MATH 314 LINEAR ALGEBRA, SPRING 2024, MIDTERM 2

Problem 1 [5 points] Let V be a finite dimensional vector space over a field F. Suppose U is a subspace of V such that $dim(V/U) = 1$. Prove that there exists a $\phi \in L(V, F)$ such that $ker(\phi) = U$.

Problem 2 [5 points] Suppose V is finite-dimensional and U and W are subspaces of V.

- Show that $(U + W)^0 = U^0 \cap W^0$.
- Show that $(U \cap W)^0 = U^0 + W^0$.

Problem 3 [5 points] Suppose T is a function from V to W. The graph of T is the subset of $V \times W$ defined by

$$\text{graph of T} = \{(v, Tv) \in V \times W : v \in V\}.$$

Prove that T is a linear map if and only if the graph of T is a subspace of $V \times W$.

Problem 4[5 points] Suppose V is finite-dimensional, U is a subspace of V, and $S \in L(U, V)$. Prove that there exists an invertible linear map T from V to itself such that $Tu = Su$ for every $u \in U$ if and only if S is injective.

Problem 5[5 points] Let V be a finite dimensional vector space over F. Suppose ϕ,$\psi \in L(V, F)$. Prove that $ker(\phi) \subseteq ker(\psi)$ if and only if there exists $c \in F$ such that $\psi = c\phi$.

Date: Tuesday 4^{th} June, 2024.