

## MATH 370 ALGEBRA, SPRING 2024, HOMEWORK 6

Problem 1 and 2 are reading exercises and will be graded for completion.

**Problem 1** Read the proof of Second Sylow theorem from book and summarize key ideas used in the proof in your own words.

**Problem 2** Read the proof of Third Sylow theorem from book and summarize key ideas used in the proof in your own words.

**Problem 3** Let  $G$  be a finite group of order 21. Find the smallest  $n$  such that there is an injective group homomorphism from  $G$  to  $S_n$ . Justify your answer. (You can use textbook to see classification of all groups of order 21.)

**Problem 5** Let  $G_1 \subseteq G_2$  be groups whose orders are divisible by  $p$ , and let  $H_1$  be a Sylow  $p$ -subgroup of  $G_1$ . Prove that there is a Sylow  $p$ -subgroup  $H_2$  of  $G_2$  such that  $H_1 = H_2 \cap G_1$ .