Mathematics 172 Homework

The solution for these problems are after the last problem. Recall that an ***equilibrium point*** of the system is a point where both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

1. For the system of differential equations

$$\frac{dx}{dt} = .4x\left(\frac{100 - x - .4y}{100}\right)$$
$$\frac{dy}{dt} = .6x\left(\frac{200 - x - .8y}{200}\right)$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x-y plane) showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

2. For the system of differential equations

$$\frac{dx}{dt} = .23x\left(\frac{100 - x - 1.5y}{100}\right)$$
$$\frac{dy}{dt} = .07x\left(\frac{150 - x - 4y}{150}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

3. For the system of differential equations

$$\frac{dx}{dt} = .15x\left(\frac{300 - x - .7y}{300}\right)$$
$$\frac{dy}{dt} = .2x\left(\frac{250 - x - 4y}{250}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

4. For the system of differential equations

$$\frac{dx}{dt} = .14x\left(\frac{100 - x - 3y}{100}\right)$$
$$\frac{dy}{dt} = .3x\left(\frac{80 - x - .4y}{80}\right)$$

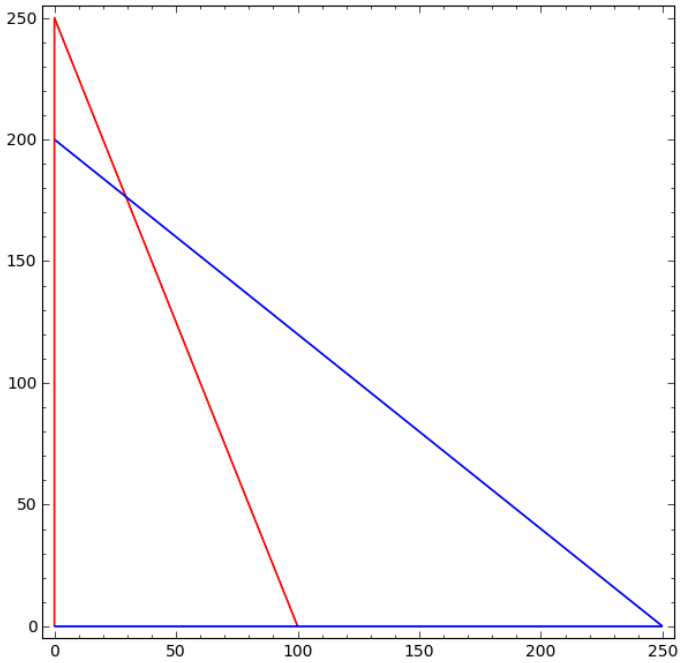
draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

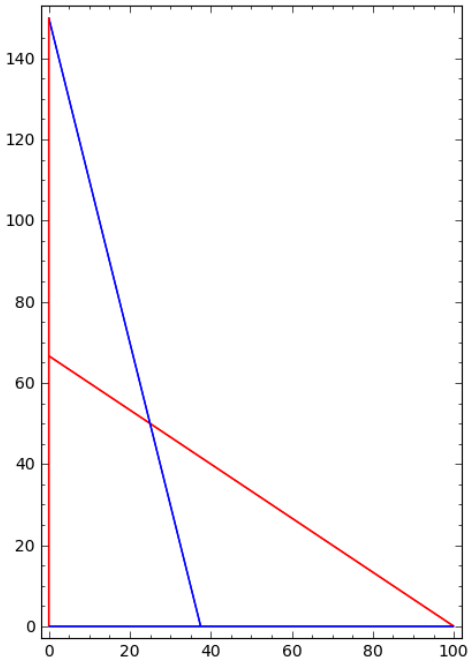
(c) The coordinates of all the equilibrium points in the first quadrant.

Solution to 1:



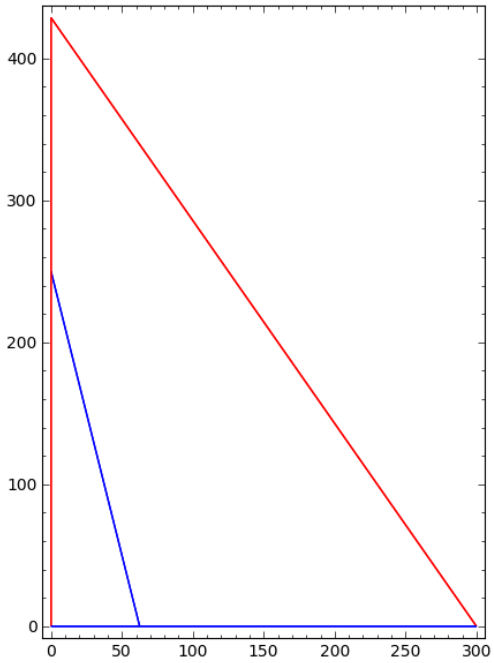
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 200)$, and $(29.41, 176.5)$.

Solution to 2:



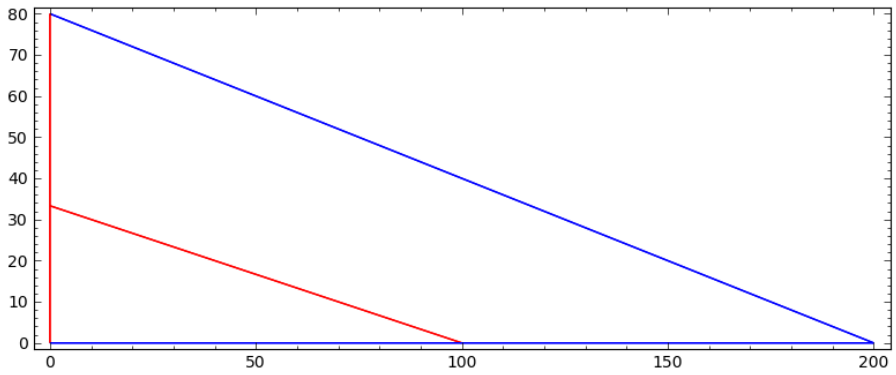
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 150)$, and $(25.00, 50.00)$.

Solution to 3:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(300, 0)$, and $(0, 250)$.

Solution to 4:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$,$(100, 0)$, and $(0, 80)$.