《 概 率 统 计 》 期 末 试 题 (B 卷)

(考 试 形 式 ： 闭 卷 考 试 时 间 ：**2** 小 时)

警 示 《中山大学授予学士学位工作细则》第六条

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1. (10')
Suppose $P(A) = 0.6$, $P(C) = 0.4$, $P(AC) = 0.3$, $P(A'B) = 0.2$ and $P(A'B'C') = 0.1$.
Determine $P((A \bigcup B)C')$. Hereafter A' denotes the complement event of A.

2. (10')
Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:
a. P(all of the next three vehicles inspected pass);
b. P(exactly one of the next three vehicles inspected passes);
c. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass?

3. (15')

Let an urn contain two coins: A_1, a fair coin with probability $\dfrac{1}{2}$ of falling H; and A_2, a biased coin with probability $\dfrac{1}{3}$ of falling H. A coin is drawn at random and tossed. Suppose that it fall head. We ask for the probability that the fair coin was selected.

4 (15')
The joint probability mass function (pmf) of X and Y appears in the accompanying tabulation as follows.

X \ Y	1	2	3
1	1/6	1/9	1/18
2	1/3	α	β

a). Assume that X and Y are independent, what is the value of α and β.

b). Compute the expected value and the standard deviation of X and Y, respectively.

c). Assume that Z = X+Y. Obtain the probability mass function (pmf) of Z.

5. (15')

Suppose the probability density function (pdf) of X is

$$f(x;\theta) = \begin{cases} (\theta+1)x^\theta & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$, where $\theta > -1$. A random sample x_1=0.92, x_2=0.79,

x_3=0.90, x_4=0.65, x_5=0.86, x_6=0.47, x_7=0.73, x_8=0.97, x_9=0.94, x_{10}=0.77.

a. Use the method of moments to obtain an estimator of θ, and then compute the estimate for this data.

b. Obtain the maximum likelihood estimator of θ, and then compute the estimate for the given data.

6. (15')

Consider the test below

$$H_0: \quad u = 100$$
$$H_1: \quad u \ne 100$$

where u denotes the mean value of a normal population with variance $\sigma^2 = 25$. Assume that $\bar{x} = 94$ is the mean value of a sample of size $n = 16$. Determine

a. Whether we should reject the null hypotheses under the significance level $\alpha = 0.1$;

b. Compute the probability of type II error when the true mean value is 98.

7. (20')

Let X and Y be two continuous random variables with joint probability density function (pdf) as follows

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & , otherwise \end{cases}$$

Compute the probability density function (pdf) of max(X, Y) and min(X, Y), respectively.

P.S. Table A.3. in the next page may be useful for your computations.

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \le z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998