A GENERAL THEORY OF ALMOST CONVEX FUNCTIONS.

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Abstract. Let $\Delta_m = \{(t_0, \dots, t_m) \in \mathbf{R}^{n+1} : t_i \geq 0, \sum_{i=0}^m t_i = 1\}$ be the standard m-dimensional simplex. Let $\varnothing \neq S \subset \bigcup_{m=1}^\infty \Delta_m$, then a function $h\colon C \to \mathbf{R}$ with domain a convex set in a real vector space is S-***almost convex*** iff for all $(t_0, \dots, t_m) \in S$ and $x_0, \dots, x_m \in C$ the inequality

$$h(t_0x_0 + \cdots + t_mx_m) \leq 1 + t_0h(x_0) + \cdots + t_mh(x_m)$$

holds. A detailed study of the properties of S-almost convex functions is made. It is also shown that if S contains at least one point that is not a vertex, then an extremal S-almost convex function $E_S\colon \Delta_n \to \mathbf{R}$ is constructed with the properties that it vanishes on the vertices of Δ_m and if $h\colon \Delta_n \to \mathbf{R}$ is any bounded S-almost convex function with $h(e_k) \leq 0$ on the vertices of Δ_n, then $h(x) \leq E_S(x)$ for all $x \in \Delta_n$. In the special case $S = \{(1/(m+1), \dots, 1/(m+1))\}$ the barycenter of Δ_m very explicit formulas are given for E_S and $\kappa_S(n) = \sup_{x \in \Delta_n} E_S(x)$. These are of interest as E_S and $\kappa_S(n)$ are extremal in various geometric and analytic inequalities and theorems.

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