}}{2}i \\
(n=1), Z_1 &= e^{i}(\frac{\pi}{3} + \frac{2\pi}{3}) \\
&= \cos\pi + i\sin\pi \\
&= -1 + (i)(0) = -1 \\
(n=2), Z_2 &= e^{i}(\frac{\pi}{3} + \frac{4\pi}{3}) \\
&= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} \\
&= \frac{1}{2} - \frac{\sqrt{3}}{2}i
\end{aligned}
\tag{7}
$$

**d).** $8^{\frac{1}{6}}$

$$
\begin{aligned}
8^{\frac{1}{6}} &= (8e^{i2\pi n})^{\frac{1}{6}} \\
&= 8^{\frac{1}{6}}e^{i\frac{\pi}{3}n} \\
(n=0), Z_0 &= 8^{\frac{1}{6}}e^{0} = 8^{\frac{1}{6}} \\
(n=1), Z_1 &= 8^{\frac{1}{6}}e^{i\frac{\pi}{3}} \\
&= 8^{\frac{1}{6}}(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 8^{\frac{1}{6}}(\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\
(n=2), Z_2 &= 8^{\frac{1}{6}}e^{i\frac{2\pi}{3}} \\
&= 8^{\frac{1}{6}}(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = 8^{\frac{1}{6}}(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\
(n=3), Z_3 &= 8^{\frac{1}{6}}e^{i}\pi \\
&= 8^{\frac{1}{6}}(\cos\pi + i\sin\pi) = 8^{\frac{1}{6}}(-1) = -8^{\frac{1}{6}} \\
(n=4), Z_4 &= 8^{\frac{1}{6}}e^{i\frac{4\pi}{3}} \\
&= 8^{\frac{1}{6}}(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}) = 8^{\frac{1}{6}}(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) \\
(n=5), Z_5 &= 8^{\frac{1}{6}}e^{i\frac{5\pi}{3}} \\
&= 8^{\frac{1}{6}}(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}) = 8^{\frac{1}{6}}(\frac{1}{2} - \frac{\sqrt{3}}{2}i)
\end{aligned}
\tag{8}
$$

**e).** $(-8-8\sqrt{3i})^{\frac{1}{4}}$

$$\begin{aligned}(-8-8\sqrt{3i})^{\frac{1}{4}} &= [\sqrt{64+(64)(3)}e^{i\tan^{-1}(\frac{-8\sqrt{3}}{-8})}]^{\frac{1}{4}} \\ &= [16e^{i(\frac{\pi}{3}+2\pi n)}]^{\frac{1}{4}} \\ &= 2e^{i(\frac{\pi}{12}+\frac{\pi}{2}n)}(n=0,n=1,n=2,n=3)\end{aligned} \tag{9}$$

**6.** Demuestre que:

**a).** $\log(-ie) = 1-\frac{\pi}{2}i$

$$\begin{aligned}\log(ee^{i(\frac{3\pi}{2}+2\pi n)}) &= \ln(|e|)+i(-\frac{\pi}{2}+2\pi n) \\ &= 1-\frac{\pi}{2}i,(n=0)\end{aligned} \tag{10}$$

**b).** $\log(1-i) = \frac{1}{2}\ln(2)-\frac{\pi}{4}i$

$$\begin{aligned}\log(\sqrt{2}e^{i(-\frac{\pi}{4}+2\pi n)}) &= \ln(\sqrt{2}-i\frac{\pi}{4}) \\ &= \frac{1}{2}\ln(2)-\frac{\pi}{4}i,(n=0)\end{aligned} \tag{11}$$

**c).** $\log(e) = 1+2n\pi i$

$$\begin{aligned}\log ee^{i2\pi n} &= \ln(e)+i2\pi n \\ &= 1+i2\pi n\end{aligned} \tag{12}$$

**d).** $\log(i) = (2n+\frac{1}{2})\pi i$

$$\begin{aligned}\log(1e^{i(\frac{\pi}{2}+2\pi n)}) &= \ln(1)+i(\frac{\pi}{2}+2\pi n) \\ &= \pi i(2n+\frac{1}{2})\end{aligned} \tag{13}$$