# Math 242 Test 2.

*This is due on Friday, October 23 at midnight. You are to work alone in it. You can look up definitions and the statements of theorems we have covered in class. And if there is an integral where you want to use a computer, your calculator, or a source such as Wolfram Alpha to compute it, that is fine, but say that this is what you did. (For example "I computed $\int x^2 e^x\,dx$ using the program Maple".) Needless to say (but I will say it anyway) no use of online help sites such as Stack Overflow or Chegg for direct help on the problems.*

*You have plenty of time to do this exam, so please write it so as to be easily readable. By this I do not just mean hand writing (I can not criticize anyone for bad hand writing), but writing it so that all steps are shown. You have time to do a problem and copy it over, so I would just a soon not get papers where a bunch of work is crossed out and then started over.*

**Problem** 1. On a homework students were ask to find the general solution to

$$x^2y'' - 2xy' + 2y = x.$$

Alice gave the answer as

$$y = -x\ln(x) + C_1x + C_2x^2$$

while Bob gave the answer as

$$y = x - x\ln(x) + C_1x + C_2x^2.$$

Write a few sentences explaining why both answers are correct. □

**Problem** 2 (5 points). There is some standard terminology about differential equations that you should know. You can read this as well as I can and so I do not see much point in lecturing on it. So here I am going to bribe you with some test points to learn it on your own. Read Section 0.3, pages 17–19 in the text and understand the meaning of the following terms:

- ***Ordinary differential equation*** (often abbreviated as ODE),
- ***Partial differential equation*** (abbreviation PDE),
- The ***order*** of a differential equation.

(a) For the following say if the equation is an ODE or PDE and give its order.

(i) $y'' - x^3y' + \sin(x)y = \cos(x)$.

(ii) $\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = x + y$.

(iii) $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$. □

**Problem** 3 (15 points). For the differential equation

$$y' = \frac{y(y-5)(y-10)(y-20)}{100+y^4}.$$

(a) Find the critical points.
(b) Sketch the critical solutions and the solution with $y(0) = 3$, $y(0) = 8$, $y(0) = 15$, and $y(0) = 22$ all on the same axis.
(c) Which of the critical points are stable and which are unstable?
(d) For the solution with $y(0) = 15$ compute $\lim_{x\to\infty} y(x)$.
(e) For the solution with $y(0) = 3$ estimate $y(100)$. □

Each of you will have your own "magic number" for use on this test. It is computed as follows. Start with your student ID number. This is letter followed by 8 digits. Say it is Q78946532, take the last four digits (in this case 6532) call this $d$. Then the magic number is the first four digits of

$$m = \text{the first four digits of the decimal expansion of } \frac{2000+d}{5000+d}$$

so in the example here we have

$$\frac{2000+6532}{5000+6532} = 0.739854318418314$$

and so the magic number is

$$m = .7398$$

For anther example if the student number is H985271418 then $d = 1418$,

$$\frac{2000+d}{5000+d} = \frac{2000+1418}{5000+1418} = 0.532564661888439$$

and thus

$$m = .5325$$

**Problem** 4 (5 points). Compute your magic number and write its value here (but do not write your student ID number on the test). □.

**Problem** 5 (10 points). For the differential equation

$$\frac{dy}{dx} = \frac{1+x^2+y^2}{1+x^2y^2}$$

and $m$ your magic number.
(a) If $y(2) = m$ compute $y'(2)$ to four decimal places.
(b) Use your answer to part (a) to estimate $y(2.01)$. □

**Problem** 6 (35 points). In this problem we are still using $m$ as your magic number. Give the general solution to the following differential equations.

(a) $\dfrac{du}{dt} - mu = 1 + x.$
(b) $y' = \dfrac{6x^2 - 4x + m}{12y^3 + 4y + 2m}.$
(c) $xy' + 4y = 4mx^2\sqrt{y}.$

(d) $\dfrac{dy}{dx} = \dfrac{4x^2 + xy + y^2}{x^2}$. $\square$

**Problem** 7 (10 points). Solve the following initial value problems.

(a) $t\dfrac{du}{dt} + \dfrac{3u}{2} = 1 + t, \quad u(1) = m.$

(b) $x\dfrac{dy}{dx} = e^{2y}, \quad y(1) = m.$ $\square$

**Problem** 8 (10 points). A snow ball is brought into a warm room. It starts with a radius of 4 inches. After 20 minutes it has a radius of 3.5 inches. Assume that it loses volume at a rate proportional to its surface area.

(a) Find a formula for the radius $t$ minutes.
(b) How long until it vanishes? □

**Problem** 9 (10 points). A tanks initially contains 5 pounds of salt dissolved in 100 gallons of water. Starting at time $t = 0$, water which contains 1/3 pound of salt per gallon is pumped into the tank a rate of 2 gallons per hour and (well mixed) water is pumped out of the tank at the same rate. Let $Q(t)$ be the number of pounds of salt in the tank after $t$ hours.

(a) Find the differential equation satisfied by $Q(t)$.
(b) Give a formula for $Q(t)$
(c) Find $\lim_{t\to\infty} Q(t)$. □