

# Mathematics 122

Quiz #28

Name: Key

*You must show your work to get full credit.*

When you sneeze your wind pipe constricts. The speed,  $v$ , of the air in the sneeze depends on the radius,  $r$ , of the windpipe. If  $R$  is the normal (non-constricted) radius then  $0 \leq r \leq R$  and the speed is given by

$$v = a(R - r)r = aRr - ar^2$$

for a positive constant  $a$ . (Note that  $R$  is also constant.)

1 pt (1) What is  $\frac{dv}{dr}$ ?  $\frac{dv}{dr} = \underline{aR - 2ar}$

1 pt (2) What are the critical point(s) of  $v$ ?  $\underline{\frac{R}{2}}$   
 set  $aR - 2ar = 0$   
 $-2ar = -aR$   
 $r = \frac{-aR}{-2a} = \frac{R}{2}$

1 pt (3) Where is  $v$  increasing?  $\underline{0 \leq r \leq \frac{R}{2}}$   
 $\frac{dv}{dr} = aR - 2ar$   
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1 pt (4) What value of  $r$  maximizes  $v$ ?  $\underline{r = \frac{R}{2}}$

1 pt (5) What is the maximum value of  $v$ ?  $\underline{\frac{aR^2}{4}}$   
 $v \Big|_{r = \frac{R}{2}} = a(R - \frac{R}{2})(\frac{R}{2}) = \frac{aR^2}{4}$