## Frank W. Warner

# FOUNDATIONS of DIFFERENTIABLE MANIFOLDS and LIE GROUPS

With 57 Illustrations



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