Mathematics 172 Homework.

Back to Euler's method for the initial value problem:

$$y' = f(y), \qquad y(0) = y_0.$$

Let $h > 0$ be a small positive number which we will refer to as the ***step size***. Set $t_0 = 0$. Now define

$$\begin{aligned} t_1 &= t_0 + h \\ y_1 &= y_0 + f(y_0)h. \end{aligned}$$

This is our first step in Euler's method. The second step is to set

$$\begin{aligned} t_2 &= t_1 + h \\ y_2 &= y_1 + f(y_1)h. \end{aligned}$$

The third step is

$$\begin{aligned} t_3 &= t_2 + h \\ y_3 &= y_2 + f(y_2)h \end{aligned}$$

and (as you have no doubt already figured out) the fourth step is

$$\begin{aligned} t_4 &= t_3 + h \\ y_4 &= y_3 + f(y_3)h. \end{aligned}$$

In general once we have taken k steps (so that we have computed t_k and y_k) the next step is

$$\begin{aligned} t_{k+1} &= t_k + h \\ y_{k+1} &= y_k + f(y_k)h. \end{aligned}$$

It is not hard to get a formula for t_k: taking k steps of size k covers a distance of kh and thus

$$t_k = kh$$

Now let use get the calculator to do this. We now need to set up the calculator to work with tables. To do a concrete example let us use the simple equation

$$y' = -.5y + 3 \qquad y(0) = 2.$$

To start press the MODE key. This should open up a screen that looks something like this (some of the highlighted boxes may be in different places):

$\boxed{\text{NORMAL}}$ SCI ENG
$\boxed{\text{FLOAT}}$ 0 1 2 3 4 5 6 7 8 9
$\boxed{\text{RADIAN}}$ DEGREE
$\boxed{\text{FUNC}}$ PAR POL SEQ
$\boxed{\text{CONNECTED}}$ DOT
$\boxed{\text{SEQUENTIAL}}$ SIMUL
$\boxed{\text{REAL}}$ a+bi re^θi
$\boxed{\text{FULL}}$ HORIZ G-T

Use the cursor key to move down to the forth line and over to SEQ and press enter to change from FUNC mode to SEQ mode. The screen will now look like:

$\boxed{\text{NORMAL}}$ SCI ENG
$\boxed{\text{FLOAT}}$ 0 1 2 3 4 5 6 7 8 9
$\boxed{\text{RADIAN}}$ DEGREE
FUNC PAR POL $\boxed{\text{SEQ}}$
$\boxed{\text{CONNECTED}}$ DOT
$\boxed{\text{SEQUENTIAL}}$ SIMUL
$\boxed{\text{REAL}}$ a+bi re^θi
$\boxed{\text{FULL}}$ HORIZ G-T

Now press 2ND TABLESET and edit until it looks like

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt : $\boxed{\text{Auto}}$ Ask
Depend: $\boxed{\text{Auto}}$ Ask

For this example we will use a step size of

$$h = .1$$

Let us store this the in H register. To do this go to the main screen and enter .1 then push STO followed by ALPHA and H.

We next enter the equation. Press the Y= bottom. The screen will now look something like

Plot1 Plot2 Plot2
nMin=
\u(n)=
u(nMin)=
\v(n)=
v(nMin)=
\w(n)=
w(nMin)=

In calculator notation we use u for the dependent variable y. So to enter a that a step in Euler's method looks like

$$y_{k+1} = y_k + f(y_k)h = y_k + (-.5y_k + 3)h$$

The calculator is happier with this after doing the change of variable $n = k + 1$, so that $k = n - 1$.

$$y_n = y_{n-1} + (-.5y_{n-1} + 3)h.$$

There are some tricks involved in entering the data:

- Where there is an n use the X,T, θ, n key.
- For u, v, and w use 2ND u (over the 7 key), 2ND v (over the 8 key), and 2ND w (over the 9 key).
- For H use press ALPHA H. (When we run the program the calculator is smart enough to use our stored value h.)

```
 Plot1 Plot2 Plot2
 nMin=0
\u(n)=u(n-1) + (-.5 u(n-1) + 3) H
 u(nMin)=2
\v(n)=
  v(nMin)=
\w(n)=
  w(nMin)=
```

And we are now almost done. Press 2ND TABLE and you get output that looks like

n	$u(n)$
0	2
1	2.1
2	2.39
3	2.5705
4	2.742
5	3.9049
6	3.0596

You can now scroll down and find that

$$u(10) = 3.6051$$

In terms of our original problem this tells use that if we take 10 steps of size $h = .1$ we get the approximation

$$y(1) \approx 3.6051$$

to the solution to the initial value problem $y' = -.5y + 3$, with $y(0) = 2$.

Problem 1. To get a more accurate approximation to this initial value problem we could take 20 Euler steps of size $h = .05$. Do this. Then approximate $y(.45)$ by taking 45 Euler steps of size $h = .01$.

Solution. The only we need to change is to store .05 in the H register. You then get $u(20) = 3.5892$ which will be a better approximation that our original 3.0596.

To do the approximation of $y(.45)$ store .01 in the H and then find that $u(45) = 2.8077$ which will be pretty close to the exact solution to the problem. □

Let us return to the logistic equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right).$$

Let us consider the care where

$$r = .15, \qquad K = 100.$$

Store .015 in the R register (that is enter .015 on the main screen, then push STO then ALPHA R. Likewise store 100 in the K register. And let us use a step size of $h = .1$, so store .1 in the H register. Enter the equation and initial

```
Plot1 Plot2 Plot2
nMin=0
\u(n)=u(n-1) + R u(n-1)( H
u(nMin)=2
```

condition as

Problem 2. With this set up estimate $P(2.5)$.

Solution. Since we are using a step size of $h = .1$ to get to 2.5 we need 25 steps. Doing this gives

$$u(25) = 99.806$$

as the approximate of $P(25)$. □