Name: _____ Key _____

You must show your work to get full credit.

1. Show that $5\sqrt{2} + 9$ is irrational. (You may assume that $\sqrt{2}$ is irrational.)

Assume $5\sqrt{2} + 9$ is rational, say

$$5\sqrt{2} + 9 = \frac{a}{b} \quad \text{with } a, b \text{ integers}$$

Then $5\sqrt{2} = \frac{a}{b} - 9 = \frac{a - 9b}{b}$

$$\sqrt{2} = \frac{a - 9b}{5b} = \frac{\text{integer}}{\text{integer}}$$

which implies $\sqrt{2}$ is rational, a contradiction.

2. Show that if 4 divides n, then 4 does not divide $n + 1$.

Towards a contradiction assume $4 \mid n$ and $4 \mid (n+1)$. Then for some integers k, ℓ

$$n = 4k, \quad n + 1 = 4\ell$$

But then $1 = (n+1) - n$
$$= 4\ell - 4k$$
$$= 4(\ell - k)$$

which gives the contradiction that $4 \mid 1$.