Quiz 20   Name:____Key_____

You must show your work to get full credit.

1. Let x be a real number. Define $|x|$.

$$|x| = \begin{cases} x, & x \geq 0; \\ -x, & x < 0. \end{cases}$$

2. Let a be a positive real number and x any real number. Show that $|ax| = a|x|$. *Hint:* There are three cases $x = 0$, $x > 0$ and $x < 0$. You can use the facts that positive times positive is positive and that positive times negative is negative.

Proof   As hinted we consider 3 cases.

Case 1. $x = 0$.   Then $ax = a \cdot 0 = 0$ and so
$|ax| = |0| = 0 = a|0|$.

Case 2. $x > 0$. Then $|x| = x$ and $|a| = a$ (as both ≥ 0).
Then $ax > 0$ (as positive times positive is positive)
so $|ax| = 0$.   Thus
$$|ax| = ax = a|x|.$$

Case 3. $x < 0$. Then $|x| = -x$, and $|a| = a$.
Also $ax < 0$ (as positive times negative is negative)
so $|ax| = -ax$.   Thus
$$|ax| = -ax$$
$$= a(-x)$$
$$= a|x|.$$

So   in all cases $|ax| = a|x|$ done