Numerical Analysis Day 3

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1 Finding Roots: the Secant Method

Sometimes we cannot compute the derivative. We can still use the Secant Method, which takes two points and interpolates a line between them, using the secant to approximate the tangent. We can start with a pair of points on the curve $(x_0, f(x_0))$ and $(x_1, f(x_1))$ and iterate using the formula

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Take steps to compute the square root of 2 using the Secant Method and the function $f(x) = x^2 - 2$ with the initial points (0, -2), (2, 2).

$(x_{i-1}$	$f(x_{i-1}))$	$(x_i$	$f(x_i))$	$(x_{i+1}$	$f(x_{i+1}))$
0	-2	2	2		
2	2				

2 Regula Falsi

Regula Falsi uses the same equation to compute the intersection of a secant line. Rather than using the points in order, we pick the most recent two points which bracket the x-axis.

Take steps to compute the square root of 2 using the Regula Falsi and the function $f(x) = x^2 - 2$ with the initial points (0, -2), (2, 2).

$(x_{i-1}$	$f(x_{i-1}))$	$(x_i$	$f(x_i))$	$(x_{i+1}$	$f(x_{i+1}))$
0	-2	2	2		

3 Ill-Conditioned Problems

The condition number at a root r, $\frac{1}{|f'(r)|}$, can predict some problems that will lead a root finder to make large errors.

What other problems can cause ill-conditioned functions?

4 Aitken Acceleration

Given a sequence $x_0, x_1, x_2, \ldots$ that converges to limit p. If the convergence is linear, we can compare the error $|p_n - p|$ at each step.

$$\frac{p_{n+1} - p}{p_n - p} = \frac{p_{n+2} - p}{p_{n+1} - p}$$

We can solve for p and get

$$p = p_n - \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$$

It will be useful to define

$$\Delta p_n = p_{n+1} - p_n$$

$$\Delta^2 p_n = \Delta p_{n+1} - \Delta p_n$$

Note that

$$\Delta^2 p_n = (p_{n+2} - p_{n+1}) - (p_{n+1} - p_n) = p_{n+2} - 2p_{n+1} + p_n$$

Using this, we can rewrite our result as

$$p = p_n - \frac{(\Delta p_n)^2}{\Delta^2 p_n}$$

p_n	Δp_n	$\Delta^2 p_n$	$p_n - \frac{(\Delta p_n)^2}{\Delta^2 p_n}$
0			
0.5			
0.75			
0.875			
0.9375			