Mathematics 172 Test 1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (15 points) The zoo sets up a large tank for South American fish. At some point they add some plants to the tank and the plants have 40 grams of brown algae on them. After 3 weeks there is 100 grams of the algae in the tank.

(a) What is the intrinsic growth rate of the algae? Include units in your answer.

Let $P(t)$ = grams of algae in week t. $r =$ .3054 (grams/wk)/gram

Then $P(t) = P_0 e^{rt} = 40e^{rt}$

$P(3) = 40e^{3r} = 100$

$e^{3r} = 100/40$

$3r = \ln(100/40)$

$r = \frac{\ln(100/40)}{3} = .3054$

(b) If $P(t)$ is the number of grams of brown algae in the tank t weeks after the plants are added, then give a formula for $P(t)$.

$P(t) = P_0 e^{rt} \longrightarrow$ $P(t) =$ $40e^{.3054t}$

(c) How long until there is 500 grams of brown algae in the tank?

Time to 500 grams is. 8.270 weeks

Solve

$$40e^{.3054t} = 500$$

$$e^{.3054t} = 500/40$$

$$.3054t = \ln(500/40)$$

$$t = \frac{\ln(500/40)}{.3054} = 8.270 \text{ weeks}$$

2. (10 points) Water hyacinth is sometimes used a method to reduce excesses of nitrites and nitrates in polluted water. The mangers of a a badly polluted pond find that the water is of such bad quality that water hyacinth has an intrinsic growth rate of $-.05$ (kg/week)/kg. They wish to keep a stable population of a metric ton (that is 1,000 kg) of water hyacinth in the pond. At what rate should they stock it?

The stocking rate is 50 kg/week

Let $P(t)$ be number of kg of water hyacinth in week t. Then if S is stocking rate

$$\frac{dP}{dt} = -.05P + S$$

we want $P = 1000$ to be an equilibrium point so

$$0 = -.05(1000) + S$$

$$S = .05(1000) = 50$$

3. (20 points) Algae is growing in a large bucket of water. Let $W(t)$ be the weight in grams of the algae in the bucket after t days. Assume that W satisfies the rate equation

$$\frac{dW}{dt} = .05W\left(1 - \frac{W}{60}\right)\left(\frac{W}{10} - 1\right).$$

(a) If $W(4) = 40$, what is $W'(4)$? $W'(4) =$ 2 grams / day

$W'(4) = .05\, W(4)\left(1 - \frac{W(4)}{60}\right)\left(\frac{W(4)}{10} - 1\right)$

$= .05(40)\left(1 - \frac{40}{60}\right)\left(\frac{40}{10} - 1\right) = 2$

(b) What are the equilibrium points of this equation?

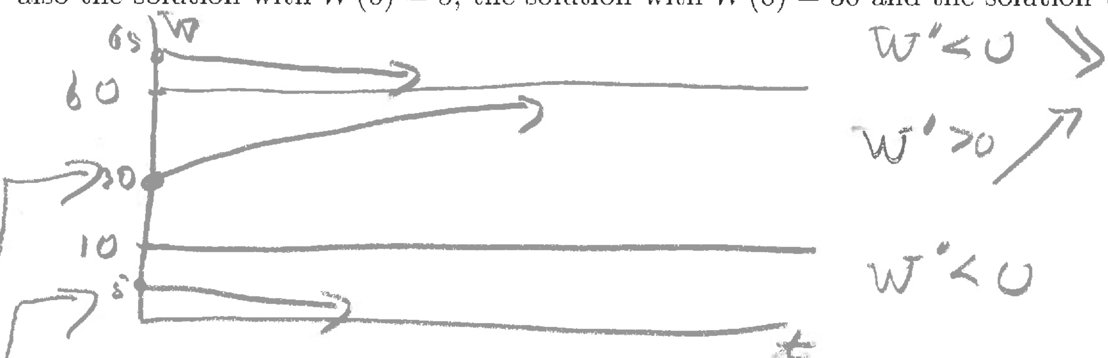
Solve

The equilibrium points are: 0, 10, 60

$.05W\left(1 - \frac{W}{60}\right)\left(\frac{W}{10} - 1\right) = 0$

To get 0, 10, 60

(c) Draw a picture (graph of W as a function of t) which shows the equilibrium solutions and also the solution with $W(0) = 5$, the solution with $W(0) = 30$ and the solution with $W(0) = 65$.



(d) Which of the equilibrium points are stable?

The stable points are: 0, 60

(e) If $W(0) = 5$, estimate $W(100)$. $W(100) \approx$ 0

The solution starting at 5 goes down to 0

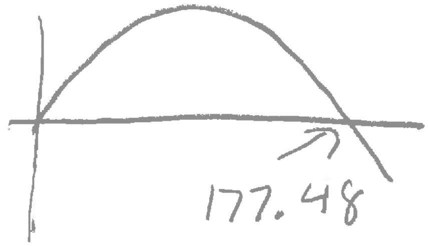
(f) If $W(0) = 30$, estimate $W(93)$. $W(93) \approx$ 60

The solution starting at 30 goes up to 60

4. (15 points) For the rate equation

$$\frac{dy}{dt} = .5y\left(1 - \frac{y^{1.2}}{500}\right)$$

(a) Use your calculator to find the equilibrium points *Hint:* Using Xmin=0 and Xmax=200 is a good choice. \Y1 = .5X(1 − X^1.2/500)



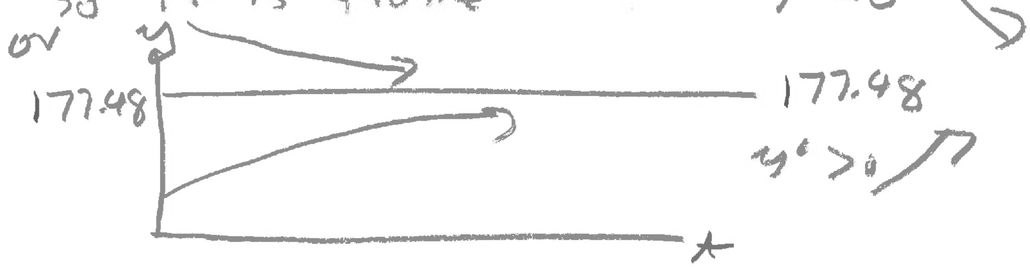
Equilibrium points are $\underline{\quad 0,\ 177.48 \quad}$

0 is clearly an equilibrium point
2nd calc 2:zero to find

(b) Which of the equilibrium points are stable?

The stable points are $\underline{\quad 177.48 \quad}$

The graph is going down hill at 177.48 so it is stable
or



(c) If $y(0) = 150$ compute $y'($[redacted]$)$ and use this to estimate $y(0.1)$.

$y'(0) = \underline{\quad 13.708 \quad}$ $\qquad$ $y(0.1) \approx \underline{\quad 151.3708 \quad}$

$$y'(0) = .5\,y(0)\left(1 - \frac{y(0)^{1.2}}{500}\right)$$
$$= .5(150)\left(1 - \frac{150^{1.2}}{500}\right)$$
$$= 13.708$$

$$y(0.1) \approx y(0) + y'(0)(.1)$$
$$= 150 + 13.708\,(.1)$$
$$= 151.3708$$

5. (10 points) A population of annual plants is introduced to an island. Originally there are 15 of the plants. After 4 years there are 75. What are the growth ratio and per capita growth rate?

$\lambda = \underline{\quad 1.521 \quad}$ $\qquad$ $r = \underline{\quad .521 \quad}$

Let N_t = number of plants. Then

$N_{t+1} = \lambda N_t$ so $N_t = N_0 \lambda^t = 15\lambda^t$

$N_4 = 15\lambda^4 = 75$

$\lambda^4 = \frac{75}{15}$

$\lambda = \left(\frac{75}{15}\right)^{\frac{1}{4}} = 1.521$

$r = \lambda - 1$

6. (15 points) A population of duckweed is growing logistically in a pond with an intrinsic growth rate of $r = 2.5$ (lbs/lb)/week and a carrying capacity of $K = 100$ pounds.

(a) The owner of the pond wishes to get rid of the duckweed. What is the least rate she can harvest it so that it eventually is eradicated. Write a sentence or two and include a picture explaining how you got the answer.

Logistic equation is

$\frac{dP}{dt} = rP(1-\frac{P}{K})$

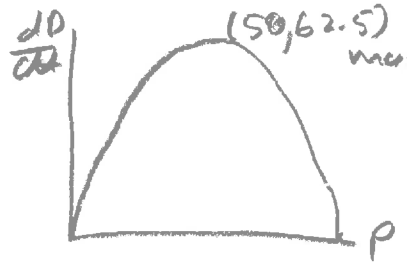
$= 2.5P(1-\frac{P}{100})$

$\backslash Y1 = 2.5X(1-X/100)$

Xmin = 0

Xmax = 100

Harvesting rate is (include units) 62.5 lbs/week



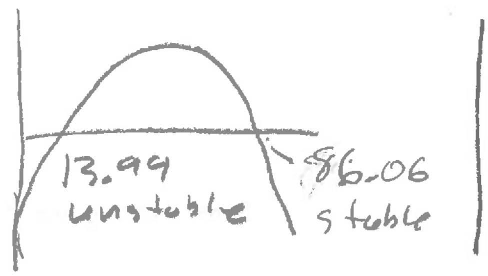
max $\frac{dP}{dt} = 62.5$

so harvesting at 62.5 or a bit more should kill of the duck weed

(b) What happens to the duckweed population if it is harvested at the rate of 30 lbs/week?

This time

$\frac{dP}{dt} = 2.5P(1-\frac{P}{100}) - 30$



so duckweed stabilizes at 86.06 pounds

7. (15 points) A population of annual cicadas are living on a small island. Let N_t be the size of the size of the population in year t and assume that

$$N_{t+1} = N_t e^{1.2(1-N_t/500)}.$$

(a) If $N_{10} = 450$ what are N_{11} and N_{12}? Give your answer to two decimal places.

$N_{11} =$ 507.37 $N_{12} =$ 498.47

(b) What are the equilibrium points of this system? (This can be done without the calculator, but if you use it I suggest letting Xmin=0 and Xmax=600)

Equilibrium points are 0, 500