

## Mathematics 172 Homework, February 25, 2019.

We saw in class today that for the Leslie matrix

$$L = \begin{bmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{bmatrix}$$

then the eigenvalue (which for us is the **growth ratio** of the stable population distribution) is the positive solution of the **Euler-Lotka equation**

$$\frac{f_1}{\lambda} + \frac{p_1 f_2}{\lambda^2} + \frac{p_1 p_2 f_3}{\lambda^3} = 1.$$

Then the **per capita growth rate** of the stable population distribution is given by

$$r = \lambda - 1.$$

The stable age distribution is given by

$$\begin{bmatrix} \frac{1}{n} \\ \frac{p_1 f_2}{n \lambda} \\ \frac{p_1 p_2 f_3}{n \lambda^2} \end{bmatrix}$$

where

$$n = 1 + \frac{p_1}{\lambda} + \frac{p_1 p_2}{\lambda^2}$$

1. For the Leslie matrix

$$L = \begin{bmatrix} 0 & 3.1 & 23 \\ .05 & 0 & 0 \\ 0 & .9 & 0 \end{bmatrix}$$

(a) Find the growth ratio  $\lambda$ . *Solution:* By solving the Euler-Lotka equation on your calculator find that  $\lambda = 1.06257$ .

(b) What is the per capita growth rate of the stable distribution? *Solution:* It is  $r = \lambda - 1 = .06257$ .

(c) What is the stable age distribution? *Solution:* By plugging into the formulas above we find:

$$\text{Proportion in stage 1} = .920038$$

$$\text{Proportion in stage 2} = .043293$$

$$\text{Proportion in stage 3} = .036669$$

2. Compute the same things for the Leslie matrix

$$L = \begin{bmatrix} .08 & 5.6 & 49 \\ .02 & 0 & 0 \\ 0 & .85 & 0 \end{bmatrix}$$

*Solution:* Plug and chug gives

$$\lambda = 1.00908$$

$$r = .00908$$

$$\text{Proportion in stage 1} = .96477$$

$$\text{Proportion in stage 2} = .01912$$

$$\text{Proportion in stage 3} = .01610$$