MATH 350, SPRING 2023

HOMEWORK 2, DUE FEBRUARY 6

(1) Find a solution to $289x + 323y = 14$.

(2) Find all solutions to $89654685435677987384983 0x + 2578763287334234232y = 5$.

(3) Find all solutions to $3939x + 10403y = 909$.

(4) Let a, b, and c be natural numbers. Write down a reasonable definition for $\gcd(a, b, c)$. If you wish, you may look up the definition online or in the exercises to your textbook (which gives two equivalent definitions, the latter of which perhaps more useful), but only after attempting to formulate your own definition. Now prove that $\gcd(a, b, c) \mid \gcd(a, b)$.

(5) State and prove the analogous theorem for $\operatorname{lcm}(a, b, c)$.

(6) Find all solutions to $15x + 35y + 21z = 1$.

(7) Prove that if $\gcd(a, b) = \operatorname{lcm}(a, b)$, then $a = \pm b$.

(8) Prove that if $c^2 \mid a^2$, then $c \mid a$.

(9) Prove that if $\gcd(a, b) = 1$, then $\gcd(a^2, b^2) = 1$.

(10) *PAR problem.* Let a and b be positive integers, and $d = \gcd(a, b)$. Suppose $a = dm$ and $b = dn$. Prove that $\gcd(m, n) = 1$.