Mathematics 172 Homework, February 2, 2019.

Here are some of the details on the unconstrained discrete population growth. The basic set up is if P_t is the population size in year t, and r is the ***per capita growth rate***, then the change in the population size is

$$P_{t+1} - P_t = rP_t$$

which can also be written as

$$P_{t+1} = P_t + rP_t = (1+r)P_t.$$

It is convenient let

$$\lambda = 1 + r$$

so that we don't have to keep writting $1 + r$. The number λ is called the ***growth factor***. The solution to

$$P_{t+1} = \lambda P_t$$

is

$$P_{t+1} = P_0 \lambda^t.$$

1. A population of annual cicadas is introduced into an island. Assume the initial population is 34 and that the per capita growth rate is .8 cicadas/cicada.

(a) What is a formula for the number of cicadas after t years? *Solution:* This is just a matter of plugging into the formulas. Here $r = .8$, $P_0 = 34$ and $\lambda = 1 + r = 1.8$.

$$P_t = P_0 \lambda^t = 34(1.8)^t.$$

(b) How many cicadas are there after 25 years? *Solution:* Anther plug in problem

$$P_{25} = 34(1.8)^{25} = 8.1901441 \times 10^7 = 81{,}901{,}441.$$

(c) How long until there are a million cicadas? *Solution:* We wish to solve

$$34(1.8)^t = 10^6.$$

This has solution

$$t = \ln(10^6/34)/\ln(1.8) = 17.5049 \text{ years}.$$

As cicadas only bread once a year, we should round up to 18 years.

2. Owls breed once a year. Assume that a species of endangered owl has 8 of its kind released in the Congaree National Park. Five years later a count is taken and there are 14 of the owls.

(a) What is the per capita growth rate of the owl population? *Solution:* Assuming that the growth rate is unconstrained, we have that the population size is given by

$$P_t = P_0 \lambda^t = 8\lambda^t.$$

We still have to find λ. We know $P_5 = 14$. Thus

$$P_5 = 8\lambda^5 = 14.$$

Solving this for λ gives

$$\lambda = (14/5)^{1/5} = 1.2287 \text{owls/owl}$$

and therefore the per capita growth rate is

$$r = \lambda - 1 = .2287 \text{owls/owl}.$$

3. Here is a problem that combines a couple of the ideas we have been using recently. Largemouth bass breed once a year. Assume that a lake in a park has a population of largemouth bass. Because of overfishing the per capita growth rate of the population is $r = -.2$ (bass/bass).

(a) What happens to the bass population in the long run? *Solution:* Here the growth factor is $\lambda = 1 + r = 1 + (-.2) = .8$. Therefore the population size, P_t stratifies

$$P_{t+1} = \lambda P_t = .8P_t.$$

The solution to this is

$$P_t = P_0(.8)^t$$

and therefore the population size decreases to 0 at an exponential rate.

(b) To keep the bass from dying out, the management of the part starts stocking the lake at the rate of 1,000 bass/year. Let P_t be the size of the bass population t years after the stocking starts. Give a formula for P_{t+1} in terms of P_t. *Solution:* Before the stocking starts the equation is $P_{t+1} = .8P_t$. Once the stocking starts 1,000 fish get added in each year and so the new equation is

$$P_{t+1} = .8P_t + 1{,}000.$$

(c) If there are $P_0 = 1{,}500$ bass in the lake when the stocking starts, find P_1, P_2, P_3 and P_4. *Solution:* Using the equation from part (b) find

$$P_1 = 2200.0 P_2 \qquad = 2760.0 P_3 = 3208.0 P_4 \qquad = 3566.4$$