Mathematics 172 Homework

Consider two species, victims and predators. We assume that the only food available to the predators is the victims. And we assume that the victim population is small enough that its growth can be modeled with a constant per capita growth rate. Thus $V = V(t)$ is the number of victims, then without any predators, V satisfies the rate equation

$$\frac{dV}{dt} = rV$$

where r is the per capita growth rate. Likewise we assume that without any victims the predator population size has a constant per capita death rate, that is

$$\frac{dP}{dt} = -dP$$

where d is the per capita death rate. When both victims and predators are present model the interaction between the two as

$$\frac{dV}{dt} = rV - \alpha VP \tag{1}$$

$$\frac{dP}{dt} = -dP + \beta VP \tag{2}$$

where the positive constants α and β depend on how the species interact. (Very very roughly α measures how efficient the predators are, and β measures how nutritious the victims are.)

1. Show that the system (1), (2) of equation has two rest points. $(V, P) = (0, 0)$ and $(V, P) = (\widehat{V}, \widehat{P})$ where

$$\widehat{V} = \frac{d}{\beta}$$

$$\widehat{P} = \frac{r}{\alpha}$$

We will call $\widehat{V}$ the ***average number*** of victims and $\widehat{P}$ the ***average number*** of prey.

2. Show that the phase space (that is the V-P plane) for this system looks like Figure 1.

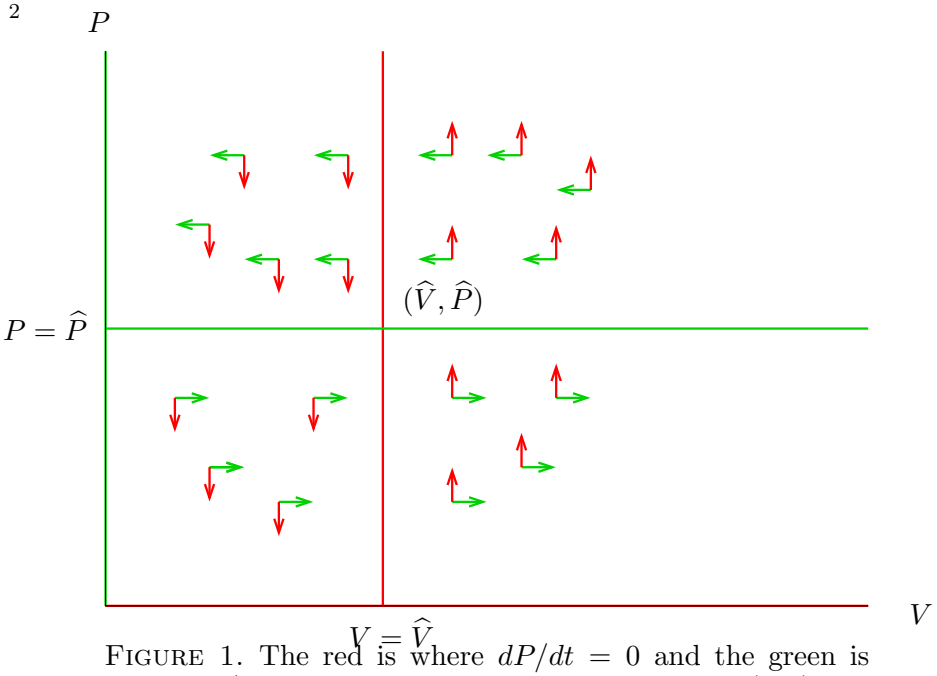

FIGURE 1. The red is where $dP/dt = 0$ and the green is where $dV/dt = 0$. Thus there are two rest points: $(0, 0)$ and $(\widehat{V}, \widehat{P})$.

3.

Foe the predator-victim system

$$\frac{dV}{dt} = .01V - .002VP$$
$$\frac{dP}{dt} = -.1P + .001VP$$

(a) What is the average number of victims? *Answer:* $\widehat{V} = .1/.001 = 100$

(b) What is the average number of predators? *Answer:* $\widehat{P} = .01/.002 = 5$

(c) If we start with 130 victims and 7 predators, what are $V'(0)$ and $P'(0)$? *Answer:* $V'(0) = -.56$, $P'(0) = .21$

(d) Base on your answer to the last question, is V initially increasing or decreasing. Is P initially increasing or decreasing. *Answer:* V is decreasing and P is increasing.

(e) Using the data from part 3 estimate $V(.2)$ and $P(.2)$. Likewise estimate $V(2)$ and $P(2)$. *Answer:*

$$V(.2) \approx V(0) + V'(0).2 = 130 + (-.56)(.2) = 129.888$$
$$P(.2) \approx P(0) + P'(0).2 = 7 + (.21)(.2) = 7.042$$

$$V(2) \approx V(0) + V'(0)2 = 130 + (-.56)(2) = 128.88$$
$$P(2) \approx P(0) + P'(0)2 = 7 + (.21)(2) = 7.42$$

4. What happens to the average number of victims if the death rate, $d = .1$ of the prey is doubled to $d = .2$ and the other constants are kept the same? *Answer:* The new $\widehat{V}$ is $\widehat{V} = .2/.001 = 200$, so it is doubled.

What is not so obvious for this system is that the points in the phase space move in closed ovals.