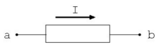
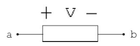
Chapter 1

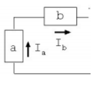
Current:



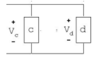
Voltage



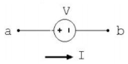
Series: Same Current



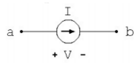
Parallel: Same Voltage



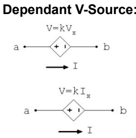
Independant V-Source: Fixed Voltage/Polarity



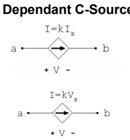
Independant C-Course: Fixed Current/Direction



Dependant V-Source:



Dependant C-Source:



KVL: Around closed loop

$$\sum V_{\text{rise}} = \sum V_{\text{drop}}$$

$$\sum V_{\text{drop}} - \sum V_{\text{rise}} = 0$$

KCL: At any node

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Absorbed Power:

$P = \frac{dW}{dt} = \frac{dW}{dq}\frac{dq}{dt} = VI$

if > 0, *absorbed power*
if < 0, *injects/delivers power*

assume SRS
Current enters $\oplus$ *Terminal*

Energy Conservation::

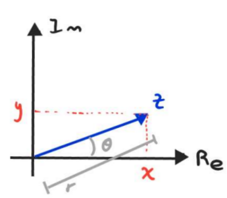
$\sum P = 0$

sum of absorbed power in a circuit = 0

Resistor(R): **[Ω]**	$P_{power} = VI = RI^2 = V^2/R$ *if* $R = 0$, *acts as a short* *if* $R = \infty$, *acts as an open circuit*
Inductor(L): **[H]**	$V_{inductor} = L\frac{di}{dt}$ $P_{power} = V_{ind}I = [L\frac{di}{dt}]i = Li\frac{di}{dt}$ $P_{energy\ stored} = \frac{d}{dt}[\frac{1}{2}Cv^2]$
Capacitor(C): **[F]**	$I_{capacitor} = C\frac{dv}{dt}$ $P_{power} = VI_{cap} = V[C\frac{dv}{dt}] = vC\frac{di}{dt}$ $P_{energy\ stored} = \frac{d}{dt}[\frac{1}{2}Li^2]$

Phasors

$Z = x + jy$
$x = Re\{Z\} = r \cdot cos(\theta)$
$y = Im\{Z\} = r \cdot sin(\theta)$
$r = |Z| = \sqrt{x^2 + y^2}$
$\theta = angle\ of\ Z$



Special cases:

$e^{j0} = 1$ $\quad e^{\pm j\pi} = -1$ $\quad e^{\pm j\pi/2} = \pm j$

Euler's identity: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$$\Rightarrow \quad \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$Z_1 \pm Z_2 = (a_1 + jb_1) \pm (a_2 + jb_2) = (a_1 \pm a_2) + j(b_1 \pm b_2)$$
$$Z_1Z_2 = (r_1e^{j\theta_1})(r_2e^{j\theta_2}) = (r_1r_2)e^{j(\theta_1+\theta_2)}$$
$$\frac{Z_1}{Z_2} = \frac{r_1e^{j\theta_1}}{r_2e^{j\theta_2}} = \left(\frac{r_1}{r_2}\right)e^{j(\theta_1-\theta_2)}$$
$$Z_1^n = (r_1e^{j\theta_1})^n = r_1^ne^{jn\theta_1}$$
$$Z_1^* = (r_1e^{j\theta_1})^* = r_1e^{-j\theta_1} = (a_1 + jb_1)^* = a_1 - jb_1 \quad \text{Complex conjugate}$$

Chapter 2

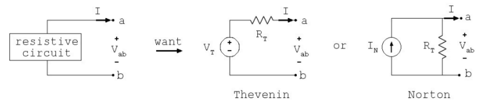
VDR	$\frac{R_i}{R_{eq}} \cdot V_{ab} \rightarrow in\ series\ only$
CDR	$\frac{R_{eq}}{R_i} \cdot I_R \rightarrow in\ parallel\ only$

Node Voltage Method- L5, Feb 1

Loop Current Method-L6, Feb 2

Superposition Method-L7, Feb 3

Thevenin & Norton Eq-L8, Feb 5-8



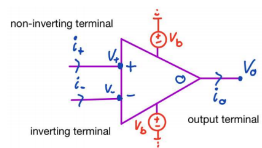
Available Power: RESISTOR NETWORK ONLY

$P_a = \frac{V_T^2}{4R_T} \rightarrow only\ possible\ if\ R_L = R_T > 0$

Operational Amplifier(op-amp):

Ideal Op Amp Conditions $\quad i_+ = i_- = 0$, $\quad V_+ = V_-$

It is a good idea to use KCL at the inverting terminal



Chapter 3

Inductor(L): **[H]** *Current(I) through inductor is continuous*	$V_{inductor} = L\frac{di}{dt}$ $i(t) = \frac{1}{L}\int_{-\infty}^{t} v(s)ds = \frac{1}{L}\int_{-\infty}^{0} v(s)ds + \frac{1}{L}\int_{0}^{t}$ $\Rightarrow i(t) = i_0 + \frac{1}{L}\int_0^t v(s)ds \quad i_0 =$ *initial st*
Capacitor(C): **[F]** *Voltage(V) through capacitor is continuous*	$I_{capacitor} = C\frac{dv}{dt}$ $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(s)ds = \frac{1}{C}\int_{-\infty}^{0} i(s)ds + \frac{1}{C}\int_{0}^{t}$ $\Rightarrow v(t) = v_0 + \frac{1}{C}\int_0^t i(s)ds \quad v_0 =$ *initial s*

RALPH BALITA NOTES(RALPHSCALVES)

Linearity-L12, Feb 12

Linear combination of inputs -> Linear combination of outputs

Must be Zero-State Linear and Zero-Input Linear

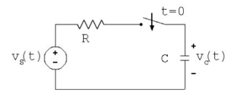
$$f_1(t) \rightarrow \boxed{\text{zero-state linear}} \rightarrow y_{ZS,1}(t), \text{ and}$$
$$f_2(t) \rightarrow \boxed{\text{zero-state linear}} \rightarrow y_{ZS,2}(t), \text{ then}$$
$$f_3(t) = k_1f_1(t) + k_2f_2(t) \rightarrow \boxed{\text{zero-state linear}} \rightarrow y_{ZS,3}(t) = k_1y_{ZS,1}(t) + k_2y_{ZS,2}(t)$$

Time-Invariance-L13, Feb 13

Delayed inputs cause equally delayed outputs...t_d

$$f_1(t) \rightarrow \boxed{\text{time-invariant}} \rightarrow y_1(t), \text{ then}$$
$$f_2(t) = f_1(t - t_d) \rightarrow \boxed{\text{time-invariant}} \rightarrow y_2(t) = y_1(t - t_d),$$

First Order RC Circuits



– For $t > 0$

$$\Rightarrow \frac{v_s(t)}{RC} = \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t)$$

– Generic first order ordinary differential equation (ODE) with constant coefficients

1st Order ODE RC Circuit -Feb 15

$$y' + \frac{1}{RC}y = \frac{V_s}{RC}$$

$$y(t) = \underbrace{\frac{K}{a}}_{=y_p(t)} + \underbrace{\left(y(0^+) - \frac{K}{a}\right)e^{-at}}_{=y_h(t)}$$

$y_p(t) = particular\ solution$ $y_h(t) = homogenous\ soluti$	$K = \frac{V_s}{RC} \quad a = \frac{1}{RC}$ $y(0^+) = v_c(0) = v_c(0^-)$

$$v_c(t) = V_s + (v_c(0^-) - V_s)e^{-\frac{t}{RC}}$$

$$v_{capacitor}(t) = V_s + [v_{cap}(0^-) - V_s]e^{-\frac{t}{RC}}$$

$$v_{cap}(t) = B + Ae^{-\frac{t}{\tau}}$$

$$B = v_{cap}(\infty) = V_s$$

$$B + A = v_{cap}(0^+) = v_{cap}(0^-)$$

$\tau = RC \ \uparrow\tau = slower\ decay,\ \downarrow = faster$

$$y(t) = y_{ZS}(t) + y_{ZI}(t) = \underbrace{\frac{K}{a}(1 - e^{-at})}_{=y_{ZS}} + \underbrace{y(0^+)e^{-at}}_{=y_{ZI}}$$

1st Order ODETransient and Steady State: -L17, Feb 23

Transient Response [$y_{tr}(t)$]:

Part of y(t) that approaches 0, as $t \rightarrow \infty$

$$y_{tr}(t) \xrightarrow{t\to\infty} 0$$

Steady State Response [$y_{ss}(t)$]:

Part of y(t) that does not approach 0, as $t \rightarrow \infty$

$y_{ss}(t) = y(t) - y_{tr}(t)$

Chapter 4

Phasors Feb 24

$$f(t) = A\cos(\omega t + \theta) = Re\left\{Ae^{j(\omega t+\theta)}\right\}$$

$$Re\left\{Ae^{j\theta}e^{j\omega t}\right\} = Re\left\{\underbrace{Ae^{j\theta}}_{:=F}e^{j\omega t}\right\} = Re\left\{Fe^{j\omega t}\right\}$$

Impedance Mar 1

$$g(t) = \frac{d}{dt} f(t) \quad \Rightarrow \quad G = j\omega F$$

$$V \ = \ ZI \quad \text{Ohm's law}$$

$$\text{Impedance, } Z \ = \ \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \frac{1}{j\omega C} = \frac{-j}{\omega C} & \text{capacitor} \end{cases}$$

Average Absorbed Power Mar 5

- **AVE POWER: Periodic signals**

$$P \ = \ \frac{1}{T}\int_T p(t)dt \ = \frac{1}{T}\int_T v(t)i(t)dt$$

- **AVE POWER: Cosine Signals**

$$v(t) = Re\{Ve^{j\omega t}\} \text{ and } i(t) = Re\{Ie^{j\omega t}\}$$

$$P \ = \ \frac{1}{2}Re\{VI^*\} \ = \ \frac{1}{2}Re\{V^*I\}$$

- **AVE POWER: Resistor**

$$P \ = \ \frac{1}{2}Re\{VI^*\} \ = \ \frac{1}{2}Re\left\{V\left(\frac{V}{R}\right)^*\right\} \ = \ \frac{|V|^2}{2R} \ = \ \frac{R|I|^2}{2}$$

- **AVE POWER: INDUCTOR**

$$P \ = \ \frac{1}{2}Re\{VI^*\} \ = \ \frac{1}{2}Re\{(j\omega LI)I^*\} \ = \ 0 \text{ W}$$

- **AVE POWER: CAPACITOR**

$$P \ = \ \frac{1}{2}Re\{VI^*\} \ = \ \frac{1}{2}Re\left\{\left(\frac{I}{j\omega C}\right)I^*\right\} \ = \ 0 \text{ W}$$

Available Power: RLC CIRCUIT Mar 8

$$P_a \ = \ \frac{|V_T|^2}{8R_T}$$ **, R_T = Re{ r + aj }**

Max Power achieved with Matched Load

$$Z_L = Z_T^* = R_T - jX_T$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance Frequency:

At w_0, Max current in RLC SERIES circuit:

$$I = \frac{V_i}{R}$$

At w_0, Max Voltage in RLC PARALLEL circuit:

$$V = RI_s$$

Chapter 5

Frequency Response of LTI Systems - Mar 9

$$f(t) = |F|\cos(\omega t + \angle F) \ \rightarrow$$

$$\boxed{H(\omega)} \rightarrow \ y(t) = \underbrace{|F||H(\omega)|}_{=|Y|}\cos\left(\omega t + \underbrace{\angle F + \angle H(\omega)}_{=\angle Y}\right)$$

- $$|Y| \ = \ |F||H(\omega)|$$

- $$\angle Y \ = \ \angle F + \angle H(\omega)$$

Frequency Response of LTI Systems - Mar 10

$$f(t) = Re\{Fe^{j\omega t}\} \ \rightarrow \boxed{\text{LTI}} \rightarrow \ y(t) = Re\{Ye^{j\omega t}\}$$

$$F \ \rightarrow \boxed{H(\omega)} \rightarrow \ Y = FH(\omega).$$

$$H(\omega) \ = \ \frac{Y}{F} \ = \ \frac{\text{output phasor}}{\text{input phasor}}$$

Lowpass

$$|H(\omega)| \approx \frac{1}{\sqrt{1+\omega^2}} \quad |H(0)| \approx 1 \quad |H(\infty)| \approx 0 \quad |H(\omega)| \searrow \text{ as } \omega \to \infty$$

Highpass

$$|H(\omega)| \approx \frac{\omega}{\sqrt{1+\omega^2}} \quad |H(0)| \approx 0 \quad |H(\infty)| \approx 1 \quad |H(\omega)| \nearrow \text{ as } \omega \to \infty$$

Bandpass

$$|H(\omega)| \approx \frac{\omega}{\sqrt{\omega^2+(1-\omega^2)^2}} \quad |H(0)| \approx 0 \quad |H(\infty)| \approx 0 \quad |H(\omega)| \nearrow \text{ and then } \searrow \text{ as } \omega \to \infty$$

A RALPH BALITA NOTES(RALPHSCALVES)

Chapter 6

Exponential Fourier Series

$$f(t) \ = \ \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

Trigonometric Fourier Series

$$f(t) \ = \ \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t))$$

$$a_0 = 2F_0 \qquad a_n = F_n + F_{-n} \qquad b_n = j(F_n - F_{-n})$$

$$F_0 = \frac{a_0}{2} \qquad F_n = \frac{a_n - jb_n}{2} \qquad F_{-n} = \frac{a_n + jb_n}{2}$$

Compact Fourier Series

$$f(t) \ = \ \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n\cos(n\omega_0 t + \theta_n)$$

$$c_0 = 2F_0 \qquad c_n = 2|F_n| \qquad \theta_n = \angle F_n$$

$$F_0 = \frac{c_0}{2} \qquad F_n = \frac{c_n}{2}e^{j\theta_n} = F_{-n}^*$$

Periodic LTI

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \ \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = \sum_{n=-\infty}^{\infty} H(n\omega_0)F_n e^{jn\omega_0 t}$$

$$\Rightarrow \quad F_n \ \longrightarrow \boxed{H(\omega)} \longrightarrow Y_n = H(n\omega_0)F_n$$

Parseval's Theorem

$$P \ = \ \frac{1}{T}\int_T |f(t)|^2 dt \ = \ \sum_{n=-\infty}^{\infty}|F_n|^2 \ = \ \frac{c_0^2}{4} + \sum_{n=1}^{\infty}\frac{c_n^2}{2}$$

Total Harmonic Distortion

$$y(t) = \frac{c_0}{2} + c_1\cos(\omega_0 t) + c_2\cos(2\omega_0 t) + \ldots$$

$$THD = \frac{\text{power in harmonics}}{\text{power in } \omega_0} \ = \ \frac{\frac{c_2^2}{2} + \frac{c_3^2}{2} + \ldots}{\frac{c_1^2}{2}} \ = \ \frac{P_y - \frac{c_0^2}{4} - \frac{c_1^2}{2}}{\frac{c_1^2}{2}}$$

Chapter 7

Fourier Transforms - Mar 22

Fourier Transform

$$F(\omega) \ = \ \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \ \in \ \mathbb{C}$$

Inverse Fourier Transform

$$f(t) \ = \ \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

Fourier Transforms for LTI Systems

$$F(\omega) \ \longrightarrow \boxed{\text{LTI}} \longrightarrow Y(\omega) = H(\omega)F(\omega)$$

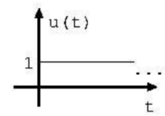
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \ \longrightarrow \boxed{\text{LTI}}$$

$$\boxed{\text{LTI}} \longrightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t}d\omega$$

Time Scaling for Fourier Transforms xA: Mar 23

$$f(t) \leftrightarrow F(\omega) \qquad\qquad f(at) \leftrightarrow \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$$

Unit Step Function:



$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{else.} \end{cases}$$

Symmetry- Mar 29

$$f(t) \leftrightarrow F(\omega) \qquad\qquad F(t) \leftrightarrow 2\pi f(-\omega)$$

f (t - b)	**b > 0, right**	**b < 0, left**
f (a*t)	**a > 1, shrink**	**0<a<1, expand**
f (a*t)	**a < 0, flipped y-axis**	
f(at+b)	**f(a(t + (b/a))**	
f(- t - b)	**f (- (t + b)**	**b > 0, left**
f(-t + b)	**f (- (t - b)**	**b > 0, right**

Energy content

$$W \ = \ \int_{-\infty}^{\infty}|f(t)|^2 dt \ = \ \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega$$

$|F(\omega)|^2$ is called the *energy spectrum* of $f(t)$

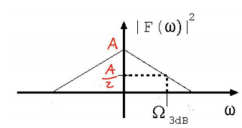
Low Pass Signal - Mar 30

3dB bandwidth (Ω_{3dB}): smallest $\omega > 0$ such that

$$10\log_{10}\left(\frac{|F(\omega)|^2}{|F(0)|^2}\right) = -3$$

or equivalently

$$\frac{|F(\omega)|^2}{|F(0)|^2} = \frac{1}{2}$$

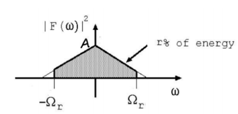


r% bandwidth: $\Omega_r > 0$ such that

$$\frac{1}{2\pi}\int_{-\Omega_r}^{\Omega_r}|F(\omega)|^2 d\omega = \frac{r}{100}W$$

or

$$\frac{1}{2\pi}\int_{0}^{\Omega_r}|F(\omega)|^2 d\omega = \frac{r}{100}\frac{W}{2}$$

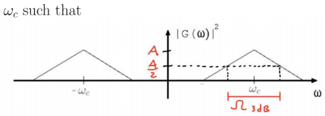


BandPass Signals - Mar 31

3dB bandwidth (Ω_{3dB}):

- First find smallest $\omega > \omega_c$ such that

$$\frac{|F(\omega)|^2}{|F(\omega_c)|^2} = \frac{1}{2}$$

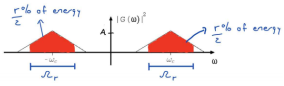


- Then $\Omega_{3dB} = 2\omega$

r% bandwidth (Ω_r):

- First find smallest $\hat{\omega}$ such that

$$\frac{1}{2\pi}\int_{\omega_c-\hat{\omega}}^{\omega_c+\hat{\omega}} |F(\omega)|^2 d\omega = \frac{r}{100}\frac{W}{2}$$



- Then $\Omega_r = 2\hat{\omega}$

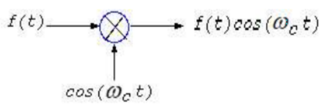
Frequency Shift

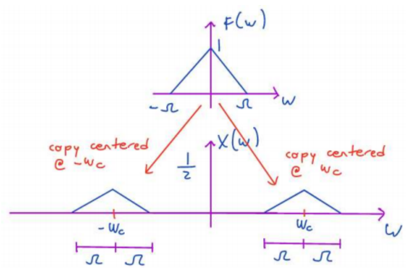
$$f(t) \leftrightarrow F(\omega) \quad \Rightarrow \quad f(t)e^{j\omega_c t} \leftrightarrow F(\omega - \omega_c)$$

Chapter 8

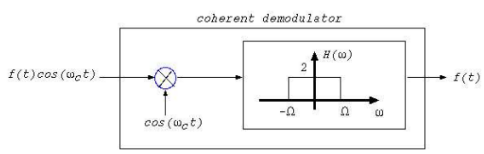
Modulation- Apr 2

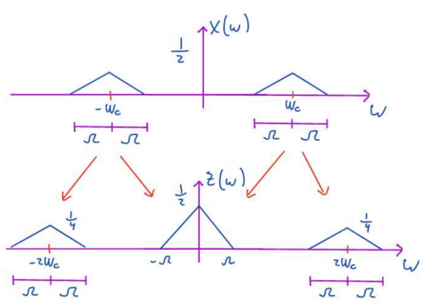
$$x(t) = f(t)\cos(\omega_c t) \leftrightarrow X(\omega) = \frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$$



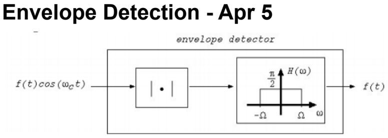


Demodulation





Envelope Detection - Apr 5



Output of full-wave rectifier, assuming $f(t) \geq 0$, is:

$$f(t)|\cos(\omega_c t)| = \frac{2}{\pi}f(t) + \sum_{n=1}^{\infty} c_n f(t)\cos(n2\omega_c t + \theta_n)$$

for some constants $c_n,\ \theta_n$.

If $f(t)$ is both negative and positive, then need to add a D.C. component, α, large enough so that $f(t) + \alpha \geq 0$

Superheterodyne AM Receiver with Envelope Detection

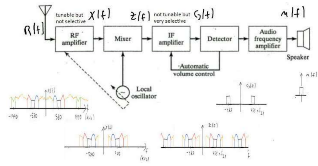
Envelope detection only works if signal is isolated

Will do in three steps:

* Pre-selector filter at f_c to remove part of the other signals. Not sharp but tunable.
  - Mainly want to remove the image station, which would otherwise land on top of our desired signal. The frequency of the image station is

$$f_{IM} = 2f_{IF} + f_c$$

  - This allows for a filter of bandwidth $\Omega < 2f_{IF} \approx$ 1MHz, compared to the 10kHz of a single signal
* Local oscillator mixer at f_{LO} to heterodyne (move) signal to f_{IF}
  - Use $f_{LO} = f_c + f_{IF}$ instead of $f_{LO} = f_c - f_{IF}$ for easier and cheaper implementation
* Intermediate frequency filter at f_{IF} to isolate signal. Very sharp but not tunable.



Chapter 9

Convolution - Apr 6

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = f(t) * h(t)$$

To convolve $f(t)$ and $h(t)$:

* Change the parameter of $f(t)$ and $h(t)$ from t to τ to get $f(\tau)$ and $h(\tau)$:

$$f(t) \to f(\tau) \qquad h(t) \to h(\tau)$$

* Time-reverse $h(\tau)$ and then shift it to the right by t (left if $t < 0$) to get $h(t-\tau)$:

$$h(\tau) \to h(t-\tau) = h(-(\tau - t))$$

* Multiply $f(\tau)$ and $h(t-\tau)$ for each τ:

$$f(\tau)h(t-\tau)$$

* Integrate the resulting product for all time τ:

$$\int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$$

Prop of Convolution: Commutative - Apr 7

$$y(t) = f(t) * h(t) = h(t) * f(t)$$

$$y(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)f(t-\tau)d\tau$$

Prop of Convolution: Time Shift

If one of the convoluted functions are time shifted, the output shifts by the same amount

$$f(t-t_0) * h(t) = y(t-t_0) = f(t) * h(t-t_0)$$

If both of the functions are time shifted, then the output shifts by the sum of the individual shifts

$$f(t-t_0) * h(t-t_1) = y(t-t_0-t_1)$$

Prop of Convolution: Distributive

$$f(t) * [g(t) + h(t)] = f(t) * g(t) + f(t) * h(t)$$

Prop of Convolution: Start Point

if $f(t) = 0$ for $t < t_{s,f}$ and $h(t) = 0$ for $t < t_{s,h}$, then

$$y(t) = f(t) * h(t) = 0 \text{ for } t < t_{s,f} + t_{s,h}$$

Prop of Convolution: End Point

if $f(t) = 0$ for $t > t_{e,f}$ and $h(t) = 0$ for $t > t_{e,h}$, then

$$y(t) = f(t) * h(t) = 0 \text{ for } t > t_{e,f} + t_{e,h}$$

Prop of Convolution: Width

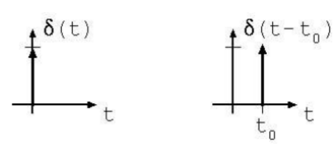
if $f(t)$ has width $T_f = t_{e,f} - t_{s,f}$ and $h(t)$ has width $T_h = t_{e,h} - t_{s,h}$, then

$$y(t) = f(t) * h(t) = 0 \text{ has width } T_y = T_f + T_h$$

Impulse - Apr 12

$$f(t) * \delta(t) = f(t)$$

$$f(t) * \delta(t-t_0) = f(t-t_0)$$



Prop of Impulse: Energy

$$W_{\delta(t)} = \infty$$

Prop of Impulse: Symmetry

$$\delta(t) = \delta(-t)$$

Prop of Impulse: Sifting

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$

Integrating against an impulse: = f(t_0) if impulse is in boundary of integration

$$\int_a^b f(t)\delta(t-t_0)dt = \begin{cases} f(t_0) & a < t_0 < b \\ 0 & else. \end{cases}$$

Prop of Impulse: Area

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

Prop of Impulse: Sampling

$$f(t)\delta(t) = f(0)\delta(t)$$

Or... $f(t)\partial(t - t_0) = f(t_0)\partial(t - t_0)$

Prop of Impulse: Time Scaling

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Prop of Impulse: Definite Integral Apr 14

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t) \quad \text{unit-step}$$

Prop of Impulse: Unit Step Derivative

$$\frac{d}{dt}u(t) = \delta(t)$$

RALPH BALITA NOTES(RALPHSCALVES)

Prop of Impulse: Fourier Transform

$$\mathcal{F}\{\delta(t)\} = 1$$

Prop of Impulse: Inverse Fourier

$$\mathcal{F}^{-1}\{\delta(\omega)\} = \frac{1}{2\pi}$$

Prop of Impulse: Doublet

$$\delta'(t) = \frac{d}{dt}\delta(t) \Rightarrow f(t) * \delta'(t) = f'(t)$$

Prop of Impulse: Impulse Response

$$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y_{ZS}(t) = \delta(t) * h(t) = h(t)$$

Prop of Impulse: Unit Step Response

$$u(t) \longrightarrow \boxed{h(t)} \longrightarrow y_{ZS}(t) = u(t) * h(t)$$

By properties of convolution

$$h(t) = \frac{d}{dt}y_{ZS}(t) \quad \text{only if } y_{ZS}(t) \text{ is the unit-step response}$$

Chapter 10

BIBO Stability- Apr 19

In a bounded input-bounded output (BIBO) stable system

$$\text{bounded input } f(t) \longrightarrow \boxed{\text{system}} \longrightarrow \text{bounded output } y(t)$$

for **all** bounded $f(t)$

Bounded $f(t)$ means $|f(t)| \leq C$ for some real-valued constant C and all t

BIBO may apply to any type of system, but if the system is LTI with impulse response $h(t)$, then

$$\text{BIBO} \leftrightarrow h(t) \text{ is absolutely integrable} : \int_{-\infty}^{\infty} |h(t)|dt < \infty$$

Boundedness of $h(t)$ does not imply BIBO, only absolute integrability does

System is BIBO Stable if

1) **assuming f(t) is bounded, y(t) is bounded**
2) **h(t) is absolutely integratable from $-\infty, \infty$**

Causality - Apr 20

$$\text{system is causal} \longleftrightarrow h(t) = 0 \text{ for } t < 0$$

System is Causal if

1) **Output y(t) of system does not depend on future of input f(t)**
2) **For LTI, impulse response h(t) has h(t)=0 for t < 0**

3) **For Signal, f(t) is casual if f(t) has f(t) = 0 for t < 0**

Chapter 11

LaplaceTransforms- Apr 21

$$\hat{F}(s)=\int_{0^-}^{\infty} f(t)e^{-st}dt$$

$s=\sigma+j\omega$, is a complex number, and so is $\hat{F}(s)$

Prop of Laplace: ROC: Region of Convergence, ROC > s on complex plane

Region in the s-plane to the right of the rightmost pole .

• Determine the Laplace transform of $f(t)=e^t u(t)$

f(t) not A.I.

$F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty e^t e^{-st}dt=\int_0^\infty e^{t(1-s)}dt$

$=\frac{e^{t(1-s)}}{1-s}\Big|_0^\infty$

$e^{t(1-s)}=e^{t(1-(\sigma+j\omega))}=e^{t(1-\sigma)}e^{-j\omega t}$, $\to 0$ if $1-\sigma<0$

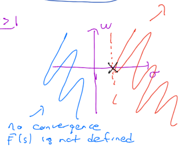
region of convergence

$F(s)=\frac{0-e^0}{1-s}=\frac{1}{s-1}$ only if $\sigma>1$

Region of convergence (ROC) is $\{s\in\mathbb{C}: s=\sigma+j\omega,\ \sigma>1\}$

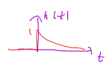
Notice $F(s)\in\mathbb{C}$

s=1 is a pole of F(s) because $|F(s)|\to\infty$ as $s\to 1$



no convergence F(s) is not defined

• Determine the Laplace transform of $h(t)=e^{-t}u(t)$



h(t) is A.I. $\to \hat{h}(\omega)$ exists

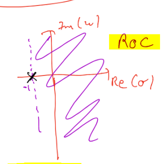
$H(s)=\int_0^\infty h(t)e^{-st}dt=\int_0^\infty e^{-t}e^{-st}dt=\int_0^\infty e^{-t(1+s)}dt$

$=\frac{e^{-t(1+s)}}{-(1+s)}\Big|_0^\infty=\frac{0-1}{-(1+s)}=\frac{1}{s+1}$ if $\sigma>-1$

$e^{-t(1+\sigma)}e^{-j\omega t}$ need $\to 0$ $\to$ ($\sigma>-1$)

ROC is $\sigma>-1$

pole @ $s=-1$



if $\sigma=0$ (Im axis) is inside ROC then $H(\omega)=\hat{H}(j\omega)$

Prop of Laplace: Pole

location where $|\hat{F}(s)|\to\infty$

Impulse Response of an LTIC System:

$\hat{H}(s)=\int_{0^-}^{\infty} h(t)e^{-st}dt$ is the *transfer function* of the system

$f(t)\longrightarrow$ LTIC with $h(t)\leftrightarrow\hat{H}(s)$ $\longrightarrow y_{ZS}(t)=f(t)*h(t)$

$\hat{F}(s)\longrightarrow$ LTIC with $h(t)\leftrightarrow\hat{H}(s)$ $\longrightarrow \hat{Y}_{ZS}(s)=\hat{F}(s)\hat{H}(s)$

$\Rightarrow \hat{H}(s)=\frac{\hat{Y}_{ZS}(s)}{\hat{F}(s)}$

$$e^{st}\longrightarrow\boxed{\hat{H}(s)}\longrightarrow y(t)=e^{st}\hat{H}(s)$$

Prop of Laplace: Hidden Poles

Hidden poles are poles at $\pm\infty$

Prop of Laplace: BIBO Stability

LTIC with impulse response h(t) is BIBO iff

$\hat{H}(s)$ has all of its poles on the left-half plane,

(after zero-pole cancellation)

Prop of Laplace: Time Shift

if $f(t)$ is causal and $t_0\geq 0$

$$g(t)=f(t-t_0)\overset{\mathcal{L}}{\longleftrightarrow}\hat{G}(s)=\hat{F}(s)e^{-st_0}$$

Prop of Laplace: Time Derivative

$$g(t)=\frac{d}{dt}f(t)\overset{\mathcal{L}}{\longrightarrow}\hat{G}(s)=s\hat{F}(s)-f(0^-)$$

Prop of Laplace: General Time Derivative Apr 28

$$x(t)=\frac{d^n}{dt^n}f(t)\overset{\mathcal{L}}{\longrightarrow}$$

$$\overset{\mathcal{L}}{\longrightarrow}\hat{X}(s)=s^n\hat{F}(s)-s^{n-1}f(0^-)-s^{n-2}f'(0^-)-\cdots-f^{(n-1)}(0^-)$$

Prop of Laplace: Common Laplace Transforms

$\delta(t)\leftrightarrow 1$
$e^{pt}u(t)\leftrightarrow\frac{1}{s-p}$
$te^{pt}u(t)\leftrightarrow\frac{1}{(s-p)^2}$
$t^n e^{pt}u(t)\leftrightarrow\frac{n!}{(s-p)^{n+1}}$

Prop of Laplace: Partial Fraction (distinct poles)

HEAVISIDE METHOD
THUMB RULE

$$=\frac{A_1}{s-p_1}+\frac{A_2}{s-p_2}+\cdots+\frac{A_n}{s-p_n},$$

$$A_k=\left[\hat{F}(s)(s-p_k)\right]\Big|_{s-p_k}$$

Prop of Laplace: Partial Fraction (repeated poles)

HEAVISIDE METHOD
THUMB RULE

$$=\frac{N(s)}{(s-p_1)^n}=\frac{A_1}{s-p_1}+\frac{A_2}{(s-p_1)^2}+\cdots+\frac{A_n}{(s-p_1)^n}$$

$$A_{n-m}=\frac{1}{m!}\left[\frac{d^m}{ds^m}\left(\hat{F}(s)(s-p_1)^n\right)\right]\Big|_{s=p_1}$$

RALPH BALITA NOTES(RALPHSCALVES)

Common Denominator: Characteristic Polynomial Apr 30

$$s^n+a_1s^{n-1}+\cdots+a_n$$

Roots: Characteristic Poles

$$p_1,\ p_2,\ \ldots,\ p_n$$

these are before any cancellation from possible zeros.

Characteristic Modes

$$e^{p_1t},\ e^{p_2t},\ \ldots,\ e^{p_nt}$$

Multiplicity of Characteristic Modes

$$e^{p_rt},\ te^{p_rt},\ \ldots,\ t^{m-1}e^{p_rt}$$

Asymptotically Stable (it is dissipative) May 3

$$\lim_{t\to\infty} y_{ZI}(t)=0$$

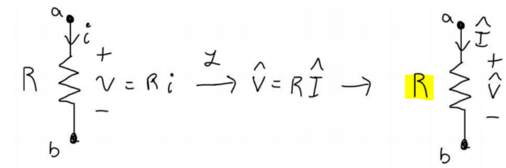
System is asymptotically stable iff its Characteristic Poles (before pole-zero cancellation) are on the left half of the plane

Marginally Stable

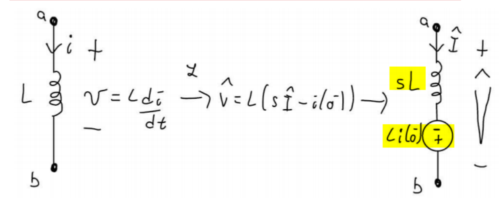
$$\lim_{t\to\infty} y_{ZI}(t)\neq 0\quad\text{but}\quad |y_{ZI}(t)|\leq C\quad\text{for some } C$$

System is Marginally Stable iff it has bounded non-transient zero-input response

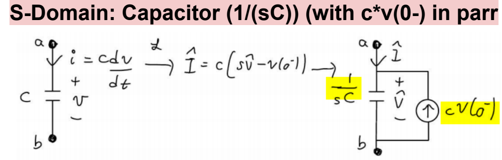
S-Domain: Resistor (R)



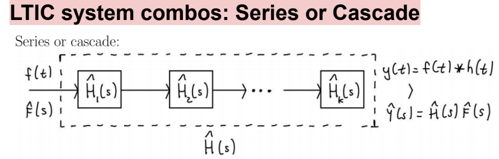
S-Domain: Inductor (sL)(with L*i(0-) in series



S-Domain: Capacitor (1/(sC)) (with c*v(0-) in parr

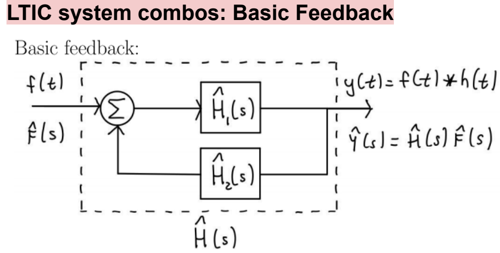


LTIC system combos: Series or Cascade



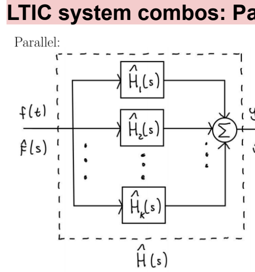
$\hat{H}(s)=\hat{H}_1(s)\hat{H}_2(s)\ldots\hat{H}_k(s)\quad\longleftrightarrow\quad h(t)=h_1(t)*h_2(t)*\cdots*h_k(t)$

LTIC system combos: Basic Feedback



$$\hat{H}(s)=\frac{\hat{H}_1(s)}{1-\hat{H}_1(s)\hat{H}_2(s)}$$

LTIC system combos: Parallel



$\hat{H}(s)=\hat{H}_1(s)+\hat{H}_2(s)+\cdots+\hat{H}_k(s)\quad\longleftrightarrow\quad h(t)=h_1(t)+h_2(t)+\cdots+h_k(t)$