

MATH 3500 Spring 2023 (lecture 1)

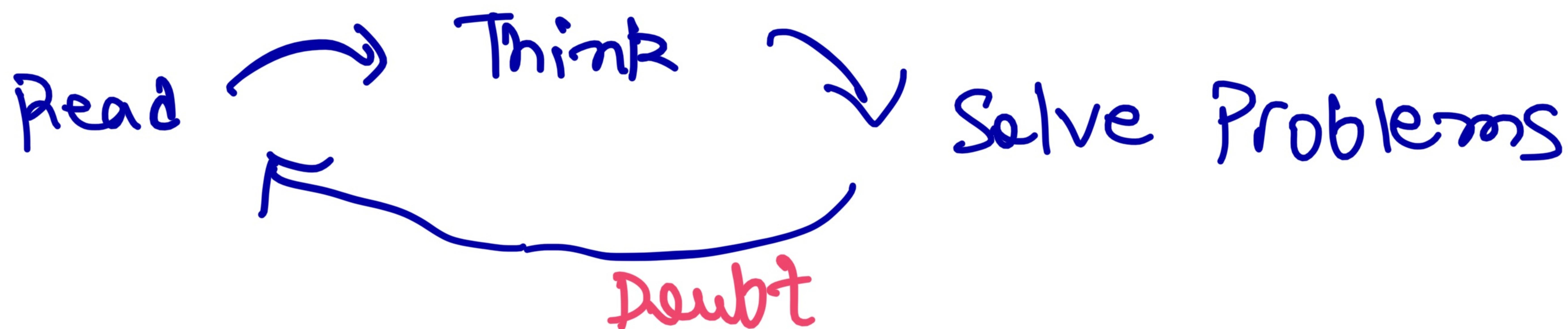
Reasons to Study Number Theory

- "Fascination with numbers"; this is how most of the subject developed in ancient times
- Nowadays Number Theory has a lot of applications in data security, data science, Cryptography etc.

How do you learn by reading a text book?

Read → Think → Repeat

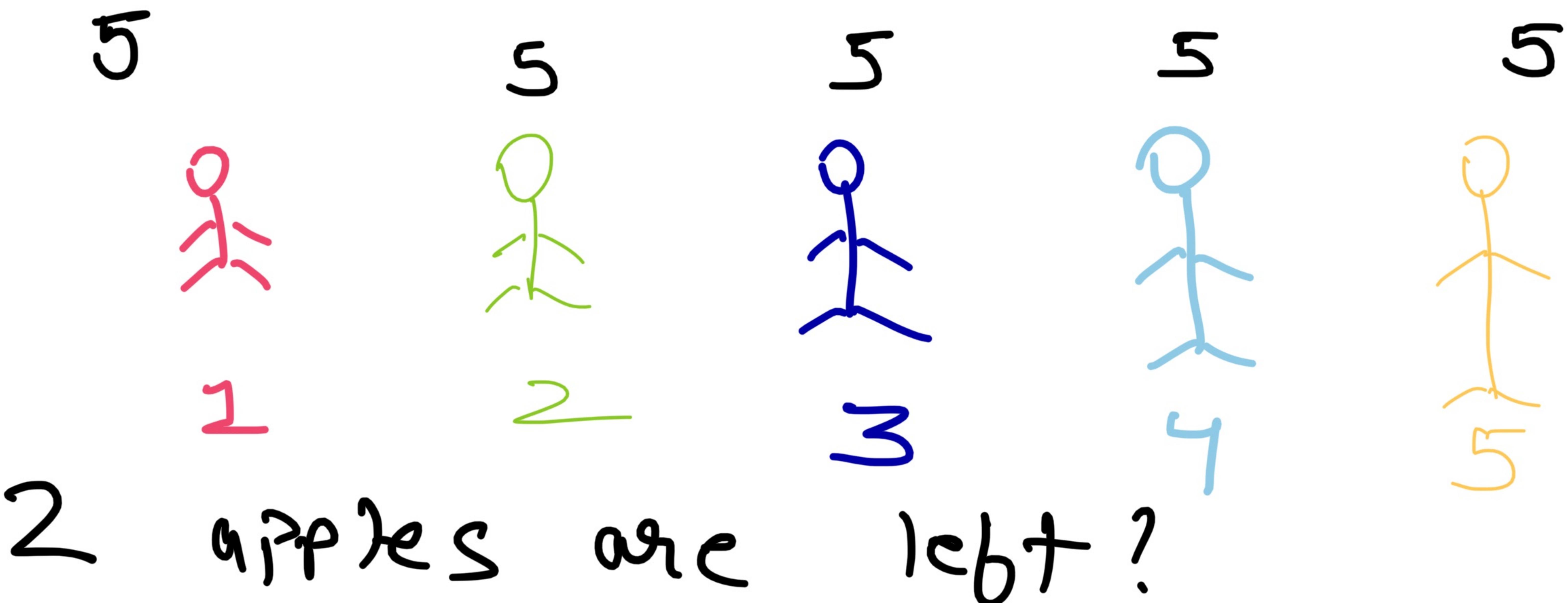
How many times do you do that?
Till you feel confident to attempt exercises.



Division

Intuitive idea behind division is
distribution.

Context: Divide 27 apples among
5 children.



$$27 = \underbrace{5(5)}_{\text{quotient}} + 2 \leftarrow \text{remainder}$$

Proposition: For two integers a and b ,
 $b > 0$ and $b < a$ there exists two
integers q, r where $0 < r < b$ such that

$$a = qb + r$$

Proof:

$$0 \leq \frac{a}{b} - \left\lfloor \frac{a}{b} \right\rfloor < 1$$

Multiply by b

$$0(b) \leq b\left(\frac{a}{b}\right) - b\left\lfloor \frac{a}{b} \right\rfloor < b$$

$$0 \leq a - b\left\lfloor \frac{a}{b} \right\rfloor < b$$

Call $\downarrow r$

$$a - b\left\lfloor \frac{a}{b} \right\rfloor = r$$

or

$$a = b\overbrace{\left\lfloor \frac{a}{b} \right\rfloor}^2 + r$$

If $r=0$, we say b divides a
or a is a multiple of b .

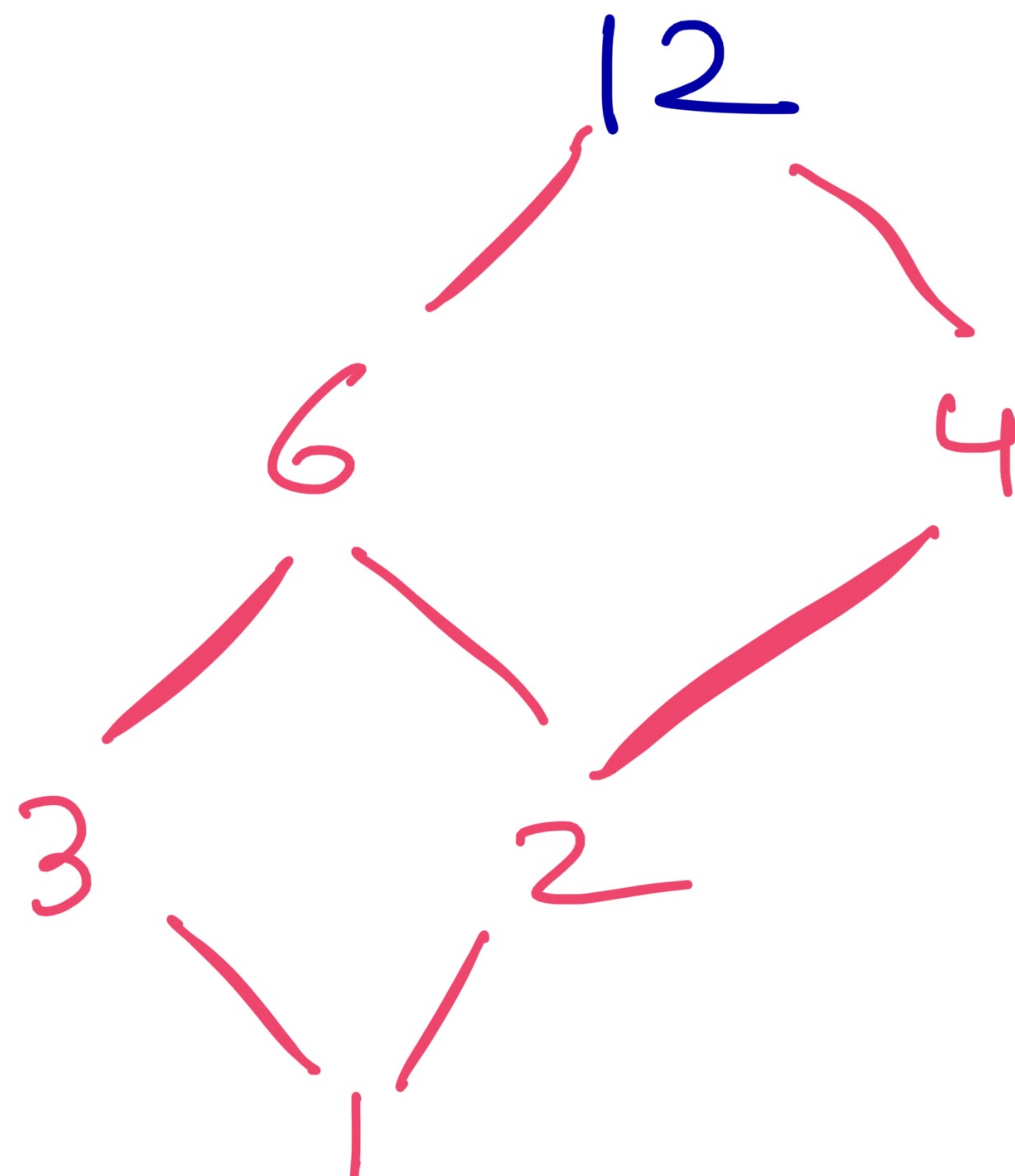
and write it as $b|a$.

Be Careful of this notation

$4|8$ but $8|4$
is incorrect.

If $r \neq 0$ then we write $b \nmid a$.

Hasse Diagram



Reflexive
Property

for every integer x

$$x|x. \quad (\text{Why?})$$

Anti Symmetric
Property

If $x|y$ & $y|x$, then
 $x = \pm y$. (Why?)

Transitive
Property

If $x|y$ and $y|z$, then
 $x|z$.

Problem 1:

find all integers x

such that $x \mid (x+6)$.

The only observation that you need to make here is that

$$x \mid (x+6) \iff x \mid 6$$

will discuss
this symbol in
class

The only factors of 6 are

$$\{\pm 1, \pm 2, \pm 3, \pm 6\}.$$

Problem 2:

Are there any integer
solutions to $7x^2 + 11 = 2ly$?

No, because if x & y are integers
and satisfy $7x^2 + 11 = 2ly$ then 7 must
divide 11 which is not possible.