

MATH 314 LINEAR ALGEBRA, SPRING 2024, HOMEWORK 3

Problem 1 [3 points] Let V be a finite dimensional vector space and let V_1 and V_2 be subspaces of V . If $\dim(V_1 + V_2) = \dim(V_1 \cap V_2) + 1$, show that $V_1 + V_2$ is either V_1 or V_2 and $V_1 \cap V_2$ is correspondingly V_2 or V_1 .

Problem 2 [3 points] Give an example of a vector space V and three subspaces W_1, W_2, W_3 of it such that

$$W_1 \cap (W_2 + W_3) \neq (W_1 \cap W_2) + (W_1 \cap W_3).$$

Problem 3 [3 points] Let W_1 and W_2 be nontrivial subspaces of a vector space V , that is, neither 0 nor V . Show that there exists an element $\alpha \in V$ such that $\alpha \notin W_1$ and $\alpha \notin W_2$. Show further that there exists a basis of V such that none of the vectors in the basis is contained in either W_1 or W_2 . Is this true for more than two nontrivial subspaces?

Problem 8 section 1.5, Problem 6 section 1.6