Name:

You must show your work to get full credit.

We are look at Euler's method for a system of rate equations with initial conditions:

$$\frac{dx}{dt} = f(x, y), \qquad x(t_0) = x_0$$

$$\frac{dy}{dt} = g(x, y), \qquad y(t_0) = y_0.$$

It will be based on the basic approximations that we already know and love:

$$x(t+h) \approx x(t) + x'(t)h$$
$$y(t+h) \approx y(t) + y'(t)h$$

which holds when h is small.

To start we choose a small number h, the **step size**. Let $k \ge 0$ be an integer and assume that we have computed t_k , x_k , and y_k in such a way that

$$x_k \approx x(t_k)$$

 $y_k \approx y(t_k)$.

To be explicit about what this notation means here x_k is our approximation to the value of the true solution x(t) at the point where $t = t_k$. Then by our basic approximations we have

$$x(t_{k+1}) = x(t_k + h) \approx x(t_k) + x'(t_k)h \approx x_k + x'(t_k)h$$

$$y(t_{k+1}) = y(t_k + h) \approx y(t_k) + y'(t_k)h \approx y_k + y'(t_k)h$$

But from the differential equations for x and y we have

$$x'(t_k) = f(x(t_k), y(t_k)) \approx f(x_k, y_k),$$

$$y'(t_k) = g(y(t_k), y(t_k)) \approx g(x_k, y_k).$$

Putting these approximation together gives

$$x(t_{k+1}) \approx x_k + f(x_k, y_k)h,$$

$$y(t_{k+1}) \approx y_k + g(x_k, y_k)h.$$

So to summarize here is Euler's method for the system:

Initial Step: Set

$$t_0 = t_0$$
$$x_0 = x_0$$
$$y_0 = y_0.$$

Euler Step from k to k+1:

$$t_{k+1} = t_k + h$$

$$x_{k+1} = x_k + f(x_k, y_k)h$$

$$y_{k+1} = y_k + g(x_k, y_k)h$$

It is not hard to see that after n steps we have that

$$t_n = t_0 + nh.$$

Then x_n and y_n will be good approximations to the true values $x(t_n)$ and $y(t_n)$.

Let us now do an example on the calculator. As a sample system we use

$$\frac{dx}{dt} = 2x - 3y \qquad x(0) = 4$$

$$\frac{dy}{dt} = -x + 2y \qquad y(0) = 1$$

and we will approximate x(2) and y(2) by taking 20 steps of size h = .1. The scheme for this is

$$t_0 = 0$$
$$x_0 = 4$$
$$y_0 = 1$$

and taking an Euler step looks like

$$t_{k+1} = t_k + .1$$

$$x_{k+1} = x_k + (2x_k - 3y_k)(.1)$$

$$y_{k+1} = y_k + (-x_k + 2y_k)(.1)$$

To set the calculator up to deal with this go to the MODE screen and edit to look like

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^\thetai
FULL HORIZ G-T
```

Store the value of the step size in the H register: at the main screen and type .1 STO ALPHA H Press 2ND TABLESET and edit until it looks like

```
TABLE SETUP

TblStart=0

\DeltaTbl=1

Indpnt: Auto Ask

Depend: Auto Ask
```

To enter the equations go to the Y= window and (where we will use u for x and v for y) and enter:

```
Plot1 Plot2 Plot2

nMin=0

\u(n)=u(n-1)+(u(n-1)-3v(n-1))H

u(nMin)=4

\v(n)=v(n-1)+(-u(n-1)+2v(n-1))H

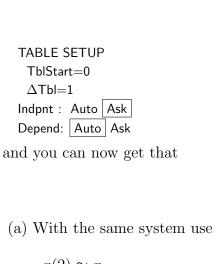
\v(nMin)=10

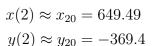
\w(n)=

\w(nMin)=
```

Here n is entered with the $[X,T,\theta,n]$ bottom, u is entered with 2ND u (which is the 7 key), v is entered with 2ND v (which is above the 8 key), and H is entered with ALPHA H.

Now 2ND TABLE will give you the first first several values for x_k and y_k . To easily get access to more values go back to 2ND TABLESET and edit until it looks like:





1.	(a)	With the same system	use 40 steps of size h	= .05 to approximate $x($	2)	and y	(2)).

(b) Get a still better approximation of x(2) and y(2) by taking 200 steps of size h = .01.

2. For the initial value problem

$$\frac{dy}{dt} = .05x \left(\frac{10 - x - .2y}{10}\right)$$

$$\frac{dy}{dt} = .03y \left(\frac{20 - .5x - y}{20}\right)$$

$$y(0) = 2$$

(a) Use 20 steps of size h = .1 to estimate x(2) and y(2).

(b) Get a better approximation of x(2) and y(2) by taking 40 steps of size h = .05.

(c) Get a still better approximation of x(2) and y(2) by taking 200 steps of size h = .01.