MATH 370 ALGEBRA, SPRING 2024, MIDTERM 3

By $\mathbb{Q}_p$, we mean the field of p-adic numbers.
Problem 1 [10 points] Show that $\sqrt{3} \notin \mathbb{Q}_7$.

Problem 2 [10 points] Is the following statement true or false? If your answer is true, please provide a proof else provide a counterexample.
Let p be a prime. All triangles in $\mathbb{Q}_p$ are equilateral.

Problem 3 [5+5 points] Show that $\mathbb{Z}[\omega]$ where ω is $e^{\frac{2\pi\iota}{3}}$ and $\mathbb{Z}[\sqrt{-2}]$ are Euclidean domains.

Problem 4 [3+3+4 points] Determine:

- the monic irreducible polynomials of degree 3 over $\mathbb{F}_3$,
- the monic irreducible polynomials of degree 2 over $\mathbb{F}_5$,
- the number of irreducible polynomials of degree 3 over the field $\mathbb{F}_5$.

Problem 5 [4+3+3 points]
In each of the following cases, state yes if the extension $\mathbb{Q}(\alpha)$ is Galois over $\mathbb{Q}$ and no otherwise. In each case, $f(x)$ is the minimal polynomial of α. You do not have to justify your answer.

- $f(x) = x^n - 2$, $\alpha = 2^{1/n}$, $n \in \mathbb{N}$
- $f(x) = x^2 + x + 1$, $\alpha = e^{\frac{2\pi\iota}{3}}$
- $f(x) = x^2 - 5$, $\alpha = \sqrt{5}$

Date: Tuesday 4^{th} June, 2024.