Mathematics 172 Homework, February 8, 2019.

Here are some problems that look very much like what we did for the continuous logistic equation with harvesting.

1. Assume that a population grows with a discrete logistic law with per capita growth rate of $r = 1.6$ and a carrying capacity of $K = 600$. Let P_t be the population size after t years.

(a) What is the discrete dynamical system satisfied by P_t? *Solution:* It is

$$P_{t+1} = P_t + 1.6P_t \left(1 - \frac{P_t}{600}\right).$$

(b) If we start harvesting 40% of the population each year, what is the new dynamical satisfied by P_t?*Solution:* It is

$$P_{t+1} = P_t + 1.6P_t \left(1 - \frac{P_t}{600}\right) - .4P_t.$$

(c) What are the equilibrium points of the new system? *Solution:* Solve the equation

$$P + 1.6P \left(1 - \frac{P}{600}\right) - .4P = P$$

to get the two points

$$P_* = 0 \quad \text{and} \quad P_* = 450.$$

(d) Which of these are stable? *Solution:* At $P_* = 0$ we have that the slope is $dy/dx = 2.6$, so it is unstable. At $P_* = 450$ the slope is $dy/dx = .2$, so this point is stable.

(e) What is the new stable population size? *Solution:* It is 450.

2. Again assume that we have a population discrete logistic law with per capita growth rate of $r = 1.6$ and a carrying capacity of $K = 600$. But this time assume that we harvest 200 organisms a year.

(a) What is the new equation this time? *Solution:*

$$P_{t+1} = P_t + 1.6P_t \left(1 - \frac{P_t}{600}\right) - 200.$$

(b) What are the new equilibrium points? *Solution:* Solve

$$P + 1.6P \left(1 - \frac{P}{600}\right) - 200 = P$$

to get the points

$$P_* = 177.5 \quad \text{and} P_* = 422.5$$

(c) Which of these points are stable? *Solution:* At $P_* = 177.5$ we have $dy/dx = 1.65$ so this point is unstable. At $P_* = 422.5$ the slope is $dy/dx = .347$ and thus this point is stable.

(d) What is the new stable population size? *Solution:* It is 422.5.