Numerical Analysis Day 2

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Our problem tonight is finding the roots (zeros) of a continuous function $f(x)$

1 Bisection

If we know points a and b such that $f(a) < 0$ and $f(b) > 0$ we can bisect the interval $[a, b]$ and get a smaller interval with a similar property.

Start with the function $f(x) = x^2 - 2$

a	c = (a+b)/2	b	f(a)	f(c)	f(b)
0	1	2	-2	-1	2
1	1.5	2			

2 Heron's Method

The find the square root of R, we start with an estimate $x = x_0$ and iterate $x_1 = (x_0 + R/x_0)/2$

Let's try this with $x_0 = 1$ and $R = 2$

x	(x + 2/x)/2
1	(1 + 2)/2 = 1.5
1.5	

3 Fixed Point Iteration

Look for fixed points of the iteration $x = \sqrt{x}$ and $x = x^2$

x	$\sqrt{x}$
0.1	

x	x^2
0.9	

4 Three Examples

To find the roots of the function $f(x) = x^3 + x - 1$ we try iterating three functions $x = g(x)$

$$x = 1 - x^3$$

$$x = \sqrt[3]{1 - x}$$

$$x = (1 + 2x^3)/(1 + 3x^2)$$

Conclusion: an iteration converges when: __________________

5 Newton's Method

Newton's method iterates the function $g(x)$ defined as

$$g(x) = x - \frac{f(x)}{f'(x)}$$

Compute $g'(x)$
Fails to converge when: __________________ __________________
For roots of order m, such as $f(x) = (x - 1)^2(x + 1)^3$, iterate

$$g(x) = x - m\frac{f(x)}{f'(x)}$$