Mathematics 172 Homework, January 16, 2019.

1. If

$$\frac{dA}{dt} = .18A$$

find the doubling time of A. *Solution:* The doubling time is $t = 3.8508$.

2. If

$$P' = rP, \quad P(0) = 50, \quad P(3) = 70$$

find

(a) r *Solution:* $P(t) = 50e^{rt}$ and $P(3) = 50e^{3r} = 70$. Solve for r to get $r = \ln(70/50)/3 = 0.1122$.

(b) the doubling time. *Solution:* $P(t) = 50e^{.1122t}$, so we want to solve $P(t) = 50e^{.1122t} = 2(50)$. This give $t = \ln(2)/.1122 = 6.1778$ for the doubling time.

(c) the time when P becomes 2,000. *Solution:* This time we want to solve $P(t) = 50e^{.1122t} = 2{,}000$. The result is $t = \ln(2000/50)/.1122 = 32.88$.

3. Let $A(t)$ have exponential decay with a half life of 17.3. Then find the r so that $A' = rA$. *Solution:* We know $A(t) = A_0e^{rt}$ and that $A(17.3) = A_0e^{17.3r} = 2A_0$. This gives the equation $e^{17.3r} = 2$, which has the solution $r = \ln(2)/17.3 = .040066$.

4. Duckweed (which is the worlds smallest flowering plant[1]) will grow exponentially when it is first introduced into a pond. Assume that $A(0) = .4$ lbs of duckweed is introduced into a pond and that 4 days later there is 2.3 lbs of duckweed. Let $A(t)$ be the number of lbs of duckweed after t days.

(a) Find the r so that $A' = rA$. *Solution:* We have $A(t) = .4e^{rt}$ and $A(4) = .4e^{4r} = 2.3$. Solve to get $r = \ln(2.3/.4)/4 = .4373$.

(b) What is the doubling time for the duckweed in the pond? *Solution:* We wish to solve $A(t) = .4e^{.4373t} = 2(.4)$, that is $e^{.4373t} = 2$. This has solution $t = \ln(2)/.4373 = 1.585$.

(c) How much duckweed is there after a week (that it $t = 7$)? *Solution:* We know $A(t) = .4e^{.4373t}$. So $A(7) = .4e^{.4373(7)} = 8.540$ lbs.

(d) How long until there is a ton (2,000 pounds) of duckweed? This time we wish to solve $A(t) = .4e^{.4373t} = 2{,}000$. The solution is $t = \ln(2000/.4)/.4373 = 19.48$ days.

[1] See `https://en.wikipedia.org/wiki/Lemnoideae`