

You must show your work to get full credit.

1. Some of the values of a function $f(x)$ are given by the following table.

| | | | | | |
|--------|---|----|----|----|----|
| x | 0 | 10 | 20 | 30 | 40 |
| $f(x)$ | 5 | 6 | 8 | 11 | 15 |

- (a) Fill in the following table for the estimates of the derivative.

| | | | | |
|---------|----|----|----|----|
| x | 5 | 15 | 25 | 35 |
| $f'(x)$ | .1 | .2 | .3 | .4 |

$$\frac{6-5}{10-0} = .1 \quad \frac{8-6}{20-10} = .2 \quad \frac{11-8}{30-20} = .3 \quad \frac{15-11}{40-30} = .4$$

- (b) Is the second derivative positive, negative, or does it change sign? Write a sentence or two to explain your answer.

The first derivative, $f'(x)$, is increasing so $f'' > 0$. Thus the second derivative is positive.

2. Compute the following derivatives. Here a and b are constants.

(a) $y = 3x^4 - 2x^3 - 9$

$$\frac{dy}{dx} = 12x^3 - 6x^2$$

(b) $A(r) = 3r + \frac{4}{r^2} = 3r + 4r^{-2}$
 $A'(r) = 3 - 8r^{-3}$

$$A'(r) = \frac{3 - 8r^{-3}}{-3 - \frac{8}{r^3}}$$

(c) $w = a\sqrt{z} + \pi z^2 = az^{\frac{1}{2}} + \pi z^2$
 $\frac{dw}{dz} = \frac{1}{2}az^{-\frac{1}{2}} + 2\pi z$

$$\frac{dw}{dz} = \frac{\frac{1}{2}az^{-\frac{1}{2}} + 2\pi z}{1}$$

(d) $f(t) = a^2 + t(t+1) = a^2 + t^2 + t$
 $(a^2)' = 0$ as a is constant

$$f'(t) = 2t + 1$$

(e) If $f(x) = \frac{1}{x^2}$ what is $f'(2)$

$$f(x) = x^{-2} \quad f'(x) = -2x^{-3} \quad f'(2) = -2(2)^{-3} = -\frac{2}{2^3} = -\frac{2}{8} = -\frac{1}{4}$$

$$f'(2) = -\frac{1}{4} = -.25$$

3. Find the second derivative of $f(x) = x^2 + \frac{1}{x}$.

$$f(x) = x^2 + x^{-1} \quad f'(x) = 2x - x^{-2} \quad f''(x) = 2 + 2x^{-3}$$

$$f''(x) = \frac{2 + 2x^{-3}}{1} = 2 + \frac{2}{x^3}$$