Mathematics 172 Homework, January 19, 2019.

We have seen that if $P(t)$ is the size of a population at time t with constant intrinsic growth rate r, then P satisfies the rate equation

$$\frac{dP}{dt} = rP$$

and that the solution to this rate equation is

$$P(t) = P_0 e^{rt}.$$

Here is an example of using this model. Assume that in a bucket of water in a backyard a population of paramecium is introduced and assume that they have an intrinsic growth rate of $r = .7$ paramecium/day.

1. What is the rate equation starisfied by $P(t)$? *Solution:* In this case $r = .7$ and so the rate equation is $P' = .7P$

2. Find the doubling time of this population. *Solution:* Here $P(t) = P_0 e^{.7t}$ and so we wish to solve $P(t) = P_0 e^{.7t} = 2P_0$. That is $e^{.7t} = 2$. This has the solution $t = \ln(2)/.7 = .9902$ days. $\square$

Now assume that some rotifers are added to the bucket and that they eat 50% of the paramecium population per day.

3. What is the new rate equation satisfied by $P(t)$, the size of the paramecium population. *Solution:* The original rate equation is $P' = .7P$. But now the rotifers are subtracting out 50% of the population each day. That is they subtract out $.5P$ of the pollution each day. So the new rate equation is

$$\frac{dP}{dt} = .7P - .5P = .2P.$$

That is the new intrinsic growth rate is $r = .2$. $\square$

4. Find the doubling time of the paramecium population after the rotifers have been added. *Solution:* This time the solution to the rate equation is $P(t) = P_0 e^{.2t}$ and so we solve $P_0 e^{.2t} = 2P_0$, thus $e^{.2t} = 2$ and $t = \ln(2)/.2 = 3.466$ days is the doubling time. $\square$

5. Now assume that instead of eating 50% of the paramecium population each day, that the rotifers eat 85%. What is the new rate equation? *Solution:* With the same reasoning as in Problem 3, the new rate equation is

$$\frac{dP}{dt} = .7P - .85P = -.15P.$$

Thus this time the new intrinsic growth rate is $r = -.15$. So that rather than exponential growth, there is exponential decay. $\square$

6. What is the half life of the paramecium population? *Solution:* We wish to solve $P(t) = P_0 e^{-.15t} = \frac{1}{2}P_0$. This gives $t = \ln(1/2)/(-.15) = 4.6210$ as the half life. $\square$