

# The Area Theorem

Piotr T. Chruściel,<sup>1</sup>

Erwann Delay<sup>2</sup>

Gregory J. Galloway<sup>3</sup>

Ralph Howard<sup>4</sup>

**Abstract:** We prove that the area of sections of future event horizons in space-times satisfying the null energy condition is non-decreasing towards the future under two circumstances: 1) the horizon is future geodesically complete; 2) the horizon is a black hole event horizon in a globally hyperbolic space-time and there exists a conformal completion with a “ $\mathcal{H}$ -regular”  $\mathcal{I}^+$ . (Some related results under less restrictive hypotheses are also established.) This extends a previous result of Hawking, in which piecewise smoothness of the event horizon seems to have been assumed. We prove smoothness or analyticity of the relevant part of the event horizon when equality in the area inequality is attained — this has applications to the theory of stationary black holes, as well as to the structure of compact Cauchy horizons. In the course of the proof we establish several new results concerning the differentiability properties of horizons.

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Horizons</b>	<b>7</b>
<b>3</b>	<b>Sections of horizons</b>	<b>13</b>
<b>4</b>	<b>Positivity of <math>\theta_{\mathcal{AI}}</math></b>	<b>21</b>
4.1	Causally regular conformal completions . . . . .	21
4.2	Complete generators . . . . .	27
<b>5</b>	<b>Propagation of Alexandrov points along generators, optical equation</b>	<b>30</b>
<b>6</b>	<b>Area monotonicity</b>	<b>36</b>
<b>7</b>	<b>Conclusions</b>	<b>51</b>
<b>A</b>	<b>The Geometry of <math>C^2</math> Null Hypersurfaces</b>	<b>53</b>
<b>B</b>	<b>A comment on the area theorem of Hawking and Ellis</b>	<b>61</b>
<b>C</b>	<b>Proof of Theorem 5.6</b>	<b>63</b>
<b>D</b>	<b>Proof of Proposition 6.6</b>	<b>64</b>

---

<sup>1</sup>Département de Mathématiques, Faculté des Sciences, Parc de Grandmont, F37200 Tours, France. [Chrusciel@Univ-Tours.fr](mailto:Chrusciel@Univ-Tours.fr), Supported in part by KBN grant # 2 P03B 130 16.

<sup>2</sup>Département de Mathématiques, Faculté des Sciences, Parc de Grandmont, F37200 Tours, France. [Delay@gargan.math.Univ-Tours.fr](mailto:Delay@gargan.math.Univ-Tours.fr) *Current adress:* Department of Mathematics, Royal Institute of Technology, S-10044 Stockholm.

<sup>3</sup>Department of Mathematics and Computer Science, University of Miami, Coral Gables FL 33124, USA. [Galloway@math.miami.edu](mailto:Galloway@math.miami.edu) Supported in part by NSF grant # DMS-9803566.

<sup>4</sup>Department of Mathematics, University of South Carolina, Columbia S.C. 29208, USA. [Howard@math.sc.edu](mailto:Howard@math.sc.edu) Supported in part by DoD Grant # N00014-97-1-0806