MATH 350, SPRING 2023

HOMEWORK 10, DUE APRIL 24

(1) Sketch range topographs (about 12 values is a decent-sized sketch) containing the triads: $\{1,2,3\}$; $\{-1,0,1\}$; $\{0,0,3\}$; $\{1,1,1\}$.
(2) If $Q(x,y) = ax^2 + bxy + cy^2$, then the *opposite* of Q is $ax^2 - bxy + cy^2$ and the *associate* of Q is $cx^2 + bxy + ay^2$. Show that Q is equivalent to both its opposite and its associate, and that the opposite and associate are properly equivalent to one another. (In general, Q will be improperly equivalent to its opposite and associate, but there are forms for which the equivalence is proper. You may find it enlightening to try to think of examples of forms that are properly equivalent to their opposites and associates.)
(3) Use the fact that

$$ax^2 + bxy + cy^2 = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

to give another proof of the arithmetic progression rule (Theorem 9.27 in the text).
(4) Find the minimum nonzero value for $41x^2 - 60xy + 22y^2$.
(5) Find a solution to the Diophantine equation $x^2 - 11y^2 = 1$ in which $x > 1$
(6) Classify all the isometries of $Q(x,y) = x^2$.
(7) What was the most interesting homework problem this semester? Justify your answer in no more than two sentences.