Mathematics 242 Homework.

Here are some practice problems. I took some of these problems from the online text *Elementary Differential Equations* by William F. Trench which can be found at

`http://ramanujan.math.trinity.edu/wtrench/texts/TRENCH_FREE_DIFFEQ_I.PDF`

It is a good source for worked examples.

Problem 1. Is $y = e^x$ a solution to $y' = \frac{1}{y^2}$? Explain how you got your answer. □

Problem 2. Assume

$$\frac{dy}{dx} = \frac{1+x-y}{1+x^2+y^2}$$

(a) If $y(0) = 2$ what is $y'(0)$?
(b) If $y(2) = 4$ compute $y'(2)$ and use it to estimate $y(2.05)$. □

Problem 3. Find the critical points of the equation

$$y' = .1y(y-5)(15-y).$$

and sketch graphs of the critical solutions along with the solutions with $y(0) = 20$, $y(0) = 10$, $y(0) = 3$, and $y(0) = -2$ all on the same axis. If $y(0) = 3$ estimate $y(100)$. □

Problem 4. Find the general solution to the following:

(a) $\dfrac{du}{dt} - ku = 0$ where k is constant.
(b) $y' + 3y = 1$.
(c) $y' + (\tan x)y = \cos x$.
(d) $y' = \dfrac{3x^2+2x+1}{2y-3}$.
(e) $x^2yy' = (y^2-1)^{3/2}$.
(f) $7xy' - 2y = -\frac{x^2}{y^6}$.
(g) $y' - xy = x^3y^3$.
(h) $y' = \dfrac{y+x}{x}$.
(i) $y' = \dfrac{y^2+2xy}{x^2}$.

Problem 5. Find the solutions to the following initial value problems.

(a) $y' = \dfrac{xy+y^2}{x^3}, \quad y(-1) = 2.$
(b) $\dfrac{du}{dt} - tu = tu^{3/2}, \quad u(1) = 4.$
(c) $y' + ky = 1, \quad y(0) = 1$ where $k > 0$ is a constant.
(d) $y' + x(y^2+y) = 0, \quad y(2) = 1.$

Problem 6. A ceramic insulator is baked at 400°C and cooled in a room in which the temperature is 25°C. After 4 minutes its temperature is 200°C. What is its temperature after 8 minutes? □

Problem 7. A tank initially contains 40 pounds of salt dissolved in 600 gallons of water. Starting $t = 0$, water that contains 1/2 pound of salt per gallon is poured into the tank at the rate of 4 gal/min and the mixture is drained from the tank at the same rate.

(a) Find a differential equation for the quantity $Q(t)$ of salt in the tank at time $t > 0$ and solve the equation to determine $Q(t)$.

(b) Find $\lim_{t\to\infty} Q(t)$. □