Mathematics 242 Homework.

Problem 1. For the equation

$$mx'' + kx = F_0 \sin(\omega t)$$

we divide by m to get

$$x'' + \omega_0^2 x = \frac{F_0}{m} \sin(\omega t)$$

where we have set

$$\omega_0 = \sqrt{\frac{k}{m}}.$$

(a) Derive the general solution to this equation in the case where $\omega_0^2 \neq \omega^2$.
(b) Derive the general solution to this equation in in the case $\omega_0^2 = \omega^2$. □

Problem 2. (a) Find the general solution to

$$9x'' + 4x = \sin(3t/2).$$

(which it the equation for a mass of 9 grams on spring with spring constant 4).
(b) show that all of these solutions become unbounded. □

Problem 3. Find the solution to initial value problem

$$4x'' + 4x' + 5x = 0, \qquad x(0) = 3, \quad x'(0) = 0.$$

Is this spring system over or under damped?

Problem 4. Let us add a forcing function to equation of the last problem. Find the solution to

$$4x'' + 4x' + 5x = .1\cos(t/2), \qquad x(0) = 3, \quad x'(0) = 0.$$

Is this spring system over or under damped?