

Mathematics 141: Review for Test 1.

- (1) We started out by talking a bit about functions, their domains, ranges, graphs, and composition of functions. As to concrete functions we reviewed a bit about trigonometric functions and exponential and logarithmic functions. Here are some practice problems related to these ideas.

- Section 1.1, Problems 1, 3, 5, 25.
- Section 1.2, Problems 1, 5, 7.
- You should know the values of the basic trigonometric functions at the standard values. That is $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ for $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2$.
- Section 1.5, Problems 1, 3, 9, 11, 13, 15.
- Section 1.6, Problems 45, 55, 57.

- (2) The next topic we covered was limits. Many of the main ideas for limits are summarized in Section 2.2 of the book, and in particular *limit laws* in Theorem 1 of Section 2.2. I am not so interested in you having these memorized as in being able to use them. The same is true for Theorems 2 and 3 of the same section. Here are some practice problems.

- Section 2.2, 11–49 odd, 43, 45, 63.

We also talked about one sided limits. Related to this discussion was the limit

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1.$$

Know this limit as its statement will be on the test. As will be at least one problem about using it. Some practice problems are

- Section 2.4, Problems 1, 3, 11, 21–39 odd.

- (3) We have talked briefly about when a function f is continuous. The definition is that f is **continuous** at the point $x = a$ if and only if

$$\lim_{x \rightarrow a} f(x) = f(a).$$

You should know this definition. The book has several theorems about continuous function, the most important being the *Intermediate Value Theorem*, (Theorem 11). Since we did not talk about it in class, don't worry about it for this test. Some practice problems:

- Section 2.5, Problems 13, 15, 21, 43.

- (4) Section 2.6 is about limits as x approaches infinity and infinite limits. This is mostly a somewhat highbrow of talking about horizontal and vertical asymptotes of the graph of a function. Practice problems:

- Section 2.6, Problems 9–23 odd, 37–51 odd.

- (5) This brings us to Chapter 3 and the derivative. The derivative is one of the main concepts of the course. If $f(x)$ is a real number valued

function of real numbers, then the *derivative* of $f(x)$ at $x = a$ is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

This definition will be on the test so have it memorized. The number $f'(a)$ has the interpretation of being the slope of the tangent line to the graph $y = f(x)$ at the point $(a, f(a))$. There will be at least one problem where you have to compute the derivative from the definition. Here is an example. Use the definition of the derivative to find $f'(c)$ where $f(x) = 3x^2$.

$$\begin{aligned} f'(c) &= \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(c+h)^2 - 3c^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(c^2 + 2ch + h^2) - 3c^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3c^2 + 6ch + 3h^2 - 3c^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6ch + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6c + 3h)}{h} \\ &= \lim_{h \rightarrow 0} (6c + h) \\ &= 6c. \end{aligned}$$

For many similar functions we have formulas which let us write down the derivative of a function without having to go through all this announce with the derivative. Here is a table of some of these

$f(x)$	$f'(x)$	Comments
c	0	(c is a constant.)
cx	c	(c a constant.)
cx^n	ncx^{n-1}	(n and c constants.)
e^x	e^x	
uv	$u'v + uv'$	
uvw	$u'vw + uv'w + uvw'$	
$\frac{u}{v}$	$\frac{u'v - uv'}{v^2}$	

Quite a few of the questions on the test will be computing derivatives. Some problems to look at:

- Section 3.1 Problems 1–21 odd (on these you can use the derivative rules rather than the definition to find the derivatives.)

- Section 3.2 Problems 1–9 odd, 13, 15, 23–25 odd. (And again you can use the derivative rules here.)
- Section 3.3 Problems 1–45 odd, 53.