# Math 350 Midterm III

## Instructions and Information

1. Read these instructions carefully, and sign the Statement on Academic Integrity below once you have completed the exam.
2. This test contains a cover page and 4 problems.
3. Each question is worth the same number of points.
4. In addition to accuracy, you will be graded on the **correctness, completeness, clarity, and neatness of your accompanying work and explanations.** While we appreciate if you type your responses using LaTeX, you are not required to do so, and it is better to turn in good hand-written work than hastily-prepared solutions written in TeX.
5. This midterm is due on April 27.
6. You may consult your notes, the official course text, and any class recordings posted on Canvas. You may not use any other external resources, such as calculators.
7. Please print this cover sheet, sign the academic integrity statement, and upload it to Canvas along with your completed exam. If you prefer not to print the cover sheet, you may include a typed or handwritten sheet of paper on which you copy and sign the academic integrity statement below.

## Statement on Academic Integrity

I acknowledge that this exam is my own work, completed with the aid of only those sources permitted above. My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

_____

Name

_____ _____

Signature Date

1. Let $Q(x, y)$ be an integral binary quadratic form, and $\bar{v}$ and $\bar{w}$ a pair of vectors such that $\{\pm\bar{v}, \pm\bar{w}\}$ is a lax basis. If $Q(\bar{w}) > 0$ and $Q(\bar{v} - \bar{w}) > Q(\bar{v})$, show that $Q(\bar{v} - 2\bar{w}) > Q(\bar{v})$.

2. Does $x^2 + 6y^2 = 7,516,800$ have an integer solution? Show work to justify your answer.

3. Prove that the quadratic forms $x^2 + xy + 4y^2$ and $23x^2 - 13xy + 2y^2$ are not properly equivalent.

4. Prove that every positive definite binary integral quadratic form of discriminant $-19$ is properly equivalent to $x^2 + xy + 5y^2$. You may take for granted any statements which are proven in the textbook. (But your argument should not rely on any claims from class or the book that were provided without proof, unless you provide the proofs yourself).