Boundary and Lens Rigidity of Lorentzian Surfaces

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Abstract

Let g be a Lorentzian metric on the plane $\mathbb{R}^2$ that agrees with the standard metric $g_0 = -dx^2 + dy^2$ outside a compact set and so that there are no conjugate points along any time-like geodesic of $(\mathbb{R}^2, g)$. Then $(\mathbb{R}^2, g)$ and $(\mathbb{R}^2, g_0)$ are isometric. Further, if (M, g) and (M^*, g^*) are two dimensional compact time oriented Lorentzian manifolds with space–like boundaries and so that all time-like geodesics of (M, g) maximize the distances between their points and (M, g) and (M^*, g^*) are "boundary isometric" then there is a conformal diffeomorphism between (M, g) and (M^*, g^*) and they have the same areas. Similar results hold in higher dimensions under an extra assumption on the volumes of the manifolds.

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