On fine differentiability properties of horizons and applications to Riemannian geometry

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Abstract

We study fine differentiability properties of horizons. We show that the set of end points of generators of a n-dimensional horizon $\mathcal{H}$ (which is included in a $(n+1)$-dimensional space-time M) has vanishing n-dimensional Hausdorff measure. This is proved by showing that the set of end points of generators at which the horizon is differentiable has the same property. For $1 \leq k \leq n+1$ we show (using deep results of Alberti) that the set of points where the convex hull of the set of generators leaving the horizon has dimension k is "almost a C^2 manifold of dimension $n+1-k$": it can be covered, up to a set of vanishing $(n+1-k)$-dimensional Hausdorff measure, by a countable number of C^2 manifolds. We use our Lorentzian geometry results to derive information about the fine differentiability properties of the distance function and the structure of cut loci in Riemannian geometry.