THE COSMOLOGICAL TIME FUNCTION

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Abstract

Let (M, g) be a time oriented Lorentzian manifold and d the Lorentzian distance on M. The function $\tau(q) := \sup_{p<q} d(p, q)$ is the **cosmological time function** of M, where as usual $p < q$ means that p is in the causal past of q. This function is called **regular** iff $\tau(q) < \infty$ for all q and also $\tau \to 0$ along every past inextendible causal curve. If the cosmological time function τ of a space time (M, g) is regular it has several pleasant consequences: (1) It forces (M, g) to be globally hyperbolic, (2) every point of (M, g) can be connected to the initial singularity by a rest curve (i.e., a timelike geodesic ray that maximizes the distance to the singularity), (3) the function τ is a time function in the usual sense, in particular (4) τ is continuous, in fact locally Lipschitz and the second derivatives of τ exist almost everywhere.

1991 *Mathematics Subject Classification.* Primary: 53C50 Secondary: 83C75.

Key words and phrases. Cosmological time function, globally hyperbolic, initial singularity.