MATH 212 **Due November 28, 2017**

PROJECT III

Interacting Species

Experiment.

The data below gives the percentage of total catch of selachian in the Mediterranean ports during 1914–1923. Selachian (sharks, skates, etc.) are not desirable fish for food. They are predators while food fish are their prey.

year	1914	1915	1916	1917	1918
%	11.9	21.4	22.1	21.2	36.4

year	1919	1920	1921	1922	1923
%	27.3	16.0	15.9	14.8	10.7

One immediately observes the large increase in the percentage of selachian during World War I (1914–1918). Is there an explanation? The model below offers one.

Development of the Model. Let x be the prey population, and y the predator population. Both populations are functions of time t. The governing differential equations for two species interaction can be written as

$$\begin{aligned} x'(t) &= f(x(t), y(t)) \\ y'(t) &= g(x(t), y(t)) \end{aligned}$$

In our model, we assume that in the absence of predators, prey will grow unlimited according to the equation

$$x'(t) = \alpha x(t)$$

while in the absence of the prey, the predator population will decrease until it eventually dies out according to

$$y'(t) = -\gamma y(t)$$

where $\alpha, \gamma > 0$.

The interaction term between the two species is $-\beta x(t)y(t)$ for the prey and $\delta x(t)y(t)$ for the predator with $\beta, \delta > 0$. Thus, we obtain the following model:

$$\begin{aligned} x'(t) &= \alpha x(t) - \beta x(t)y(t) \\ y'(t) &= -\gamma y(t) + \delta x(t)y(t) \end{aligned} \tag{1}$$

or in matrix form

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & -\gamma \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} -\beta x(t)y(t) \\ \delta x(t)y(t) \end{pmatrix}$$

Mathematical Analysis

1. Find the equilibria of system (1).

2. Choose any positive $\alpha, \beta, \gamma, \delta$. Start close to each equilibrium and find the solution numerically.

3. Plot $(t, x(t))$ and $(t, y(t))$, and the equilibria on the same graph.

4. Plot $(x(t), y(t))$, including the equilibria.

5. Give a mathematical interpretation of the results.

Results.

Write an interpretation of all results so that a non-mathematician can understand the meaning of the solution, and the relationship between the parameters and the solution.

Discuss applications of this model.

Taking Fishing into Account. Fishing will decrease the food fish population at a rate $Ex(t)$, and decrease the selachian population at a rate $Ey(t)$, where $E = cosnt.$ is the fishing effort (i.e., number of boats, nets., etc.). This leads to the modified model

$$\begin{aligned} x'(t) &= \alpha x(t) - \beta x(t)y(t) - Ex(t) \\ y'(t) &= -\gamma y(t) + \delta x(t)y(t) - Ey(t) \end{aligned} \tag{2}$$

For a moderate amount of fishing, $E < \alpha$.

1. Repeat the mathematical analysis above for model (2) for different values of E.

2. What happens to the average population of the predator and prey as E increases or decreases? (*The phenomenon you are discovering is the Volterra principle*).

Results.

Write an interpretation of all results so that a non-mathematician can understand the meaning of the solution, and the relationship between the parameters and the solution.

Can you explain the data in the experiment above?

Discuss applications of this model. For example, insecticide treatment which destroys both insect predators and insect prey.