

TUTORIAL – 4B VECTOR CALCULUS

1	Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n} dS$ <p>where $\vec{F} = (x^2 + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x+y+z=6$, lying in the first octant.</p>
2	Evaluate the surface integral $\iint_S 15xy dS$ <p>where S is the portion of the plane $x + y + z = 1$, lying in front of the yz-plane.</p>
3	Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n} dS,$ <p>where $\vec{F} = z^2\hat{i} - 12\hat{j} + 3xy\hat{k}$, and S is the part of the plane $2x + 3y + 6z = 12$, in the first octant.</p>
4	Evaluate the volume integral $\iiint_V x^2 y dV$ <p>where V is the region bounded by the planes $4x + 2y + z = 6$, $x = 0$, $y = 0$, $z = 0$.</p>
5	Evaluate the volume integral $\iiint_V (x^2 + y^2 + z^2)^2 dV$ <p>where V is the solid sphere $x^2 + y^2 + z^2 = 4$.</p>
6	Verify Green's theorem for $\oint_C [(x^2 - 2xy^2) dx + (x^2y + 1) dy]$ <p>where C is the boundary of the region enclosed by $y = x^2$ and $y = x$.</p>
7	Using Green's theorem, evaluate $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ <p>where C is the positively oriented boundary of the region bounded by $y^2 = x$ and $y = x^2$.</p>

8	<p>Using Gauss's divergence theorem, evaluate</p> $\iint_S \vec{F} \cdot \hat{n} dS,$ <p>where $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and S is the surface of the cube $0 \leq x, y, z \leq 1$.</p>
9	<p>Apply the Gauss divergence theorem to find the outward flux of $\vec{F} = (x+y)\hat{i} + (y+z)\hat{j} + (z+x)\hat{k}$ across the surface of the sphere $x^2 + y^2 + z^2 = a^2$.</p>
10	<p>Using Stokes' theorem, evaluate</p> $\oint_C \vec{F} \cdot d\vec{r}$ <p>where $\vec{F} = (y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$ and C is the boundary of the triangle cut from the plane $x + y + z = 1$, by the coordinate planes.</p>