PARUL UNIVERSITY
Faculty of Engineering & Technology
Department of Applied Sciences and Humanities
1ST SEMESTER B.Tech PROGRAMME (CSE, IT)
CALCULUS(03019101BS01)
ACADEMIC YEAR – 2025-26

Assignment-03

1.	Determine the domain and range of the following functions (i) $f(x) = x^2$ (ii) $f(x) = \sqrt{x+2}$ (iii) $f(x) = \frac{1}{x-3}$ (iv) $f(x) = \sqrt{x-3}$ (v) $f(x) = \frac{1}{x-2}$ (vi) $f(x) = x^2 + 4$ (vii) $f(x) = \sin x$ (viii) $f(x) = \cos x$
2.	Determine whether the following functions are even, odd, or neither. (i) $f(x) = x^3 - x$ (ii) $f(x) = x^2 + 1$ (iii) $f(x) = x^4 - 2x^2$ (iv) $f(x) = sinx$ (v) $f(x) = x^2 + x$ (vi) $f(x) = cos(x)$ (vii) $f(x) = log\, x$ (viii) $f(x) = e^x$
3.	Evaluate (i) $\lim_{x\to 2}(x^2 + 3x - 4)$ (ii) $\lim_{x\to 2}\frac{x^2-4}{x-2}$ (iii) $\lim_{x\to 0}\frac{sinx}{x}$ (iv) $\lim_{x\to 3}(2x^2 - 5x + 1)$ (v) $\lim_{x\to 2}\frac{x^2+5x+6}{x^2-4}$ (vi) $\lim_{x\to 0}\frac{e^x-1}{x}$ (vii) $\lim_{x\to 2}\frac{x^3-8}{x-2}$ (viii) $\lim_{x\to 0}\frac{\tan(3x)}{\sin(5x)}$
4.	Discuss the continuity of the function f given by $f(x) = \begin{cases} x^2 - 1, & if\ x < 1 \\ 2, & if\ x \geq 1 \end{cases}$.
5.	Discuss the continuity of the function f given by $f(x) = \begin{cases} x + 1, & if\ x \leq 0 \\ x^2, & if\ x > 0 \end{cases}$.
6.	Discuss the continuity of the function f given by $f(x) = tanx$, at $x = \frac{\pi}{2}$.
7.	Test continuity of $f(x) = \begin{cases} x^2 - 1, & if\ x < 2 \\ 3, & if x = 2 \\ x + 1, & x > 2 \end{cases}$ at x=2.
8.	Find the derivative of $f(x) = x^2$ at $x = 1$ by the first principle of derivative.
9.	Find $\frac{df}{dx}$ if $f(x) = \sin x$ by definition.
10.	Find the derivative of $f(x) = \frac{1}{x^2}$ using the first principle of derivative.
11.	Find the derivative of $f(x) = \sqrt{x}$. Also, show that $f'(x)$ at $x = 0$ doesn't exist.
12.	Differentiate (i) $y = (5x^3 - x^4)^7$ (ii) $y = \log(2x + 3)$ (iii) $y = e^{\tan x}$ (iv) $y = \sin(x^2)$ (v) $y = cos^2(x)$ (vi) $y = e^{3x^2}$ (vii) $y = \sqrt{1 + x^2}$ (viii) $y = sin(x^2 + x)$
13.	Evaluate $\frac{dy}{dx}$ for (i) $y = x^5 - \log x + 7$ (ii) $y = x^2 \cos x$

	(iii) $y=\frac{x^2+1}{x-3}$ (iv) $y=\frac{\sin x}{x^2}$
14.	If $x^3+y^3=3axy$, find $\frac{dy}{dx}$.
15.	If $x^2+xy+y^2=7$, find $\frac{dy}{dx}$.
16.	If $x=sint$ and $y=cost$, find $\frac{dy}{dx}$.
17.	Find the local maximum and local minimum values of the function f given by $f(x)=3x^4+4x^3-12x^2+12$.
18.	State Rolle's Theorem. Give one example of a function satisfying the hypotheses of Rolle's theorem.
19.	Does Rolle's Theorem apply to $f(x)=\|x\|$ on [-1,1]? Give reasons.
20.	Check if Rolle's Theorem is applicable for $f(x)=x^2-1$ on the interval [-1,1].
21.	Find the point c that satisfies Rolle's Theorem for $f(x)=x^3-3x$ on [-√3, √3].
22.	Show that LMVT is not applicable for $f(x)=1/x$ on [-1,1].
23.	Find the value of c given by LMVT for $f(x)=\sqrt{x}$ on [1,4].
24.	Using LMVT, prove that $\|sin\,x-sin\,y\|\leq\|x-y\|$ for all real x,y.
25.	Verify LMVT for $f(x)=\ln x$ on the interval [1, e].
26.	Evaluate the definite integral as a limit of sum $\int_a^b x\,dx$.
27.	Evaluate the definite integral as a limit of sum $\int_0^3(x^2+2x+1)\,dx$.
28.	Find the area bounded by $f(x)=2-x^2$ and $g(x)=x$.
29.	Find the area bounded by $f(x)=\frac{x^2}{8}$ and $g(x)=\frac{x+8}{2}$.
30.	Find the length of the arc of the parabola $x^2=4ay$ measured from the vertex to one extremity of the latus rectum.
31.	Find the length of the curve $(y-1)^3=x^2$ on the x - interval $[0,8]$.
32.	Find the length of the arc of the curve $f(x)=2x^{\frac{3}{2}}$ over the interval [0,1] .
33.	Find the surface area of the solid generated by revolving the curve $f(x)=x^3$ on the interval $[0,1]$ about the x – axis.
34.	Find the area of the surface formed by the revolution of $x=y^3/3$ about y – axis which lies between $y=0$ and $y=1$.
35.	Find the volume of the solid generated by the curve $y=\sqrt{x}$ on the interval $[0,1]$, about x – axis.
36.	The line segment $x=1-y$, $0\leq y\leq 1$ is revolved about the y – axis to generate the cone. Find its lateral surface area.
37.	Find the volume of the reel-shaped solid formed by the revolution about the y – axis, of the part of the parabola $y^2=4ax$ cut off by the latus rectum.
38.	Find the area of the surface formed by the revolution of $y=x^2$ about y – axis which lies between y= 0 and $y=4$.
39.	Find the volume of cone generated by revolving the triangle in the first quadrant bounded by $x+y=2$ about y – axis.
40.	Evaluate $\lim_{(x,y)\to(3,-2)}\frac{2x^2-y}{x^2+y^2+3}$.
41.	Applying the definition of limit, show that $\lim_{(x,y)\to(0,0)}\frac{4x^2y}{x^2+y^2}=0$.

42.	Evaluate limit by path method, if exists (i) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$ (ii) $\lim_{(x,y)\to(0,0)} \frac{x^2y+xy^2}{x^2+y^2}$ (iii) $\lim_{(x,y)\to(0,0)} \frac{5xy}{x^2+y^2}$
43.	Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2-y^2}{\sqrt{x^2+y^2}} & ,(x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at the point (0,0).
44.	Check whether the given function $f(x,y) = \begin{cases} \frac{x+y}{\sqrt{x}-\sqrt{y}}, & (x,y) \neq (0,0) \\ -1 & (x,y) = (0,0) \end{cases}$ is continuous at origin or not, if yes then find point of continuity.
45.	Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ at the point (0, 0).
46.	If $f(x,y,z) = 2x^2yz - x^4z^3 + 2y^4$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ at $(1,1,1)$.
47.	Find the values of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$ at the point $(1,2)$ for $f(x,y) = x^2 + 3xy + y - 1$.
48.	If $u = x^2y + y^2z + z^2x$, find the values of (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ (ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$.
49.	If $u = e^{ax} \sin by$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
50.	If $u = \log(\tan x + \tan y + \tan z)$, then show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$.
51.	If $u(x,y,z) = e^{3xyz}$, show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (3 + 27xyz + 27x^2y^2z^2)e^{3xyz}$.
52.	If $u(x,y,z) = \ln(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.
53.	If $z = x + y^x$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.
54.	If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, prove that $x\frac{\partial u}{\partial x} + \mathrm{y}\frac{\partial u}{\partial y} + \mathrm{z}\frac{\partial u}{\partial z} = -u$.
55.	Is $u = x^2 + y^2 + 1$ a homogeneous function? Justify your answer.
56.	If $u = \frac{1}{x^3} + \frac{1}{x^2 y}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
57.	If $u = \log\left(\frac{x^4-y^4}{x^3+y^3}\right)$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
58.	If $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin 2u}{2}$. (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$.
59.	If $u = x^3 e^{-\frac{x}{y}}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
60.	Find $\frac{du}{dt}$ when $u = \frac{x}{y}, \quad x = e^t, \quad y = t$.
61.	Find $\frac{du}{dt}$ when $u = xy, \quad x = \cos t, \quad y = \sin t$.

62.	If $z = x^2 + y^2, x = rcos\theta\ , y = rsin\theta$. Find $\frac{dz}{d\theta}$.
63.	Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + logs, z = 2r$.
64.	If $u = f\left(\frac{1}{x} - \frac{1}{y}, \frac{1}{x} - \frac{1}{z}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.
65.	If $u = f(x, y)$ where, $x = e^s \cos t$ and $y = e^s \sin t$, show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$.
66.	If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
67.	If $u = f(x^2 + 2yz, y^2 + 2xz)$, prove that $(y^2 - xz)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$.
68.	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
69.	Find $\frac{dy}{dx}$ when (i) $x^2 + \ln y = 0$ (ii) $x^2y + y^3 = 1$.
70.	If $u = 2xy,\ v = x^2 - y^2$ and $x = rcos\theta,\ y = rsin\theta$ then, evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$.
71.	Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x - y, v = x + y$. Also, verify $J.J' = 1$.
72.	Find the equations of tangent plane and normal line to the surface $z = x^2 + 3y^2 - 4$ at (1,1,0).
73.	Find the equations of tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at $(2,1,-3)$.
74.	Find the stationary points for function $3x^2 - y^2 + x^3$.
75.	Discuss the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3axy$.
76.	Find the minimum distance from the origin to the plane $3x + 2y + z = 12$.
77.	Find the expression for the Lagrange multiplier λ for the given function $f(x, y) = xy$ subject to the constraint $g(x, y) = x + 2y - 6 = 0$.
78.	Find the expression for the Lagrange multiplier λ for the given function $f(x, y) = x^2y$ subject to the constraint $g(x, y) = x + y - 3 = 0$.
79.	Find the points on the surface $z^2 = x^2 + y^2$, that are closed to P(1,1,0).
80.	Divide 24 into three parts such that the product of the first, square of the second and cube of the third is maximum.
81.	Find the points on the surface $z^2 = xy + 1$ nearest to the origin. Also, find the distance.
82.	Expand $f(x, y) = e^x \cos y$ in powers of x and y upto third degree.
83.	Expand $f(x, y) = x^2 + y^2$, about the point (1,1) upto the second order terms.