Mathematics 172

Quiz 27 Name: Key

You must show your work to get full credit.

Consider the predator-victim system

$$\frac{dV}{dt} = .1V\left(1 - \frac{V}{600}\right) - .01VP$$
$$\frac{dP}{dt} = -.4P + .002VP$$

1. The point $(V, P) = (0, 0)$ is a rest point of this system. Write a sentence or two explaining the biology of this rest point. If we start with no victors or predators, then the population of both stays at 0.

2. If there are no predators present, that is if $P = 0$, then the equation for victims becomes

$$\frac{dV}{dt} = .1V\left(1 - \frac{V}{600}\right)$$

which is the logistic equation with intrinsic growth rate $r = .1$ and carrying capacity $K = 600$. Thus $(V, P) = (600, 0)$ is a rest point. Write a sentence or two explaining the biology of this rest point. If there are no predators, the population of victims stabilizes at its carrying capacity of $K = 600$.

3. We now look for a rest point with $P \neq 0$ and $V \neq 0$. Using this in the equations $\frac{dV}{dt} = \frac{dP}{dt} = 0$ leads to the system of equations

$$(*) \quad .1\left(1 - \frac{V}{600}\right) - .01P = 0$$
$$-.4 + .002V = 0.$$

Solve the second equation for V and then use this in the first equation to find the third rest point.

$.002V = .4$

$V = \frac{.4}{.002} = 200$

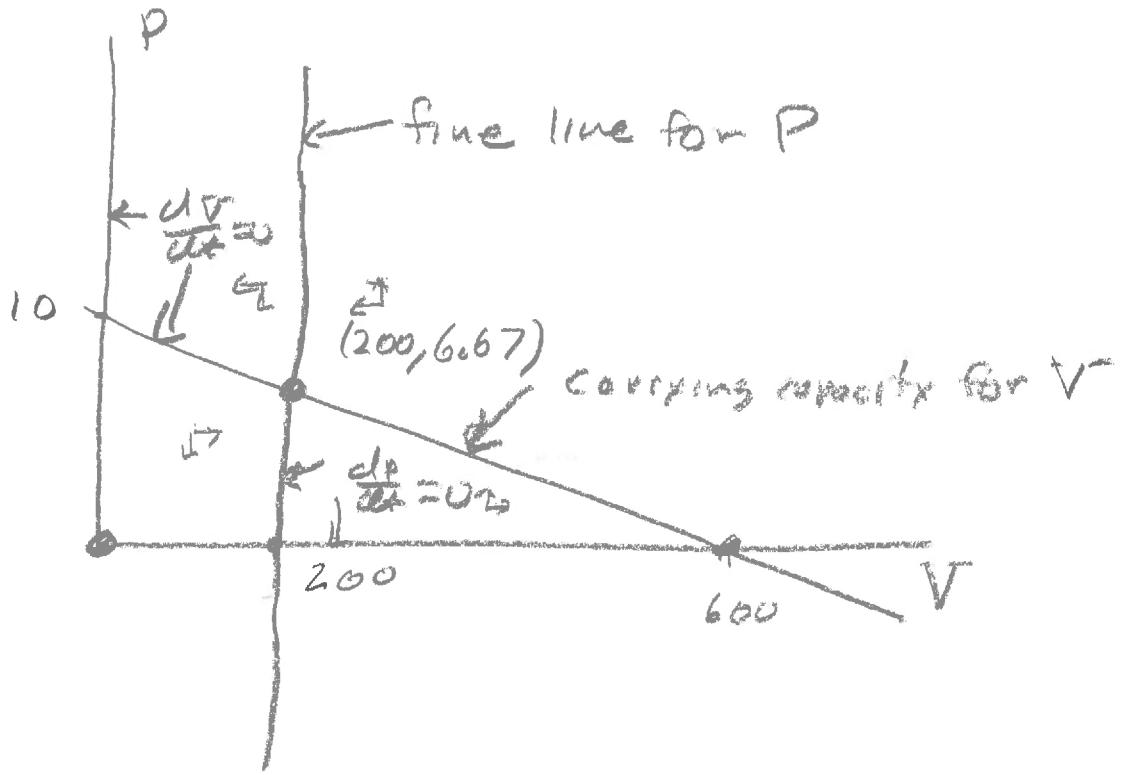
use this in (*)

The third rest point is $(200, 6.67)$

$.1\left(1 - \frac{200}{600}\right) - .01P \Rightarrow P = \frac{.1\left(1 - \frac{1}{3}\right)}{.01} = 6.67$

4. It turns out that this last rest point is stable. Write a sentence or two explaining the biology of this case.

5. Use the back of this paper to draw the V-P phase space and label the rest points, and line $\frac{dV}{dt} = 0$ (which corresponds to the carrying capacity) and the line where $\frac{dP}{dt} = 0$, which corresponds to the famine line for the predator.


P
fine line for P
$\frac{dV}{dt}=0$
10
(200, 6.67)
carrying capacity for V
$\frac{dP}{dt}=0$
200
600
V