CHARACTERIZATION OF EIGENFUNCTIONS BY BOUNDEDNESS CONDITIONS

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Abstract

Suppose $\{f_k(x)\}_{k=-\infty}^{\infty}$ is a sequence of functions on $\mathbb{R}^n$ with $\Delta f_k = f_{k+1}$ (where Δ denotes the Laplacian) that satisfies the growth condition: $|f_k(x)| \leq M_k(1+|x|)^a$ where $a \geq 0$ and the constants have sublinear growth $\frac{M_k}{k} \to 0$ as $k \to \pm\infty$. Then $\Delta f_0 = -f_0$. This characterizes eigenfunctions f of Δ with polynomial growth in terms of the size of the powers $\Delta^k f$, $-\infty < k < \infty$. It also generalizes results of Roe (where $a = 0$, $M_k = M$, and $n = 1$) and Strichartz (where $a = 0$, $M_k = M$, for n). The analogue holds for formally self-adjoint constant coefficient linear partial differential operators on $\mathbb{R}^n$.

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