Mathematics 122

Quiz 37 Name: Key

You must show your work to get full credit.

1. Use the Fundamental Theorem of Calculus to find the exact value of $\int_1^2 \frac{1}{4x^3}\,dx$. Show your work and write the answer as a fraction rather than a decimal (that is 3/4 rather than .75).

$$\int_1^2 \frac{1}{4} x^{-3} dx = \frac{1}{4}\frac{1}{(-2)} x^{-2}\Big|_1^2$$
$$= \frac{-1}{8x^2}\Big|_1^2$$
$$= -\frac{1}{8(2)^2} - \left(\frac{-1}{8(1)^2}\right) = \frac{-1}{32} + \frac{1}{8} = \frac{-1+4}{32} = \frac{3}{32}$$

$\int_1^2 \frac{1}{4x^3}\,dx =$ $\frac{3}{32}$

2. Let c be a constant. Compute $\int_1^c (x-c)\,dx$. $\int_1^c (x-c)\,dx =$ $-\frac{c^2}{2} - \frac{1}{2} + c$

$$\int_1^c (x-c)dx = \left(\frac{x^2}{2} - cx\right)\Big|_1^c$$
$$= \left(\frac{c^2}{2} - c^2\right) - \left(\frac{1^2}{2} - c(1)\right)$$
$$= -\frac{c^2}{2} - \frac{1}{2} + c$$

3. A student invests \$500 is invested at 8% simple interest.

(a) Give a formula for the principal after t years.

$P(t) =$ $500(1.08)^t$

(b) How long until the principal reaches \$20,000?

Time to \$20,000 47.93 years.

Solve

$P(t) = 500(1.08)^t = 2000$

$(1.08)^t = \frac{20000}{500} = 40$

$t \ln(1.08) = \ln(40$

$t = \ln(40)/\ln(1.08) = 47.93$

4. If $f(4) = 9$ and $f'(4) = -1.5$ estimate the following

$f(4.15) \approx$ 8.775

$f(4.15) \approx f(4) + f'(4)(.15)$

$= 9 - 1.5(1.5)$

$= 8.775$

$f(3.95) \approx$ 9.075

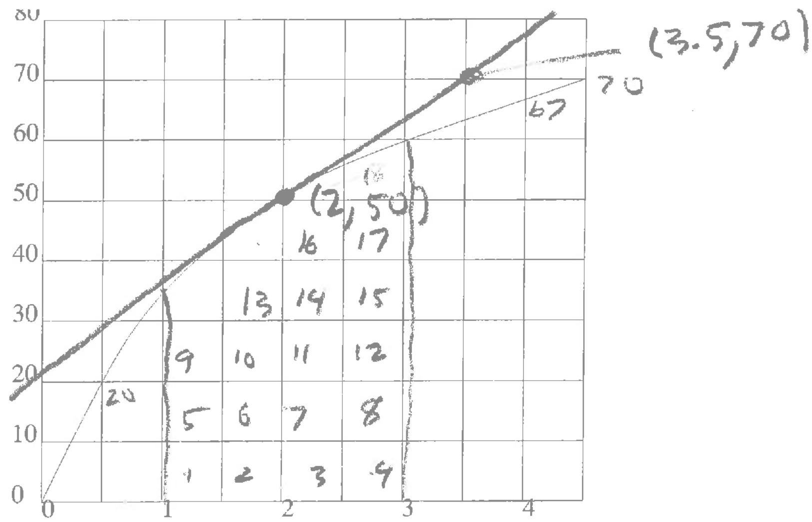
$f(3.95) = f(4 + (-.05))$

$\approx f(4) + f'(4)(-.05)$

$= 9 + (-1.5)(-.05)$

$= 9.075$

5. The following graph gives the average distance, s, a car has traveled t seconds after the brakes hare applied.



(a) What is the average speed between $t = 0$ and $t = .5$?

Avenge speed is 40 ft/sec

$\frac{\Delta s}{\Delta t} = \frac{20-0}{.5-0} = \frac{20}{.5} = 40$

(b) What is the average speed between $t = 4$ and $t = 4.5$?

Average speed is 6 ft/sec

$\frac{\Delta s}{\Delta t} = \frac{70-67}{4.5-4} = \frac{3}{.5} = 6$

(c) Draw the tangent line at the point where $t = 2$, label two points on the line showing their coordinates and use these points to estimate $s'(2)$.

$s'(2) \approx$ 13.333..

(Your answer may differ a bit)

$\frac{\Delta s}{\Delta t} = \frac{70-50}{3.5-2} = \frac{20}{1.5} = 13.333..$

6. Use the same graph as in the last problem but this time assume that it is the speed of train in miles per hour t hours after it leaves the station. Estimate the distance the train covered between $t = 1$ and $t = 3$ hours.

Distance covered $\approx$ 97.5 miles

Each box is

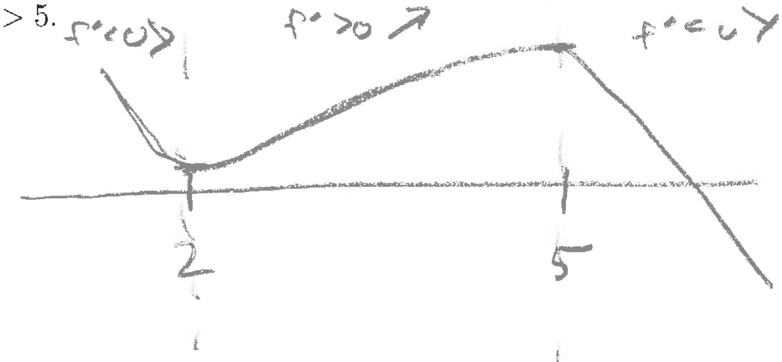
(.5 hours)(10 miles/hour) = 5 miles

I counted $\approx$ 19.5 boxes

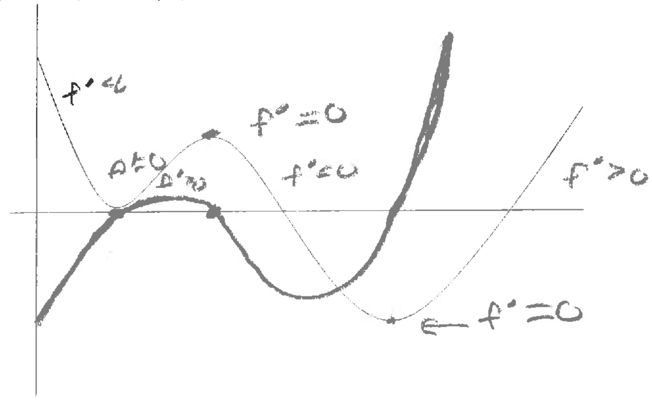
so distance = 5(19.5) = 97.5 miles

7. Draw the graph of a function $y = f(x)$ that satisfies $f'(x) > 0$ for $2 < x < 5$ and $f'(x) < 0$ for $x < 2$ and $x > 5$.

$f'<0$ ↘ $f'>0$ ↗ $f'<0$ ↘



8. The following is the graph of $y = h(x)$. Draw the graph of $y = g'(x)$ on the same axis.



9. Let G be the number of watts of power it takes to run a computer for t hours. Then $G = f(t)$ for some function t.

(a) In the equation $f(10) = 57$ what are the units of 10 and 57?

Units of 10 hours — Units of 57 watts

$f' = \frac{dG}{dt}$

(b) In $f'(10) = 4$ what are the units of 10 and 4/

Units of 10 hours — Unit of 4 watts / hour.

(c) If $f(10) = 57$ and $f'(10) = 4$ estimate $f(10.25)$. — $f(10.25) \approx$ 58

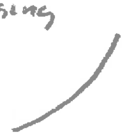
$f(10.25) \approx f(10) + f'(10)(.25)$

$= 57 + 4(.25) = 58$

10. Draw graphs of a functions $y = f(x)$ that satisfy:

(a) $f'(x) > 0$ and $f''(x) > 0$

Increasing concave up



(b) $f'(x) > 0$ and $f''(x) < 0$

Increasing concave down

(c) $f'(x) < 0$ and $f''(x) > 0$

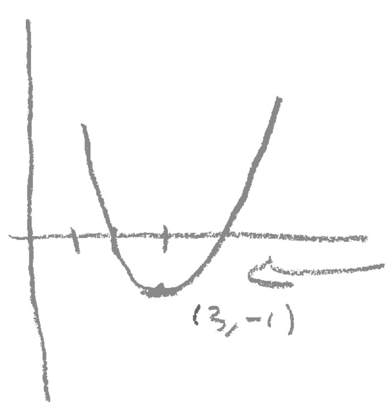
decreasing concave up

(d) $f'(x) < 0$ and $f''(x) < 0$

decreasing concave down

11. Draw a graph of a function with $f(3) = -1$, $f'(3) = 0$ and $f''(x) > 0$.

concave up



so a minimum

12. Some of the values of a function $f(x)$ are given by the following table.

$\Delta x = 5$

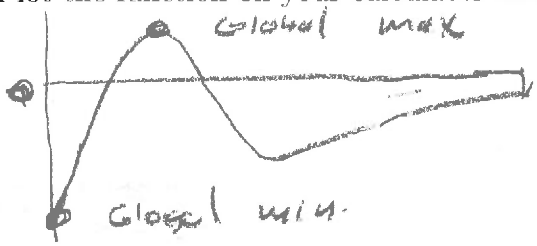
x	0	5	10	15	20
$f(x)$	12	10	12	15	19

(a) Fill in the following table for the estimates of the derivative.

x	2.5	7.5	12.5	17.5
$f'(x)$	$-.4$	$.4$	$.6$	$.8$
	$\frac{10-12}{5}$	$\frac{12-10}{5}$	$\frac{15-12}{5}$	$\frac{19-15}{5}$

13. For the function $f(x) = \dfrac{x - x^2}{1 + .5x^4}$ on the interval $-1 \le x \le 5$

(a) Plot the function on your calculator and make a sketch of the result here:



$Y1 = (X - X^2)/(1 + .5X^4)$

Xmin = -1

Xmax = 5

ZoomFit

(b) For $f(x)$ on the interval $-1 \le x \le 5$ what are

2nd calc 4:maximum

Global maximum $\underline{.2432}$ Global maximiser $\underline{.4741}$

Global minimum $\underline{-1.3333\ldots}$ Global minimizer $\underline{-1}$

since it is clear from graph that min. at $x=-1$, do 2nd calc 1:value

14. Let a be a positive constant and set $f(x) = x^3 - 3a^2x$.

(a) What are the first and second derivatives of $f(x)$.

$f'(x) = \underline{3x^2 - 3a^2}$ $f''(x) = \underline{6x}$

(b) What are the critical points of $f(x)$?

solve $f'(x) = 3x^2 - 3a^2 = 0$ The critical points are $\underline{a, -a}$

$x^2 = a^2$

$x = a, -a$

(c) Use the second derivative test to determine which of the critical points are a local maximizers and which are local minimizers.

Local maximizers $\underline{-a}$ Local minimizers $\underline{a}$

$f''(a) = 6a > 0$ concave up ∪ so local min

$f''(-a) = -6a < 0$ concave down ∩ so local max