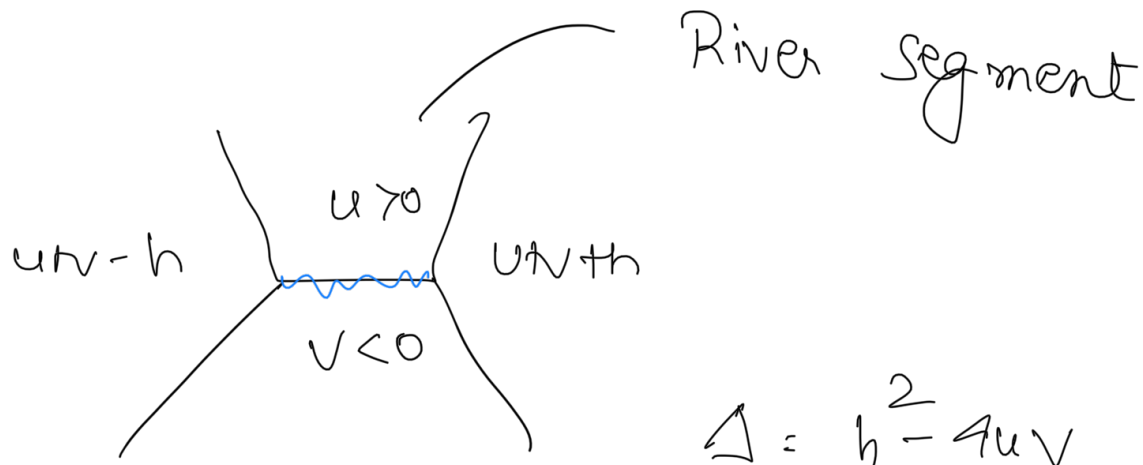


Rivers in topograph



$$\Delta = h^2 - 4uv$$

If Q is indefinite

$$\Delta > 0$$

$$h^2 > 4uv$$

$$-4uv > 0$$

$$0 \leq h^2 \leq \Delta$$

$$0 < |u| |v| \leq \Delta/4$$

→ There are finitely many possibilities of distinct river segments.

→ If range topograph of Q contains an endless river, then there is a repetition.

Corollary: $Q(x, y)$

$A/Q) > 0$
non-square.

Let $N \neq 0 \in \mathbb{Z}$.

If $Q(x, y) = N$ has one solution, then
it has infinitely many primitive
solutions.

Pf: Suppose $Q(x_0, y_0) = N$

$$(x_0, y_0) = \gcd(x_0, y_0) (u, v)$$

\downarrow
This is a
primitive
soln

$$N = \gcd(x_0, y_0)^2 \quad (n)$$

\downarrow
this appears in
range to graph of
 Q once \Rightarrow
appears infinitely many
times

Say $Q(a, b) = n$

$$\boxed{Q(\gcd(x_0, y_0) a, \gcd(x_0, y_0) b) = N}$$

Pell's Equation $\nexists N > 0$ not a square

then $x^2 - Ny^2 = 1$ has infinitely many solutions.

Examples: 1) $x^2 - 3y^2 = 1$

Prn: Find a solution to Diophantine equation $2x^2 + 5xy + y^2 = 13$ or prove that there is no solution.

Ques: What are all the rational solutions of $x^2 - 3y^2 = 1$?

Real solutions?

Complex solutions?

$$x^2 - dy^2 = 1$$

$$x = \frac{1 + dm^2}{1 - dm^2}$$

$$y = \frac{2m}{1 - dm^2}$$