

MATH 370 ALGEBRA, SPRING 2024, HOMEWORK 9

Problem 1 Prove that a positive integer n that is not an integer square is not the square of a rational number.

Problem 2 Let a and b be relatively prime integers. Prove that there are integers m and n such that $a^m + b^n$ is 1 modulo ab .

Problem 3 Compute the greatest common divisor of the polynomials $x^6 + x^4 + x^3 + x^2 + x + 1$ and $x^5 + 2x^3 + x^2 + x + 1$ in $\mathbb{Q}[x]$.

Problem 4

- Factor $x^9 - x$ and $x^9 - 1$ in $\mathbb{F}_3[x]$.
- Factor $x^{16} - x$ in $\mathbb{F}_2[x]$.