

You must show your work to get full credit.

1. The sequence $a_1, a_2, a_3 \dots$ is defined by

$$a_n = 2a_{n-1} - a_{n-2} \quad \text{for } n \geq 3$$

and $a_1 = 1, a_2 = 4$. Write the first five terms in the sequence.

$$a_1 = 1$$

$$a_2 = 4$$

$$a_3 = 2(4) - (1) = 7$$

$$a_4 = 2(7) - (4) = 10$$

$$a_5 = 2(10) - (7) = 13$$

$$a_1 = \underline{1}$$

$$a_2 = \underline{4}$$

$$a_3 = \underline{7}$$

$$a_4 = \underline{10}$$

$$a_5 = \underline{13}$$

2. Show that $a_n = 1 + 2^n$ is a solution to

$$a_{n+2} = 3a_{n+1} - 2a_n \quad \text{for } n \geq 0$$

and $a_0 = 2$ and $a_1 = 3$

$$a_0 = 1 + 2^0 = 1 + 1 = 2$$

$$a_1 = 1 + 2^1 = 1 + 2 = 3$$

$$\begin{aligned} 3a_{n+1} - 2a_n &= 3(1 + 2^{n+1}) - 2(1 + 2^n) \\ &= 3 + 3 \cdot 2^{n+1} - 2 - 2^{n+1} \\ &= 1 + 2 \cdot 2^{n+1} \\ &= 1 + 2^{n+2} \\ &= a_{n+2} \end{aligned}$$