

# Exam 2 Review

## COSC 320: Advanced Data Structures and Algorithm Analysis

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1. All homework, labs, reviews, exams, and lecture notes since exam 1.
2. What is the binary search tree property?
3. What is the Max-Heap property? What is the analogous Min-Heap property?
4. Describe the heap operations and their respective asymptotic runtimes in terms of the number,  $n$ , of heap elements.
5. Let  $A[1 \cdots n]$  be an  $n$ -element array. An *inversion* of  $A$  is a pair  $(i, j)$  such that  $A[i] > A[j]$ . Suppose that the elements of  $A$  form a uniform random permutation of the numbers  $1, 2, \cdots n$  and use indicator random variables to compute the expected number of inversions.
6. What are the five defining properties of a red-black tree?
7. Draw a valid red-black tree with 7 internal nodes that has exactly 3 red nodes. Then give each internal node a value and show the result of inserting 2 more nodes into the tree. Illustrate each “step” that modifies the tree (coloring, rotation, etc.).
8. Show that any  $n$ -node BST can be transformed into any other  $n$ -node BST by using only  $O(n)$  rotations. Hint: first show that  $n - 1$  right rotations suffice to transform the tree into a right-going chain.
9. Write a dynamic-programming solution to compute the  $n$ th Fibonacci number using at most  $O(n)$  operations.
10. Consider a variation of the rod-cutting problem where each cut incurs a fixed cost  $c$ . The revenue is then the sum of the prices fetched from each piece, minus the total cost of cutting. Show a dynamic programming solution to compute the optimal profit.
11. Consider the variant of the matrix multiplication problem where the goal is to maximize the number of scalar multiplications. Does this problem exhibit optimal sub-structure? Why or why not?