??

1 True or False (5 parts, 12 points)

Indicate whether each statement below is true or false by circling the correct answer.

1.1 [2 points]

A 4-D hypercube has 8 nodes.

True **False**

Answer: False. 16 nodes

1.2 [2 points]

GPUs use extensive branch prediction to improve performance.

True **False**

Answer: False.

1.3 [2 points]

CUDA supports global synchronization.

True **False**

Answer: False.

Indicate whether each statement below is true or false by circling the correct answer, and justify your answer in at most one or two sentences. **Your justification is worth more than your true-false designation.**

1.4 [3 points]

Consider the `scale()` function:

```
void scale(double *A, double *B, double c, size_t n)
{
  for (size_t i = 0; i < n; ++i)
    A[i] += c * B[i];
}
```

One can parallelize the `scale()` function by simply adding a `#pragma omp parallel for` above the loop.

True **False**

Answer: False: If `A` and `B` are aliased, then the parallel version may contain races.

1.5 [3 points]

Suppose that the running time of a recursive program satisfies the recurrence $T(n) = 2T(n/2) + \Theta(n^2)$ (with a base condition of $T(n) = \Theta(1)$ for sufficiently small n). Optimizing the leaves of the recursion should result in a significant speedup.

True **False**

Answer: False. Looking at the recursion, there will be $\Theta(n^{\log_2 2}) = \Theta(n)$ leaves (each performing some constant-time task), but $\Theta(n^2)$ work done at each internal node. Therefore, most of the work done in this tree is not being done at the leaves, and optimizing them is most likely a waste of time.

2 Multiple Choice (4 parts, 8 points)

2.1 [2 points]

Consider the snippet of code below that computes the matrix-vector product of a matrix A of size $n \times n$ and a vector x of size n.

```
// Computes: y <- y + A . x
for (i = 0; i < n; ++i)  // Loop 1
  for (j = 0; j < n; ++j)  // Loop 2
    y[i] += A[i,j] * x[j];
```

Which for-loops may be **safely** converted into parallel-for loops.

A Only loop 1
B Only loop 2
C Both loops 1 and 2
D Neither loop 1 nor 2

Answer: A

2.2 [2 points]

How much memory does the SUMMA algorithm need compared to the 1-D matrix multiply algorithm discussed in class?

A More than 1D
B Less than 1D
C Same as 1D

Answer: A, B, C

2.3 [2 points]

What is the **diameter** defined as the maximum number of message hops between processors in a $N \times N$ processor mesh, with $N = \sqrt{P}$?

A $\sqrt{P}$
B $2(\sqrt{P} - 1)$
C $2\sqrt{P}$
D P
E P^2

Answer: B

2.4 [2 points]

Ben is worried about races and uses a critical section to synchronize the code snippet below.

```
#pragma omp parallel for
for (i = 0; i < n; ++i) {
  #pragma omp critical
  sum += a[i];
}
```

The code does not achieve any speedup. (Circle all that apply)

A The code still has a race and the race slows down the program
B The critical section only allows one iteration to execute the body of the loop at any time
C Ben could have used OpenMP reduction to achieve the same result and could have been faster
D All of the above

Answer: B, C.

3 Analysis of Parallelism (4 parts, 20 points)

3.1 [5 points]

Five students have implemented recursive Fibonacci programs, where the base case of each program returns 1 if the program input is $\mathtt{n} = 0$ or $\mathtt{n} = 1$. For $\mathtt{n} > 1$, the various students calculate Fibonacci using the code snippets for the recursive cases shown below:

a:

```
x = fib(n-1);
y = fib(n-2);
```

b:

```
x = spawn fib(n-1);
y = spawn fib(n-2);
sync;
```

c:

```
x = fib(n-1);
y = spawn fib(n-2);
sync;
```

d:

```
y = spawn fib(n-2);
x = fib(n-1);
sync;
```

e:

```
x = spawn fib(n-1);
y = fib(n-2);
sync;
```

Assume that the overhead of spawning a function is about 10 times the cost of an ordinary function call. Rank these codes in order of the performance you would expect on a 32-core machine (e.g., fastest $>$ second fastest $> \cdots >$ slowest):

__________ $>$ __________ $>$ __________ $>$ __________ $>$ __________

Answer: d > e > b > a > c

For each of the multiple-choice questions below, circle the letter corresponding to the correct answer. **You need to explain your answers.**

3.2 [5 points]

Consider the following multithreaded function, which implements a dot product of two vectors, each of size n, in parallel:

```
double dot_product(double *A, double *B, int n)
{
  if (n == 1)
    return *A * *B;
  else
  {
    int mid = n / 2;
    double p1 = spawn dot_product(A, B, mid);
    double p2 = dot_product(A + mid, B + mid, n - mid);
    sync;
    return p1 + p2;
  }
}
```

What is the parallelism of the `dot_product` function?

A $\Theta(\lg n)$
B $\Theta(n/\lg n)$
C $\Theta(n)$
D $\Theta(n^2/\lg n)$
E None of the above

Answer: B, $\Theta(n/\lg n)$

$T(\mathrm{n}) = 2 \cdot T(\mathrm{n}/2) + \Theta(1)$. So, by case 1 of the Master Theorem ($\mathrm{n}^{\log_2 2} = \omega(1)$), the work is $T(\mathrm{n}) = \Theta(\mathrm{n})$.

$S(\mathrm{n}) = S(\mathrm{n}/2) + \Theta(1)$. So, by case 2 of the Master Theorem ($\mathrm{n}^{\log_2 1} = \Theta(1)$), the span is $S(\mathrm{n}) = \Theta(\log \mathrm{n})$.

Parallelism = Work/Span = $\Theta(n/\lg n)$

3.3 [5 points]

A matrix L is ***lower triangular*** if $L(i,j) = 0$ for $i < j$. A lower-triangular matrix can be stored compactly in a one-dimensional array by storing row i, which has length $i+1$ starting in location $i(i+1)/2$.

Consider the following function that computes the matrix-vector product of a lower triangular matrix L of size $n \times n$ and a vector X of size n:

```
void lt_matrix_vector_product(double *L, double *X, double *B, int n)
{
  parallel for (int i = 0; i < n; i++)
  {
    int offset = (i * (i + 1)) / 2;
    B [i] = dot_product(L + offset, X, i + 1);
  }
}
```

For the purposes of analysis, assume that the grain size for the `parallel for` is 1. What is the parallelism of `lt_matrix_vector_product` function?

A $\Theta(n/\lg^2 n)$
B $\Theta(n)$
C $\Theta(n^2/\lg^2 n)$
D $\Theta(n^2/\lg n)$
E None of the above

Answer: D, $\Theta(n^2/\lg n)$

To calculate the work, we simply compute the running time of its serialization, which we obtain by replacing the parallel for loop with ordinary for loop. Thus, we have $T(n) = \Theta(n^2)$.

A compiler can implement a parallel for loop as a divide-and-conquer subroutine using nested parallelism. Thus $S(n) = \Theta(\log \mathrm{n}) + \Theta(\log \mathrm{n}) = \Theta(\log \mathrm{n})$

Parallelism = Work/Span = $\Theta(n^2/\lg n)$

3.4 [5 points]

Now consider a different way of computing the matrix-vector product for a lower-triangular matrix:

```
void new_lt_matrix_vector_product(double *L, double *X, double *B, int n)
{
  for (int i = 0; i < n; i++)
  {
    int offset = (i * (i + 1)) / 2;
    B [i] = spawn dot_product(L + offset, X, i + 1);
  }
  sync;
}
```

What is the parallelism of `new_lt_matrix_vector_product` function?

- **A** $\Theta(n/\lg n)$
- **B** $\Theta(n)$
- **C** $\Theta(n^2/\lg^2 n)$
- **D** $\Theta(n^2/\lg n)$
- **E** None of the above

Answer: B, $\Theta(n)$

Work is the same as 5.3.

The span is $\Theta(n)$ because each iteration of the inner parallel dot product contains n iterations of the outer (serial) for loop.

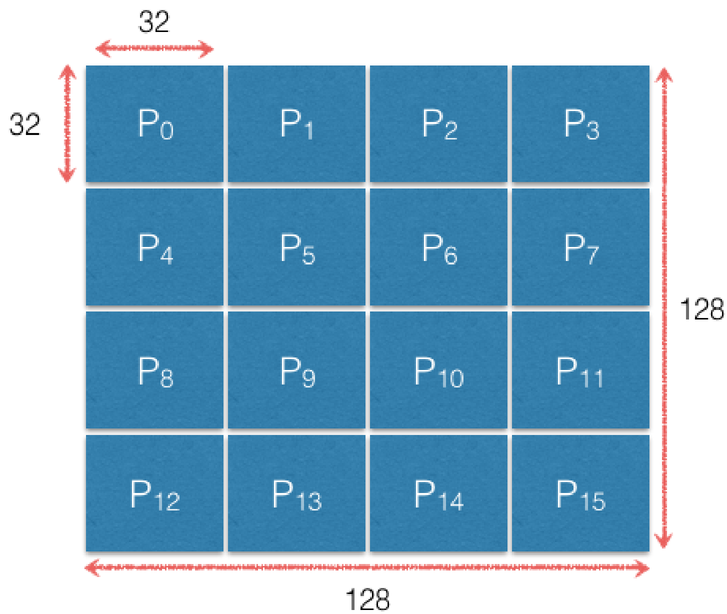
Parallelism = Work/Span = $\Theta(n^2/n) = \Theta(n)$

4 Distributed Memory (3 parts, 10 points)

4.1 [2 points]

Draw a picture to show how a 128 x 128 matrix A would be distributed among 16 processors using a 2-D block-cyclic distribution with a block size of 32 x 32 .

Answer:



4.2 [4 points]

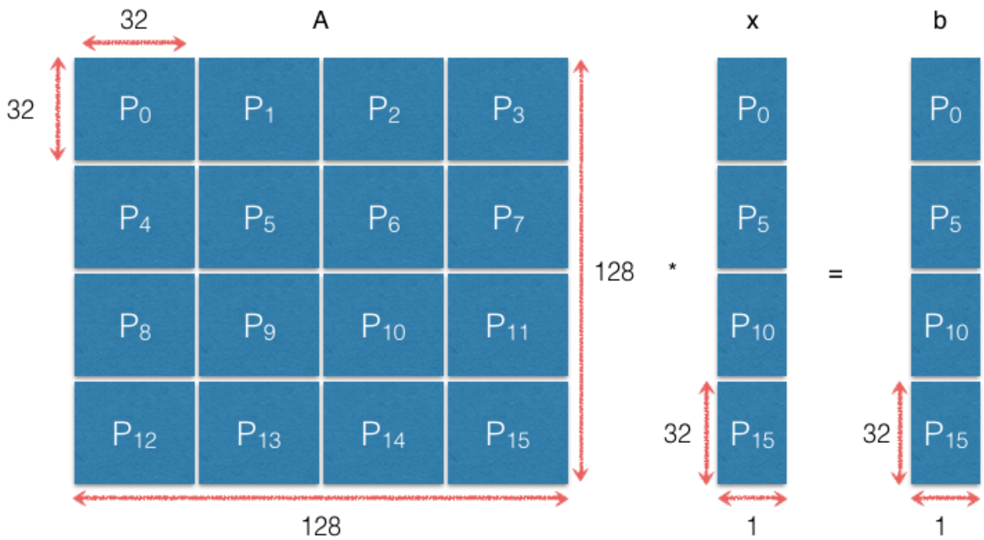
Sketch and describe an algorithm for multiplying the matrix with a 128 x 1 vector x using collective communication algorithms. You may assume that the vector x and the result vector b are stored on the diagonal processors.

Answer:

1. Broadcast x along processor columns. For example, P_0 broadcasts it's 32×1 block of x to processors P_4, P_8, and P_{12}.

2. All processors multiply the 32×32 block of A they own with the received 32×1 block of x.

3. Reduce along processor rows at P_0, P_5, P_{10}, and P_{15} for the final result vector b.



4.3 [4 points]

Write expressions for the computation and communication costs of your algorithm.

Answer:

$$
\begin{aligned}
T_{comp} &= 2 * (32)^2 \\
T_{comm} &= T_{tree\_bcast} + T_{reduce} \\
&= \left(\alpha + \frac{32}{\beta}\right) * \log 4 + \left(\alpha + \frac{32}{\beta}\right) * \log 4
\end{aligned}
$$