

Mathematics 172 Homework

The solution for these problems are after the last problem.

1. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .4x \left(\frac{100 - x - .4y}{100} \right) \\ \frac{dy}{dt} &= .6x \left(\frac{200 - x - .8y}{200} \right)\end{aligned}$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x - y plane) showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

2. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .23x \left(\frac{100 - x - 1.5y}{100} \right) \\ \frac{dy}{dt} &= .07x \left(\frac{150 - x - 4y}{150} \right)\end{aligned}$$

draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

3. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .15x \left(\frac{300 - x - .7y}{300} \right) \\ \frac{dy}{dt} &= .2x \left(\frac{250 - x - 4y}{250} \right)\end{aligned}$$

draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

4. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .14x \left(\frac{100 - x - 3y}{100} \right) \\ \frac{dy}{dt} &= .3x \left(\frac{80 - x - .4y}{80} \right)\end{aligned}$$

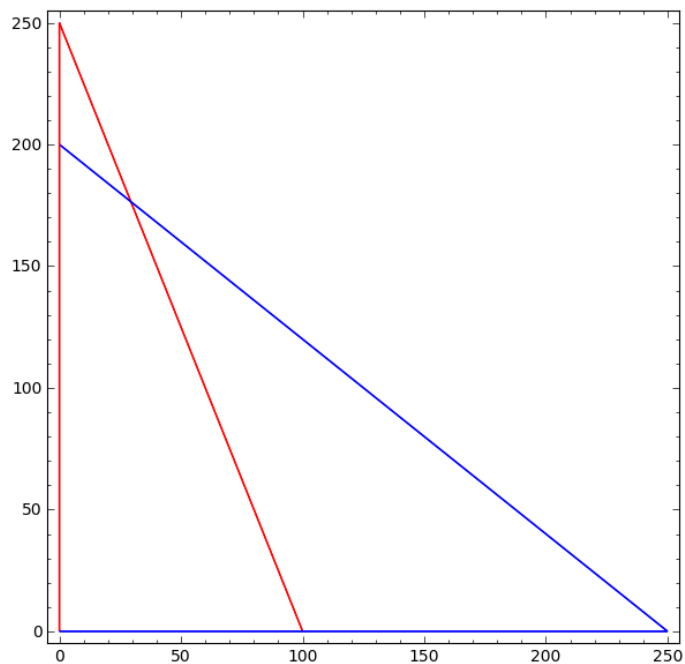
draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

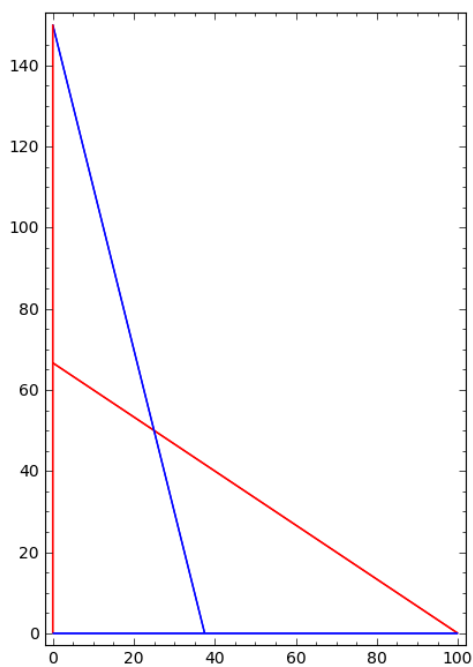
(c) The coordinates of all the equilibrium points in the first quadrant.

Solution to 1:



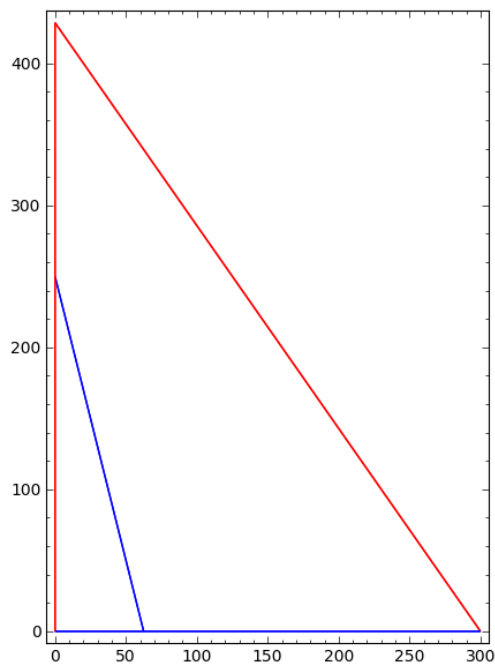
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are (0, 0), (100, 0), (0, 200), and (29.41, 176.5).

Solution to 2:



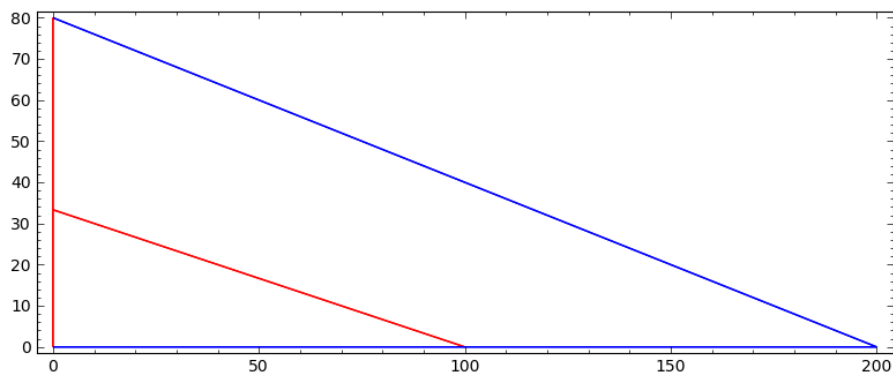
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are (0, 0), (100, 0), (0, 150), and (25.00, 50.00).

Solution to 3:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are (0, 0), (300, 0), and (0, 250).

Solution to 4:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are (0, 0), (100, 0), and (0, 80).