CS 405/6001: Game Theory and Algorithmic Mechanism Design

Autumn 2024
Check course webpage for registration policy

Swaprava Nath

IIT Bombay

Let us play a game: Neighboring King(Queen)dom’s Dilemma

Each kingdom can invest either in Agriculture or War – but not both

- If both choose Agri – happiness is 5 for each
- If both choose War – happiness is 1 for each
- If one chooses Agri, but the other War – the Agri kingdom stand to lose everything and War kingdom gets happiness more than 5

What should the King / Queen do?

What is collectively best?

(Agriculture, Agriculture)

	Agriculture	War
Agriculture		
War		

Decisions are simultaneous

War is a Dominant Strategy for Queen as well as King

(War, War) is a Dominant Strategy Equilibrium

Professor's Dilemma

- More effort in setting difficult exam papers
- Attentive in class leads to better learning
- Gives a positive feedback to the class
- Attentive = Positive feedback
- Not attentive = Negative feedback

Equilibrium

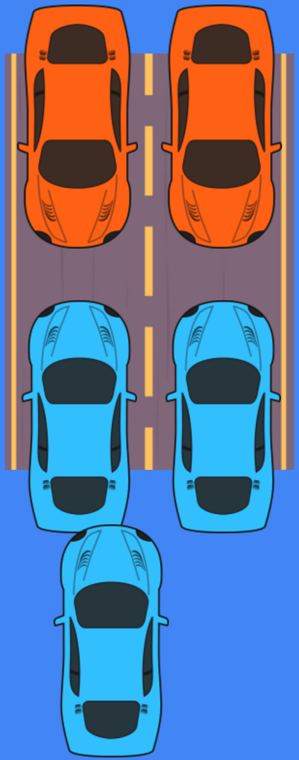
	Negative feedback	Positive feedback
Exams: hard		
Exams: easy		

Decisions are simultaneous

Easy exam is a Dominant Strategy for professor

Being attentive is a Dominant Strategy for student

Another game: Traffic Movement



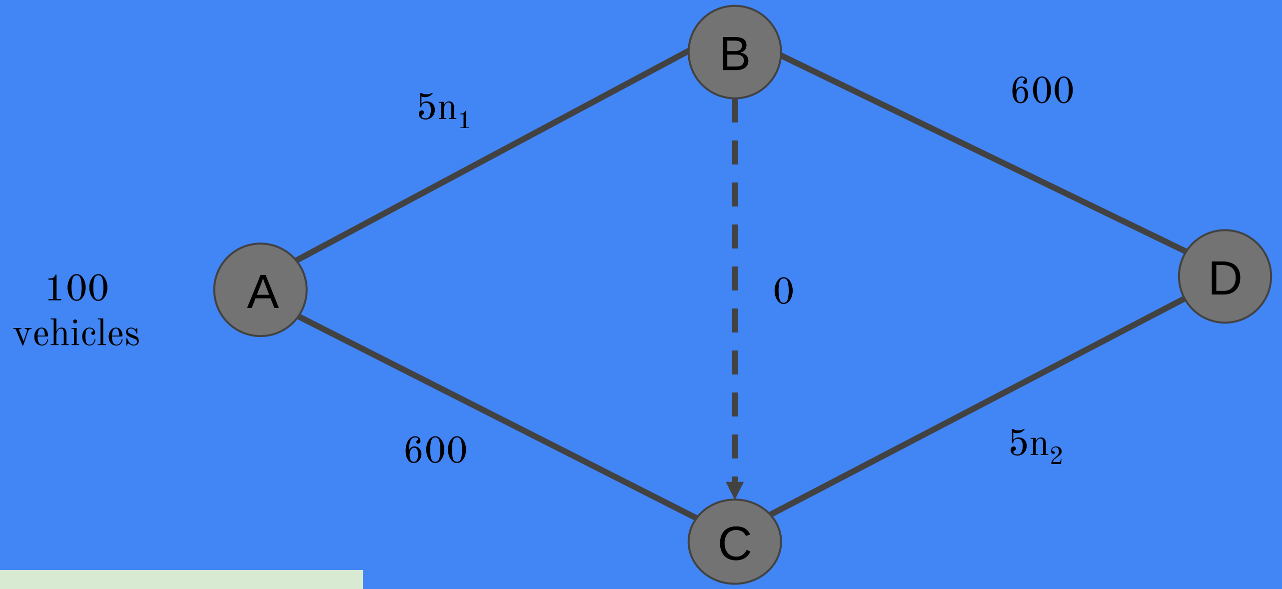
Does this game have a dominant strategy equilibrium?

The Nash Equilibrium (John Nash, 1951)

	Left	Right
Left		
Right		

Equilibrium here is a strategy profile from where no player wants to unilaterally deviate

Adding resources (blindly) does not improve the society



Design is important

n_k : number of vehicles on road k, $k = 1, 2$

Equilibrium (original)

50 vehicles each on paths ABD and ACD

Time for each vehicle = 850

Equilibrium (after)

100 vehicles on path ABCD

Time for each vehicle = 1000

Example: Auction

Question: how to allocate the object to the individual who (truly) values it the most?

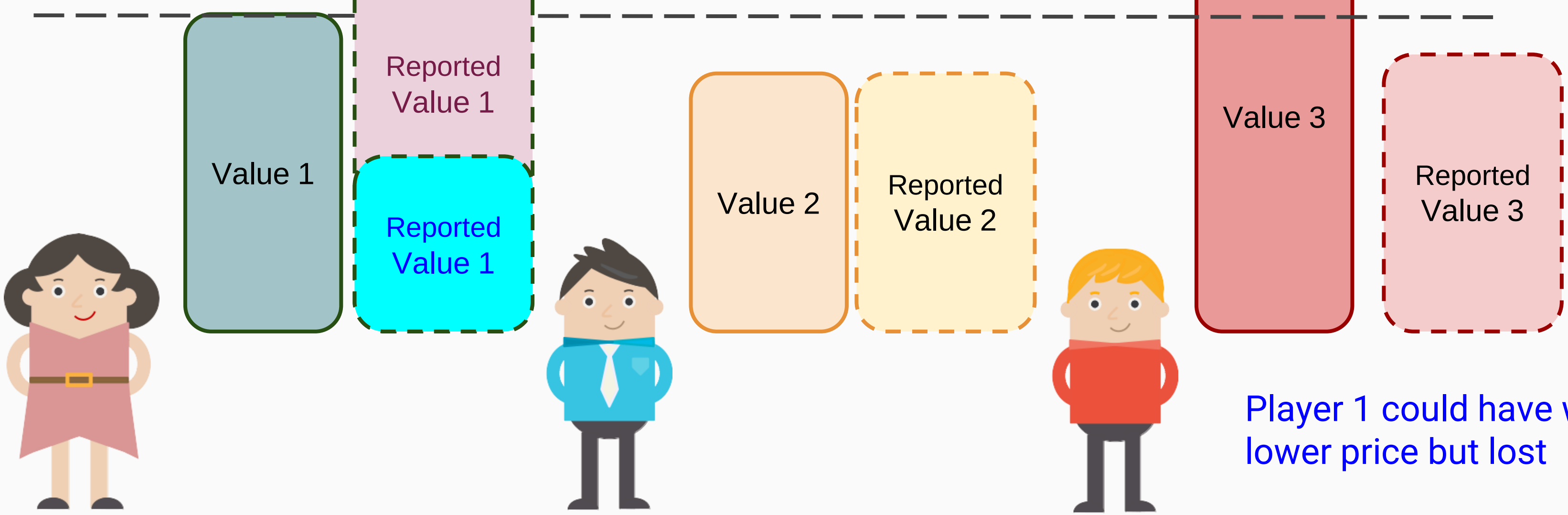
Agents want to maximize:
value - payment

- Ask everyone to bid
- Highest bidder wins the object
- Pays **second** highest bid

Second price auction

Claim: agents will be truthful **regardless of what other agents do**

Player 1 wins but pays value 3 > her true value for the object



Player 1 could have won at a lower price but lost

Can we design algorithms for a better society?

Fair division

Equal amount may have different values for an agent

Heterogeneous

Any fractional allocation is feasible

Divisible

Differing preferences

Different agents may have different preferences for the same piece

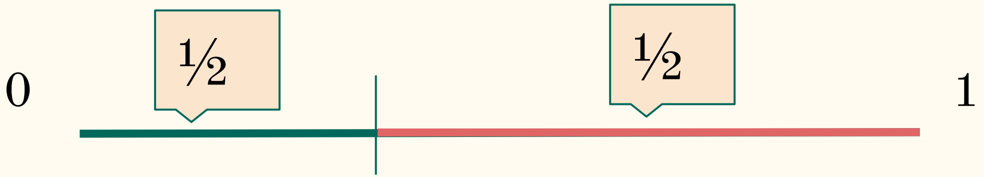
Proportional division

For each agent i

$v_i(A_i) \geq 1/n$

Each agent gets at least the average share

Normalization: for each i, $v_i([0,1]) = 1$



"I cut, you choose" algorithm

Envy free division

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j)$

Each agent likes her own share than others

Proportional?

Yes, agent 1 cuts ½, and agent 2 picks the larger

Envy-free?

Yes, agent 1 gets ½, which is the same as the other piece in his view
agent 2 picks first, can't envy the other piece

Fair division of indivisible objects

All are indivisible objects

Faculty retires and wants to give away his/her belongings to the department staff / existing faculty

Items:

1. Books
2. Shelves
3. Furnitures
4. Wall decor
5. Table decor
6. Electronic gadgets
7. Many more …

Notice that Envy-free allocation is no longer possible

Consider a single item and two agents

Envy free upto one good allocation

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j \setminus x_j)$, for some $x_j \in A_j$

Each agent likes her own share upto all but one item of every other agent

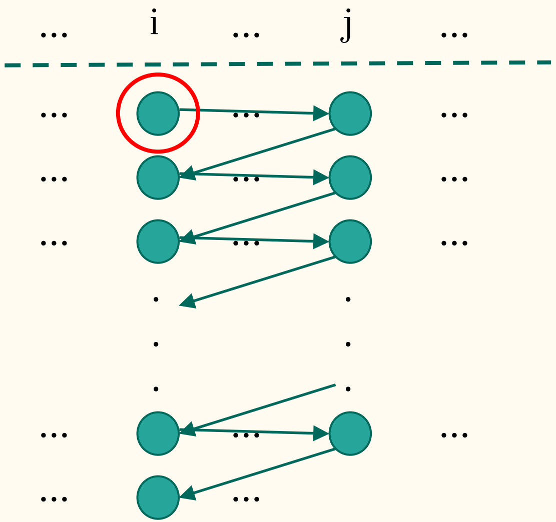
Envy free upto one good (EF-1)

All are indivisible objects

Always exists and computable in polynomial time!

Example for additive valuations: Round-Robin Algorithm

Place the agents in any arbitrary order, and ask them to pick their favorite remaining item



i does not envy j, since it picks before j

j may envy i, but not if the first item i picked is dropped

Round-Robin achieves EF-1 for additive valuations

Envy free upto one good allocation

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j \setminus x_j)$, for some $x_j \in A_j$

Each agent likes her own share upto all but one item of every other agent

- A game is an interaction between agents who want to maximize their utilities
- Game theory predicts the outcome of a game
- This is a predictive approach
- Mechanism design tries to design the game with desirable outcomes
- This is a prescriptive approach

Applications

- Online advertising – google, facebook, etc.
- Stable matching
- Kidney exchanges
- Automated priority scheduling
- Peer-grading
- Airlines scheduling
- Many more …

What you will learn in this course

- You will be equipped with a general purpose tool to analyze strategic behavior in multi-agent interaction
- Mathematically capture the situations of strategic agent modeling and interaction
- Design protocols / mechanisms that satisfy desirable economic and computational properties
- Applications in multi-agent environment like sponsored advertisements, crowdsourcing, social networks, internet-based trade

What you will need to follow this course

- Familiarity with formal mathematical reasoning
- Probability theory (detailed)
- Calculus
- Basics of computational complexity
- Moderate familiarity with computer programming (in any language)

Evaluation

- CS 405: Two quizzes -- 20% weightage for each, no project
- CS 6001: One course project (groups of size ≤ 3) -- 30% weightage, no quizzes
- One midsem and one endsem exam -- 30% weightage for each (for CS 405), 35% weightage for each (for CS 6001)
- Offline exams – as scheduled by the institute

Exam Schedule

- August 28: Quiz 1 – during class times
- Midsem: Institute scheduled (between 14 - 22 September)
- October 25: Quiz 2 – during class times
- Endsem: Institute scheduled (between 11 - 23 November)

Problem sets

- Will be given about a week before every exam (2 quizzes and 2 exams)
- You are supposed to solve all the problems
- The problem sets are NOT exhaustive – should look for other problems of similar kind from books, lecture notes, internet resources, past question papers, etc.

Tutorials

- On some Saturday/Sunday before each exam
- You may ask to revisit some topics or problem solving from the problems sets / otherwise
- If the problem is outside problem sets, post them on Piazza at least two days before bringing them to tutorials (failing this, such problems will not be discussed)
- Before asking the TA to solve a problem, you should show till what distance you have got

Team – so far ...

Swaprava Nath
Lead instructor

Ramsundar
Anandanarayanan, TA

Drashthi Doshi
TA

Sayantika Mandal
TA

Isha Arora
TA

Karan Godara
TA

Ameya Vikrama Singh
TA

Course content delivery

- The course will be taught in a **regular** classroom mode
- Lecture videos will be posted on the course website as additional resources
- Any additional reading materials will be posted on the course webpage
- Online discussion forum to clarify the doubts on the topics discussed

https://www.cse.iitb.ac.in/~swaprava/cs6001_07_2024.html

Piazza access code: `cs6001_2024`

Content sources

- Game Theory — Michael Maschler, Eilon Solan, Shmuel Zamir
- Multiagent Systems — Y. Shoham and K. Leyton Brown, Cambridge University Press
- Game Theory and Mechanism Design — Y. Narahari, World Scientific and IISc Press
- Lecture notes on Theory of Mechanism Design, by Debasis Mishra, Indian Statistical Institute, New Delhi
- Lecture notes on Individual and Collective Choice, by Arunava Sen, Indian Statistical Institute, New Delhi
- Preprints of Introduction to Economics and Computation, David C Parkes and Sven Seuken
- Several research papers
- Lecture notes (non-reviewed) on the course webpage