PARUL UNIVERSITY
Faculty of Engineering & Technology
Department of Applied Sciences and Humanities
1ST SEMESTER B.Tech PROGRAMME (CSE, IT)
CALCULUS(03019101BS01)
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Tutorial 2A: Multivariate Calculus

Q. 1 Evaluate the following limits, if exists:

a) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$ b) $\lim\limits_{(x,y)\to(0,1)} \frac{x^2+y^2+1}{3+x^2+3y^2}$ c) $\lim\limits_{(x,y)\to(0,0)} \frac{x+y}{x^2+y^2}$ d) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+2y^3}{x^2+4y^2}$

Q. 2 Check whether the given function is continuous or not, if yes then find point of continuity.

a) $f(x,y) = \begin{cases} \frac{x^2y^2}{2x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ b) $f(x,y) = \begin{cases} \frac{xy}{x^2+xy+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$.

Q. 3 If $u = x^3y + e^{xy^2}$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

Q. 4 If $= (x^2+y^2+z^2)^{-\frac{1}{2}}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

Q. 5 If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$.

Q. 6 If $u = cosec^{-1}\left(\sqrt{\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{12}\tan u\left[\frac{13}{12} + \frac{1}{12}\tan^2 u\right]$.

Q. 7 Use the Chain Rule to find $\frac{dw}{dt}$ for $= xe^{y/z}, x = t^2, y = 1-t, z = 1+2t$.

Q. 8 If $y\,log\,(cosx) = xlog(siny)$, find $\frac{dy}{dx}$.

Q. 9 If $z = f(x,y)$ where $x = e^u + e^{-v}, y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial v}$.

Q. 10 If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.