Mathematics 172

You must show your work to get full credit.

A population of 15 roaches colonizes a dorm. It undergoes exponential growth, and after two months there are 50 roaches. Let $R(t)$ be the number of roaches t months after they colonize the dorm.

(1) Find the intrinsic growth rate r.

$$\frac{dR}{dt} = rR \quad, \quad R(0) = 15, \quad R(2) = 50$$

$r =$ _____.60198_____

$$R(t) = R(0)e^{rt} = 15e^{rt}$$

$$\Rightarrow r = \ln(50/15)/2 \nearrow$$

$$\text{But} \quad R(2) = 15e^{r2} = 50$$

$$e^{2r} = 50/15$$

$$2r = \ln(50/15)$$

(2) What are the units of r?

$$r = \frac{1}{R}\frac{dR}{dt}$$

The units are: _(Roaches/month)/Roaches._

units of $\frac{dR}{dt}$ are Roaches/month

so   units of $r = \frac{1}{R}\frac{dR}{dt}$ are $\left(\text{Roaches/month}\right)/\text{Roach}$

(3) How long until there are 100,000 roaches?

We want to solve

$t =$ _____14.6265 months._____

$$R(t) = 15 e^{.60198 t} = 100,000$$

$$e^{.60198 t} = 100,000/15$$

$$.60198 t = \ln(100,000/15)$$

$$t = \ln(100,000/15)/.60198$$