

《SE-203 概率统计》期末试题参考答案(A)

1. (15)

$$\bar{x} = 5$$

$$\tilde{x} = 5.5$$

$$f_s = \frac{6+6}{2} - \frac{3+4}{2} = 2.5$$

$$s^2 = 5.714286$$

$$s = 2.390457$$

2. (15)

$$(a) \quad P(W) = 0.6 \times 0.4 + 0.4 \times 0.3 = 0.36;$$

$$(b) \quad P(H|W) = \frac{0.6 \times 0.4}{0.36} = \frac{2}{3} = 0.667.$$

3. (10)

(a) P(in a given minute the number of arrivals will equal 3)

$$= \frac{1^3}{3!} \cdot e^{-1} = \frac{1}{6e} = 0.061313;$$

(b) P(at most 2 people arrive in a 2-minute period)

$$= \frac{2^0}{0!} \cdot e^{-2} + \frac{2^1}{1!} \cdot e^{-2} + \frac{2^2}{2!} \cdot e^{-2} = \frac{5}{e^2} = 0.676676.$$

4. (15)

$$(a) \quad P\{X \geq 16.7\} = 1 - \Phi\left(\frac{16.7 - 16}{0.35}\right) = 1 - \Phi(2) = 0.02275.$$

(b) Suppose that the mean required is  $\mu_0$ .

$$\text{Since } 0.005 = P\{X \geq 16.7\} = 1 - \Phi\left(\frac{16.7 - \mu_0}{0.35}\right),$$

$$\text{we have } \mu_0 = 16.7 - \Phi^{-1}(0.995) * 0.35 = 15.79846.$$

(c) Suppose that the mean required is  $\sigma_0$ .

Since  $0.005 = P\{X \geq 16.7\} = 1 - \Phi\left(\frac{16.7 - 16}{\sigma_0}\right)$ ,

we have  $\sigma_0 = \frac{0.7}{\Phi^{-1}(0.995)} = 0.271757$ .

5.(15)

(a)  $A=1$ ;

(b)  $EX = \int_0^1 \int_0^1 x(x+y) dx dy = \frac{7}{12}$ ,

$$EY = \int_0^1 \int_0^1 y(x+y) dx dy = \frac{7}{12},$$

$$EXY = \int_0^1 \int_0^1 xy(x+y) dx dy = \frac{1}{3},$$

$$\text{Cov}(X, Y) = EXY - EXEY = -\frac{1}{144}.$$

(c) Since  $\text{Cov}(X, Y) \neq 0$ ,  $X$  and  $Y$  are not independent.

6. (20)

(a)  $EX^2 = \int_0^{+\infty} x^2 \cdot 2\lambda x \cdot e^{-\lambda x^2} dx = \frac{1}{\lambda}$ ,

$$\hat{\lambda}_{ME} = \frac{1}{\frac{1}{n} \sum_{i=1}^n X_i^2}.$$

(b) Likelihood function:  $L = \prod_{i=1}^n 2\lambda x_i \cdot e^{-\lambda x_i^2}$ ,

$$\ln L = n \ln 2 + n \ln \lambda - \lambda \sum_{i=1}^n x_i^2,$$

Likelihood equation:  $\frac{\partial}{\partial \lambda} \ln L = \frac{n}{\lambda} - \sum_{i=1}^n x_i^2 = 0$ ,

The solution is  $\lambda = \frac{n}{\sum_{i=1}^n x_i^2}$ ,

The resulting MLE is  $\hat{\lambda}_{MLE} = \frac{n}{\sum_{i=1}^n X_i^2}$ .

7. (10)

$$\left( \bar{x} - t_{\frac{\alpha}{2}, n} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2}, n} \cdot \frac{s}{\sqrt{n}} \right) = \left( 1.67 - 2.093 \cdot \frac{0.32}{\sqrt{20}}, 1.67 + 2.093 \cdot \frac{0.32}{\sqrt{20}} \right) = (1.520237, 1.819763)$$