Baby Food

$\int_a^b c\,dx = c(b-a)$	$\int_a^b x\,dx = \frac{b^2}{2} - \frac{a^2}{2}$	$\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$

Fundamental Theorems of Baby Food

Fund.Theorem.Calc.1	**Fund.Theorem.Calc.2**
$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$	$\int_a^b f(x)\,dx = F(b) - F(a)$

Integrals of Trig

$\int \sin(x)\,dx = -\cos(x) + C$	$\int \cos(x)\,dx = \sin(x) + C$
$\int \tan(x)dx = \ln\lvert\sec(x)\rvert + C$	$\int \cot(x)\,dx = \ln\lvert\sin(x)\rvert + C$
$\int \sec^2(x)\,dx = \tan(x) + C$	$\int \csc^2(x)dx = -\cot(x) + C$
$\int \sec(x)\,dx = \ln\lvert\sec(x) + \tan(x)\rvert$	$\int \csc(x)\,dx = -\ln\lvert\csc(x) + \cot(x)\rvert$
$\int \sec(x)\tan(x)\,dx = \sec(x) + C$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

Integrals of Inverse Trig

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}(\frac{x}{a}) + C$	$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$$

Area Under Curve

Top-to-bottom $A(x) = \int_a^b [\,f(x) - g(x)\,]dx$	**Right-to-left** $A(y) = \int_c^d [\,f(y) - g(y)\,]dy$
Vol. by Rotation (x-axis) $V = \int_a^b \pi\,[\,R(x)\,]^2 dx$	**Vol. by Rotation (y-axis)** $V = \int_c^d \pi\,[\,R(y)\,]^2 dy$
Vol. by Washer Rotation ***Outer-inner (X-axis)*** $V(x) = \int_a^b \pi\,[\,R(x)\,]^2 - [\,r(x)\,]^2 dx$	**Vol. by Washer Rotation** ***Outer-inner (Y-Axis)*** $V(y) = \int_a^b \pi\,[\,R(y)\,]^2 - [\,r(y)\,]^2 dy$
Shell Method ***About Y-Axis*** $V = \int_a^b 2\pi\,[Radius][Height]\,dx$ $V = \int_a^b 2\pi\,[x][f(x)]\,dx$	**Shell Method** ***About X-Axis*** $V = \int_a^b 2\pi\,[Radius][Height]\,dx$ $V = \int_c^d 2\pi\,[y][f(y)]\,dx$
Arc Length $L(x) = \int_a^b \sqrt{1 + [f'(x)]^2}dx$	**Arc Length** $L(y) = \int_c^d \sqrt{1 + [g'(y)]^2}dy$
Surface Area ***Around X-Axis*** $S(x) = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}dx$	**Surface Area** ***Around Y-Axis*** $S(y) = \int_a^b 2\pi f(y)\sqrt{1 + [g'(y)]^2}dy$

Physics Applications

Work Done Along a Line $W = \int_a^b F(x)dx$ $[W = force \times (x),\ given\ x = distance]$	**Hooke's Law for Springs** $W = \int_a^b kx\,dx$ $a \to b = \Delta length(spring)$
Lifting Objects $W = \int_a^b weight \cdot (height - x)\,dx$	**Pumping Liquid** $W = \int_a^b weight \cdot (height - x) \cdot \Delta Volume\,dx$
Fluid Force on Constant Surface $F = weight \cdot height \cdot area$	**Fluid Force on a Vert Plate** $W = \int_a^b weight \cdot (strip\ depth) \cdot (length\ of\ strip)\,dx$
Center of Mass $= \frac{moment}{mass}$ $\bar{x} = \frac{\int_a^b (weight) \times (x) \times [f(x) - g(x)]\,dx}{\int_a^b [f(x) - g(x)]\,dx}$	**Center of Mass** $= \frac{moment}{mass}$ $\bar{y} = \frac{\int_a^b (weight) \times (\frac{1}{2}) \times [\,f^2(x) - g^2(x)\,]\,dx}{\int_a^b [\,f(x) - g(x)\,]\,dx}$

ln() properties

$\ln(b \cdot a) = \ln(b) + \ln(a)$	$\ln(\frac{b}{a}) = \ln(b) - \ln(a)$
$\ln(\frac{1}{x}) = -\ln(x)$	$\ln(x^r) = r \times \ln(x)$
$\frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$	$\int \frac{1}{u}du = \ln\lvert u\rvert + C$
$\int \frac{f'(x)}{f(x)}du = \ln\lvert f(x)\rvert + C$	$\int_1^x \frac{1}{t}dt = \ln(x)$

$$\int_a^b \ln(x)dx = USE\ Integration\ by\ Parts$$

More e and ln() properties

$\frac{d}{dx}e^u = e^u\frac{du}{dx}$	$e^{\ln(x)} = x$	$\ln(e^x) = x$
$\int e^u\,du = e^u + C$	$e^{-x} = \frac{1}{e^x}$	**a ralph b production**
$e^{x_1} \cdot e^{x_2} = e^{x_1 + x_2}$	$e^{x_1}/e^{x_2} = e^{x_1 - x_2}$	$(e^{x_1})^{x_2} = e^{x_1 \cdot x_2}$
$a^x = e^{x \cdot \ln(a)}$	$\frac{d}{dx}a^u = a^u\ln(a)(\frac{du}{dx})$	$\int a^u\,du = \frac{a^u}{\ln(a)} + C$

log() properties

$\log_a(b \times x) = \log_a(b) + \log_a(x)$	$\log_a(\frac{b}{x}) = \log_a(b) - \log_a(x)$
$\log_a(\frac{1}{x}) = -\log_a(x)$	$\log_a(x^r) = r \times \log_a(x)$

$$\frac{d}{dx}\log_a u = \frac{1}{\ln(a)}\frac{1}{u}\frac{du}{dx} = \frac{1}{u \times \ln(a)}\frac{du}{dx}$$

Hyperbolic properties

$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$	$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$ $\sinh(2x) = 2\sinh(x)\cosh(x)$ $\cosh^2(x) - \sinh^2(x) = 1$
$\cosh^2(x) = \frac{\cosh(2x) + 1}{2}$ $\sinh^2(x) = \frac{\cosh(2x) - 1}{2}$	$\tanh^2(x) = 1 - \mathrm{sech}^2(x)$ $\coth^2(x) = 1 + \mathrm{csch}^2(x)$	**Good luck on your exam!**
$\frac{d}{dx}\sinh(u) = \cosh(u)\frac{du}{dx}$ $\frac{d}{dx}\cosh(u) = \sinh(u)\frac{du}{dx}$	$\frac{d}{dx}\tanh(u) = \mathrm{sech}^2(u)\frac{du}{dx}$ $\frac{d}{dx}\coth(u) = -\mathrm{csch}^2(u)\frac{du}{dx}$	$\frac{d}{dx}\mathrm{sech}(u) = -\mathrm{sech}(u)\tanh(u)\frac{du}{dx}$ $\frac{d}{dx}\mathrm{csch}(u) = -\mathrm{csch}(u)\coth(u)\frac{du}{dx}$

Message from creator: *if you can memorize this sheet by heart, i can 1000% guarantee you will do fine in Calculus I. good luck. The name mother-calculator came from the person I learned from the most (my mom) and the thing I learned from the least (using my calculator).*

Integrals of Hyperbolic properties

$\int \sinh(u) = \cosh(u) + C$	$\int \cosh(u) = \sinh(u) + C$
$\int \tanh(u) = \ln[\cosh(u)] + C$	$\int \coth(u) = \ln[\sinh(u)] + C$
$\int \mathrm{csch}(u) = \ln[\tanh(\frac{u}{2})] + C$	$\int \sec(u) = \tan^{-1}[\sinh(u)] + C$
$\int \mathrm{csch}^2(u) = -\coth(u) + C$	$\int \mathrm{sech}^2(u) = \tanh(u) + C$
$\int \mathrm{csch}(u)\coth(u) = -\coth(u) + C$	$\int \mathrm{sech}(u)\tanh(u) = -\mathrm{sech}(u) + C$
$\mathrm{csch}^{-1}(x) = \sinh^{-1}(\frac{1}{x})$	$\mathrm{sech}^{-1}(x) = \cosh^{-1}(\frac{1}{x})$

$$\coth^{-1}(x) = \tanh^{-1}(\frac{1}{x})$$

Derivatives of Hyperbolic properties

$\frac{d}{dx}\sinh^{-1}(u) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$	$\frac{d}{dx}\cosh^{-1}(u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$
$\frac{d}{dx}\tanh^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$	$\frac{d}{dx}\coth^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$
$\frac{d}{dx}\mathrm{csch}^{-1}(u) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}$	$\frac{d}{dx}\mathrm{sech}^{-1}(u) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$

Integrals of Inverse Hyperbolic properties

$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}(\frac{u}{a}) + C$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}(\frac{u}{a}) + C$
$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + C$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\coth^{-1}(\frac{u}{a}) + C$
$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}\mathrm{csch}^{-1}\left|\frac{u}{a}\right| + C$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\mathrm{sech}^{-1}(\frac{u}{a}) + C$

Integration by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int u \times v' = uv - \int v \times du$$

$$\int x^2e^x\,dx.$$

With $f(x) = x^2$ and $g(x) = e^x$, we list:

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$	e^x
$2x$	$(-)$	e^x
2	$(+)$	e^x
0		e^x

Integrating Powers of cos() and sin()

Odd Powers of Cos(x) and Sin(x)

$$\sin^2(x) + \cos^2(x) = 1$$

EX. $\int \cos^5(x)dx = \int \cos^4(x)\cos(x)dx = \int [\cos^2(x)]^2\cos(x)dx$

Even Powers of Cos(x) and Sin(x)

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

EX. $\int \sin^4(x)dx = \int [\sin^2(x)]^2\,dx = \int [\frac{1}{2} - \frac{1}{2}\cos(2x)]^2\,dx$

More Integration Trig Properties

When integrating EVEN powers of sin() and cos(), use these

$cos^2(\theta) = (\frac{1}{2})[1 + cos(2\theta)]$ **OR** $sin^2(\theta) = (\frac{1}{2})[1 - cos(2\theta)]$

When integrating ODD powers of sin() and cos(), use these

$sin^2(x) + cos^2(x) = 1$

More Trig properties to remember

$sin^2(x) + cos^2(x) = 1$	$tan^2(x) + 1 = sec^2(x)$	$cot^2(x) + 1 = csc^2(x)$

$cos^2(x) = (\frac{1}{2})[1 + cos(2x)]$

$sin^2(x) = (\frac{1}{2})[1 - cos(2x)]$

$1 + cos(x) = 1 + cos(2 \times \frac{x}{2}) = 2cos^2(\frac{x}{2})$

$cos^2(2x) = \frac{1}{2}[1 + cos(4x)]$

$sinh^{-1}(\frac{x}{a}) = ln\,|\,\frac{\sqrt{a^2+x^2}}{a} + \frac{x}{a}$

For angle representation, convert the following

$\sqrt{a^2+x^2}$ let $x = a \cdot tan(\theta)$

$\sqrt{a^2-x^2}$ let $x = a \cdot sin(\theta)$

$\sqrt{x^2-a^2}$ let $x = a \cdot sec(\theta)$

& use trig identities to solve integral

$$sin(ax) \times sin(bx)dx = \frac{1}{2}(cos[(a-b)x] - cos[(a+b)x])$$
$$sin(ax) \times cos(bx)dx = \frac{1}{2}(sin[(a-b)x] + sin[(a+b)x])$$
$$sin(ax) \times sin(bx)dx = \frac{1}{2}(cos[(a-b)x] - cos[(a+b)x])$$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

Even/Odd Indentities

$\sin(-\theta) = -\sin\theta$ $\quad\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$ $\quad\cot(-\theta) = -\cot\theta$

$\csc(-\theta) = -\csc\theta$ $\quad\sec(-\theta) = \sec\theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\quad\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\frac{\pi}{2}$ radians = 90°

Sum/Difference Indentities

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$

Double Angle Identities

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half Angle Indentities

$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$

$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$

$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

Sum to Product of Two Angles

$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

Product to Sum of Two Angles

$\sin\theta\sin\phi = \frac{[\cos(\theta-\phi) - \cos(\theta+\phi)]}{2}$

$\cos\theta\cos\phi = \frac{[\cos(\theta-\phi) + \cos(\theta+\phi)]}{2}$

$\sin\theta\cos\phi = \frac{[\sin(\theta+\phi) + \sin(\theta-\phi)]}{2}$

$\cos\theta\sin\phi = \frac{[\sin(\theta+\phi) + \sin(\theta-\phi)]}{2}$

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Welcome to Calculus III

Distance Between $P_1(x_1, y_1, z_1)$ **&** $P_2(x_2, y_2, z_2)$

$$\left|P_1P_2\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Standard Equation for a Sphere

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

Length / Magnitude of Vector v

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Direction of Vector

$$\frac{v}{|v|} = \frac{v_1 i + v_2 j + v_3 k}{\sqrt{v_1^2 + v_2^2 + v_3^2}}$$

Math of Vectors

u + v = $< u_1 + v_1\, , u_2 + v_3, u_3 + v_3 >$

u * v = $u_1v_1 + u_2v_2 + u_3v_3$

u * v = $|u| * |v|\, cos\theta$

Proj of v onto u

$$proj_U V = \frac{U \cdot V}{|U|^2} < U >$$

$$\text{proj}_{\mathbf{v}}\,\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}.$$

Applications of Cross Product

Cross Product

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}.$$

Triple Scalar Product

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

A RALPH B PRODUCTION

Area of Parallelogram:
Magnitude (length) of the Cross Product of 2 vectors

Area of Triangle:
Half the area of a parallelogram

Vector Perpendicular to plane($\hat{N}$ **):**
coefficients of the plane. $< N_1, N_2, N_3 >$

Parametrization of a line:

$$\overline{P_0P} = t\,\overline{V}$$
$$< x - P_1, y - P_2, z - P_3 > \; = T < V_1, V_2, V_3 >$$

Distance from Line to Point

$$d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|}$$

Distance from Plane to Point

$$d = \left|\overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Angles between planes:
the angle between their normal vectors

Vector velocity and acceleration:
derivatives of position vector

Stinky Trig properties

$\lim_{x\to 0} \frac{sin(x)}{x} = 1$	$\lim_{x\to 0} \frac{1 - cos(x)}{x} = 0$

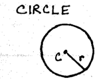
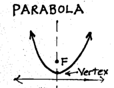
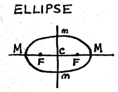
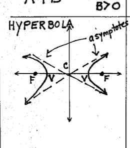
CIRCLE 	Equation $(x-h)^2 + (y-k)^2 = r^2$ Conic $Ax^2+By^2+Cx+Dy+F=0$ $A=B$ (neither is zero) Center = (h,k) radius = r	
PARABOLA 	Equation $y = \frac{1}{4p}(x-h)^2 + k$ Vertex: (h,k) Focus: (h, k+p) Directrix: y = k-p Conic A=0 or B=0	VERTICAL Equation $x = \frac{1}{4p}(y-k)^2 + h$ Vertex (h,k) Focus: (h+p, k) Directrix: x = h-p
ELLIPSE  A≠B but A>0 B>0	Equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ center: (h,k) Foci: $(h+\sqrt{a^2-b^2}, k)$ $(h-\sqrt{a^2-b^2}, k)$ Major extrema: (h+a, k) (h-a, k) minor extrema: (h, k+b) (h, k-b)	same, but b>a Center: (h,k) Foci: $(h, k+\sqrt{b^2-a^2})$ $(h, k-\sqrt{b^2-a^2})$ Major: (h, k+b) (h, k-b) minor: (h+a, k) (h-a, k)
HYPERBOLA 	Equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ center: (h,k) vertices: (h+a, k) (h-a, k) Foci: $(h+\sqrt{a^2+b^2}, k)$ $(h-\sqrt{a^2+b^2}, k)$ Asymptotes: $y-k = \pm\frac{b}{a}(x-h)$	$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2}$ Center: (h,k) Vertices: (h, k+b) (h, k-b) Foci: $(h, k+\sqrt{a^2+b^2})$ $(h, k-\sqrt{a^2+b^2})$ Asymptotes: $y-k = \pm\frac{a}{b}(x-h)$

Photo credits:
https://www.cccti.edu/asc/Documents/ConicCheatSheet.pdf

Arc Length

Arc Length: $L = s = \int_a^b |v| dt$ **or** $ds = |v|\, dt$

Message from creator: At this point in Calc, we dive into complex problems that require proofs. Never be afraid to ask your professor for proofs in maths. Also, do not memorize these. If you understand how these work, then you will be golden. I did not include U-sub or Integration by parts, since these are "METHODS", not things you memorize. Know "HOW" they work, and use this sheet as a helper.

Applications of Derivatives and proofs

$$T_{Unit\,Tangent\,Vector} = \frac{v}{|v|} = \frac{dr}{ds} = \frac{dr}{dt} \cdot \frac{dt}{ds} = v \cdot \frac{1}{|v|}$$

$$\varkappa = \frac{|v \times a|}{|v|^3}$$

$$\varkappa_{Curvature} = \left|\frac{dT}{ds}\right| = \left|\frac{dT}{dt} \cdot \frac{dt}{ds}\right| = \left|\frac{dT}{dt}\right| \cdot \frac{1}{|v|}$$

$$N_{Principal\,Unit\,Vector} = \frac{1}{\varkappa}\frac{dT}{ds} = \frac{\frac{dT}{ds}}{\left|\frac{dT}{ds}\right|} = \frac{\frac{dT}{dt} \cdot \frac{dt}{ds}}{\left|\frac{dT}{dt} \cdot \frac{dt}{ds}\right|} = \frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$$

$$v = \frac{dr}{dt} = \frac{dr}{ds}\frac{ds}{dt} = T\frac{ds}{dt}$$

$$a = a_T T + a_N N$$
$$a_T = \frac{d^2s}{dt^2} = \frac{d}{dt}|v|$$
$$a_N = \varkappa\left(\frac{ds}{dt}\right)^2 = \varkappa|v|^2$$
$$a_N = \sqrt{|a|^2 - a_T^{\,2}}$$

$$a = \frac{d}{dt}v$$
$$= \frac{d}{dt}\left(T\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}T + \frac{ds}{dt}\frac{dT}{dt}$$
$$= \frac{d^2s}{dt^2}T + \frac{ds}{dt}\left(\frac{dT}{ds} \cdot \frac{ds}{dt}\right)$$
$$= \frac{d^2s}{dt^2}T + \varkappa N\left(\frac{ds}{dt}\right)^2$$

$$B = T \times N \qquad B = \frac{r' \times r''}{|r' \times r''|}$$

$$\frac{dB}{ds} = \frac{d}{ds}(T \times N) = \; -\frac{dB}{ds} \cdot N$$

$$\tau_{Torsion} = \qquad \tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}. \qquad \tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\mathbf{v} \times \mathbf{a}|^2}$$

Some stuff

Chain Rule f(x,y): $\frac{\delta w}{\delta t} = \frac{\delta w}{\delta x}\frac{dx}{dt} + \frac{\delta w}{\delta y}\frac{dy}{dt}$

Chain Rule f(x,y,z): $\frac{\delta w}{\delta t} = \frac{\delta w}{\delta x}\frac{dx}{dt} + \frac{\delta w}{\delta y}\frac{dy}{dt} + \frac{\delta w}{\delta z}\frac{dz}{dt}$

Chain Rule f(x,y,z):

$x = g(r,s),\; y = h(r,s),\; z = k(r,s)$

$$\frac{\delta w}{\delta r} = \frac{\delta w}{\delta x}\frac{dx}{dr} + \frac{\delta w}{\delta y}\frac{dy}{dr} + \frac{\delta w}{\delta z}\frac{dz}{dr}$$
$$\frac{\delta w}{\delta s} = \frac{\delta w}{\delta x}\frac{dx}{ds} + \frac{\delta w}{\delta y}\frac{dy}{ds} + \frac{\delta w}{\delta z}\frac{dz}{ds}$$

Implicit Differentiation: $\frac{dy}{dx} = -\frac{F_x}{F_y}$

Gradient Vector: $grad(f) = \frac{\delta f}{\delta x}\, i + \frac{\delta f}{\delta y}\, j + \frac{\delta f}{\delta z}\, k$

Directional Derivative: $D_{\bar{u}} f(x,y,z) = [grad(f)]_{P_0} \cdot \bar{u}$

Largest $D_{\bar{u}} f(x,y,z)$ **in direction of u:** $\frac{\Delta f\,(x_0, y_0)}{|\,\Delta f\,(x_0, y_0)\,|}$

Smallest $D_{\bar{u}} f(x,y,z)$ **in direction of u:** $-\frac{\Delta f\,(x_0, y_0)}{|\,\Delta f\,(x_0, y_0)\,|}$

Tangent Line to The Level Curve:

$$f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) = 0$$

Tangent Plane to Curve at point P

$$P_0(x_0,y_0,z_0) = f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

Normal Line to Curve at

$x = x_0 + f_x\, t. \qquad y = y_0 + f_y\, t \qquad z = z_0 + f_z\, t$

Estimating the Change in f in the Direction of U

Plane Tangent to a Surface $z = f(x, y)$ at $(x_0, y_0, f(x_0, y_0))$

The plane tangent to the surface $z = f(x, y)$ of a differentiable function f at the point $P_0(x_0, y_0, z_0) = (x_0, y_0, f(x_0, y_0))$ is

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0. \tag{3}$$

Linearization

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

First Derivative Test for Local Extreme Values

$f_x(a,b) = 0$ and $f_y(a,b) = 0$