

ON THE SIZE OF APPROXIMATELY CONVEX SETS IN NORMED SPACES

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ABSTRACT. Let X be a normed space. A set $A \subseteq X$ is *approximately convex* if $d(ta + (1 - t)b, A) \leq 1$ for all $a, b \in A$ and $t \in [0, 1]$. We prove that every n -dimensional normed space contains approximately convex sets A with $\mathcal{H}(A, \text{Co}(A)) \geq \log_2 n - 1$ and $\text{diam}(A) \leq C\sqrt{n}(\ln n)^2$, where \mathcal{H} denotes the Hausdorff distance. These estimates are reasonably sharp. For every $D > 0$, we construct worst possible approximately convex sets in $C(0, 1)$ such that $\mathcal{H}(A, \text{Co}(A)) = \text{diam}(A) = D$. Several results pertaining to the Hyers-Ulam stability theorem are also proved.

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