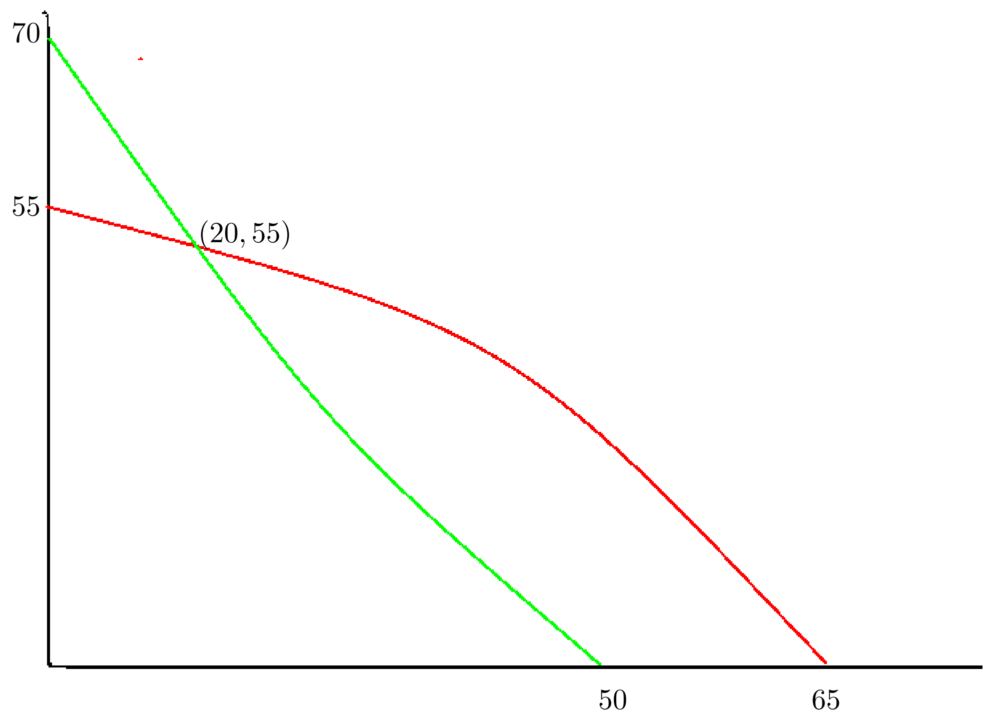
Mathematics 172 Homework, March 18, 2019.



The figure above is the phase space for the rate equations

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y)$$

1. Assume that the red curve is where $f(x,y) = 0$ and the green curve is where $g(x,y) = 0$. Also assume $f(x,y) > 0$ for points below the red curve and $g(x,y) > 0$ for points below the green curve.

(a) Find all the rest points. *Solution:* The rest points are $(0,0)$, $(65,0)$, $(0,70)$ and $(20,55)$.

(b) Draw in the arrows in the different regions showing the direction that a point will move.

(c) Can you tell which of the rest points are stable or unstable? What about long term behavior? *Solution:* The rest points $(65,0)$ and $(0,70)$ are stable. The other two are unstable. The long term behavior is ***competitive exclusion***.

(d) If there is a stable x population, is it possible for a small number of the y-species to invade the region? *Solution:* No the y-species can not invade.

(e) If $x(0) = 5$ and $y(0) = 65$ estimate $x(100)$ and $y(100)$. *Solution:* $x(100) \approx 0$ and $y(100) \approx 70$.

2. This time assume that assume that the red curve is where $g(x, y) = 0$ and the green curve is where $f(x, y) = 0$. Assume that $f(x, y) > 0$ for points below the green curve and $g(x, y) > 0$ for points below the red curve.

(a) Find all the rest points. *Solution:* The rest points are $(0, 0)$, $(50, 0)$, $(0, 55)$, and $(20, 55)$.

(b) Draw in the arrows in the different regions showing the direction that a point will move.

(c) Can you tell which of the rest points are stable or unstable? What about long term behavior? *Solution:* The only stable rest point is $(20, 55)$. The others are all unstable. This is a case of ***competitive coexistence***.

(d) If there is stable population of the y-species, then is a possible for a small number of the x-species to invade the region? *Solution:* Yes the x-species can invade.

(e) If $x(0) = 35$ and $y(0) = 45$, estimate $x(57)$ and $y(57)$. *Solution:* $x(57) \approx= 20$ and $y(57) \approx 55$.