Mathematics 174

Quiz # 19 Name: Key

You must show your work to get full credit.

1. Find the first five terms of the sequence defined by

$$a_k = 2a_{k-1} + k \quad \text{for} \quad k \geq 2$$

and $a_1 = 1$.

$a_1 = 1$

$a_2 = 3$

$a_3 = 9$

$a_4 = 22$

$a_5 = 49$

$a_1 = 1$

$a_2 = 2(1) + 2 = 3$

$a_3 = 2(3) + 3 = 6 + 3 = 9$

$a_4 = 2(9) + 4 = 18 + 4 = 22$

$a_5 = 2(22) + 5 = 44 + 49$

2. Let $t_n = 2 + n$ for all integers $n \geq 0$. Show this sequence satisfies

$$t_k = 2t_{k-1} - t_{k-2}.$$

$$\begin{aligned} 2t_{k-1} - t_{k-2} &= 2(2 + (k-1)) - (2 + (k-2)) \\ &= 2(k+1) - (k) \\ &= 2k + 2 - k \\ &= k + 2 \\ &= t_k \end{aligned}$$