Math 350 Midterm I

Instructions and Information

1. Read these instructions carefully, and sign the Statement on Academic Integrity below once you have completed the exam.
2. This test contains a cover page and 4 problems.
3. Each question is worth the same number of points.
4. In addition to accuracy, you will be graded on the **correctness, completeness, clarity, and neatness of your accompanying work and explanations.** While we appreciate if you type your responses using LaTeX, you are not required to do so, and it is better to turn in good hand-written work than hastily-prepared solutions written in TeX.
5. This midterm is due on February 22.
6. You may consult your notes, the official course text, and any class recordings posted on Canvas. You may not use any other external resources, such as calculators.
7. Please print this cover sheet, sign the academic integrity statement, and upload it to Canvas along with your completed exam. If you prefer not to print the cover sheet, you may include a typed or handwritten sheet of paper on which you copy and sign the academic integrity statement below.

Statement on Academic Integrity

I acknowledge that this exam is my own work, completed with the aid of only those sources permitted above. My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

____
Name

____ __________
Signature Date

1. Use the Euclidean Algorithm to find the greatest common divisor d of $28+13i$ and $5+i$ in the Gaussian integers. Then find all pairs of Gaussian integers (x, y) satisfying the equation $(28+13i)x+(5+i)y=d$.

2. We say a positive integer $q \neq 1$ is *primary* if whenever $q \mid ab$, either $q \mid a^n$ or $q \mid b^n$ for some n. Show that q is primary if and only if q has only one prime factor.

3. Consider the ring $\mathbb{Z}[\sqrt{-6}]=\{a+b\sqrt{-6}: a, b \in \mathbb{Z}\}$. Use the norm $N(a+b\sqrt{-6})=a^2+6b^2$ to show that $\mathbb{Z}[\sqrt{-6}]$ does not satisfy unique factorization. (You may use the fact that the norm is the square of the complex absolute value.)

4. Read Theorem 3.4 on page 81 of your textbook. Then adapt the proof to show that there are infinitely many integer triples (a, b, c) with $\gcd(a, b, c)=1$ and $a^2-2b^2=c^2$.