

# Exam 1 Review

## COSC 320: Advanced Data Structures and Algorithm Analysis

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1. All homework, labs, and lecture notes from before Exam 1.
2. Give asymptotic upper and lower bounds for each of the following recurrence relations. Assume that  $T(n)$  is constant for sufficiently small  $n$ .
  - (a)  $T(n) = 7T(n/2) + n^2$
  - (b)  $T(n) = 7T(n/3) + n^2$
  - (c)  $T(n) = 2T(n/4) + \sqrt{n}$
  - (d)  $T(n) = 3T(n/3 - 2) + n/2$
  - (e)  $T(n) = \sqrt{n}T(\sqrt{n}) + n$
3. What is the recurrence that describes the worst-case running time of Quicksort? On what input does this happen? Solve the recurrence and give the asymptotic complexity.
4. Let  $A[1, \dots, n]$  be an array of  $n$  distinct numbers. If  $i < j$  and  $A[i] > A[j]$ , then the pair  $(i, j)$  is called an **inversion** of  $A$ .
  - (a) List the five inversions of the array  $[2, 3, 8, 6, 1]$ .
  - (b) What array with elements from the set  $\{1, 2, \dots, n\}$  has the most inversions? How many does it have?
  - (c) What is the relationship between the running time of insertion sort and the number of inversions in the input array? Justify your answer.
  - (d) Given an algorithm that determines the number of inversions in any permutation on  $n$  elements in  $\Theta(n \log n)$  worst-case time. (Hint: modify merge sort).
5. Professor Diogenes has  $n$  supposedly identical integrated-circuit chips that in principle are capable of testing each other. The professor's test jig accommodates two chips at a time. When the jig is loaded, each chip tests the other and reports whether it is good or bad. A good chip always reports accurately whether the other chip is good or bad, but the professor

cannot trust the answer of a bad chip. Thus, the four possible outcomes of a test are as follows:

Chip $A$ says	Chip $B$ says	Conclusion
$B$ is good	$A$ is good	both are good or both are bad
$B$ is good	$A$ is bad	at least one is bad
$B$ is bad	$A$ is good	at least one is bad
$B$ is bad	$A$ is bad	at least one is bad

- (a) Show that if more than  $n/2$  chips are bad, the professor cannot necessarily determine which chips are good using any strategy based on this kind of pairwise test. Assume that the bad chips can conspire to fool the professor.
  - (b) Consider the problem of finding a single good chip from among  $n$  chips, assuming that more than  $n/2$  of the chips are good. Show that  $\lfloor n/2 \rfloor$  pairwise tests are sufficient to reduce the problem to one of nearly half the size.
  - (c) Show that the good chips can be identified with  $\Theta(n)$  pairwise tests, assuming that more than half of the chips are good. Give and solve a recurrence that describes the number of tests.
6. Where in a Max Heap might the smallest element reside, assuming all elements are distinct?
  7. Is an array that is in descending-sorted order a Max Heap?
  8. Show that, in an array representation of a Max Heap, the leaves are the array elements indexed by  $\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \dots, n$ .
  9. Prove that for an  $n$  element heap, the longest path from a root to a leaf (aka the “height” of the tree) is  $\lfloor \log_2 n \rfloor$ .
  10. Express the running time of the following in terms of the input,  $n$ . Simplify your answer using big- $\Theta$  notation.

(a) 1: **Function**  $f(n)$ :  
 2:  $s := 10$   
 3:  $i := 1$   
 4: **while**  $i < n^2$  **do**  
 5:      $s := s * 2$   
 6:      $j := i$   
 7:     **while**  $j > 1$  **do**  
 8:          $s := s - j$   
 9:          $j := j/2$   
 10:     **end while**  
 11:      $i := i + 5$   
 12: **end while**

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13: return  $s$ 
(b) 1: Function  $g(n)$ :
    2:  $s := 0$ 
    3:  $i := 0$ 
    4: while  $i < n^3$  do
    5:    $s := s + 2$ 
    6:    $i := i + n$ 
    7: end while
    8: return  $g(\lfloor n/2 \rfloor) + g(\lfloor n/2 \rfloor)$ 

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