MATH 314 LINEAR ALGEBRA, SPRING 2024, HOMEWORK 6

Problem 1 Suppose that v, x are two vectors in V and U, W are two subspaces of V such that $v + U = x + W$. Prove that $U = W$.

Problem 2 Fix a field F. Suppose $U = \{(x_1, x_2, x_3, \ldots) \mid x_i \in F \ \& \ x_k \neq 0 \texttt{ for finitely many k}\}$. Show that U is a subspace of F^∞ (F^∞ is the cartesian product of F with itself countably infinitely many times). Prove that F^∞ / U is infinite-dimensional.

Problem 3 Suppose that U, W are two subspaces of V such that $V = U \oplus W$. Suppose $w_1, w_2, \ldots, w_m$ is a basis of W. Prove that $w_1 + U, w_2 + U, \ldots, w_m + U$ is a basis of V/U.

Problem 4 Suppose $T \colon V \to W$ is a linear map and U is a subspace of V. Let π denote the quotient map from V onto V/U. Prove that there exists a linear map $S \colon V/U \to W$ such that $T = S \circ \pi$ if and only if $U \subseteq null(T)$.

Date: Tuesday 4^{th} June, 2024.