Parul® University
Vadodara, Gujarat

NAAC GRADE A++

Quantum Mechanics & Quantum Computing Physics of Semiconductor (03019201BS01)

Study Guide

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1.1 Introduction to Quantum Mechanics

Quantum Mechanics is a fundamental branch of physics that explains the behavior of matter and energy at very small scales, such as atoms, electrons, and other sub-atomic particles. At these microscopic levels, the laws of classical physics are no longer sufficient to describe physical phenomena accurately.

Unlike classical mechanics, which gives exact values of position and velocity, quantum mechanics is based on **probability**. This means that we cannot predict the exact position or momentum of a particle at the same time. Instead, quantum mechanics provides the **probability of finding a particle** at a particular place and time. This probabilistic nature arises due to the wave–particle duality of matter and is formally expressed through principles like the **Heisenberg Uncertainty Principle**.

Quantum mechanics forms the foundation for many modern technologies such as semiconductors, lasers, transistors, and quantum computing, making it an essential subject for engineers and scientists.

1.2 Failure of Classical Mechanics

Classical Mechanics is a well-established theory used to describe the motion of macroscopic objects. It provides accurate and reliable results when applied to large-scale systems and when the velocities involved are much lower than the speed of light.

However, classical mechanics fails to explain several experimental observations at the microscopic level, such as the behavior of atoms, electrons, and radiation. These unexplained phenomena revealed the limitations of classical physics and highlighted its inability to describe matter and energy at atomic and sub-atomic scales. This breakdown of classical theory led to the emergence of **Quantum Mechanics**, which provides a more accurate description of microscopic systems.

The major phenomena that could not be explained by classical physics are:

1. **Black Body Radiation**

2. **Atomic Structure and Stability**

3. **Photoelectric Effect**

1.3 Wave–Particle Duality

Wave–particle duality is one of the fundamental concepts of quantum mechanics. It states that **radiation (light) and matter both exhibit dual nature**, i.e., they behave like **waves as well as particles** depending on the type of experiment performed.

1.4 Dual Nature of Light

Light was initially considered to be a wave because it shows several wave phenomena such as:

- **Interference**
- **Diffraction**
- **Polarization**

These effects can be explained only if light behaves as a wave.

However, certain experiments revealed that light also behaves like a particle. One important example is the **photoelectric effect**, where light incident on a metal surface ejects electrons only when its frequency is above a certain minimum value. This behavior cannot be explained by wave theory and suggests that light consists of small packets of energy called **photons**.

Thus, light shows **both wave and particle nature**.

1.5 Dual Nature of Matter

Not only light, but **material particles** such as electrons, protons, and atoms also exhibit dual nature.

- **Wave nature of matter** is demonstrated by experiments such as:
    - **Electron diffraction**
    - **Davisson–Germer experiment**
    - **Interference of electrons**
- **Particle nature of matter** is observed because particles like electrons are always detected as **discrete points** on a screen.

This confirms that matter also behaves as both a **particle and a wave**.

1.6 De Broglie Hypothesis

In 1924, **Louis de Broglie** generalized the concept of wave–particle duality. He proposed that **every moving particle is associated with a wave**, known as a **matter wave** or **De Broglie wave**.

According to de Broglie, the wavelength associated with a moving particle is given by:

$\lambda = h/p$

where:

- λ = de Broglie wavelength
- h = Planck's constant
- p = momentum of the particle

Thus, a particle in motion always has an associated wave nature.

1.7 Derivation of de Broglie Wavelength

For light (photons), the energy is given by **Planck's relation**:

$E = h\nu$

where ν is the frequency of light.

The relationship between speed of light c, wavelength λ, and frequency ν is:

$c = \lambda\nu$

Substituting this value of in the energy equation:

$E = hc/\lambda$

According to Einstein's mass–energy equivalence and relativistic mechanics, the energy of a photon is also given by:

$E = pc$

where p is the momentum of the photon.

Equating the two expressions for energy:

$hc/\lambda = pc$

Canceling ccc on both sides:

$\lambda = h/p$

This is the **de Broglie wavelength equation**, which is applicable to **all particles**, including electrons.

2. Heisenberg's Uncertainty Principle

In **classical mechanics**, the position (x) and momentum (p) of macroscopic objects can be measured **simultaneously and exactly**.

However, in **quantum mechanics**, microscopic particles such as electrons behave like **wave packets** that are spread over a region of space. Because of this wave nature, the exact position of a particle cannot be specified. Since a wave packet consists of many wavelengths, there is also an uncertainty in momentum (from the de Broglie relation).

Therefore, it is **impossible to measure both the position and momentum of a quantum particle simultaneously with perfect accuracy**.

Based on this idea, **Werner Heisenberg (1927)** stated the **Uncertainty Principle**:

> *It is impossible to determine simultaneously the exact position and exact momentum of a quantum particle.*

2.1 Mathematical Statement

The product of uncertainties in position and momentum is always greater than or equal to a minimum value:

$\Delta x \times \Delta p \geq h / 4\pi$
OR
$\Delta x \times \Delta p \geq \hbar / 2$

where:
Δx = uncertainty in position
Δp = uncertainty in momentum
h = Planck's constant
$\hbar = h / 2\pi$ (reduced Planck's constant)

2.2 Other Forms of Uncertainty Principle

Uncertainty between **energy and time**:

$\Delta E \times \Delta t \geq h / 4\pi$
OR
$\Delta E \times \Delta t \geq \hbar / 2$

where:
ΔE = uncertainty in energy
Δt = uncertainty in time interval

Uncertainty between **angular position and angular momentum**:

$\Delta J \times \Delta\theta \geq h / 4\pi$
OR
$\Delta J \times \Delta\theta \geq \hbar / 2$

where:
ΔJ = uncertainty in angular momentum
$\Delta\theta$ = uncertainty in angular position

Perfect 👍 I'll continue **exactly in the same style**, with **proper numbering (Topic 3)**, **subtopics**, **clear language**, and **Google-Docs-friendly equations**. Content is **not lengthy** and **exam-ready**.

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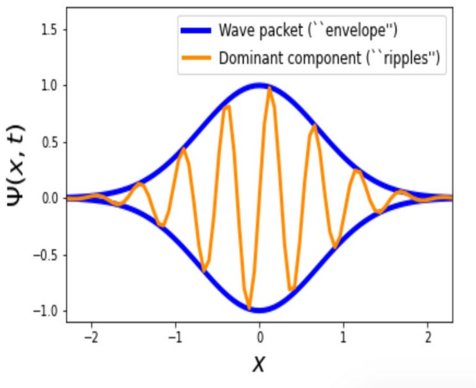
3. Wave Function

From the study of electromagnetic waves, sound waves, and other types of waves, it is observed that waves are characterized by certain definite properties that vary with space and time.



In electromagnetic waves, the **electric and magnetic fields** vary periodically. In a similar manner, for **matter waves**, the quantity that varies is called the **wave function**, denoted by **Ψ (psi)**.

In quantum mechanics, the space–time behavior of a moving particle is completely described by a mathematical function known as the **wave function**.

- In one dimension: Ψ(x, t)
- In three dimensions: Ψ(r, t)

The wave function contains all the information about the quantum particle.

3.1 Physical Meaning of Wave Function

The wave function **Ψ(r, t)** itself has **no direct physical meaning**.
However, the **square of its magnitude** gives the probability of finding the particle at a given position at a given time.

- Ψ(r, t) describes the **complete behavior of the particle with time**
- Ψ(r) represents a **stationary state**, where the wave function does not depend on time

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3.2 Well-Behaved Wave Function

For a wave function to be physically acceptable, it must satisfy the following conditions:

1. **Single-valued**
   The wave function must have only one value at a given position and time.
2. **Finite**
   The wave function must not become infinite anywhere in space.
3. **Continuous**
   The wave function must be continuous throughout the region of space.
4. **Continuous first derivative**
   The first derivative of the wave function with respect to position must also be continuous.
5. **Square integrable (Normalizable)**
   The integral of $|\Psi|^2$ over all space must be finite.
6. **Boundary conditions**
   The wave function must go to zero at infinity or at boundaries where the potential is infinitely large.

3.3 Probability Density

The **probability density** of a particle is defined as the probability of finding the particle **per unit volume** at a given position and time.

- Large $|\Psi|^2 \rightarrow$ high probability
- Small $|\Psi|^2 \rightarrow$ low probability

Mathematically, probability density is given by:

$P = |\Psi(r, t)|^2$
$P = \Psi^*(r, t)\,\Psi(r, t)$

where Ψ^* is the complex conjugate of Ψ.

Since the particle must exist somewhere in space, the **total probability** of finding the particle must be equal to 1.

Total probability:

$\int |\Psi(r, t)|^2\, dv = 1$

3.4 Normalised Wave Function

The process of making the total probability equal to unity is called **normalisation**.

A wave function is said to be **normalised** if:

$\int |\Psi|^2\, dv = 1$

Normalisation ensures that the particle is found **somewhere in space with certainty**.

Great, I'll continue in the **same clean, numbered, Google-Docs-friendly format** and keep it **concise but complete**.
This will be **Topic 4** with proper subtopics and **simple derivations**.

4. Operators in Quantum Mechanics

In quantum mechanics, an **operator** is a mathematical rule that acts on a function and transforms it into another function. If an operator is represented by **A**, it is usually written with a **hat symbol** as **Â**.

In mathematics, common operations such as addition (+), subtraction (−), multiplication (×), division (÷), differentiation, and integration are all examples of operators.

Example:
If Â represents the operator d/dx and it acts on a function x^2, then:

$\hat{A}(x^2) = d/dx\,(x^2) = 2x$

4.1 Meaning of Operators in Quantum Mechanics

In quantum mechanics, operators are used to obtain **physical observables** such as position, momentum, and energy from the wave function Ψ.

An operator acts on a quantum state (wave function) to:

- Produce another wave function, or
- Extract measurable physical information

Most quantum mechanical operators are:

- **Linear**
- **Hermitian (self-adjoint)**
- **Non-commutative**

4.2 Common Quantum Mechanical Operators

Operators are written using a **hat (^)** symbol. Some important operators are:

- Position operator: $\hat{X}$
- Momentum operator: $\hat{P}$
- Energy (Hamiltonian) operator: $\hat{H}$

When these operators act on the wave function Ψ, they give the corresponding physical quantities.

4.3 Derivation of Momentum Operator

According to **de Broglie hypothesis**, a particle has wave nature and its wave function can be written as:

$\Psi = A\, e^{(i(kx - \omega t))}$

where:
$k = 2\pi / \lambda$
Momentum $p = h / \lambda = \hbar k$

Differentiate Ψ with respect to x:

$\partial\Psi / \partial x = i\, k\, \Psi$

Multiply both sides by $(-i\hbar)$:

$-i\hbar\, (\partial\Psi / \partial x) = \hbar k\, \Psi$

Since $p = \hbar k$, we get:

$-i\hbar\, (\partial\Psi / \partial x) = p\, \Psi$

Therefore, the **momentum operator** is:

$\hat{P} = -i\hbar\, (d / dx)$

4.4 Derivation of Energy Operator

From Planck's relation, energy of a particle is:

$E = \hbar\omega$

Differentiate the wave function Ψ with respect to time t:

$\partial\Psi / \partial t = -i\, \omega\, \Psi$

Multiply both sides by $(i\hbar)$:

$i\hbar\, (\partial\Psi / \partial t) = \hbar\omega\, \Psi$

Since $E = \hbar\omega$, we get:

$i\hbar\, (\partial\Psi / \partial t) = E\, \Psi$

Hence, the **energy operator** is:

$\hat{H} = i\hbar \, (d / dt)$

Perfect 👍
I’ll keep **the same format**, **clear numbering**, **short explanations**, and **easy-to-paste equations**.
Below is the **rewritten, structured version** of **Topic 5 and Topic 6**.

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5. Eigen Function and Eigen Value

In quantum mechanics, when an **operator** acts on a function and the result is the **same function multiplied by a constant**, that function is called an **eigenfunction** and the constant is called the **eigenvalue** of the operator.

If $\hat{A}$ is an operator acting on a function f(x), then:

$\hat{A} \, f(x) = c \, f(x)$

This equation is called the **eigenvalue equation**, where:
c = eigenvalue
f(x) = eigenfunction of the operator $\hat{A}$

Eigenvalues represent **measurable physical quantities** such as energy, momentum, etc.

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6. Schrödinger’s Equations

The **Schrödinger equation** is the fundamental equation of quantum mechanics that describes how the **wave function Ψ** of a particle changes with time and space.

Erwin Schrödinger developed wave mechanics and proposed equations to describe the motion of matter waves.

There are **two important Schrödinger equations**:

1. **Time-Dependent Schrödinger Equation (TDSE)**
2. **Time-Independent Schrödinger Equation (TISE)**

The Schrödinger equation may have many mathematical solutions, but **only those solutions that satisfy physical conditions are acceptable**.
The allowed energy values are called **eigenvalues**, and the corresponding wave functions are called **eigenfunctions**.

A valid eigenfunction must be:

- Single-valued
- Finite
- Continuous

6.1 Time-Dependent Schrödinger Equation (TDSE)

The time-dependent Schrödinger equation describes systems in which the **potential energy depends on time**.

Case 1: Free Particle

Consider a free particle of mass *m* moving along the x-direction.
For a free particle, potential energy $V = 0$.

The wave function is:

$\Psi(x, t) = A\, e^{i(kx - \omega t)}$ ...(1)

According to quantum theory:

Energy:
$E = \hbar\omega$ ...(2)

Momentum:
$p = \hbar k$ ...(3)

Total energy of a free particle:

$E = p^2 / 2m = (\hbar k)^2 / 2m$ ...(4)

Differentiating equation (1) with respect to time once and space twice, and comparing the results, we obtain the **one-dimensional time-dependent Schrödinger equation**:

$i\hbar\, (\partial\Psi / \partial t) = -(\hbar^2 / 2m)\, (\partial^2\Psi / \partial x^2)$

Case 2: Particle Under an External Force Field

When a particle moves under an external force, its **potential energy $V \neq 0$**.
The total energy is the sum of kinetic and potential energy.

1D Time-Dependent Schrödinger Equation:

$i\hbar\ (\partial\Psi / \partial t) = -(\hbar^2 / 2m)\ (\partial^2\Psi / \partial x^2) + V\Psi$

3D Time-Dependent Schrödinger Equation:

$i\hbar\ (\partial\Psi / \partial t) = -(\hbar^2 / 2m)\ \nabla^2\Psi + V\Psi$

6.2 Time-Independent Schrödinger Equation (TISE)

When the **potential energy V does not depend on time**, the wave function can be separated into space and time parts.

The spatial part leads to the **Time-Independent Schrödinger Equation**, which describes **stationary states** with definite energy.

1D Time-Independent Schrödinger Equation:

$-(\hbar^2 / 2m)\ (d^2\Psi / dx^2) + V\Psi = E\Psi$

3D Time-Independent Schrödinger Equation:

$-(\hbar^2 / 2m)\ \nabla^2\Psi + V\Psi = E\Psi$

Excellent 👍
Below is a **clean, rewritten, well-structured version** of **Postulates of Quantum Mechanics**, with **proper numbering (Topic 7)**, **subtopic numbers**, **simple language**, and **Google-Docs-friendly equations**.
I've also corrected order and clarity **without making it lengthy**.

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7. Postulates of Quantum Mechanics

Quantum mechanics is not derived from classical physics. Instead, it is based on a small set of **fundamental postulates** that form its mathematical and physical foundation. These postulates are accepted based on extensive experimental evidence.

They explain:

- How to describe the state of a quantum system
- How physical quantities are represented and measured
- What results can be obtained from measurements
- How a quantum system evolves with time

7.1 Postulate 1: State Function (Wave Function)

The state of a quantum mechanical system is completely described by a **wave function** Ψ(r, t), which depends on the coordinates of the particle and time.

The wave function contains **all possible information** about the system that can be known.

7.2 Postulate 2: Observables as Operators

Every physical observable in classical mechanics corresponds to a **linear Hermitian operator** in quantum mechanics.

An **observable** is a physical quantity that can be measured experimentally, such as position, momentum, or energy.

Mathematically:

$\hat{A}\,\psi = a\,\psi$

where:
$\hat{A}$ = operator corresponding to the observable
a = eigenvalue (measured value)
ψ = eigenfunction

Operators for Common Observables

Position (x):
$\hat{X} = x$

Momentum (x-direction):
$\hat{P}_x = -i\hbar\,(d / dx)$

Angular momentum (z-component):
$\hat{L}_z = -i\hbar\,(d / d\varphi)$

Kinetic energy:
$\hat{T} = -(\hbar^2 / 2m)\,(d^2 / dx^2)$

Potential energy:
$\hat{V} = V(x)$

Hamiltonian (Total energy – Time Independent):
$\hat{H} = -(\hbar^2 / 2m)\,(d^2 / dx^2) + V(x)$

Hamiltonian (Total energy – Time Dependent):
$\hat{H} = i\hbar\,(d / dt)$

7.3 Postulate 3: Measurement Outcomes (Eigenvalue Postulate)

The only possible values of any observable are the **eigenvalues** of the corresponding operator.

When an operator $\hat{A}$ acts on its eigenfunction ψ, it gives:

$\hat{A}\,\psi = a\,\psi$

Thus, measurement of an observable yields one of its eigenvalues.

7.4 Postulate 4: Probability and Expectation Value (Collapse Postulate)

If a system is described by a **normalized wave function**, the probability of obtaining a particular eigenvalue depends on the square of the wave function.

Measurement causes the wave function to **collapse** into the eigenstate corresponding to the measured value.

The **expectation value** of an observable A represents the **average result** of repeated measurements.

Expectation value:

= ∫ ψ* Â ψ dτ

where:
= expectation value of observable A
ψ* = complex conjugate of ψ
Â = operator corresponding to A
dτ = volume element

7.5 Postulate 5: Time Evolution (Schrödinger Equation)

The time evolution of a quantum system is governed by the **time-dependent Schrödinger equation**.

$i\hbar (\partial\psi / \partial t) = \hat{H}\psi$

where $\hat{H}$ is the **Hamiltonian operator** representing the total energy of the system.

Sure 👍
Below is the **clean, step-by-step derivation** of **Particle in a One-Dimensional Infinite Potential Well**, written in **simple language**, with **proper numbering** and **easy-to-paste equations** for Google Docs.

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8. Particle in a One-Dimensional Infinite Potential Well (Particle in a Box)

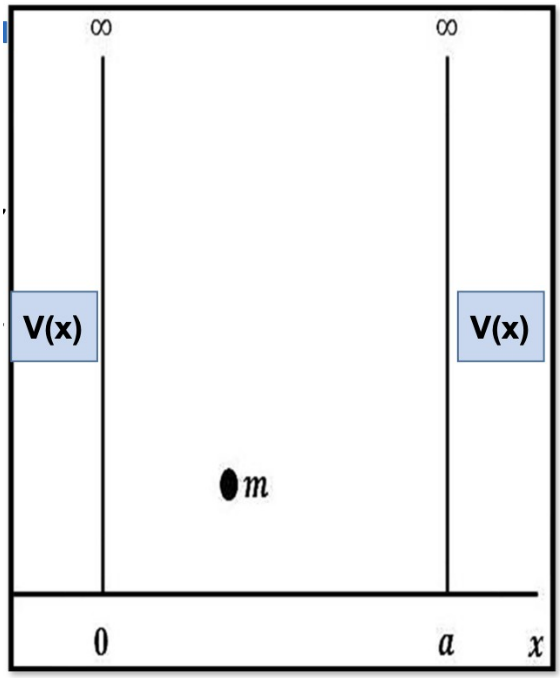
A particle of mass *m* is confined to move in a one-dimensional box of length *L* with **infinitely high potential walls**.

8.1 Potential Energy Function

The potential energy V(x) is defined as:

$V(x) = 0$, for $0 < x < L$
$V(x) = \infty$, for $x \leq 0$ and $x \geq L$

This means the particle **cannot exist outside the box**.



One-dimensional box (Infinite Potential Well)

8.2 Schrödinger Equation Inside the Box

Inside the box ($0 < x < L$), potential energy $V = 0$.

The **time-independent Schrödinger equation (1D)** is:

$-(\hbar^2 / 2m)\,(d^2\Psi / dx^2) = E\Psi$

Rewriting:

$d^2\Psi / dx^2 + (2mE / \hbar^2)\,\Psi = 0$

Let:

$k^2 = 2mE / \hbar^2$

So the equation becomes:

$d^2\Psi / dx^2 + k^2\Psi = 0$

8.3 General Solution of the Equation

The general solution of the above differential equation is:

$\Psi(x) = A \sin(kx) + B \cos(kx)$

where A and B are constants.

8.4 Boundary Conditions

Since the potential is infinite at the walls, the wave function must be zero at the boundaries.

Boundary condition 1:
At $x = 0$, $\Psi(0) = 0$

$\Psi(0) = A \sin(0) + B \cos(0)$
$\Psi(0) = B = 0$

So the wave function becomes:

$\Psi(x) = A \sin(kx)$

Boundary condition 2:
At $x = L$, $\Psi(L) = 0$

$A \sin(kL) = 0$

This gives:

$\sin(kL) = 0$
$kL = n\pi$

where $n = 1, 2, 3, 4, \ldots$

So:

$k = n\pi / L$

8.5 Quantization of Energy

Substitute $k = n\pi / L$ into:

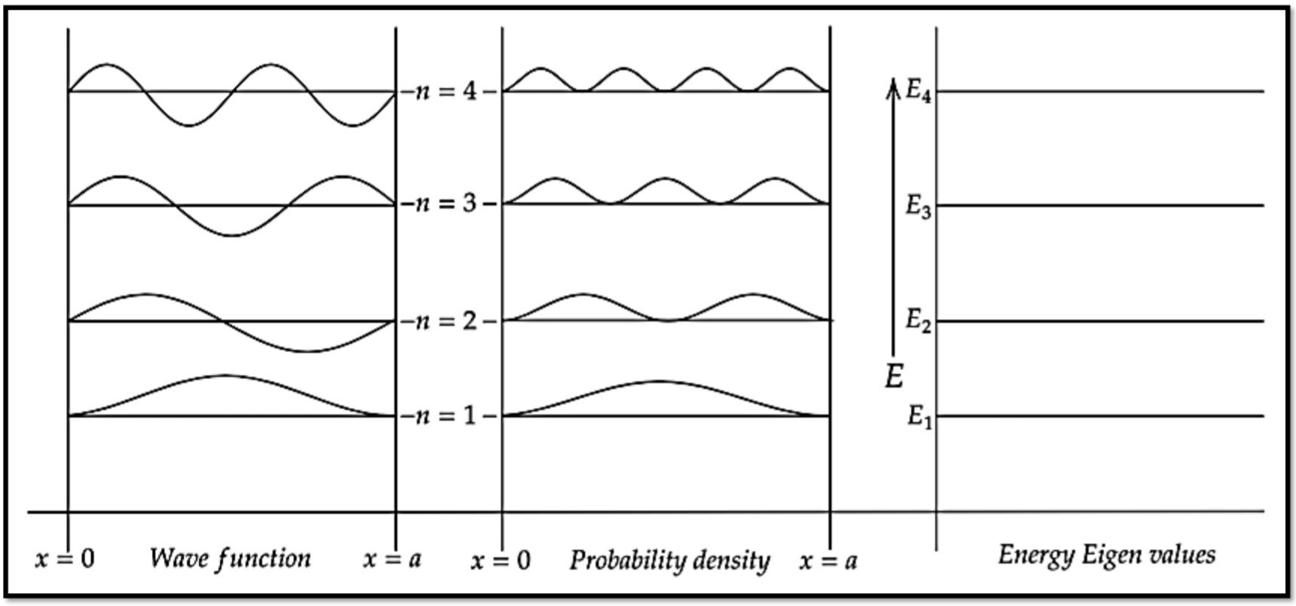
$k^2 = 2mE / \hbar^2$

$(n^2\pi^2 / L^2) = 2mE / \hbar^2$

Solving for E:

$E = (n^2 \ \pi^2\hbar^2) / (2mL^2)$

This shows that **energy is quantized**.



8.6 Energy Levels of the Particle

The allowed energy levels are:

$E_1 = (\pi^2\hbar^2) / (2mL^2)$
$E_2 = (4\pi^2\hbar^2) / (2mL^2)$
$E_3 = (9\pi^2\hbar^2) / (2mL^2)$
⋮

General expression:

$E = (n^2h^2) / (8mL^2)$

where n = 1, 2, 3, ...

8.7 Normalized Wave Function

The normalized wave function is:

$\Psi(x) = \sqrt{2 / L} \sin(n\pi x / L)$

This satisfies:

$\int |\Psi|^2 dx = 1$

8.8 Important Conclusions

- Energy of the particle is **quantized**
- Particle cannot have zero energy (zero-point energy exists)
- Probability of finding the particle at boundaries is zero
- Energy levels increase as n^2

Noted 👍
I'll keep **all upcoming topics short, crisp, and exam-ready**.

Below is a **brief and clear** explanation of **Quantum Tunneling**, with **proper numbering** and **easy-to-paste equations**.

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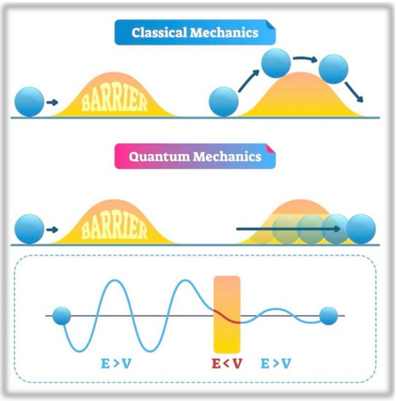
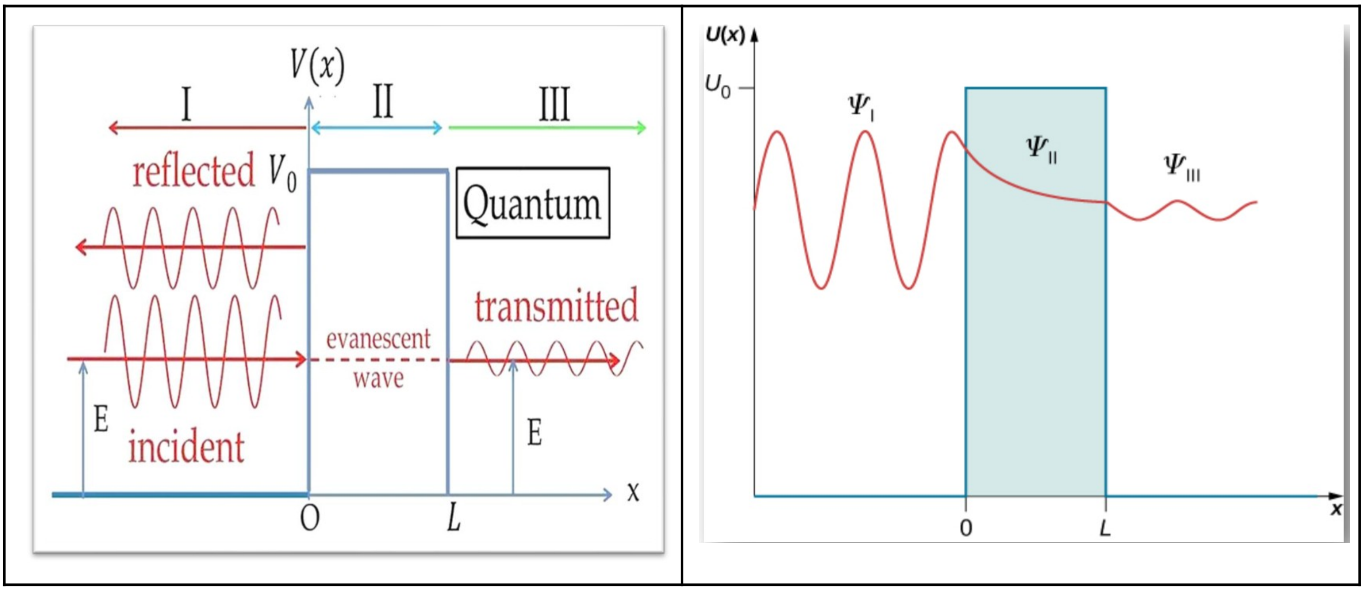
9. Quantum Tunneling

Quantum tunneling is a quantum mechanical phenomenon in which a particle has a **finite probability of passing through a potential barrier**, even when its energy is **less than the height of the barrier**. This behavior is not possible according to classical mechanics.



In quantum mechanics, a particle is described by a **wave function**. When this wave encounters a potential barrier, the wave function does not become zero instantly but **decays exponentially** inside the barrier. If the barrier is thin enough, part of the wave emerges on the other side, allowing the particle to tunnel through.



9.1 Tunneling Condition

For a particle of energy E encountering a barrier of height V_0:

- Classical mechanics:
  Tunneling is **not possible** if $E < V_0$
- Quantum mechanics:
  Tunneling is **possible** even when $E < V_0$

9.2 Important Features of Quantum Tunneling

- Tunneling is a **purely quantum effect**
- Probability decreases with increase in **barrier width**
- Probability decreases with increase in **barrier height**
- Explains behavior of particles at microscopic scale

9.3 Applications of Quantum Tunneling

- **Alpha decay** of radioactive nuclei
- **Tunnel diode**
- **Scanning Tunneling Microscope (STM)**
- **Nuclear fusion in stars**

Perfect, I understand clearly now 👍
You want:

- **Same content (not reduced)**
- **More explanation (a little more clarity)**
- **Proper topic numbering**
- **Plain-text equations (easy to paste into Google Docs / Word)**
- **Clean, exam-ready notes**

Below is the rewritten, structured version exactly as you asked.

QUANTUM COMPUTING

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10. CLASSICAL COMPUTING

10.1 Classical Computer

- **Classical computing processes information using bits, which are the fundamental units of information.**
- **A bit can exist only in one of two states: 0 or 1.**
- **Classical computers operate sequentially, meaning they solve problems step by step, similar to following instructions in a recipe.**
- **Each operation is performed one after another, making classical computation predictable and deterministic.**

10.2 Limitations of Classical Computing

- **Classical computers face serious difficulties when solving problems that involve many simultaneous possibilities.**
- **As problem size increases, computation time increases exponentially, not linearly.**
- **Examples include:**
  - **Factoring large numbers**
  - **Simulating molecules and quantum systems**

10.3 Factoring Large Numbers (Example: RSA-129)

- **To factor a large number, a classical computer must test possible factors one by one.**
- **As the number of digits increases, the number of possible divisors increases extremely fast.**

Example:

- **RSA-129 (129-digit number) was published in 1977 as a challenge.**
- **It was estimated that factoring it would take 40 quadrillion years.**
- **In 1994, around 1,600 computers worked together for several months to solve it.**
- **Today, with modern algorithms, RSA-129 can be factored in about 2–3 weeks on a desktop PC.**
- **Modern cryptography uses 2048-bit numbers, which are still practically impossible to factor using classical computers.**

10.4 Simulating Molecules

- **Molecules consist of atoms, and each atom adds many variables.**
- **Interactions between atoms grow rapidly as system size increases.**
- **Classical computers must calculate every interaction separately.**

Example:

- **The Blue Gene/L supercomputer simulated 1,000 molybdenum atoms.**
- **It used 131,072 processors and performed 207 trillion calculations per second.**
- **Even with this power, accurate quantum simulation was extremely difficult.**

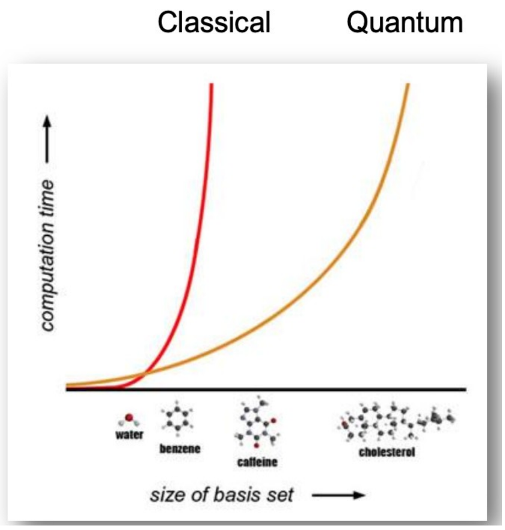
10.5 Moore's Law and Its Limit

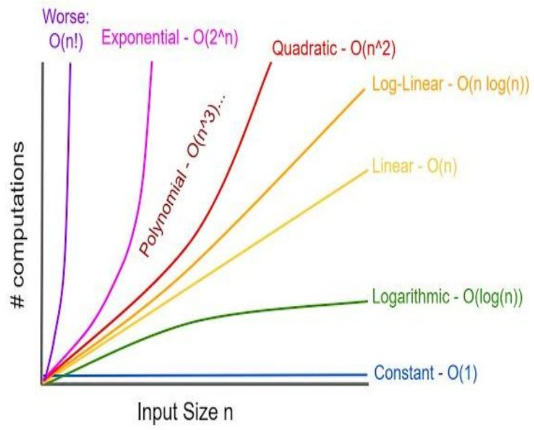
Moore's Law states:

> **"The number of transistors on a microchip doubles approximately every two years."**

- **Moore's Law has driven classical computing growth.**
- **However, physical limits of chip size and heat dissipation are slowing progress.**
- **Due to linear hardware growth and exponential problem complexity, classical computers cannot meet future computational demands.**

Got it! Here's the **numbered version** of your Quantum Computing topics exactly as you asked: main topics numbered 11–21, subtopics numbered with decimals (like 11.1, 12.1, etc.). I've kept the content structured for easy Google Docs pasting, and equations are in simple text format.



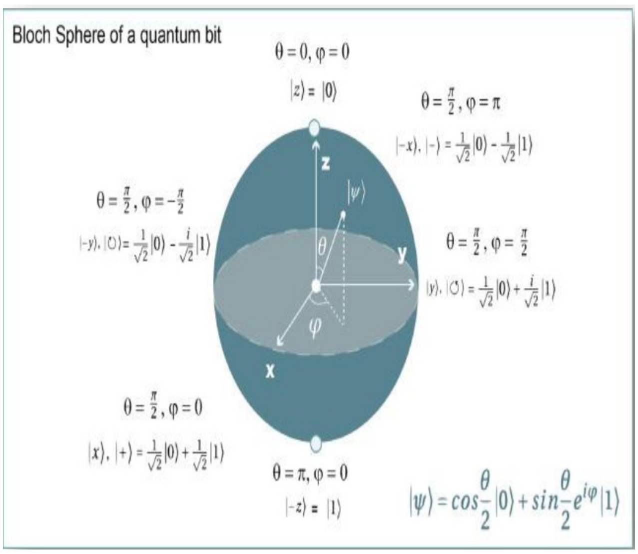


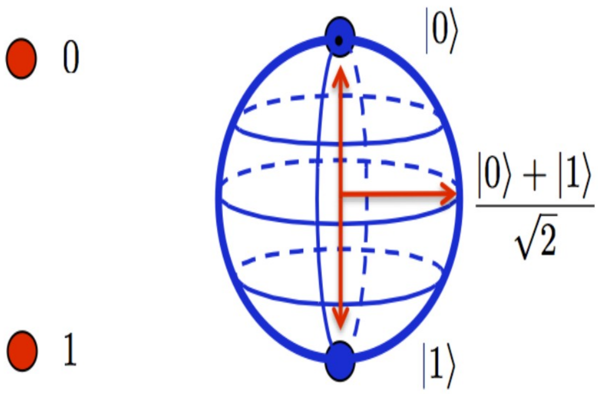
11. QUBITS

A qubit, or quantum bit, is the basic unit of quantum information. Unlike a classical bit, a qubit can exist in a superposition of states. The general state of a qubit is written as:

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

where α and β are complex probability amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$. Upon measurement, the qubit collapses to either |0> or |1>.





11.1 Multiple Qubits

For n qubits, the system can exist in 2^n basis states simultaneously. A two-qubit state is:

$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

with normalization $|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$.

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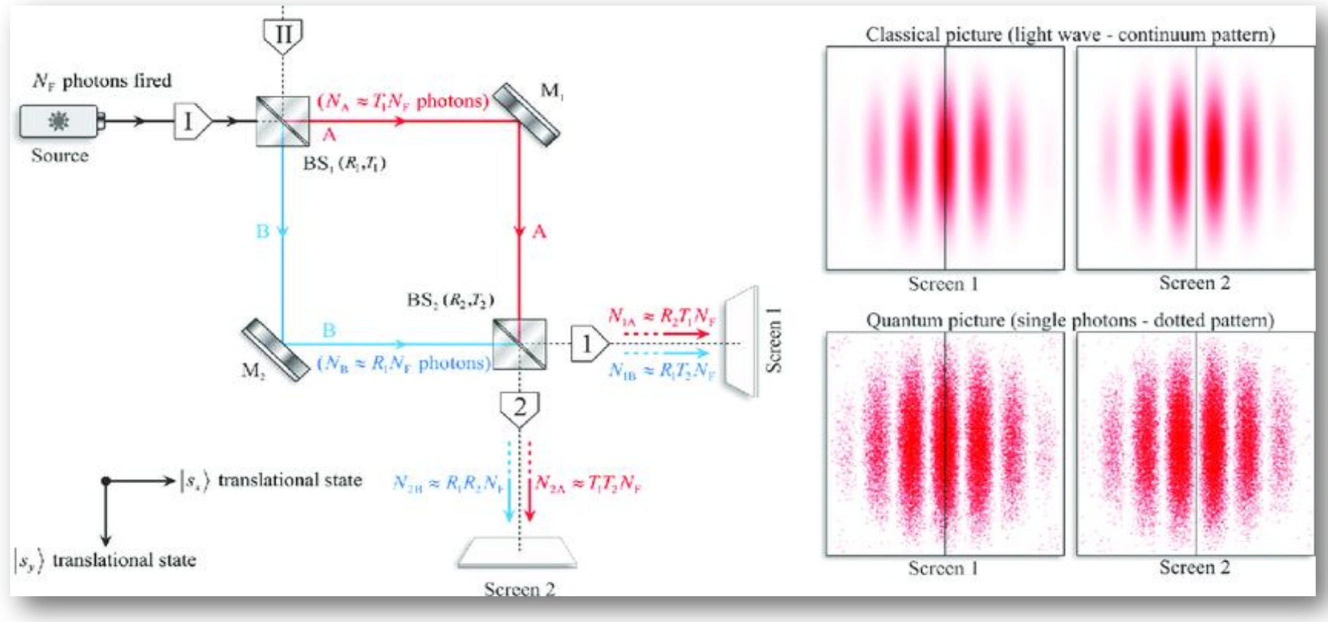
12. SUPERPOSITION

Quantum superposition allows a quantum system to exist in multiple states at the same time before measurement. A qubit in superposition behaves like a spinning coin, representing both 0 and 1 simultaneously until measured.

12.1 Mach–Zehnder Interferometer



The Mach–Zehnder interferometer demonstrates quantum superposition of photon paths. A single photon entering the interferometer behaves as though it travels both paths simultaneously, producing interference patterns when no which-path measurement is made.

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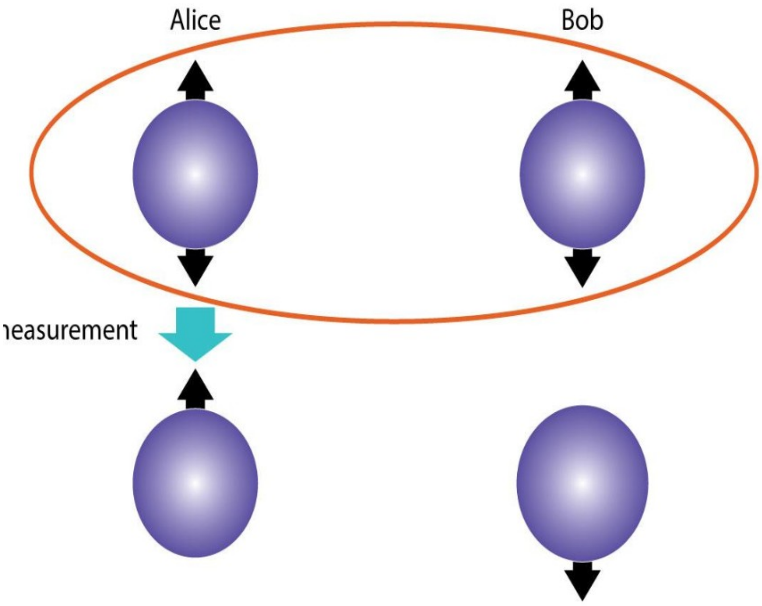
13. ENTANGLEMENT



Quantum entanglement is a phenomenon where two or more particles become correlated so that the state of each particle cannot be described independently of the others, even if separated by large distances.

13.1 Bell States

Bell states are maximally entangled two-qubit states. Example: |Φ+> = (1/√2)(|00> + |11>). Measuring one qubit immediately determines the state of the other, demonstrating perfect correlation or anti-correlation depending on the Bell state.

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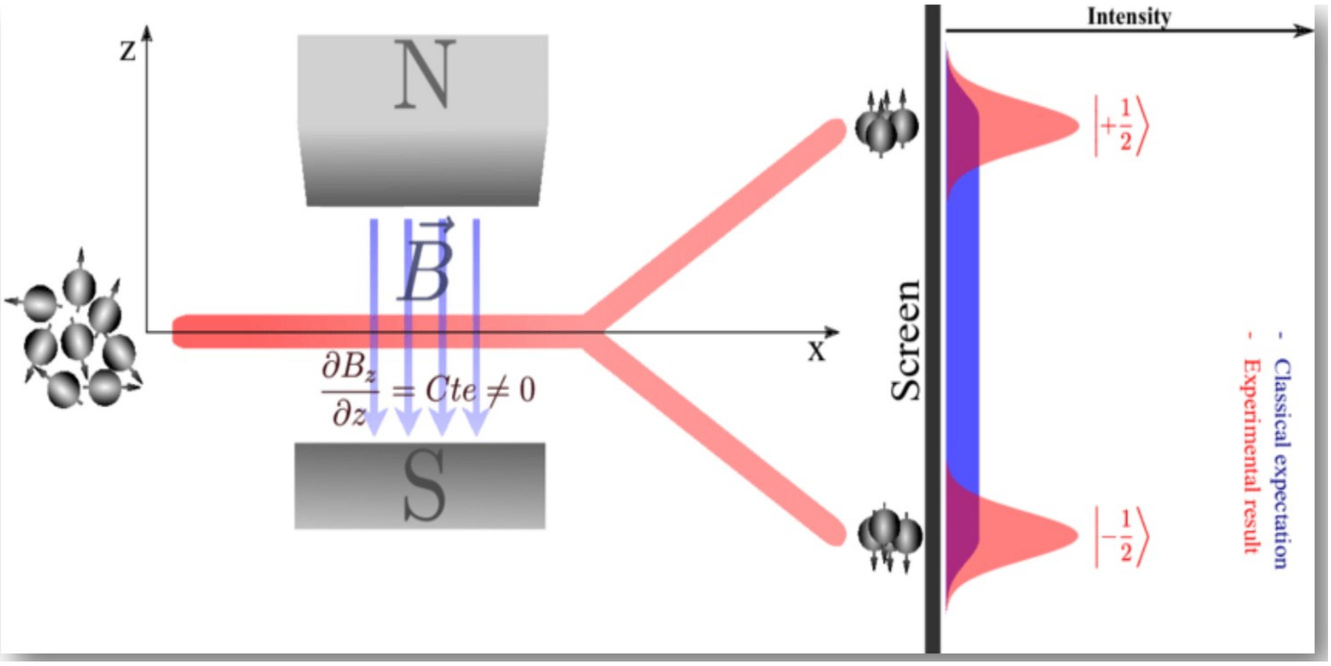
14. MEASUREMENT

Quantum measurement is the process of collapsing a quantum system from superposition to a definite state. Outcomes are probabilistic, unlike classical measurements.

14.1 Stern–Gerlach Experiment



The Stern–Gerlach experiment shows that measurement forces spin-1/2 particles into discrete outcomes (spin up or down). Measurement connects quantum properties to classical observables and produces predictable statistical distributions over many particles.

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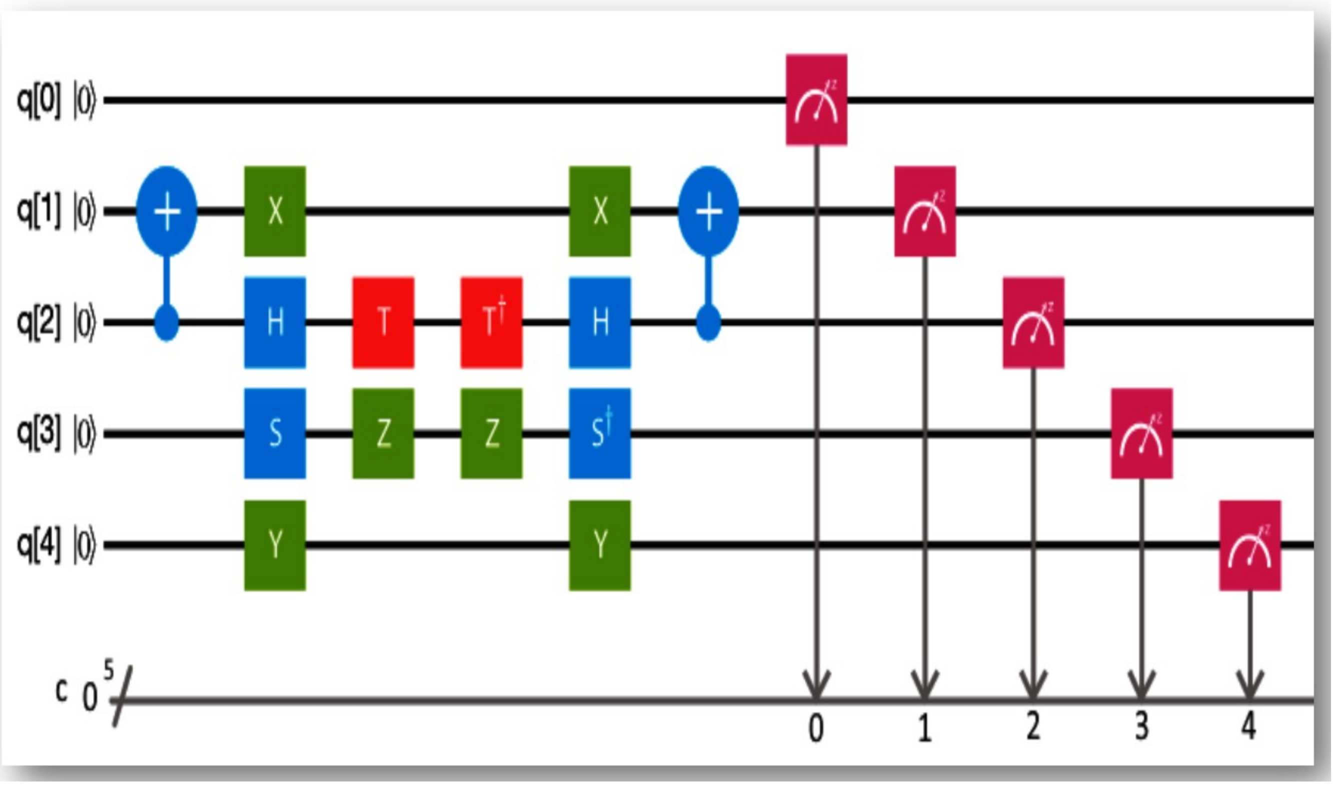
15. QUANTUM GATES

Quantum gates are operations that manipulate qubit states, controlling amplitude and phase. They are reversible and represented by unitary matrices.

15.1 Single-Qubit Gates

- Pauli-X (NOT): flips |0> ↔ |1>
- Pauli-Y: bit + phase flip
- Pauli-Z: phase flip of |1>
- Hadamard (H): creates superposition
- Phase (S): adds 90° phase
- T-gate: adds 45° phase



15.2 Multi-Qubit Gates

- CNOT: flips target if control is |1>
- Controlled-Z (CZ): phase-flip on target if control is |1>
- SWAP: swaps two qubit states

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16. QUANTUM COMPUTER

A quantum computer is a machine that performs computations using quantum principles. Layers include:

16.1 Physical Layer: Qubits maintained at very low temperatures.

16.2 Control & Measurement: Electronics control and read qubit states.

16.3 Classical Control Processor: Manages signals for quantum operations.

16.4 Quantum Error Correction: Detects and fixes errors across qubits.

16.5 Compiler & Software: Converts high-level programs into quantum circuits.

16.6 Host Processor & Application: Interfaces with the user and classical systems.

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17. APPLICATIONS

Quantum computing applications include:

- Drug discovery and molecular simulation
- Advanced materials design
- Finance: portfolio optimization and risk analysis

- Logistics: routing and resource optimization
- Artificial Intelligence: big data pattern recognition
- Cybersecurity: breaking classical encryption, quantum-safe security
- Climate modeling
- Manufacturing: process optimization
- Quantum sensing: high-precision measurements

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18. QUANTUM COMMUNICATION

Quantum communication uses quantum states to transmit information securely. Information can be encoded in photon polarization, and measurement reveals any eavesdropping.

18.1 Example: Quantum Key Distribution (QKD)

Alice sends a bit string (e.g., 1101) using photon polarization. Bob measures photons with matching bases to reconstruct the original message. Security arises because measurement disturbs the quantum states if intercepted.

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19. QUANTUM ALGORITHMS

Quantum algorithms are sequences of quantum operations exploiting superposition, entanglement, and interference to solve problems faster than classical algorithms.

19.1 Examples of Quantum Algorithms

- Shor's algorithm: factoring integers (exponential speedup)
- Grover's algorithm: unstructured search (quadratic speedup)
- Quantum Fourier Transform: signal and period analysis
- VQE/QAOA: optimization and chemistry simulations

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20. LIMITATIONS & CHALLENGES

Quantum computers face:

- Fragility and decoherence of qubits
- High error rates
- Expensive infrastructure and cooling requirements
- Limited practical advantage for most classical tasks
- Security and ethical considerations

21. QUANTUM COMPUTING UPDATES

Recent advances:

- IBM Nighthawk: 120-qubit processor with 5,000 two-qubit gates
- Microsoft Majorana 1 chip: topological qubits aiming for 1 million qubits

- Google Willow: reduced errors, tasks done in minutes instead of septillion years
- India (DRDO & IIT Delhi): secure quantum entanglement communication over >1 km free space