Mathematics 172 Homework

Before doing these problems look at previous homework and make sure you understand the first problem there, which is about the equilibrium points of the discrete logistic equation and understand when they are stable.

Here are some problem for finding equilibrium points in discrete dynamical and determining if they are stable using your calculator.

1. For the dynamical system

$$P_{t+1} = P_t + .3P_t\left(1 - \frac{P_t^{1.5}}{100}\right)$$

Find the equilibrium points and determine if they are stable.

Solution: First enter

`\Y1=X+.3X(1-X^(1.5)/100)`

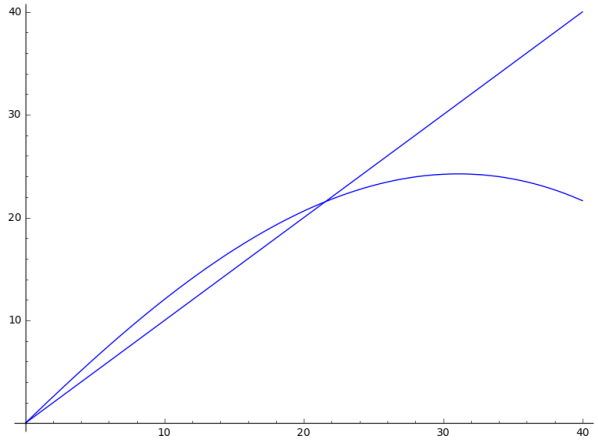
`\Y2=X`

And by some trial and error I found that good size for the window is

`Xmin=0`

`Xmax=40`

Now plot by using `ZOOM` and then `0:ZoomFit` The result should look something like:



From the picture we see that there are two equilibrium points.

To find them use `2nd CALC 5:intesect` It will now ask your `First curve`. Just hit verb+ENTER+. It will now ask `Second curve`. Again just hit verb+ENTER+. The next (and last) question is `Guess?`. This time move the cursor to be as close as possible to the intersection point you want to find and hit `ENTER`. It you moved the cursor to 0 (which is clearly and equilibrium point from the picture) we get that `x=0` and `Y=0` is an intersection point. To find if this is stable we hit `2nd CALC 6:dy/dx` which will give us the value of the derivative of the `\Y1` curve at the current x value, which is $x = 0$. This will tell use that $f'(0) = 1.3$. As $|f'(0)| = 1.3 > 1$ the point $P_* = 0$ is unstable.

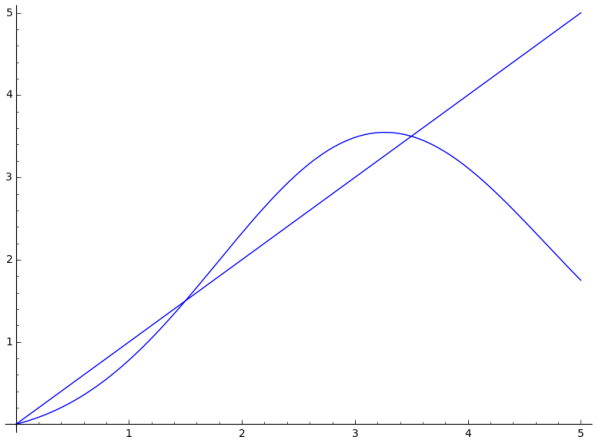
To find the second equilibrium point go through the steps above with the difference that when you are ask `Guess?`, you move the cursor to get as close as you can to the other point of intersection. This time you will get that `X=21.544347` and that `Y` has the same value. Therefore $P_* = 21.544347$ is the second equilibrium point. Now use the calculator to find the value of $f'(21.544347) = .55$ so this point is stable.

2. For the dynamical system

$$N_{t+1} = .5N_t e^{N_t - .2N_t^2}$$

(a) Plot $y = f(x) = .5xe^{x-.2x^2}$ and $y = x$ on your calculator for $0 \leq x \leq 5$.

Solution: Your picture should look like:



(b) So we see there are three equilibrium points. Now find them and determine if they are stable or unstable.

Solution: The first is $P_* = 0$. As $f'(0) = .35$ this one is stable.

The second is $P_* = 1.4995554$ and at this point $f'(1.4995554) = 1.60008883447996$ so this one is unstable.

The third is $P_* = 3.5004446$ and at this point $f'(3.5004446) = -0.400800357451433$ so this one is stable.