# ON THE SIZE OF APPROXIMATELY CONVEX SETS IN NORMED SPACES

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ABSTRACT. Let $X$ be a normed space. A set $A \subseteq X$ is *approximately convex* if $d(ta + (1-t)b, A) \leq 1$ for all $a, b \in A$ and $t \in [0,1]$. We prove that every $n$-dimensional normed space contains approximately convex sets $A$ with $\mathcal{H}(A, \mathrm{Co}(A)) \geq \log_2 n - 1$ and $\mathrm{diam}(A) \leq C\sqrt{n}(\ln n)^2$, where $\mathcal{H}$ denotes the Hausdorff distance. These estimates are reasonably sharp. For every $D > 0$, we construct worst possible approximately convex sets in $C(0,1)$ such that $\mathcal{H}(A, \mathrm{Co}(A)) = \mathrm{diam}(A) = D$. Several results pertaining to the Hyers-Ulam stability theorem are also proved.



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