**PARUL UNIVERSITY**
**Faculty of Engineering & Technology**
**Department of Applied Sciences and Humanities**
**1ST SEMESTER B.Tech PROGRAMME (CSE, IT)**
**CALCULUS(03019101BS01)**
**ACADEMIC YEAR – 2025-26**

## Assignment-04

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| 1. | Evaluate the following integrals:<br>(i) $\int_0^3\int_1^4 (2x+3y)dxdy$ (ii) $\int_1^4\int_2^5 (xy+3)dxdy$ (iii) $\int_0^1\int_1^3 (e^{x+y})dxdy$<br>(iv) $\int_0^3\int_1^4 (x+y^2)dxdy$ |
| 2. | Evaluate the following integrals:<br>(i) $\int_0^2\int_0^x xy^2\,dydx$ (ii) $\int_0^3\int_0^{\frac{x}{3}} (x^2-y^2)dydx$ (iii) $\int_0^2\int_0^{2-x} (x^2-y^2)dydx$<br>(iv) $\int_0^2\int_0^{2-x} (x^2+y^2)dydx$ |
| 3. | Evaluate the $\iint_R (x^2+y^2)dA$, where R is the region bounded by the line $y=2x$ and the parabola $y=x^2$. |
| 4. | Evaluate the $\iint_R (3x-y)dA$. where R is bounded by $x=y^2$ and the vertical line $x=4$ . |
| 5. | Evaluate the $\iint_R (x^2+y)dA$ , where R is bounded by $y=0$ and $y=\sqrt{x}$ for ,$0\le x\le 1$ . |
| 6. | Compute the double integral of the function $f(x,y)=6-x+2y$ over the region bounded by the curves $x=y^2$ and $y=2-x$ in the $x$-$y$ plane. |
| 7. | Evaluate $\iint_R (6x+2y^2)dA$, where $R$ is the region enclosed by the parabola $x=y^2$ and the line $x+y=2$. |
| 8. | Evaluate $\iint_R sin(y^2)dA$, where R is the region bounded by the lines $y=x, y=\pi, x=0$. |
| 9. | Change the order of integration of the following integrals:<br>(i) $\int_0^a\int_y^a f(x,y)dydx$ (ii) $\int_o^a\int_x^a f(x,y)dydx$<br>(iii) $\int_o^1\int_{y^2}^{y^{\frac{1}{3}}} f(x,y)dxdy$ (iv) $\int_o^1\int_x^{\sqrt{x}} f(x,y)dydx$ |
| 10. | Change the order of integration in $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$ and evaluate the same. |
| 11. | Change the order of integration in the following integral and evaluate $\int_0^{4a^2}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ |
| 12. | Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dydx$ . |
| 13. | Change the order of integration and evaluate $\int_0^a\int_x^a (x^2+y^2)\,dydx$ . |
| 14. | Change the order of integration and evaluate $\int_0^\pi\int_x^\pi \frac{\sin y}{y}dydx$ . |
| 15. | Change the order of integration and evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$ . |

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| 16. | Change the order of integration and evaluate $\int_0^{\pi}\int_x^{\pi}\frac{\cos y}{y}\,dydx$ . |
| 17. | Evaluate the following integrals:<br>(i) $\int_0^{\frac{\pi}{4}}\int_0^1 rdrd\theta$ (ii) $\int_0^{\pi}\int_0^{sin\theta} rdrd\theta$ (iii) $\int_0^{\frac{\pi}{4}}\int_0^1 rdrd\theta$ (iv) $\int_0^{2\pi}\int_0^{a} r^3drd\theta$<br>(v) $\int_0^{\frac{\pi}{2}}\int_0^{acos\theta} rsin\theta dr\,d\theta$ |
| 18. | If $u = x + y, v = x - y$, find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$. |
| 19. | If $u = x^2, v = y^2$, find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$. |
| 20. | If $x = (u + v)/2\ , y = (u - v)/2$, find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$. |
| 21. | Evaluate $\iint_R (y - x)dxdy$ over the region R in the xy-plane bounded by the straight lines $y = x - 3, y = x + 1, 3y + x = 5, 3y + x = 7$. |
| 22. | Evaluate $\iint_R \sqrt{x + y}dxdy$ where R in the parallelogram bounded by the lines $x + y = 0, x + y = 1, 2x - 3y = 0,\ 2x - 3y = 4$. |
| 23. | Evaluate $\iint_R r^3drd\theta$, over the area bounded between the circles $r = 2\cos\theta$ & $r = 4\cos\theta$. |
| 24. | Evaluate $\iint_R \frac{rdrd\theta}{\sqrt{a^2+r^2}}$ over one loop of lemniscate $r^2 = a^2\cos 2\theta$. |
| 25. | Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing into polar co-ordinates. |
| 26. | Evaluate $\iint_R\ (x^2 + y^2)\ dA$, where $R$ is the annular region between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$, by changing into polar coordinates. |
| 27. | Evaluate $\int_0^1\int_x^{\sqrt{2-x^2}}(x^2 + y^2)\ dydx$ by changing into polar co-ordinates. |
| 28. | Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. |
| 29. | Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\ dx\ dy$ over the first quadrant of the circle $x^2 + y^2 = 1$. |
| 30. | Find the volume of the region bounded by the surface $x = 0, y = 0, z = 0$ and $2x + 3y + z = 6$. |
| 31. | Find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 6$ and the paraboloid $z = x^2 + y^2$. |
| 32. | Evaluate the following integrals:<br>(i) $\int_0^{\frac{\pi}{2}}\int_0^1\int_0^r rzdzdrd\theta$ (ii) $\int_0^1\int_0^1\int_0^1 xyzdzdydx$ (iii) $\int_0^{2\pi}\int_0^2\int_0^1 rdzdrd\theta$<br>(iv) $\int_0^{2\pi}\int_0^2\int_0^3 r^3dzdrd\theta$ (v) $\int_0^1\int_0^2\int_0^e dydxdz$ |
| 33. | Evaluate the following integrals:<br>(i) $\int_0^1\int_0^{2-x}\int_0^{x-y} dzdydx$ (ii) $\int_0^1\int_0^{\sqrt{z}}\int_0^{2\pi}(r^2\theta + z^2)rd\theta drdz$<br>(iii) $\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} xyzdxdydz.$ |
| 34. | Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. |
| 35. | Use triple integration to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ between the planes $z = 1$ and $x + z = 1$. |
| 36. | Find the gradient of $\emptyset = 3x^2y - y^3z^2$ at the point $(1, -2, 1)$. |
| 37. | If $r(t) = cost\,\hat{\imath} + sint\hat{\jmath} + t\hat{k}$ , then find velocity $v(t)$ and acceleration $a(t)$ at $t = \frac{\pi}{2}$. |

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| 38. | A particle moves along a curve with position $r(t) = (2t+1)i + t^2 j - 3tk$. Find the velocity at $t = 3$. |
| 39. | Evaluate $\nabla e^{(r^2)}$, where $r^2 = x^2 + y^2 + z^2$. |
| 40. | Find a unit normal vector to the surface $x^2 + y^2 + z^2 = a^2$ $at$ the point $\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$. |
| 41. | Find the directional derivative of $\emptyset = 6x^2y + 24y^2z - 8z^2x$ at (1,1,1) in the direction of $v = 2i - 2j + k$. Hence, find the maximum value. |
| 42. | Find the directional derivative of $\varphi(x, y, z) = xy + yz + zx$ at the point $(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$. |
| 43. | In what direction from $(2,1,-1)$ is the directional derivative of $\varphi = x^2yz^3$ a maximum? Find also the magnitude of this maximum. |
| 44. | The temperature of the points in space is given by $\emptyset = x^2 + y^2 - z$. A mosquito located at the point $(1,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move. |
| 45. | If $F = x^2z\hat{\imath} - 2y^3z^3\hat{\jmath} + xy^2z\hat{k}$ then , find divergence of $F$ at $(1, -1, 1)$. |
| 46. | Determine the constant a such that $A = (ax^2y + yz)\hat{\imath} + (xy^2 + xz^2)\,\hat{\jmath} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. |
| 47. | Find div $\vec{\mathbf{F}}$, where $\vec{F} = grad\ (x^3 + y^3 + z^3 - 3xyz)$. |
| 48. | If $F = xz^3\hat{\imath} - 2x^2yz\hat{\boldsymbol{\jmath}} + 2yz^4\hat{k}$.then, find curl of F $at(1, -1, 1)$ |
| 49. | Find $div(grad\varphi)$ and $curl(grad\varphi)$ at (1,1,1) for $\varphi = x^2y^3z^4$. |
| 50. | Find curl of $A = e^{xyz}(i + j + k)$ at the point (1,2,3). |
| 51. | Find curl of $F(x, y, z) = (y^2 - z^2)i + (z^2 - x^2)j + (x^2 - y^2)k$ at the point (1,-1,2). |
| 52. | Determine the constant a and b such that $curl\ of\ \ A = (2\mathrm{xy} + 3\mathrm{yz})\hat{\imath} + (\mathrm{x}^2 + \mathrm{axz} - 4\mathrm{z}^2)\hat{\jmath} + (3\mathrm{xy} + 2\mathrm{byz})\hat{k}$ is zero. |
| 53. | Show that $F = (\mathrm{y}^2 - \mathrm{z}^2 + 3\mathrm{yz} - 2\mathrm{x})\hat{\imath} + (3\mathrm{xz} + 2\mathrm{xy})\hat{\jmath} + (3\mathrm{xy} - 2\mathrm{xz} + 2\mathrm{z})\hat{k}$ is both Solenoidal and irrotational. |
| 54. | Write the parametric equations of a circle of radius r centered at the origin in the xy -plane. |
| 55. | Find the parametric form of the line passing through the points (1,2,3) and (4,5,6). |
| 56. | Express the surface of a sphere of radius $a$ centered at the origin in parametric form. |
| 57. | Evaluate the line integral $\int_C (x^2ydx + y^2dy)$, where C is the line segment from (0,0) to (1,2). |
| 58. | Find the work done by the force field $\vec{F} = (y^2, 2xy)$ in moving a particle along the curve $y = x^2$ from (0,0) to (1,1). |
| 59. | Compute $\int_C (3\mathrm{x} - \mathrm{y})\mathrm{dx} + (\mathrm{x} + \mathrm{y})\mathrm{dy}$ where C is the boundary of the square with vertices $(0,0), (1,0), (1,1), (0,1)$ traversed counter-clockwise. |
| 60. | Evaluate the line integral of $\vec{F} = (y, -x)$ around the circle $x^2 + y^2 = a^2$. |

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| 61. | Find the flux of $F = yzj + z^2k$ outward through the surface $S$ cut from the cylinder $y^2 + z^2 = 1, z \geq 0$ by the planes $x = 0$ and $x = 1$. |
| 62. | Find the flux of the vector field $F = zi + yj + xk$ across the unit sphere $x^2 + y^2 + z^2 = 1$. |
| 63. | Evaluate $\iint_S F.\hat{n}\, dS$ when $F = x^2 i + 3y^2 k$ and $S$ is the portion of the plane $x + y + z = 1$ in the first octant. |
| 64. | Use Green's theorem to evaluate $\int_C (y^3\, dx + x^3\, dy)$ where C is the boundary of the region enclosed by $x = 0, y = 0, x = 1, y = 1$. |
| 65. | Apply Green's theorem to compute $\int_C (2x^2 - y)dx + (x + y^2)dy$ where C is the square with vertices $(0,0), (2,0), (2,2), (0,2)$. |
| 66. | Verify Green's theorem for $\int_C (ydx - xdy)$ where C is the unit circle $x^2 + y^2 = 1$. |
| 67. | Use Green's theorem to evaluate $\int_C x^2ydx + xy^2dy$ where C is the rectangle bounded by $x = 0, x = 1, y = 0, y = 2$. |
| 68. | Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem. |
| 69. | Use Green's theorem to evaluate the integral $\int_C y^2dx + x^2dy$ where C is the triangle bounded by $x = 0, x + y = 1, y = 0$. |
| 70. | Use Green's theorem to evaluate the integral $\int_C (x - y)dx + 3xydy$, where C is the boundary of the region bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$. |
| 71. | Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$ where $\vec{F} = xi + yj + zk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. |
| 72. | Use the divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$ for $\vec{F} = x^2i + y^2j + z^2k$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. |
| 73. | Verify the divergence theorem for $\vec{F} = xzi + yzj + z^2k$ over the volume bounded by $z = 0$ and $z = 1 - x^2 - y^2$. |
| 74. | Use Gauss's divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = xi + yj + z^3k$, and S is the closed surface bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. |
| 75. | Evaluate $\iiint_V div\, F\, dV$, where $F = (2x^2 - 4z)i - 2xyj - 8x^2k$ and V is bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$. |
| 76. | Use Stokes' theorem to evaluate $\int_C F.dr$ if $F = (x - y)i + (2x - z)j + (y + z)k$ and C is the boundary of the triangle (2,0,0),(0,3,0) and (0,0,6). |
| 77. | Use Stokes' theorem to evaluate $\int_C F.dr$ for $F = 2yi - xj + zk$ over the triangular surface with vertices (0,0,0),(1,0,0) and (0,1,0). |
| 78. | Verify Stokes' theorem for $F = (x + y)i + (y + z)j - xk$ and S is the surface of the plane $2x + y + z = 2$ in the first octant. |
| 79. | Evaluate by Stokes' theorem $\oint_C (e^x\, dx + 2y\, dy - dz)$ where C is the curve $x^2 + y^2 = 4$, $z = 2$. |