CIRCLES MINIMIZE MOST KNOT ENERGIES

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Let $\mathcal{U}(L)$ denote the space of continuous mappings $c\colon \mathbf{R} \to \mathbf{R}^n$ with period $L > 0$, which have *unit speed*, i.e., $\|c'(t)\| = 1$ for almost all $t \in \mathbf{R}$. Define a functional on $\mathcal{U}(L)$ by

$$E_j^p[c] := \int_0^L \int_0^L \left(\frac{1}{\|c(t) - c(s)\|^j} - \frac{1}{\min\{|t-s|, L - |t-s|\}^j} \right)^p dt\, ds \tag{0.1}$$

For embedded $C^{1,1}$ curves $c \in \mathcal{U}(L)$, $E_j^p[c] < \infty$ for $j < 2 + 1/p$. These functionals include as special cases many of the "knot energies" introduced by O'Hara and Freedman-He-Wang.

Theorem 0.1. *Suppose $j < 2 + 1/p$, and $p \geq 1$. Then for every $c \in \mathcal{U}(L)$,*

$$E_p^j[c] \geq 2\pi^{jp-1} L^{2-jp} \int_0^{\pi/2} \left(\left(\frac{1}{\sin(s)} \right)^j - \left(\frac{1}{s} \right)^j \right)^p ds,$$

with equality if and only if c traces a circle of radius $L/(2\pi)$.

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Date: December 12, 2000.
1991 *Mathematics Subject Classification.* Primary 53A04; Secondary 52A40.
Key words and phrases. knot energies, minimizers, distortion of curves.
*Supported in part by DoD Grant No. N00014-97-1-0806.