

# ISRAEL JOURNAL OF MATHEMATICS

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December 23, 2005

Ref: 3534  
Prof. R. Howard  
Department of Mathematics  
University of South Carolina  
Columbia, SC 29208  
U.S.A.

Dear Prof. Howard :


I am pleased to inform you that in view of a positive referee recommendation we accept your paper:

**Smooth convex bodies with proportional projection functions**  
by  
**R. Howard and D. Hug**

for publication in the Israel Journal of Mathematics.\*

We would greatly appreciate your sending us a copy of the manuscript as a text file via electronic mail. We hope that this will help us to speed up the process of publication. In case you cannot send us the file, please let us know as soon as possible, so we will retype the article. In any case the paper will be sent to you for final proof-reading.  
Thank you very much for your cooperation in this matter.

Sincerely yours,

  
Avinoam Mann and Shahar Mozes  
Editors in Chief  
Israel Journal of Mathematics

\* Please note the referee's remarks  
and try to make the paper  
"friendlier", especially the abstract.

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Date: Fri, 23 Dec 2005 10:23:14 +0200 (IST)  
 From: Israel Journal of Mathematics <iton@math.huji.ac.il>  
 To: the israel journal of mathematics <iton@math.huji.ac.il>  
 Subject: 3534

I have completed my investigation of the paper by Ralph Howard and Daniel Hug, "Smooth convex bodies with proportional projection functions," which had been submitted to the Israel Journal.

I am happy to report that I believe that the work presented in the paper is correct. That the result is of sufficient interest to warrant publication in the Israel Journal was never in doubt.

Let me try briefly to provide some background and imprecisely state what the authors have achieved.

Let  $G(n,i)$  be the set of all  $i$ -dimensional subspaces of Euclidean  $n$ -space. Some 80 years ago, Nakajima proved the following result in Euclidean 3-space: If  $K$  is a convex body and  $B$  is a solid ball, and if the lengths of the orthogonal projections of  $K$  and  $B$  onto each line in  $G(3,1)$  are equal and if the areas of the orthogonal projections onto each plane in  $G(3,2)$  of  $K$  and  $B$  are equal, then (assuming certain critical smoothness assumptions)  $K$  and  $B$  are translates of each other. Some 40 years ago Chakerian extended this result by replacing the requirement that  $B$  be a ball with the condition that  $B$  be a convex body that is symmetric relative to reflections about some point --- say the origin.

In the paper under review, the authors extend Chakerian's result to Euclidean  $n$ -space (where  $n > 3$ ). Specifically, the authors prove the following: Suppose  $K$  is a convex body and  $K_o$  is a convex body that is symmetric relative to reflections in the origin. Suppose also that the  $i$ -dimensional volumes of the orthogonal projections of  $K$  and  $K_o$  onto each subspace in  $G(n,i)$  are equal. Suppose further that this is true for two different values of  $i$  in the set  $\{1, \dots, n-1\}$ . Then (assuming certain critical smoothness assumptions)  $K$  and  $K_o$  are translates of each other. As the authors clearly state, their approach fails in only one important case: if the two values of  $i$  are  $n-2$  and  $n-1$ . (This is of course an important special case.)

The results of the authors were claimed in a recent (1999) paper by Haab that appeared in the Journal of Differential Geometry, however Haab's paper turned out to contain gaps. I believe that the authors' proof is the first correct proof of the  $n$ -space extension of Chakerian's 3-space result.

The paper is carefully written but is not what I would describe as "reader friendly". This can be already seen in the abstract: What is " $C^2_+$ "? Yes, every student of Schneider's book will know. Yes, the notation will become standard --- someday. I respect the authors' right to write a not necessarily reader-friendly paper, as long as it is correct. I'm convinced the paper is correct.

I am happy to recommend this paper to you for publication.