MATH 3500 Spring 2023 (Lecture 2)

WRITING PROOFS

Problem: Show that sum of two consecutive integers is an odd integer.

Understand the problem first

Consecutive integers

Defn: We say that a & b, $a<b$ are consecutive integers if and only if $b = a+1$.

Defn: We say that b is an odd integer iff $b = 2k+1$ for some integer k.

Direct Proof:

Let $a<b$ be two consecutive inregers.

So, $b = a+1$. Then $a+b = 2a+1$

which is odd by definition.

So, done.

PROOF BY CONTRADICTION

If p, then q. Assume p is true and q is false and arrive at a $\underbrace{\text{contradiction}}$

↓

a false statement

Show that sum of two consecutive integers is odd.

Assume a, b $a<b$ are consecutive.
Assume $a+b$ is even.

Since a & b are consecutive $b = a+1$
We have assumed $a+b = 2m$ for some integer m.
Also $a+b = 2a+1 = 2m$
So, $1 = 2(m-a)$ $\rightarrow\leftarrow$

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PROOF BY CONTRAPOSITIVE

Want to show $p \Rightarrow q$

In this method we show $\underset{\sim}{-q} \Rightarrow -p$

not of q

a & b are consecutive $\Rightarrow$ a+b is odd

Assume

a+b is even

$\Rightarrow$ a & b are not consecutive

$a+b$ is even

a even b even
a odd b odd

} cannot be consecutive

. - . - Try to write down complete proof using well formed sentences.

Proof by induction is used to prove results about natural numbers, see example below:

Show that $1+2+\cdots+n = \frac{n(n+1)}{2}$

1. This holds for $n=1$.
2. Assume it holds for $n=k$.
3. Now show that the statement holds for $n=k+1$

$$1+2+\cdots+k+k+1 = \frac{k(k+1)}{2} + k+1 = \frac{(k+1)(k+2)}{2}.$$

LINEAR DIOPHANTINE EQUATION

Degree

1

involves variables and coefficients

Coefficients are integers

Example:

$$7x^2 + 11xy + 21z^2 = 0$$

This is diophantine equation but not linear!

Example: (Linear Diophantine equation)

$$2x + 3y = 5$$

Solving linear diophantine equations

It is easy to show that there are no solutions than to find them all

Are there any integer solutions to

→ $12x + 15y = 5$?

No, because 3 divides LHS but not RHS

→ $2x + 3y = 1$

Yes $x = 2$, $y = -1$ and many more.

GCD of two integers

Defn: Let a, b be two integers.
We say that d is the GCD of a & b iff

(i) $d > 0$

(ii) $d \mid a$ & $d \mid b$

(iii) If c is an integer such that $c \mid a$ & $c \mid b$ then $c \mid d$.

Find GCD(2, 3)

Common divisors are 1, −1.

$\downarrow$

GCD.

Find GCD(12, 15).

Divisors of 12 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Divisors of 15 $\pm 1, \pm 3, \pm 5, \pm 15$

GCD = 3

Find $GCD(720, 134)$.

$\downarrow$

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Most effective & useful way to Compute GCD is by using Euclidean algorithm.

Next time!