Review for Test 1.

I will be referring to the homeworks on the class web page. I have made some corrections and additions to these so the numbering may differ from what you have last down loaded. So you are should refer to the current versions when something like "See Theorem 3 on Homework 4" you should look at the current versions.

We started the class with a review of some algebra. This included summation notation and the basic factoring (Homework 1, Propositions 2 and 3).

Sample Problem 1. (a) If $0 < a < b$ are integers and $n \geq 2$ such that $b^n - a^n$ is prime, then $b = a + 1$.

(b) We can refine part (a). If $0 < a < b$ are integers and $n \geq 2$ such that $b^n - a^n$ is prime, then $b = a + 1$ and n is prime. $\square$

We also did bit with the calculus of finite differences. The main result being the "Fundamental Theorem of Summation Theory" (Homework 1, Theorem 5) which is really nothing more than knowing how to sum a telescoping series. We also discussed the ***falling factorial powers***.

Sample Problem 2. Show that if

$$f(x) = \frac{1}{\sqrt{x} + \sqrt{x+1}} \qquad \text{and} \qquad F(x) = \sqrt{x}$$

then

$$\Delta F(x) = f(x).$$

Use this to find a formula for

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k} + \sqrt{k+1}}.$$

$\square$

Sample Problem 3. Find

$$\sum_{k=1}^{20} k(k-1)$$

Hint: $k(k-1) = k^{\underline{2}}$. $\square$

We then talked the basic properties of the integers (algebraic properties, order properties, and the well ordering principle). The last of these was used to principle of induction. There will likely be some type of induction proof. Look at the homework of the problems on pages 6 and 7 of Andrew's book (see class web page for link) gives some examples.

We defined the ***binomial coefficients*** and used induction to show that they are integers and also to prove the binomial theorem. *You should certainly know the statement of the binomial theorem.*

Sample Problem 4. Show

$$\sum_{k=1}^{n} 2^k \binom{n}{k} = 3^n$$

for all positive integers n. *Hint:* $3^n = (2+1)^n$. □

We then got down to basics about divisibility. You should know the definitions of all of the following a ***divides*** b ($a \mid b$), a is ***prime***, a is ***composite***, a is ***even***, a is ***odd***, etc. And you should definitely the statement of the ***division algorithm***. You should be able to do Problems 1–7 on Homework 3. *You should also be able to prove there are infinitely many primes.*

Know the definition of the ***greatest common divisor*** of two integers and how to use the Euclidean algorithm to compute $\gcd(a, b)$. Sample problems would be Homework 4, Problems 2, 4, 8, 13, and

Sample Problem 5. If a, b, c are positive integers, then show

$$\gcd(ac, bc) = c \gcd(a, b).$$

□

Related to the gcd and Bézout's Theorem you show know the statement about the solvablity of the ***linear Diophantine equation*** as stated in Theorem 1 on Homework 5.

You definitely should know the statement of ***Fundamental Theorem of Arithmetic.*** Also you should know the ***rational root test*** and how to use it to show that numbers are irrational.

You will have to prove one or more of the following:

(1) There are infinitely primes.
(2) That every ideal is principal. This includes knowing the definition of an ideal.
(3) Bézout's Theorem.
(4) Being able to use Bézout's Theorem to show if $\gcd(a, b) = 1$ and $a \mid bc$, then $a \mid c$.