Baby Food

$\int_a^b c\,dx = c(b-a)$	$\int_a^b x\,dx = \frac{b^2}{2} - \frac{a^2}{2}$	$\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$

Fundamental Theorems of Baby Food

Fund.Theorem.Calc.1	Fund.Theorem.Calc.2
$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$	$\int_a^b f(x)\,dx = F(b) - F(a)$

Integrals of Trig

$\int \sin(x)\,dx = -\cos(x) + C$	$\int \cos(x)\,dx = \sin(x) + C$
$\int \tan(x)dx = \ln\lvert\sec(x)\rvert + C$	$\int \cot(x)\,dx = \ln\lvert\sin(x)\rvert + C$
$\int \sec^2(x)\,dx = \tan(x) + C$	$\int \csc^2(x)dx = -\cot(x) + C$
$\int \sec(x)\,dx = \ln\lvert\sec(x) + \tan(x)\rvert$	$\int \csc(x)\,dx = -\ln\lvert\csc(x) + \cot(x)\rvert$
$\int \sec(x)\tan(x)\,dx = \sec(x) + C$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

Integrals of Inverse Trig

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}(\frac{x}{a}) + C$	$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$

$$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$$

Area Under Curve

Top-to-bottom $A(x) = \int_a^b [\,f(x) - g(x)\,]dx$	**Right-to-left** $A(y) = \int_c^d [\,f(y) - g(y)\,]dy$
Vol. by Rotation (x-axis) $V = \int_a^b \pi\,[\,R(x)\,]^2 dx$	**Vol. by Rotation (y-axis)** $V = \int_c^d \pi\,[\,R(y)\,]^2 dy$
Vol. by Washer Rotation ***Outer-inner (X-axis)*** $V(x) = \int_a^b \pi\,[\,R(x)\,]^2 - [\,r(x)\,]^2 dx$	**Vol. by Washer Rotation** ***Outer-inner (Y-Axis)*** $V(y) = \int_a^b \pi\,[\,R(y)\,]^2 - [\,r(y)\,]^2 dy$
Shell Method ***About Y-Axis*** $V = \int_a^b 2\pi\,[Radius][Height]\,dx$ $V = \int_a^b 2\pi\,[x][f(x)]\,dx$	**Shell Method** ***About X-Axis*** $V = \int_a^b 2\pi\,[Radius][Height]\,dx$ $V = \int_c^d 2\pi\,[y][f(y)]\,dx$
Arc Length $L(x) = \int_a^b \sqrt{1 + [f'(x)]^2}dx$	**Arc Length** $L(y) = \int_c^d \sqrt{1 + [g'(y)]^2}dy$
Surface Area ***Around X-Axis*** $S(x) = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}dx$	**Surface Area** ***Around Y-Axis*** $S(y) = \int_a^b 2\pi f(y)\sqrt{1 + [g'(y)]^2}dy$

Physics Applications

Work Done Along a Line $W = \int_a^b F(x)dx$ $[W = force \times (x),\ given\ x = distance]$	**Hooke's Law for Springs** $W = \int_a^b kx\,dx$ $a \to b = \Delta length(spring)$
Lifting Objects $W = \int_a^b weight \cdot (height - x)\,dx$	**Pumping Liquid** $W = \int_a^b weight \cdot (height - x) \cdot \Delta Volume\,dx$
Fluid Force on Constant Surface $F = weight \cdot height \cdot area$	**Fluid Force on a Vert Plate** $W = \int_a^b weight \cdot (strip\ depth) \cdot (length\ of\ strip)\,dx$
Center of Mass $= \frac{moment}{mass}$ $\bar{x} = \frac{\int_a^b (weight) \times (x) \times [f(x)-g(x)]\,dx}{\int_a^b [f(x)-g(x)]\,dx}$	**Center of Mass** $= \frac{moment}{mass}$ $\bar{y} = \frac{\int_a^b (weight) \times (\frac{1}{2}) \times [\,f^2(x) - g^2(x)\,]\,dx}{\int_a^b [\,f(x) - g(x)\,]\,dx}$

ln() properties

$\ln(b \cdot a) = \ln(b) + \ln(a)$	$\ln(\frac{b}{a}) = \ln(b) - \ln(a)$
$\ln(\frac{1}{x}) = -\ln(x)$	$\ln(x^r) = r \times \ln(x)$
$\frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$	$\int \frac{1}{u}du = \ln\lvert u\rvert + C$
$\int \frac{f'(x)}{f(x)}du = \ln\lvert f(x)\rvert + C$	$\int_1^x \frac{1}{t}dt = \ln(x)$

$$\int_a^b \ln(x)dx = USE\ Integration\ by\ Parts$$

More e and ln() properties

$\frac{d}{dx}e^u = e^u\frac{du}{dx}$	$e^{\ln(x)} = x$	$\ln(e^x) = x$
$\int e^u\,du = e^u + C$	$e^{-x} = \frac{1}{e^x}$	**a ralph b production**
$e^{x_1} \cdot e^{x_2} = e^{x_1 + x_2}$	$e^{x_1}/e^{x_2} = e^{x_1 - x_2}$	$(e^{x_1})^{x_2} = e^{x_1 \cdot x_2}$
$a^x = e^{x \cdot \ln(a)}$	$\frac{d}{dx}a^u = a^u \ln(a)(\frac{du}{dx})$	$\int a^u\,du = \frac{a^u}{\ln(a)} + C$

log() properties

$\log_a(b \times x) = \log_a(b) + \log_a(x)$	$\log_a(\frac{b}{x}) = \log_a(b) - \log_a(x)$
$\log_a(\frac{1}{x}) = -\log_a(x)$	$\log_a(x^r) = r \times \log_a(x)$

$$\frac{d}{dx}\log_a u = \frac{1}{\ln(a)}\frac{1}{u}\frac{du}{dx} = \frac{1}{u \times \ln(a)}\frac{du}{dx}$$

Hyperbolic properties

$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$	$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$ $\sinh(2x) = 2\sinh(x)\cosh(x)$ $\cosh^2(x) - \sinh^2(x) = 1$
$\cosh^2(x) = \frac{\cosh(2x) + 1}{2}$ $\sinh^2(x) = \frac{\cosh(2x) - 1}{2}$	$\tanh^2(x) = 1 - sech^2(x)$ $\coth^2(x) = 1 + csch^2(x)$	**Good luck on your exam!**
$\frac{d}{dx}\sinh(u) = \cosh(u)\frac{du}{dx}$ $\frac{d}{dx}\cosh(u) = \sinh(u)\frac{du}{dx}$	$\frac{d}{dx}\tanh(u) = sech^2(u)\frac{du}{dx}$ $\frac{d}{dx}\coth(u) = -csch^2(u)\frac{du}{dx}$	$\frac{d}{dx}sech(u) = -sech(u)\tanh(u)\frac{du}{dx}$ $\frac{d}{dx}csch(u) = -csch(u)\coth(u)\frac{du}{dx}$

Message from creator: *if you can memorize this sheet by heart, i can 1000% guarantee you will do fine in Calculus I. good luck. The name mother-calculator came from the person I learned from the most (my mom) and the thing I learned from the least (using my calculator).*

Integrals of Hyperbolic properties

$\int \sinh(u) = \cosh(u) + C$	$\int \cosh(u) = \sinh(u) + C$
$\int \tanh(u) = \ln[\cosh(u)] + C$	$\int \coth(u) = \ln[\sinh(u)] + C$
$\int csch(u) = \ln[\tanh(\frac{u}{2})] + C$	$\int sec(u) = \tan^{-1}[\sinh(u)] + C$
$\int csch^2(u) = -\coth(u) + C$	$\int sech^2(u) = \tanh(u) + C$
$\int csch(u)\coth(u) = -\coth(u) + C$	$\int sech(u)\tanh(u) = -sech(u) + C$
$csch^{-1}(x) = \sinh^{-1}(\frac{1}{x})$	$sech^{-1}(x) = \cosh^{-1}(\frac{1}{x})$

$$\coth^{-1}(x) = \tanh^{-1}(\frac{1}{x})$$

Derivatives of Hyperbolic properties

$\frac{d}{dx}\sinh^{-1}(u) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$	$\frac{d}{dx}\cosh^{-1}(u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$
$\frac{d}{dx}\tanh^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$	$\frac{d}{dx}\coth^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$
$\frac{d}{dx}csch^{-1}(u) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}$	$\frac{d}{dx}sech^{-1}(u) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$

Integrals of Inverse Hyperbolic properties

$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}(\frac{u}{a}) + C$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}(\frac{u}{a}) + C$
$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + C$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\coth^{-1}(\frac{u}{a}) + C$
$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}csch^{-1}\left|\frac{u}{a}\right| + C$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}sech^{-1}(\frac{u}{a}) + C$

Integration by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int u \times v' = uv - \int v \times du$$

$$\int x^2 e^x\,dx.$$

With $f(x) = x^2$ and $g(x) = e^x$, we list:

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$	e^x
$2x$	$(-)$	e^x
2	$(+)$	e^x
0		e^x

Integrating Powers of cos() and sin()

Odd Powers of Cos(x) and Sin(x)

$$\sin^2(x) + \cos^2(x) = 1$$

EX. $\int \cos^5(x)dx = \int \cos^4(x)\cos(x)dx = \int [\cos^2(x)]^2 \cos(x)dx$

Even Powers of Cos(x) and Sin(x)

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

EX. $\int \sin^4(x)dx = \int [\sin^2(x)]^2\,dx = \int [\frac{1}{2} - \frac{1}{2}\cos(2x)]^2\,dx$