

## Mathematics 172 Homework, March 4, 2019.

The solution for these problems are after the last problem.

1. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .4x \left( \frac{100 - x - .4y}{100} \right) \\ \frac{dy}{dt} &= .6x \left( \frac{200 - x - .8y}{200} \right)\end{aligned}$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the  $x$ - $y$  plane) showing

(a) The lines where  $\frac{dx}{dt} = 0$ ,

(b) The lines where  $\frac{dy}{dt} = 0$ ,

(c) The coordinates of all the equilibrium points in the first quadrant.

2. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .23x \left( \frac{100 - x - 1.5y}{100} \right) \\ \frac{dy}{dt} &= .07x \left( \frac{150 - x - 4y}{150} \right)\end{aligned}$$

draw the phase plane showing

(a) The lines where  $\frac{dx}{dt} = 0$ ,

(b) The lines where  $\frac{dy}{dt} = 0$ ,

(c) The coordinates of all the equilibrium points in the first quadrant.

3. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .15x \left( \frac{300 - x - .7y}{300} \right) \\ \frac{dy}{dt} &= .2x \left( \frac{250 - x - 4y}{250} \right)\end{aligned}$$

draw the phase plane showing

(a) The lines where  $\frac{dx}{dt} = 0$ ,

(b) The lines where  $\frac{dy}{dt} = 0$ ,

(c) The coordinates of all the equilibrium points in the first quadrant.

4. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .14x \left( \frac{100 - x - 3y}{100} \right) \\ \frac{dy}{dt} &= .3x \left( \frac{80 - x - .4y}{80} \right)\end{aligned}$$

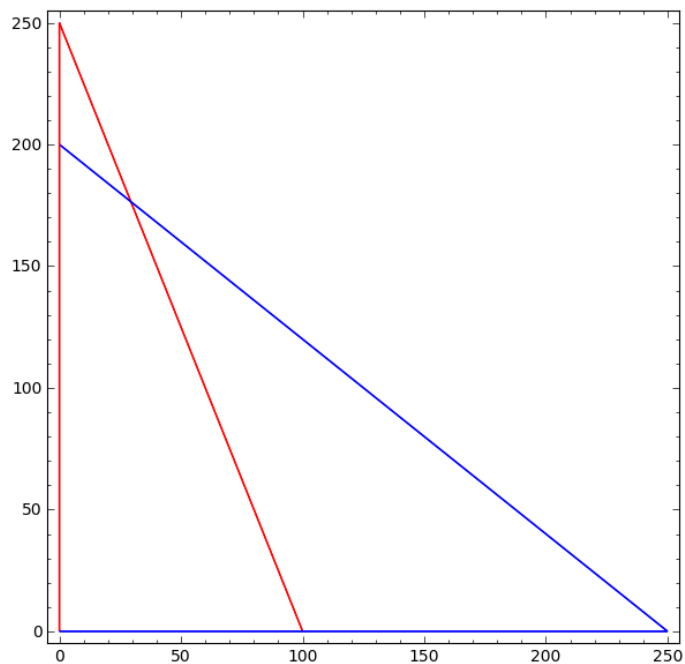
draw the phase plane showing

(a) The lines where  $\frac{dx}{dt} = 0$ ,

(b) The lines where  $\frac{dy}{dt} = 0$ ,

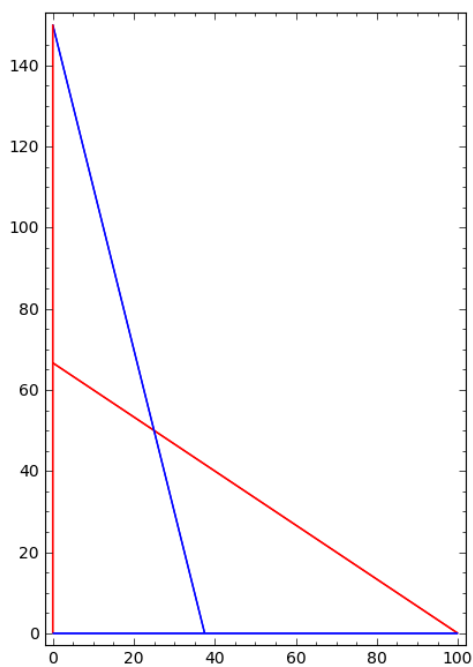
(c) The coordinates of all the equilibrium points in the first quadrant.

*Solution to 1:*



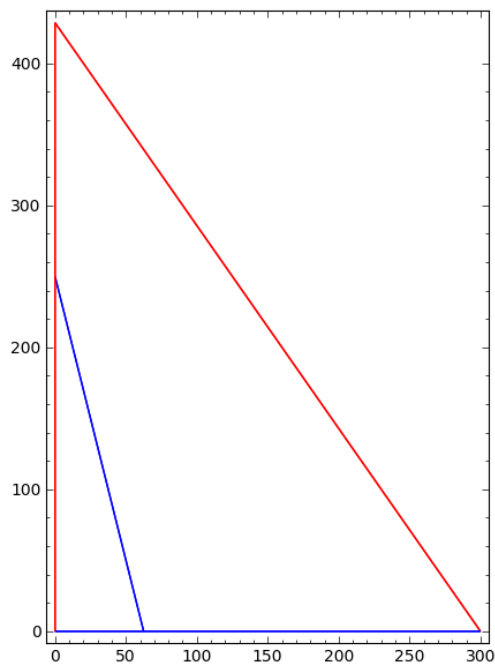
(a) The  $dx/dt = 0$  lines are in red. (b) The  $dy/dx = 0$  lines are in blue. The equilibrium points are (0, 0), (100, 0), (0, 200), and (29.41, 176.5).

*Solution to 2:*



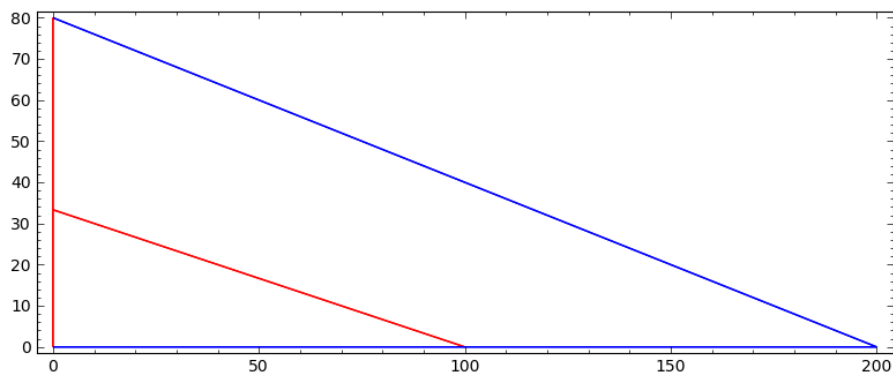
(a) The  $dx/dt = 0$  lines are in red. (b) The  $dy/dx = 0$  lines are in blue. The equilibrium points are (0, 0), (100, 0), (0, 150), and (25.00, 50.00).

*Solution to 3:*



(a) The  $dx/dt = 0$  lines are in red. (b) The  $dy/dx = 0$  lines are in blue. The equilibrium points are (0, 0), (300, 0), and (0, 250).

*Solution to 4:*



(a) The  $dx/dt = 0$  lines are in red. (b) The  $dy/dx = 0$  lines are in blue. The equilibrium points are (0, 0), (100, 0), and (0, 80).