MATH 314 LINEAR ALGEBRA, SPRING 2024, MIDTERM 3

Problem 1 [5 points] Let V be a finite dimensional vector space. Suppose $T : V \to V$ is a linear map. Prove that if dimension of null space of $T^4 = 8$ and dimension of null space of $T^6 = 9$, then dimension of null space of $T^m = 9$ for all integers $m \geq 5$.

Problem 2 [5 points] Let V be a finite dimensional vector space over $\mathbb{C}$. Suppose $P : V \to V$ is a linear map such that $P^2 = P$. Prove that the characteristic polynomial of P is $x^m(x-1)^n$, where m is nullity of P and n is the dimension of range of P.

Problem 3 [5 points] Suppose T is a linear map from V to V such that with respect to some basis of V, all entries of the matrix of T are rational numbers. Explain why all coefficients of the minimal polynomial of T are rational numbers.

Problem 4[5 points] Suppose A is an $n \times n$ matrix, and suppose c is such that $|A_{j,k}| \leq c$ for all $j, k \in \{1, \ldots, n\}$. Prove that $|detA| \leq c^n n^{n/2}$.

Problem 5[5 points] Give an example of a nonzero alternating 2-linear form α on $\mathbb{R}^3$ and a linearly independent list v_1, v_2 in $\mathbb{R}^3$ such that $\alpha(v_1, v_2) = 0$.

Date: Tuesday 4^{th} June, 2024.