Mathematics 172 Homework, April 4, 2019.

We wish to program our TI calculators to do Euler's method on the SIR system

$$\begin{aligned} S' &= -bSI \\ I' &= bSI - kI \\ R' &= kI \end{aligned}$$

The calculator does not like the variables S, I and R so we will use variables it does like, which are $u = S$, $v = I$, and $w = R$. So the system now becomes

$$\begin{aligned} u' &= -buv \\ v' &= buv - kv \\ w' &= kv \end{aligned}$$

We will be doing Euler steps of length $h = 1$. That is we will be computing the values at the integer points $0, 1, 2, 3, \ldots$. If we have the values at the value $n - 1$, then the differential equations tell us that the values of the derivatives are

$$\begin{aligned} u'(n-1) &= -bu(n-1)v(n-1) \\ v'(n-1) &= bu(n-1)v(n-1) - kv(n-1) \\ w'(n-1) &= kv(n-1). \end{aligned}$$

Euler tells us the next approximation is

$$\begin{aligned} u(n) &= u(n-1) + u'(n-1)(1) \\ v(n) &= v(n-1) + v'(n-1)(1) \\ w(n) &= w(n-1) + w'(n-1)(1) \end{aligned}$$

These can be combined to give that the next step in the approximation is

$$\begin{aligned} u(n) &= u(n-1) - bu(n-1)v(n-1) \\ v(n) &= v(n-1) + bu(n-1)v(n-1) - kv(n-1) \\ w(n) &= w(n-1) + kv(n-1). \end{aligned}$$

The goal now is to make the calculator do the work of this for us. To start use the values

$$\begin{aligned} b &= .001 \\ k &= .2 \\ u(0) &= S_0 = 990 \\ v(0) &= I_0 = 10 \\ w(0) &= R_0 = 0. \end{aligned}$$

We store the first two of these in the B and K registers. To store the first number press

.001 STO ALPHA ENTER.

(What you will see on the screen is .001→B.) You can check that this has worked by pressin sf 2ND RCL ALPHA B ENTER. Do the same steps to store .2 in the K register.

We now need to set up the calculator to work with tables. To start press the MODE key and the calculator will open up a screen that looks something like this (some of the highlighted boxes may be in different places):

[NORMAL] SCI ENG
[FLOAT] 0 1 2 3 4 5 6 7 8 9
[RADIAN] DEGREE
[FUNC] PAR POL SEQ
[CONNECTED] DOT
[SEQUENTIAL] SIMUL
[REAL] a+bi re^θi
[FULL] HORIZ G-T

Use the cursor key to move down to the forth line and over to SEQ and press enter to change from FUNC mode to SEQ mode. The screen will now look like:

[NORMAL] SCI ENG
[FLOAT] 0 1 2 3 4 5 6 7 8 9
[RADIAN] DEGREE
FUNC PAR POL [SEQ]
[CONNECTED] DOT
[SEQUENTIAL] SIMUL
[REAL] a+bi re^θi
[FULL] HORIZ G-T

Now press 2ND TABLESET and edit until it looks like

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt : [Auto] Ask
Depend: [Auto] Ask

Now press the Y= key. If you have never used the SEQ mode before it will look like

Plot1 Plot2 Plot2
nMin=
\u(n)=
u(nMin)=
\v(n)=
v(nMin)=
\w(n)=
w(nMin)=

Edit this until it look like as follows as below. There are some tricks involved in this:

- Where there is an n use the X,T, θ, n key.
- For u, v, and w use 2ND u (over the 7 key), 2ND v (over the 8 key), and 2ND w (over the 9 key).
- For B use press ALPHA B and for K press ALPHA K. (These recover our stored values of b and k.)

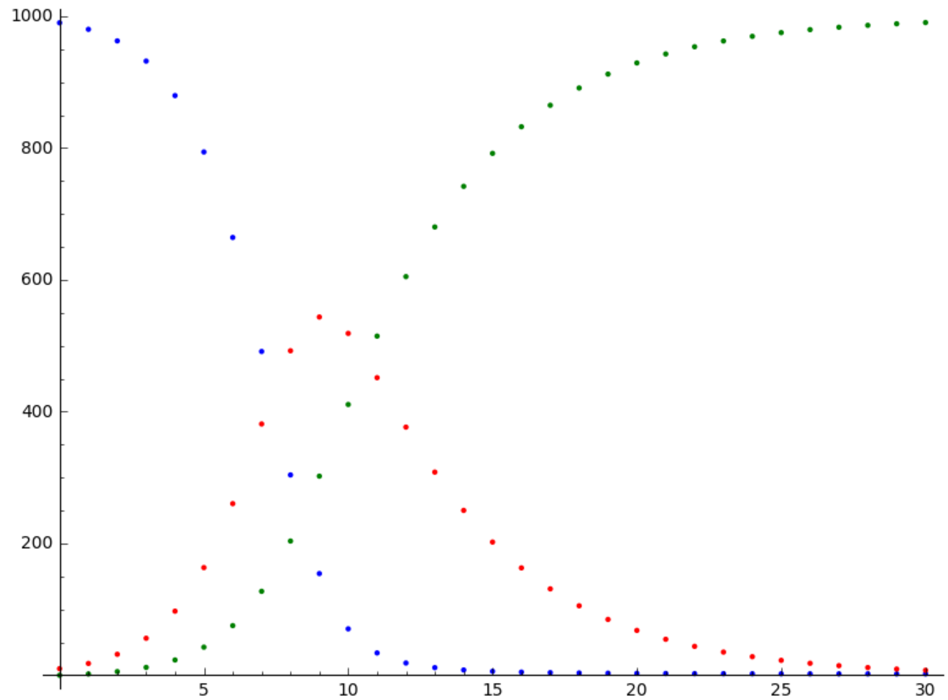
Plot1 Plot2 Plot2
nMin=0
\u(n)=u(n-1)-Bu(n-1)v(n-1)
u(nMin)=990
\v(n)=v(n-1)+Bu(n-1)v(n-1)-Kv(n-1)
v(nMin)=10
\w(n)=w(n-1) + Kv(n-1)
w(nMin)=0

And we are now pretty much done. Press sf 2ND TABLE and you get a table that looks like

n	u(n)	v(n)
0	990	10
1	980.1	17.9
2	962.56	31.864
3	931.89	56.162
4	879.55	97.266
5	794	162.36
6	664.29	260.4

You can get the w values by moving the cursor to the right. Likewise you can scroll down to get the values of u, v and w for values of n larger than 6.

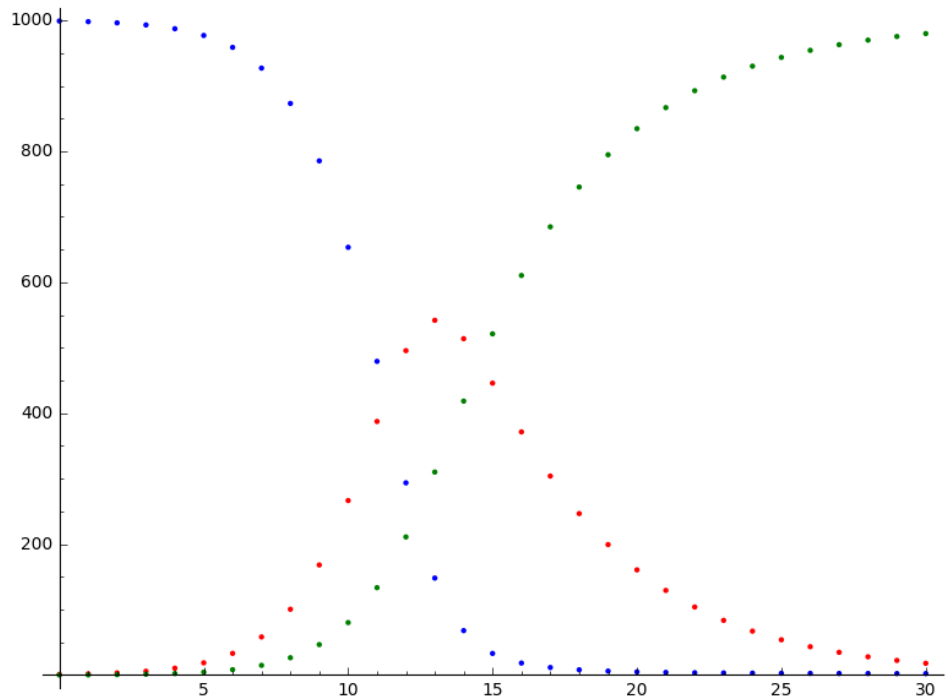
We can also graph this data. Press WINDOW and set
nMin = 0
nMax = 30
Do a ZoomFit and you should get graph that looks like



Now we can ask what happens if we change the initial values to

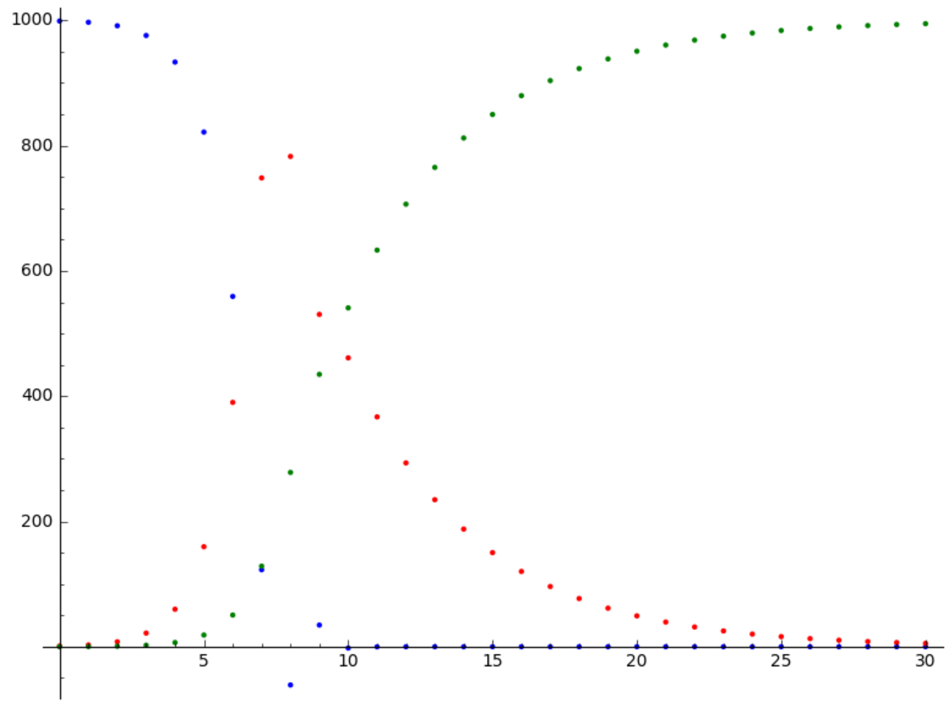
$$S_0 = 999$$
$$I_0 = 1$$
$$R_0 = 0$$

That is we only have one infected to start with. To do this you just go $\mathsf{Y=}$ and change the values of $\mathsf{u}(n\mathsf{Min})$ and $\mathsf{v}(n\mathsf{Min})$ to 999 and 1. The new graph looks like



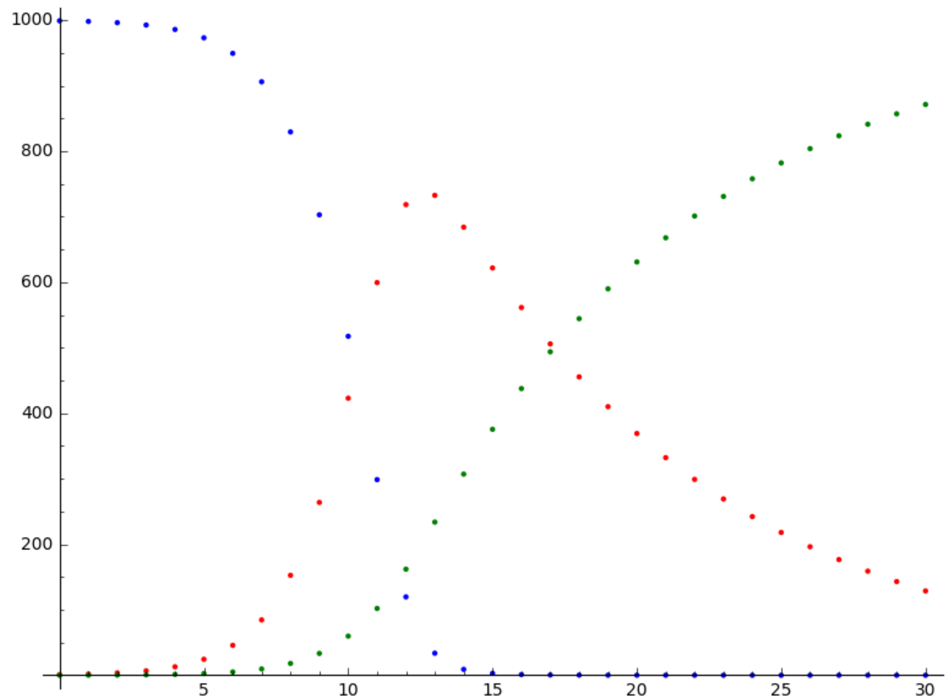
Which is not much of a change. The epidemic is just a little bit slower to get started.

What if we change b to a larger value. This would means that the infection is more contagious. Change the value of b to $b = .002$ by storing .002 in the B register. Still using $S_0 = 999$ and $I_0 = 1$ the graph is



In this case the infection spreads faster (no surprise). The negative values mean that we should have taking a smaller step size in Euler's method. But this is still good enough to see what is going on.

Let us do one more experiment. Change b back to $b = .001$. We have been using the value $k = .2$, which means that the average length of an infection is $1/k = 5$ days. If we change k to $k = .1$, so that the length of an infection is 10 days, then the graph is



Thus lengthening the length of the infection slows how fast it spreads, which I find a bit surprising.