

MATH 370 ALGEBRA, SPRING 2024, HOMEWORK 1

Problem 1 [5 points] Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Find a formula for the product A^n and prove it using induction.

Problem 2 [5 points] Show that a square matrix A is invertible if and only if $\det(A)$ is non zero.

Problem 3 [5 points] Let S be a set of cardinality n . Show that the cardinality of power set of S is 2^n .

Problem 4 [6 points] Find solutions to the system of equations $Ax = b$ where

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 3 & 0 & 0 & 4 \\ 1 & -4 & -2 & 2 \end{pmatrix}$$

and b is

$$\begin{aligned} &\bullet \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ &\bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \\ &\bullet \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \end{aligned}$$

Problem 5 [5 points] The support of a permutation is the set of points moved by that permutation. We say that two permutations $\sigma, \tau \in S_n$ are disjoint if and only if they have disjoint support. Prove that if σ and τ are disjoint permutations and $\sigma\tau = e$, then $\sigma = e$ and $\tau = e$.