Mathematics 242 Homework.

Problem 1. Here are some integration practice problems. Most, but not all, them will involve integration by parts. In all of these s is a constant.

(a) $\displaystyle\int e^{-st}t^3\,dt,$

(b) $\displaystyle\int e^{-st}\cos(3t)\,dt$

(c) $\displaystyle\int e^{-st}e^{5t}\,dt$

(d) $\displaystyle\int e^{-st}t^2e^{5t}\,dt$

A pair of functions that will come up are the ***hyperbolic sine and cosine***. These are defined by

$$\cosh(t) = \frac{e^t + e^{-t}}{2}$$
$$\sinh(t) = \frac{e^t - e^{-t}}{2}$$

Problem 2. Here are some basic properties of these function for you to prove.

(a) Their derivatives are

$$\frac{d}{dt}\cosh(t) = \sinh(t)$$
$$\frac{d}{dt}\sinh(t) = \cosh(t).$$

(b) Them satisfy the identity

$$\cosh(t)^2 - \sinh(t)^2 = 1.$$

□

Problem 3. We have seen that the general solution to the equation

$$x''(t) - x(t) = 0$$

is

$$x(t) = c_1e^t = c_2e^{-t}.$$

Show that the functions

$$x_1(t) = \cosh(t), \qquad x_2 = \sinh(t)$$

are also solutions to $x''(t) - x(t) = 0$ and this it is also possible to write the general solution as

$$x = C_1\cosh(t) + C_2\sinh(t).$$

□

Problem 4. Let s be a constant and compute the following integrals:

$$\int e^{-st}\cosh(t)\,dt$$

$$\int e^{-st}\sinh(t)\,dt$$ □

Problem 5. These problems review some properties of improper integrals. Compute the following

(a) $\displaystyle\int_0^\infty t^2 e^{-3t}\,dt$,

(b) $\displaystyle\int_0 e^{2t}e^{-st}\,dt$ where s is a constant with $s > 2$. Why is it important that $s > 2$? □