Mathematics 122

Quiz 14 Name: Key

You must show your work to get full credit.

1. Let a and b be constants. Find the following derivatives.

(a) $A = \frac{2}{r} - 2\sqrt{r} = 2r^{-1} - 2r^{\frac{1}{2}}$

$\frac{dA}{dr} = -2r^{-2} - 2(\frac{1}{2})r^{-\frac{1}{2}}$

$= -2r^{-2} - r^{-\frac{1}{2}}$

$\frac{dA}{dr} = -2r^{-2} - r^{-\frac{1}{2}}$

$= \frac{-2}{r^2} - \frac{1}{\sqrt{r}}$

(b) $V = \frac{4}{a^5} + 2\pi b r^3$

$V' = (\frac{4}{a^5})' + (2\pi b r^3)'$

$= 0 + 3 \cdot 2\pi b r^2 = 6\pi b r^2$

(constant)$' = 0$

$\frac{dV}{dr} = 6\pi b r^2$

2. Find the second derivatives of the following

(a) $f(x) = 5x^3 - 4\sqrt{x}$

$= 5x^3 - 4x^{\frac{1}{2}}$

$f'(x) = 15x^2 - 2x^{-\frac{1}{2}}$

$f''(x) = 30x + x^{-\frac{3}{2}}$

$f''(x) = 30x + x^{-\frac{3}{2}}$

$= 30x + \frac{1}{x^{3/2}}$

(b) $P(t) = t^2 + t^{-2} = t^2 + t^{-2}$

$P'(t) = 2t - 2t^{-3}$

$P''(t) = 2 + 6t^{-4}$

$P''(t) = 2 + 6t^{-4}$

$= 2 + \frac{6}{t^4}$

3. Find the tangent line to $y = \frac{12}{x}$ at the point where $x = 2$.

The equation of the tangent line is $y = -3x + 12$

The point-slope form of a line is

$y - y_0 = m(x - x_0)$

In our case

$y_0 = y(2) = \frac{12}{2} = 6$

$y = 12x^{-1}$

$y' = -12x^{-2} = \frac{-12}{x^2}$

so

$m = y'(2) = \frac{-12}{2^2} = -3$

$y - y_0 = m(x - x_0)$

$y - 6 = -3(x - 2)$

$y - 6 = -3x + 6$

$y = -3x + 12$