UCIrvine | The Henry Samueli School of Engineering

Parallel Algorithms

EECS 221: Intro to High-Performance Computing

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April 20, 2017

Inclusive Prefix-Sums (Scan)

- The prefix-sums operation takes a binary associative operator ⊕, and an array of *n* elements

  $$[x_0, x_1, \ldots, x_{n-1}]$$

  and returns the array

  $$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})]$$

- Example: If ⊕ is addition

3	1	7	0	4	1	6	3

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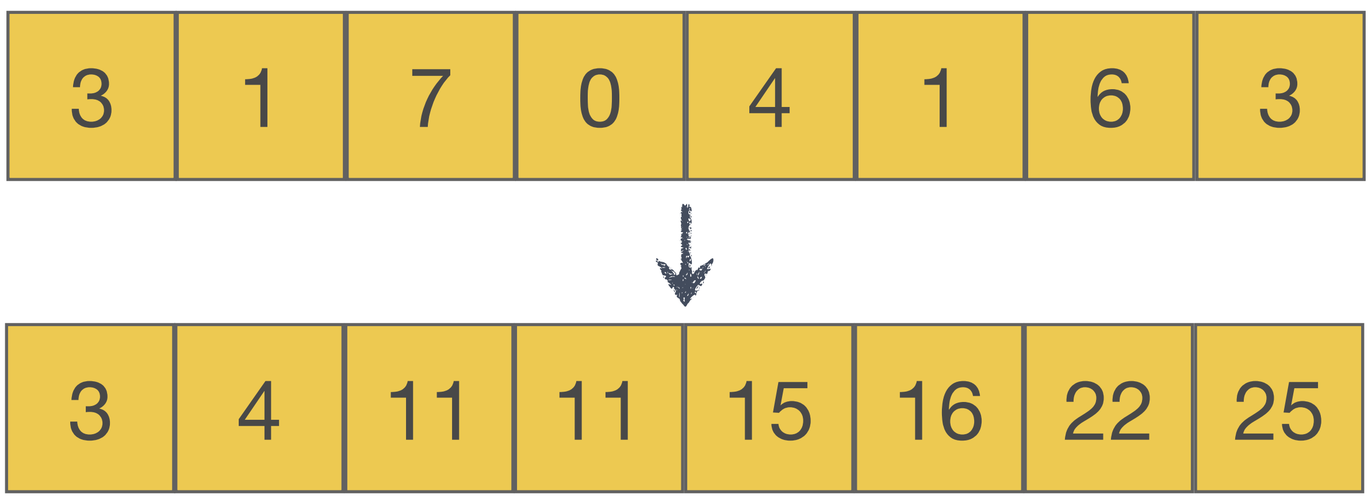

Exclusive Prefix-Sums (Scan)

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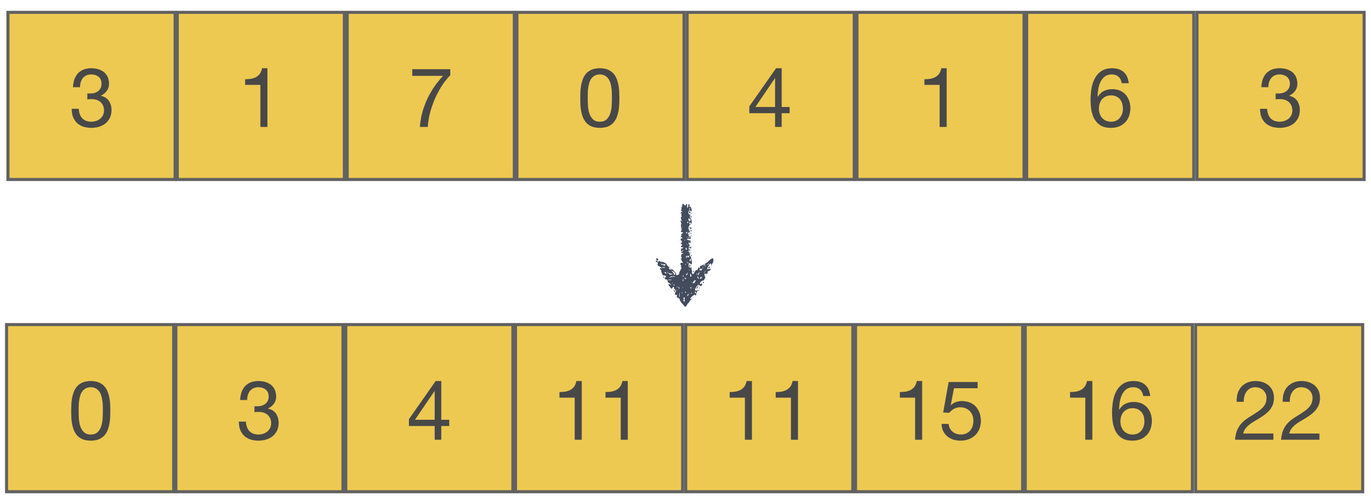

Work Efficient Sequential Scan

```
for i ← 1 to n − 1 do
    A[i] ← A[i − 1] + A[i]
end for
```

$$W_{scan}(n) = \Theta(n)$$

$$D_{scan}(n) = \Theta(n)$$

Inclusive Parallel Scan

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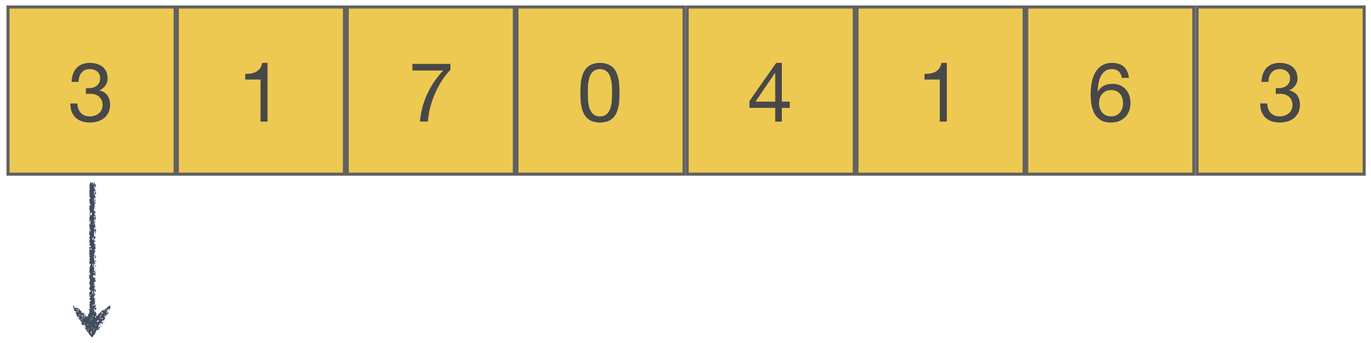
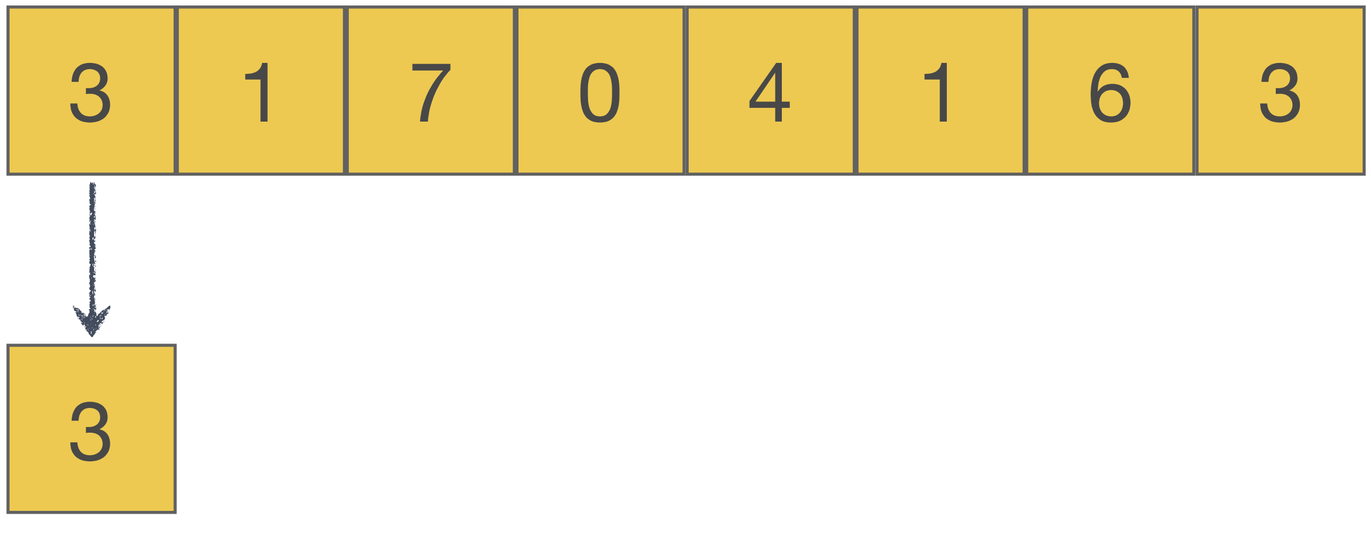


Inclusive Parallel Scan



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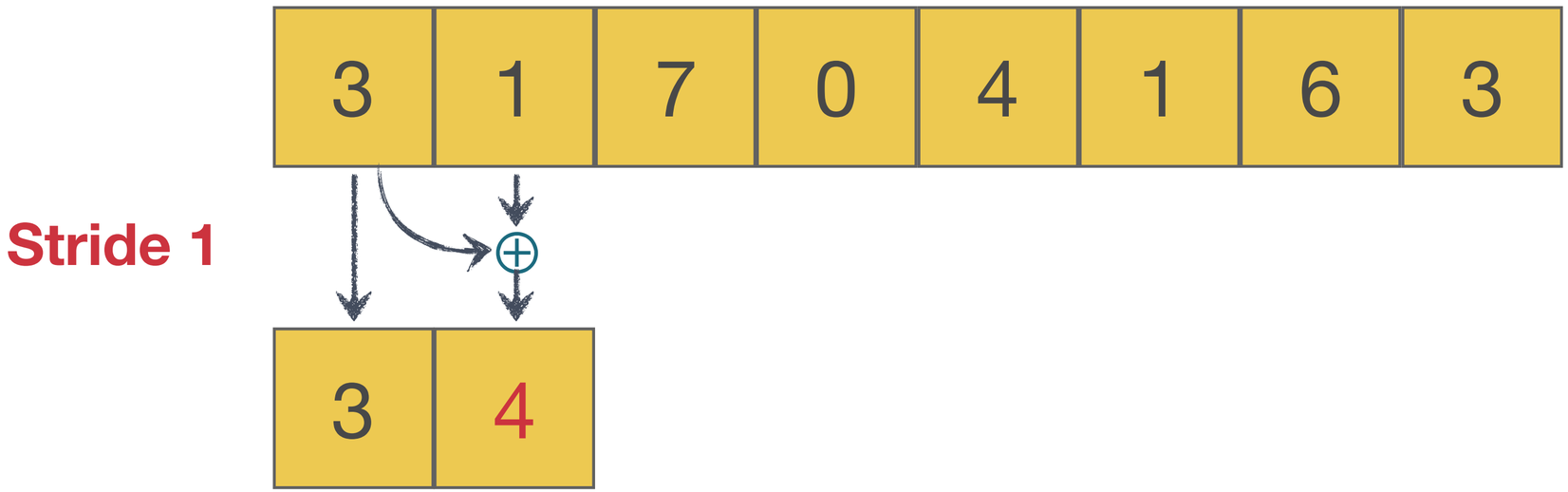

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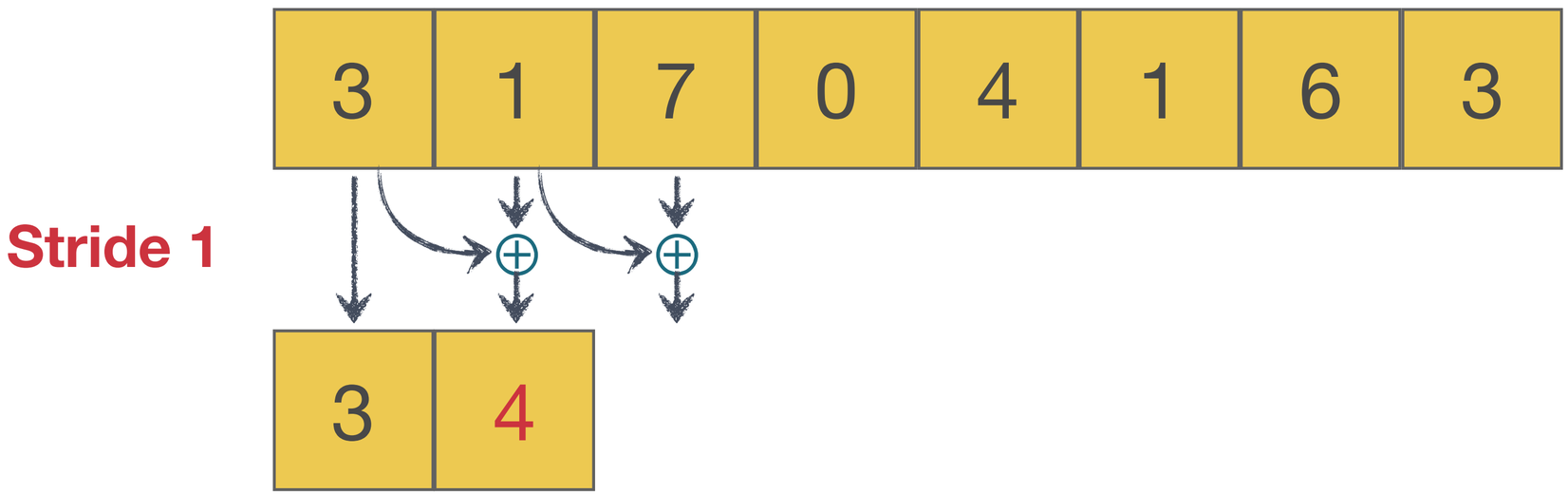

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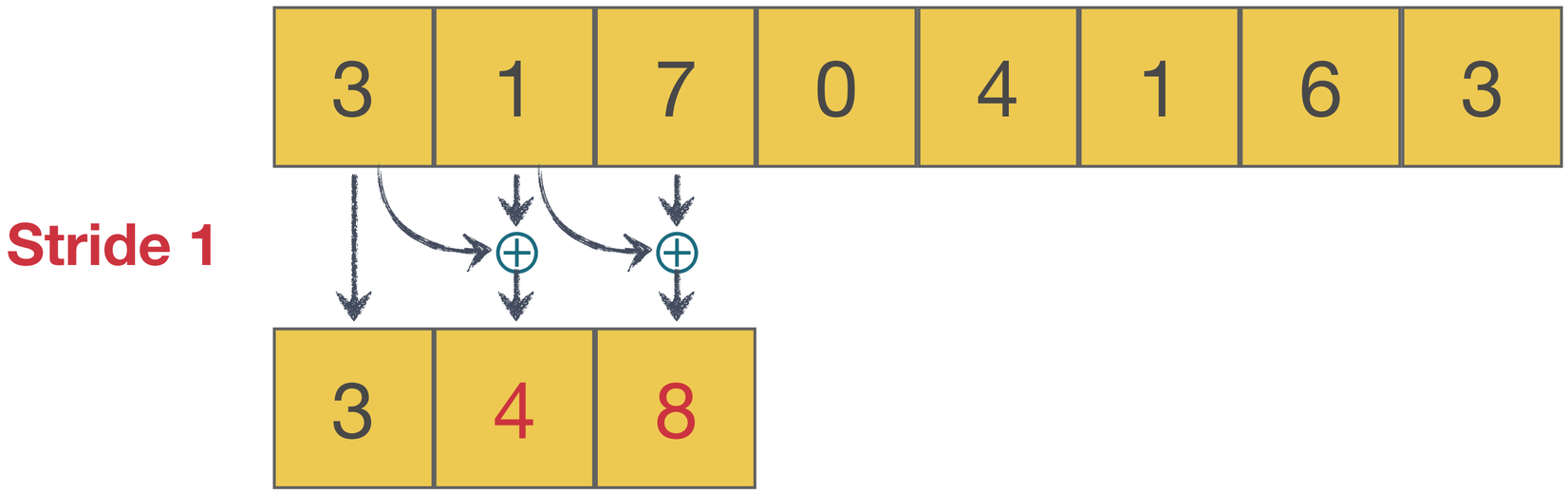

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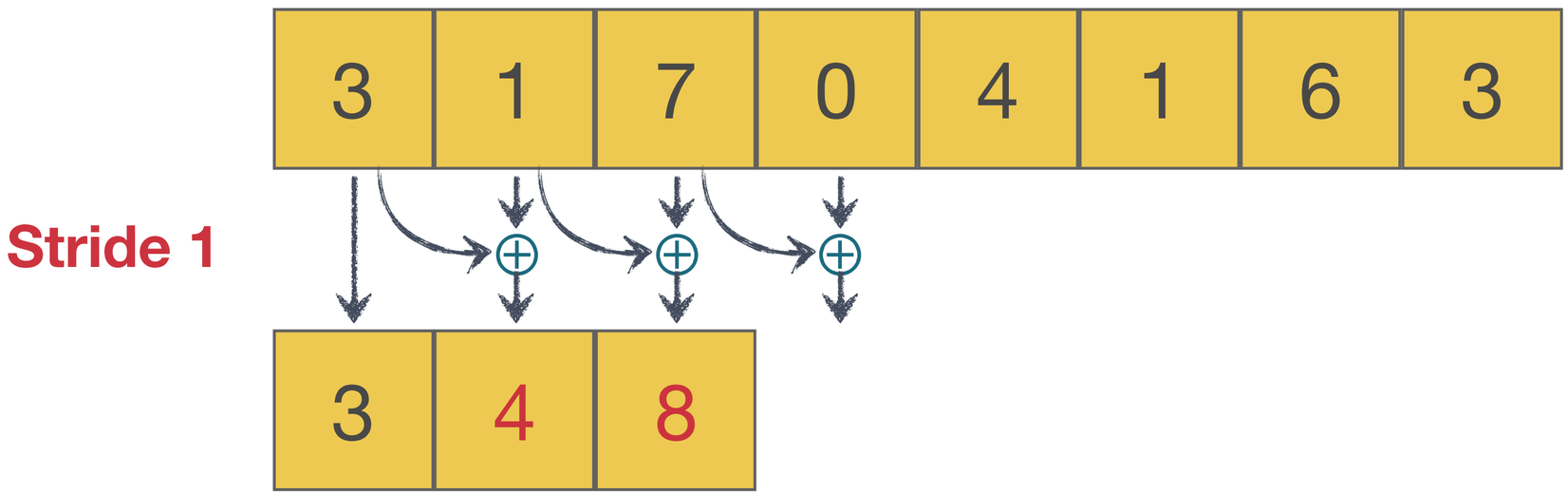

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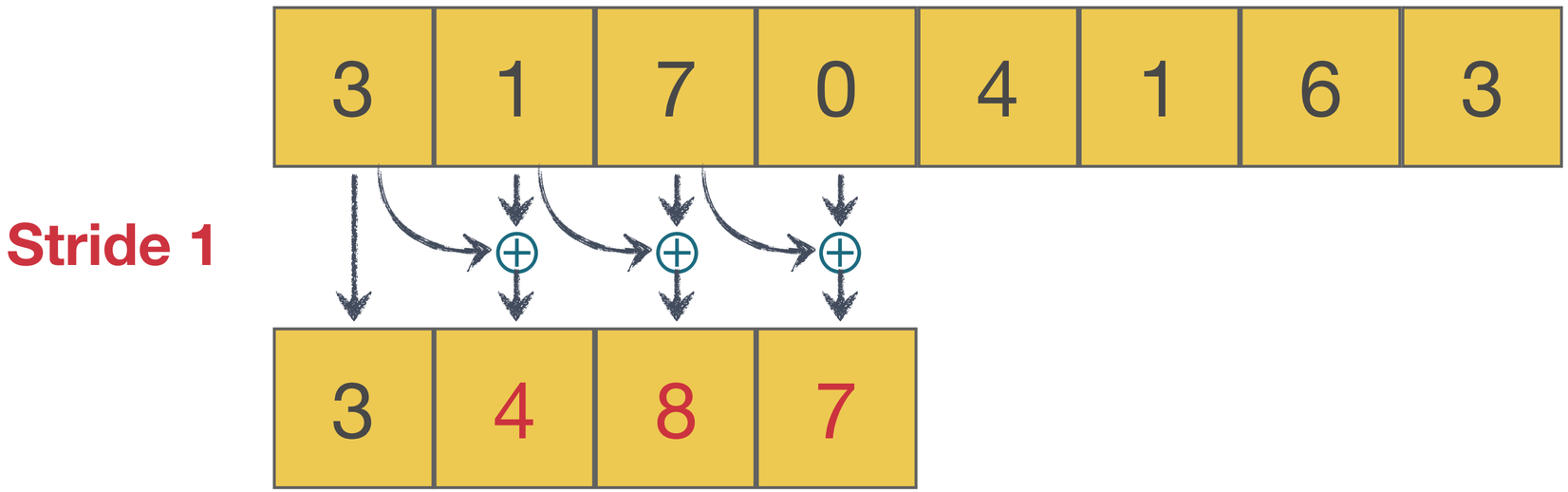

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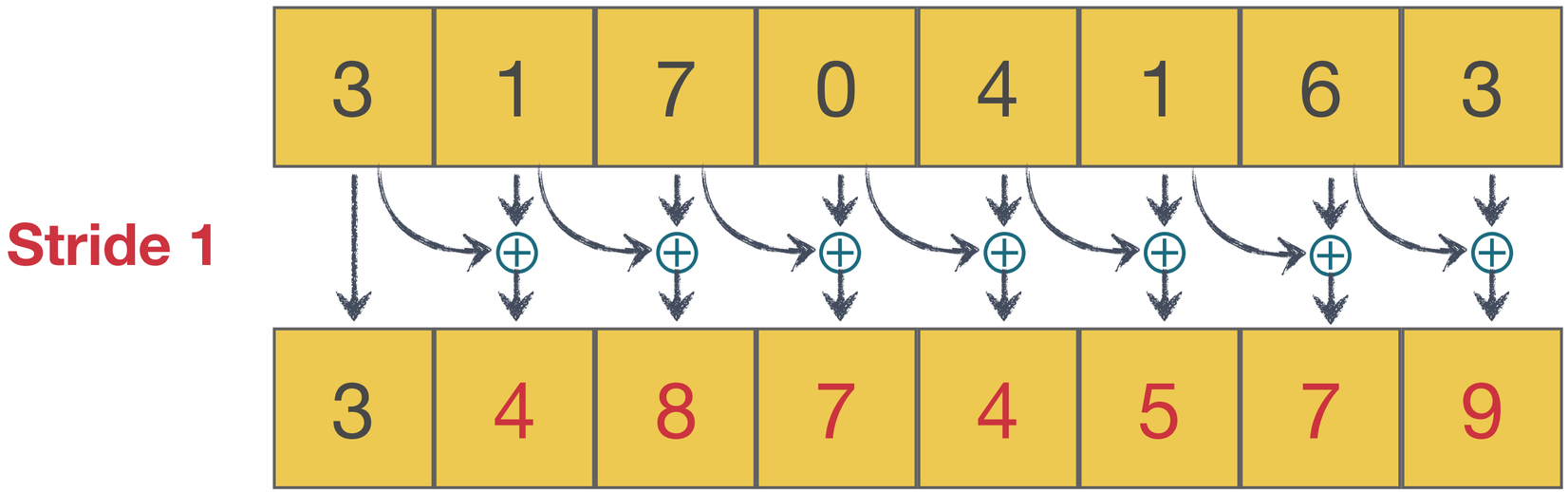

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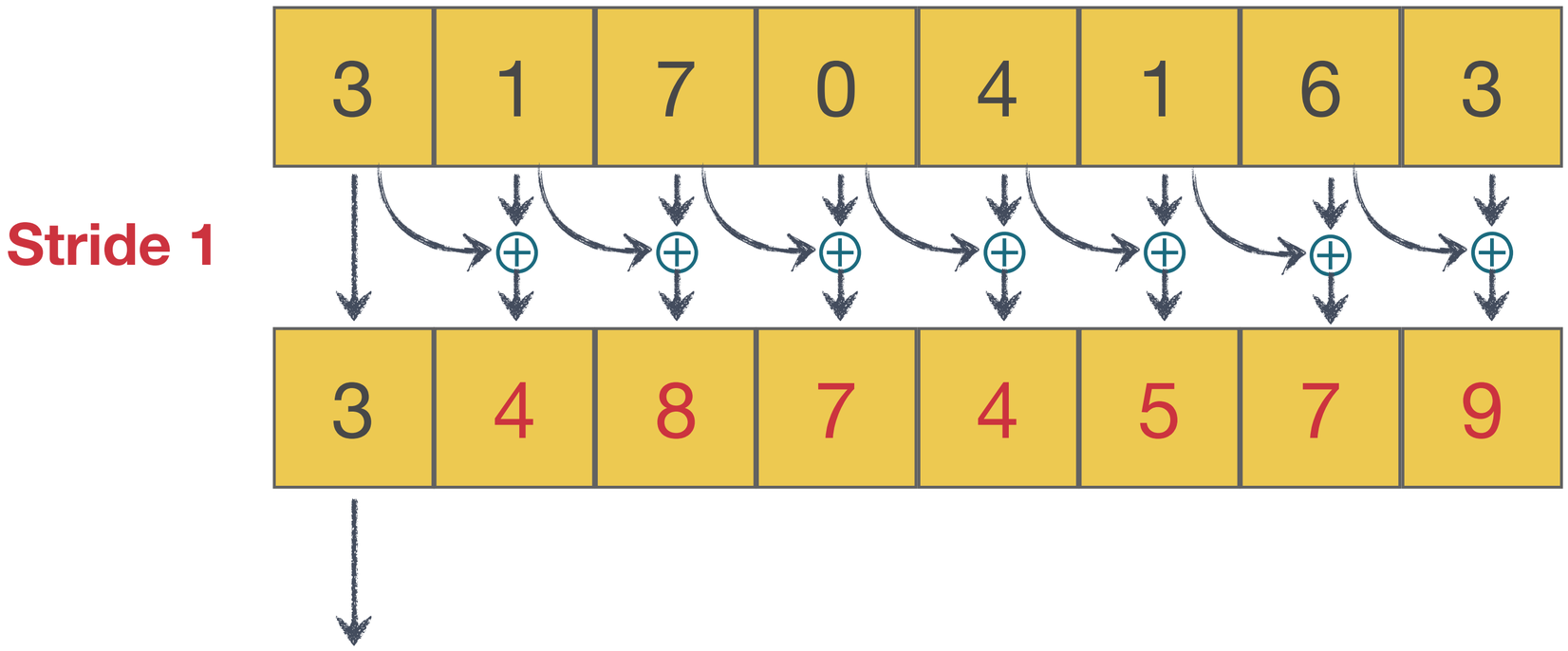

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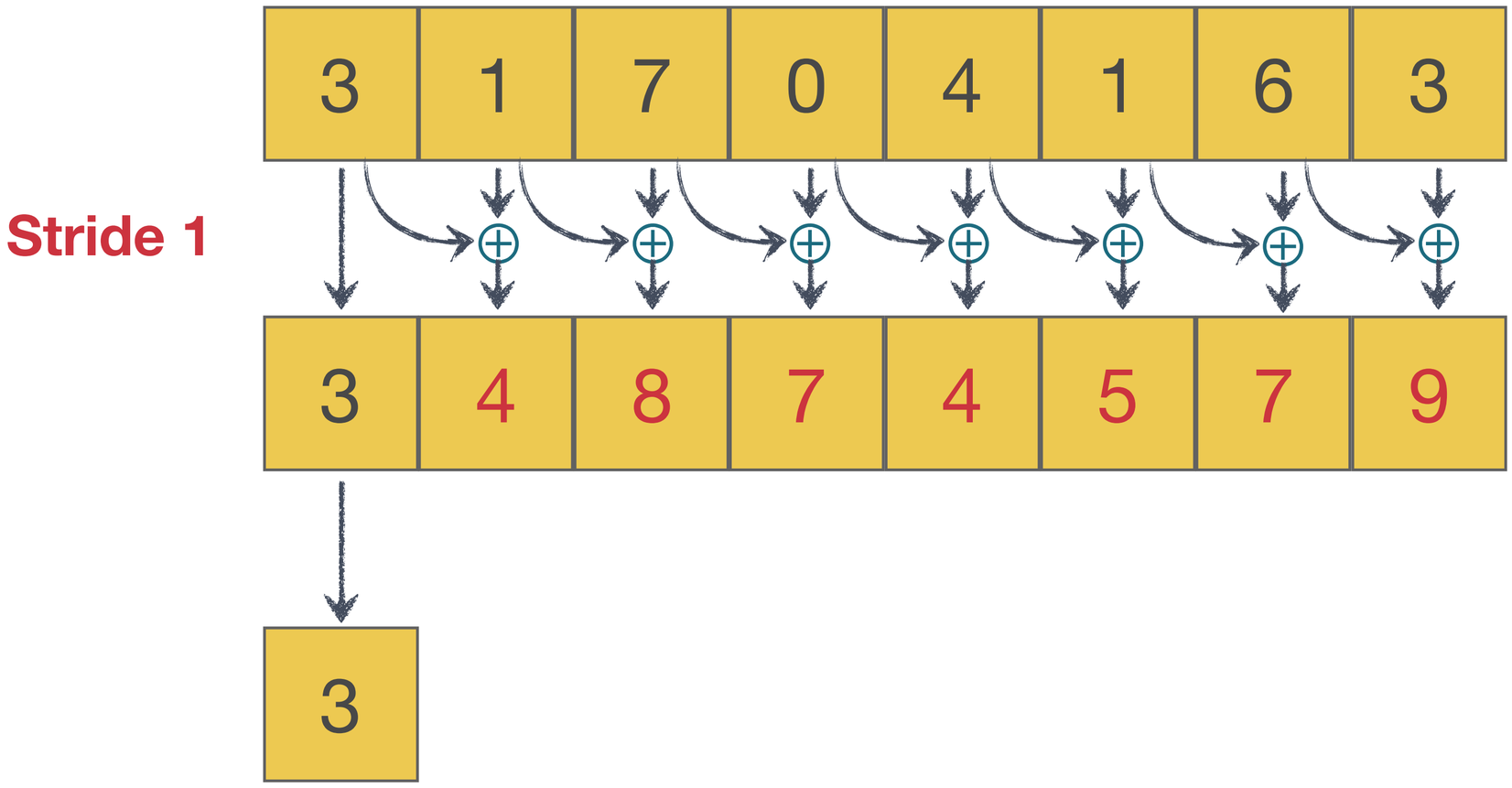

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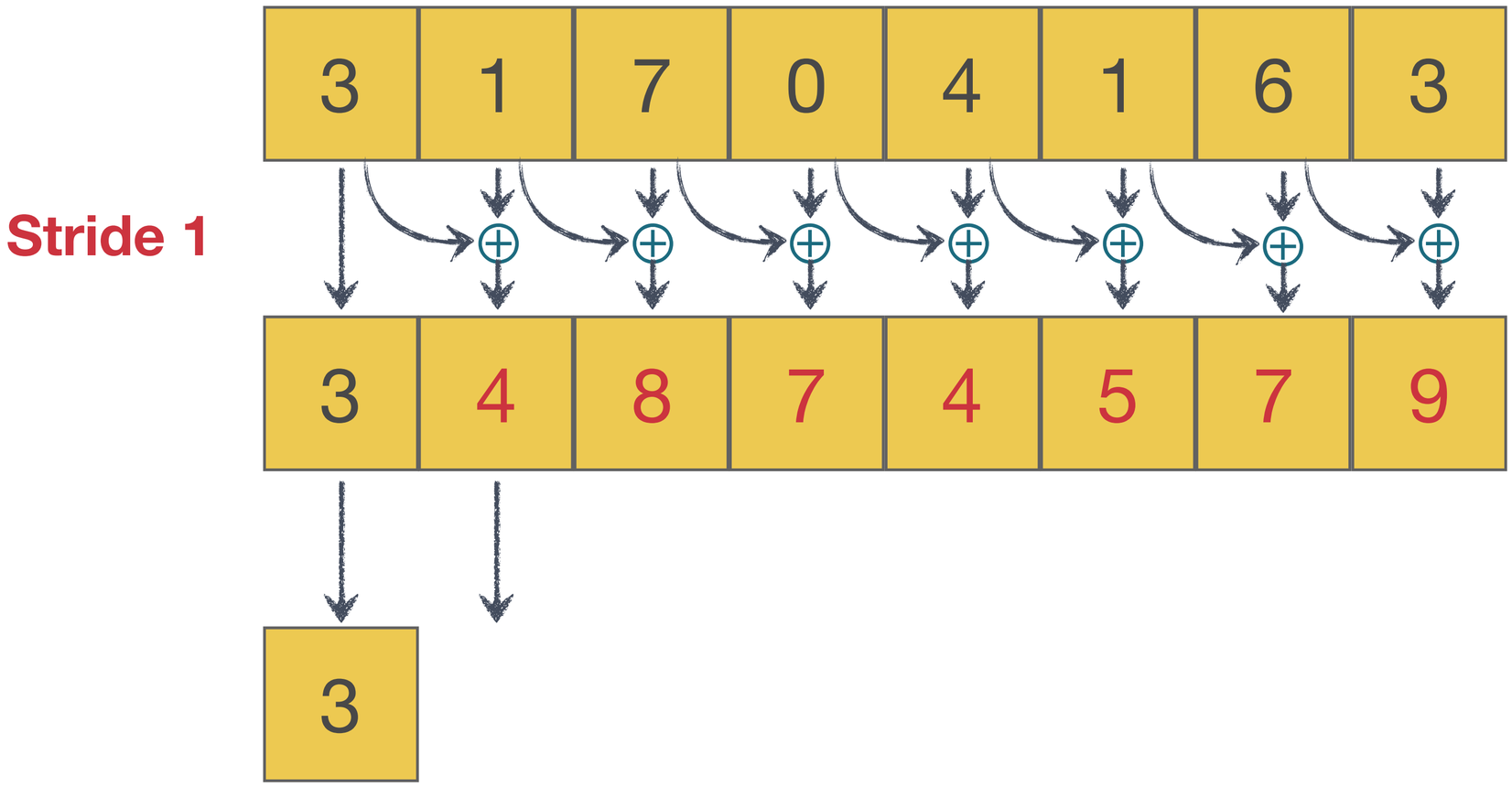

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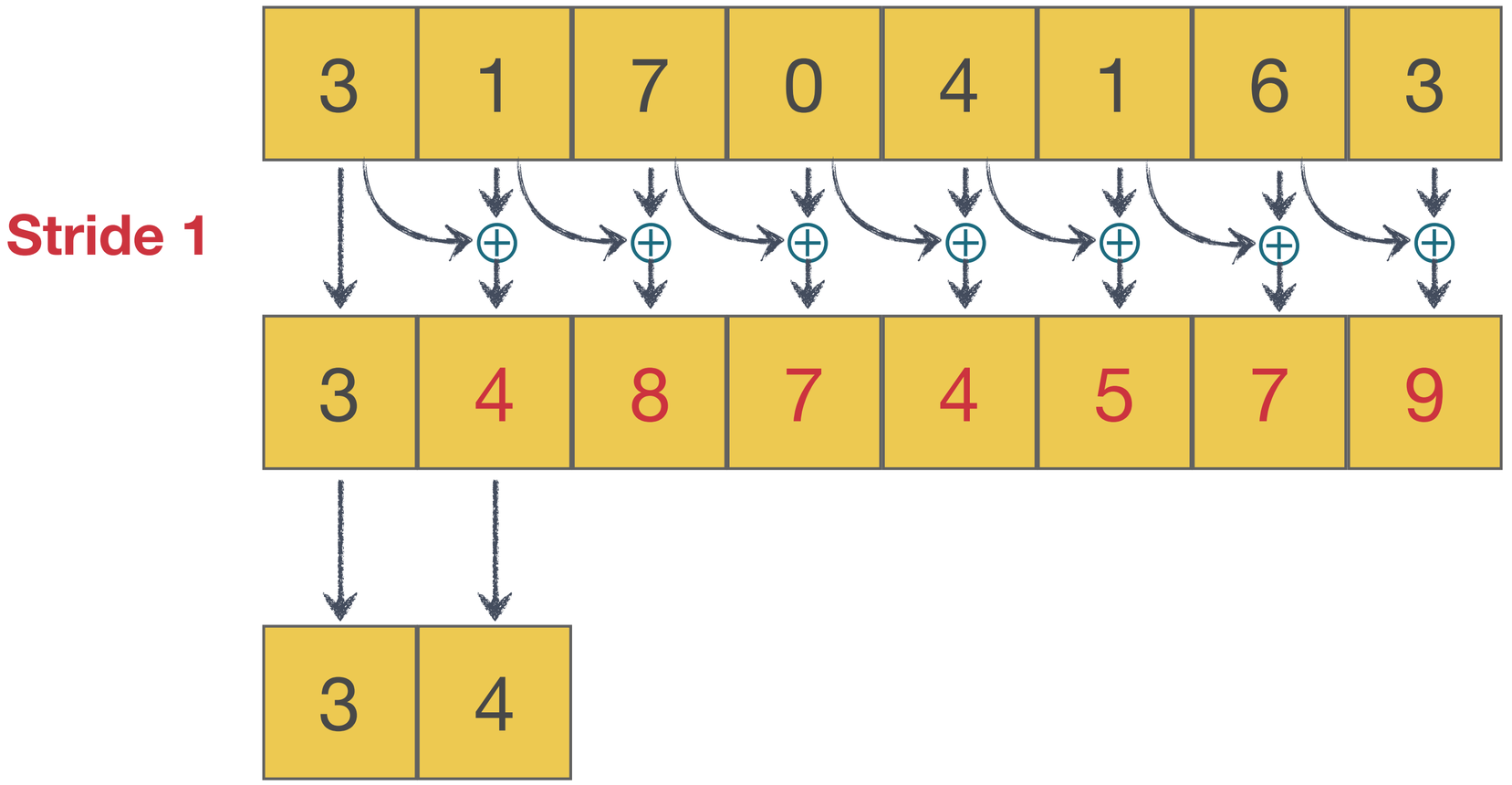

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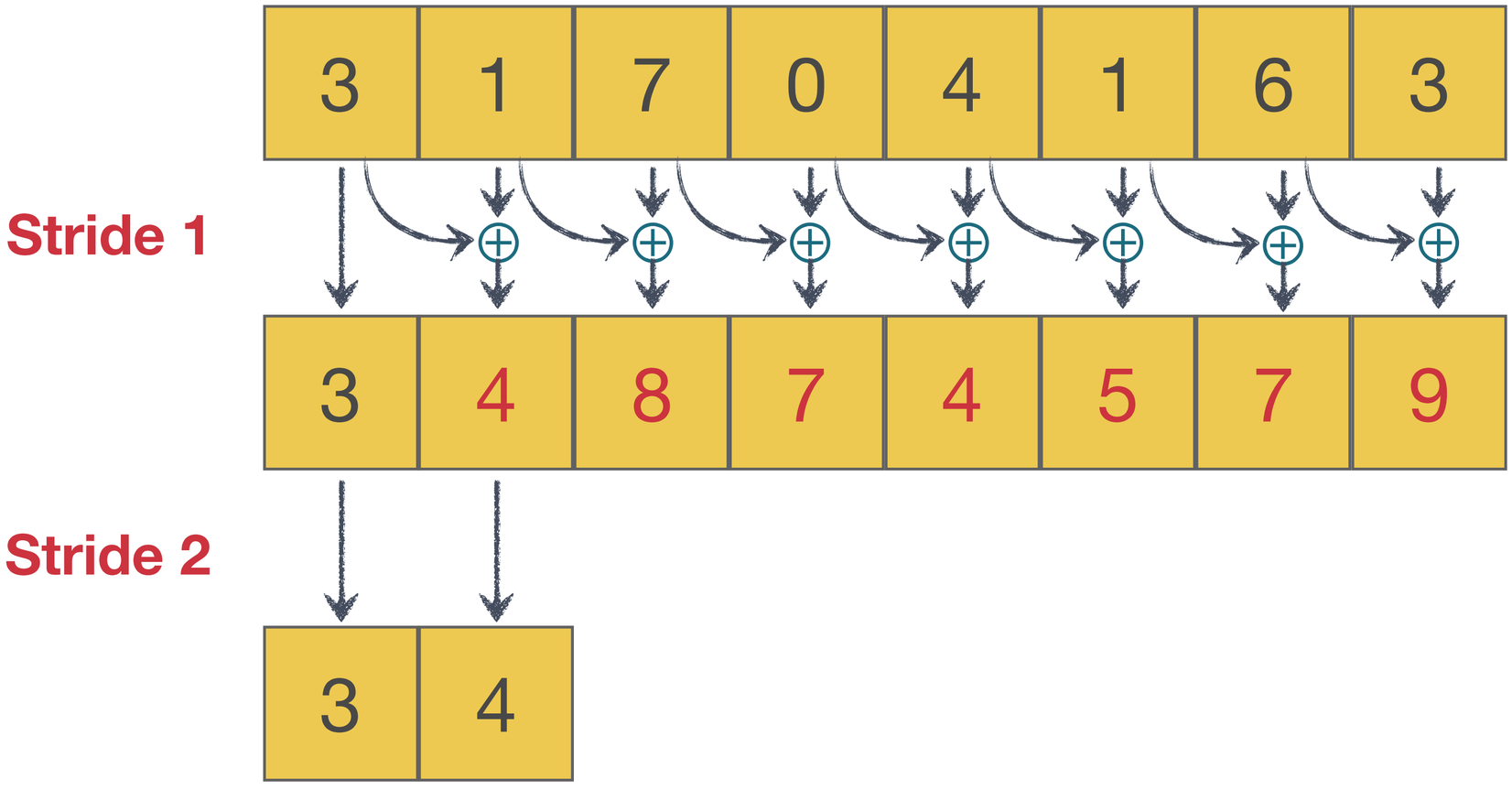

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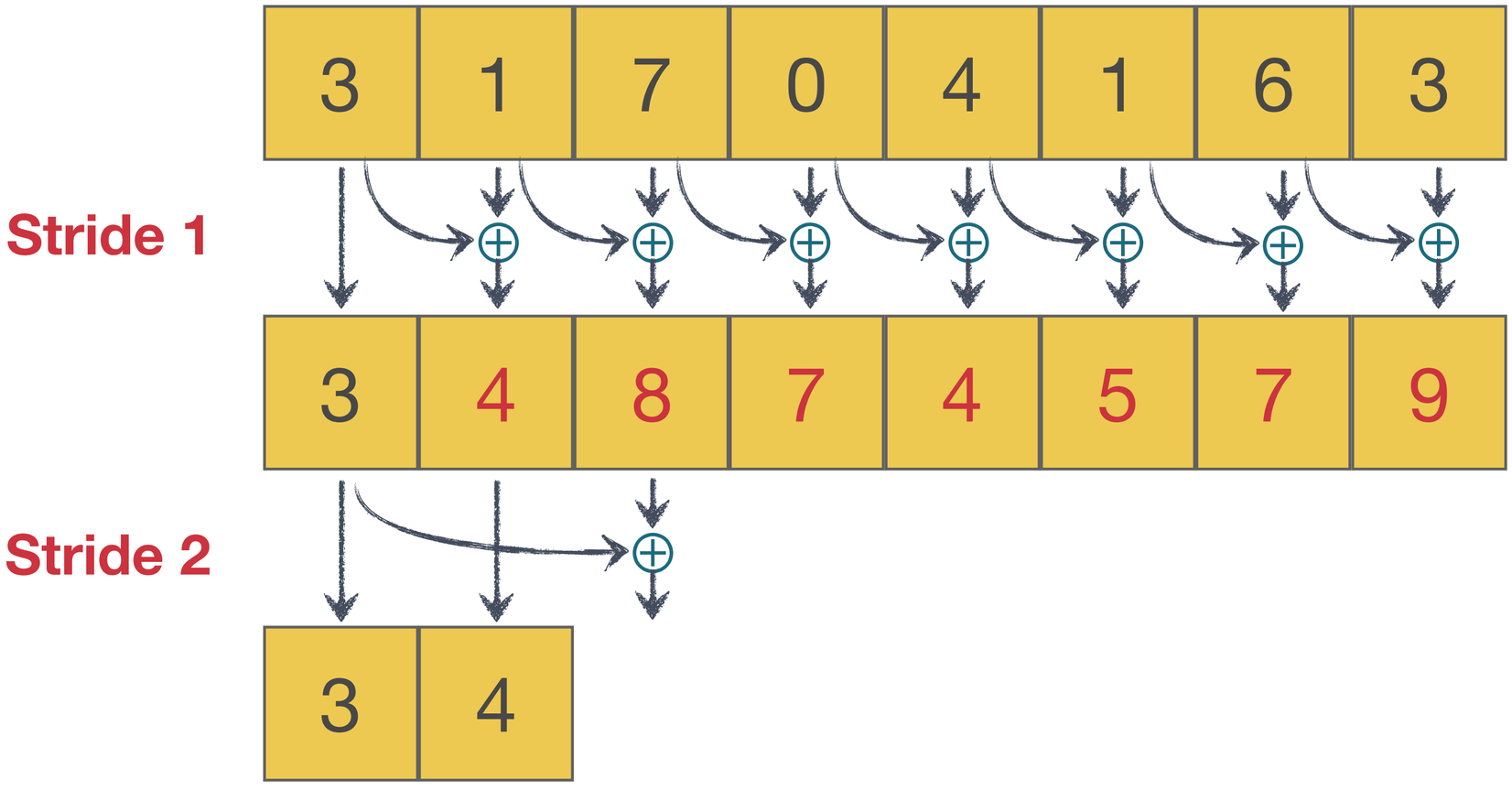

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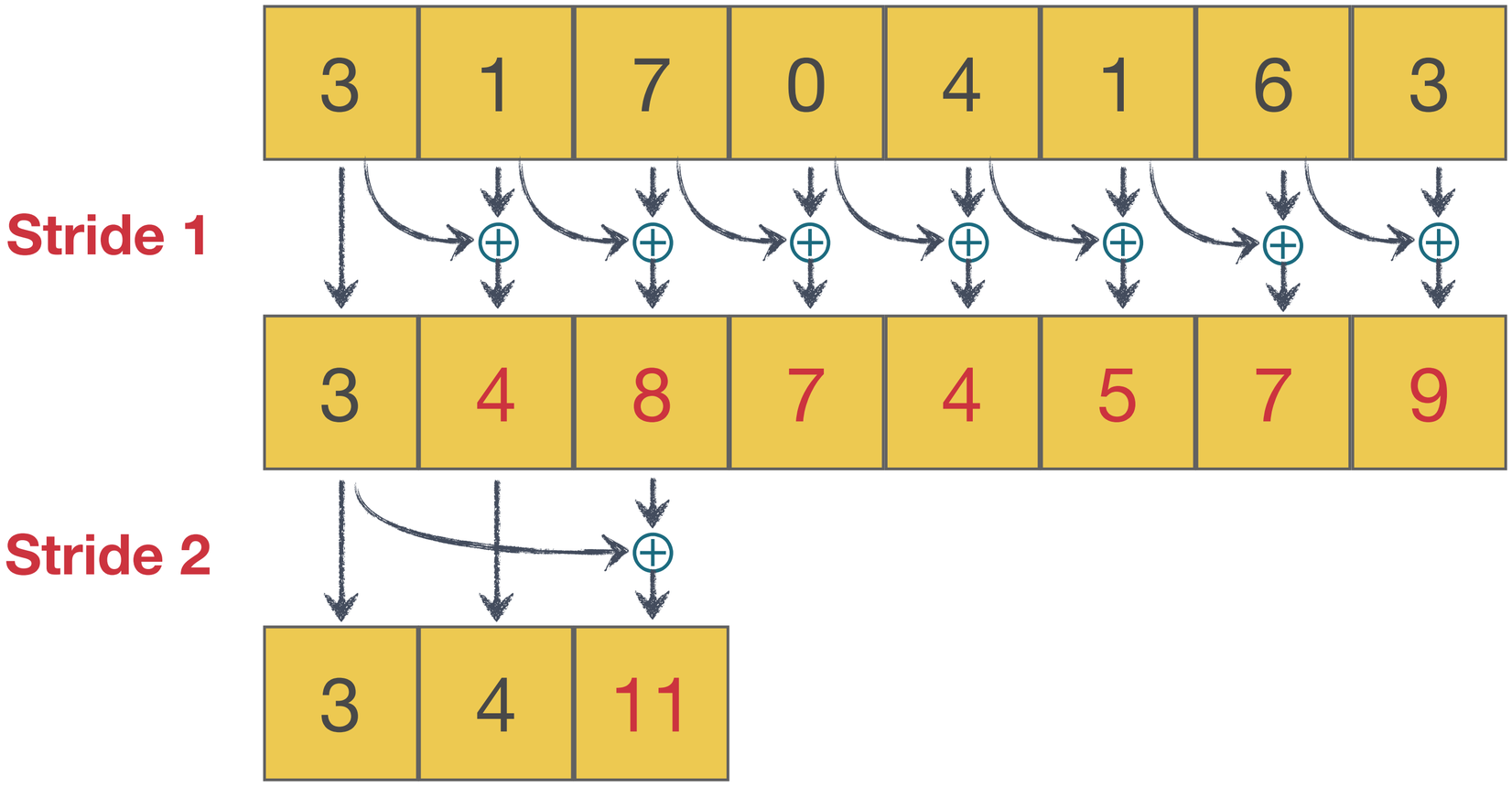

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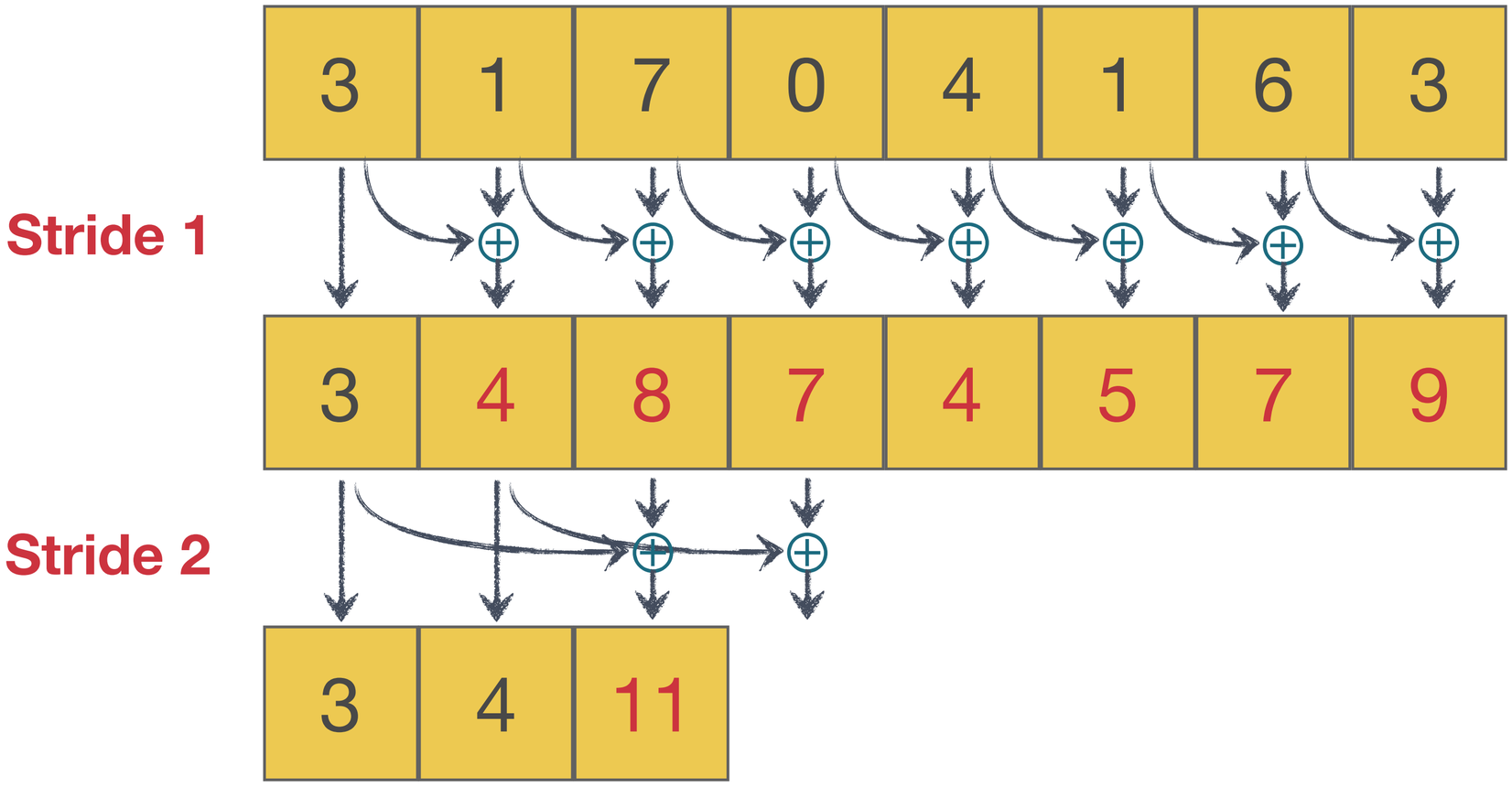

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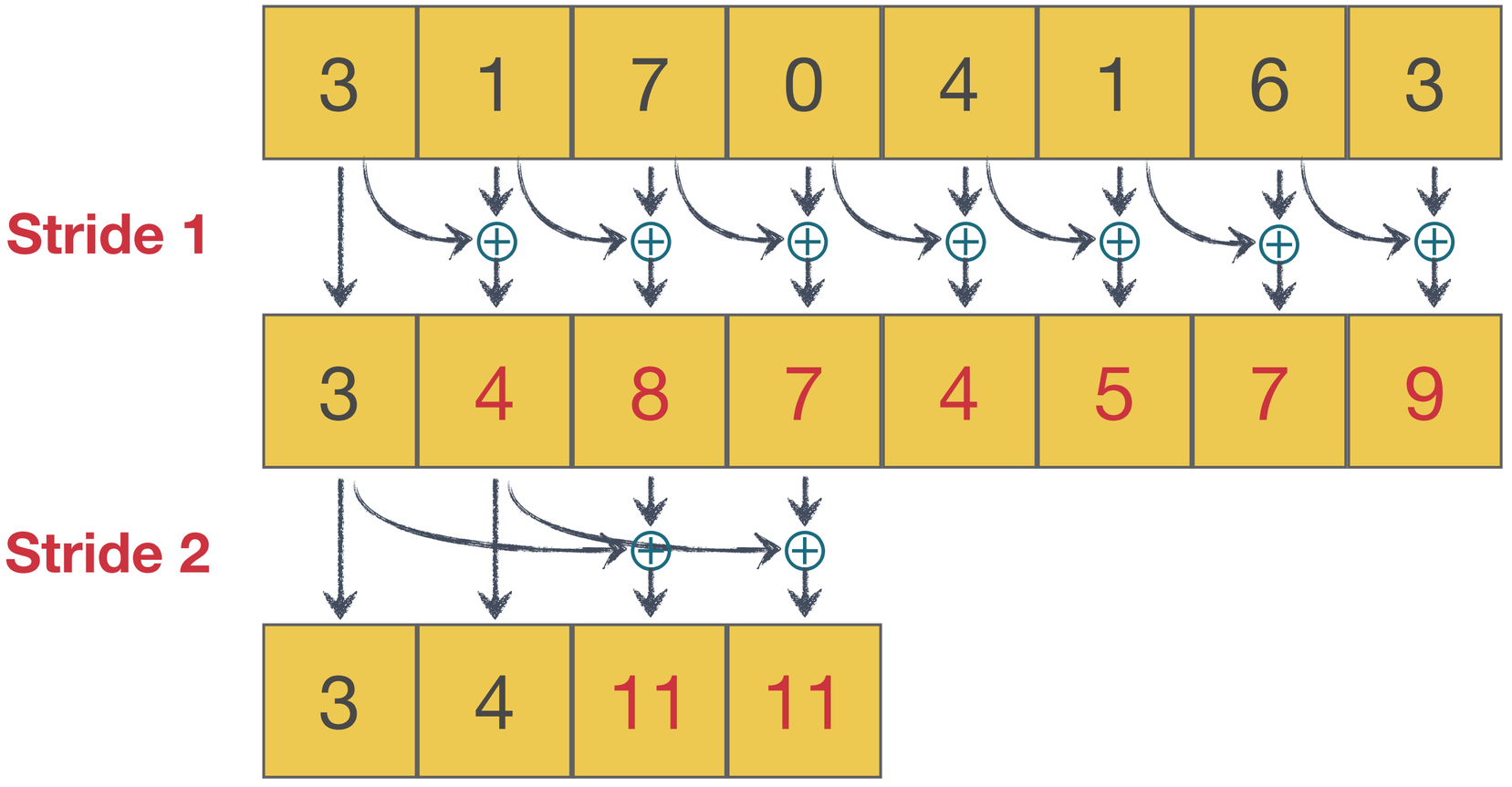

Inclusive Parallel Scan

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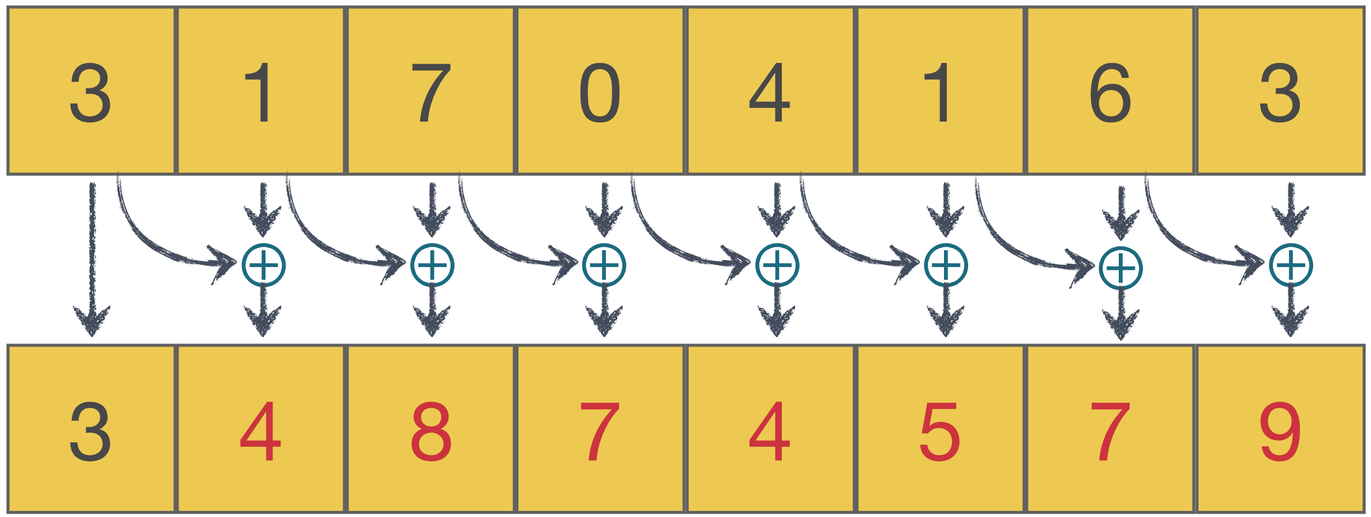

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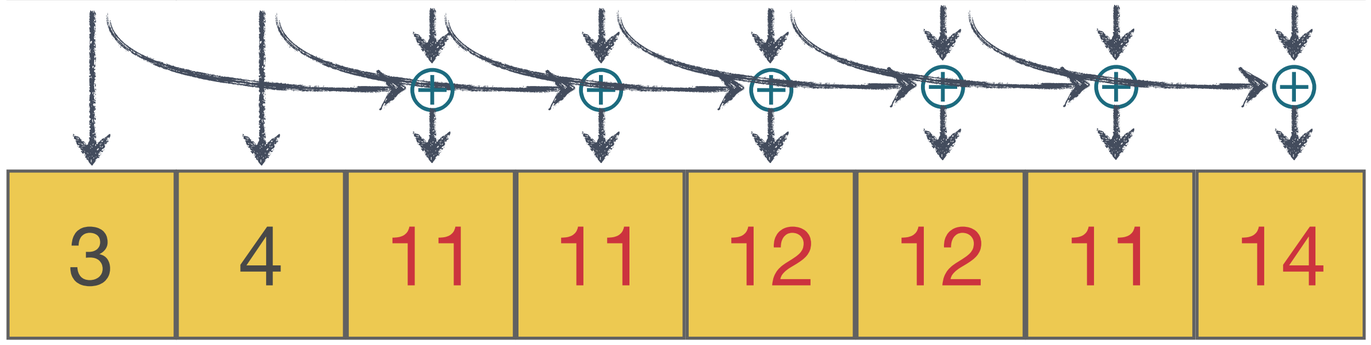

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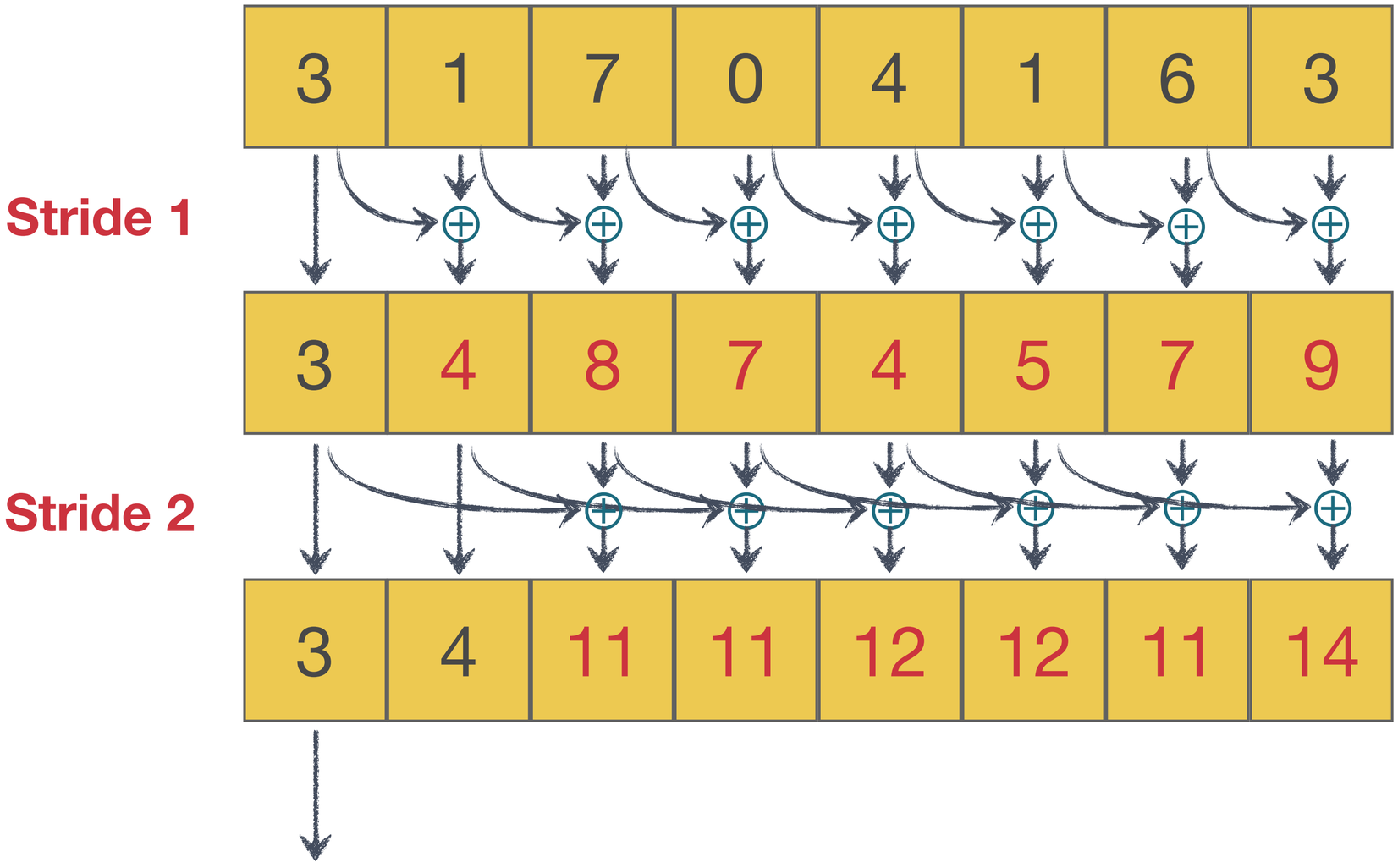

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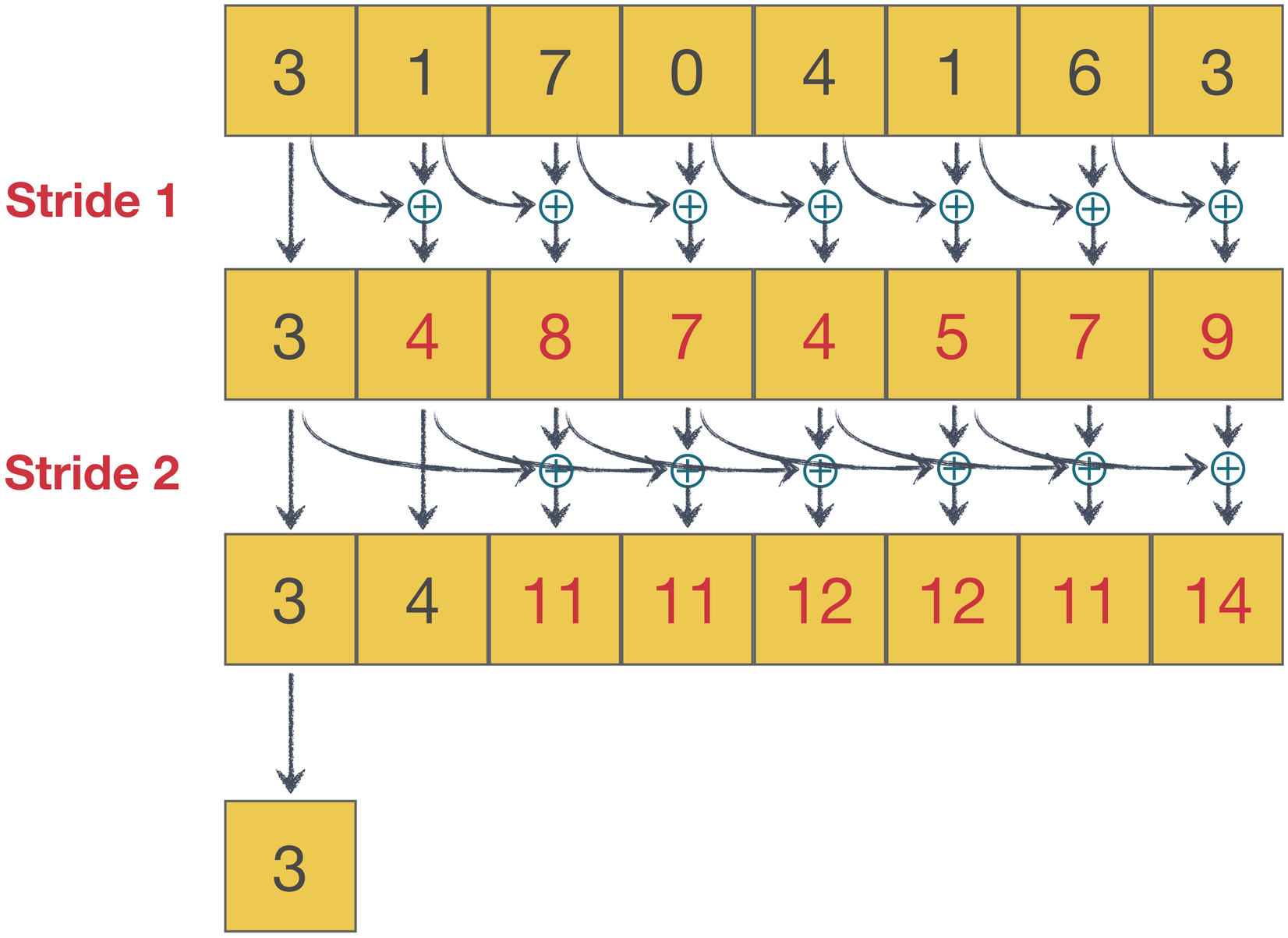

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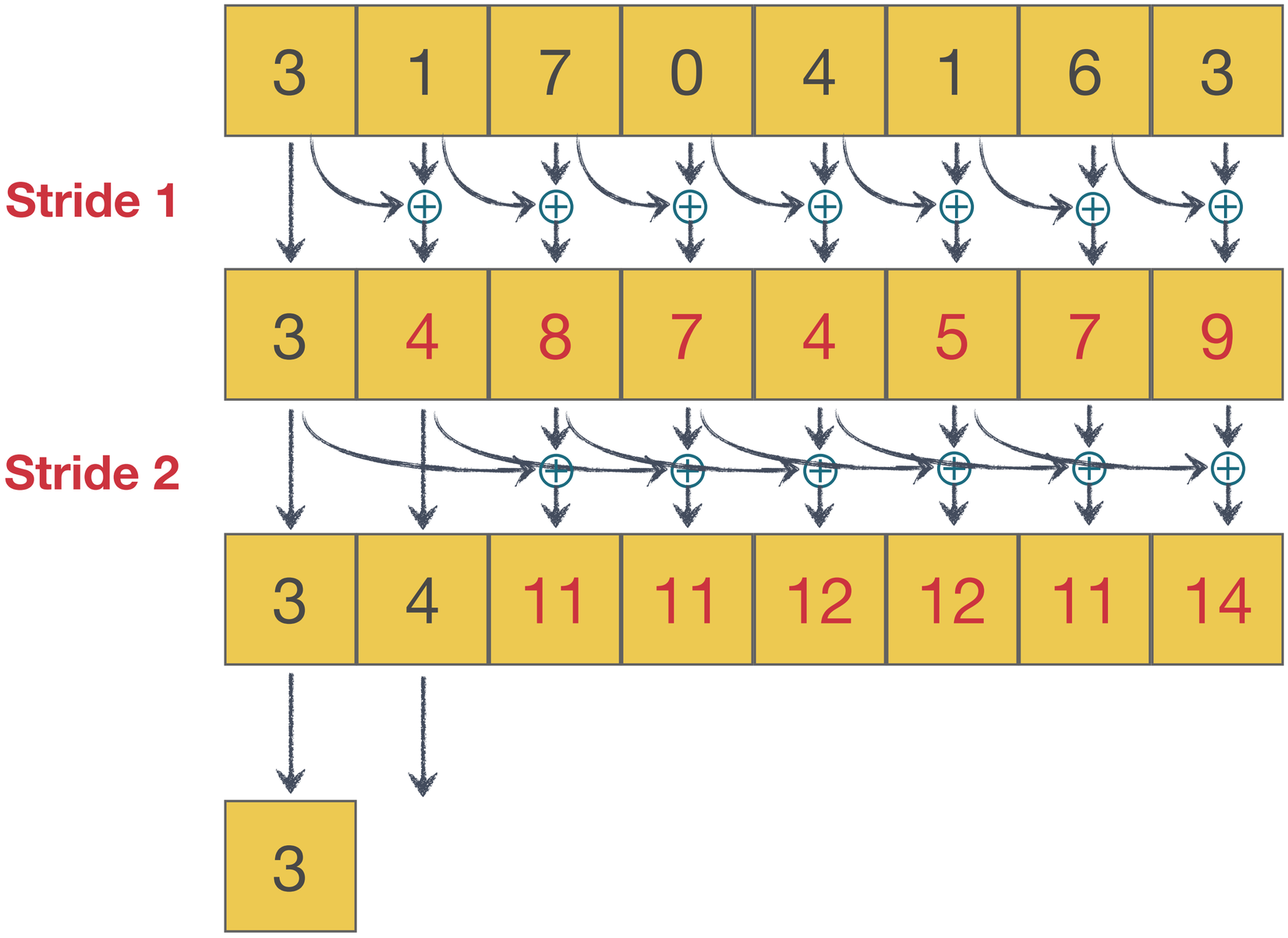

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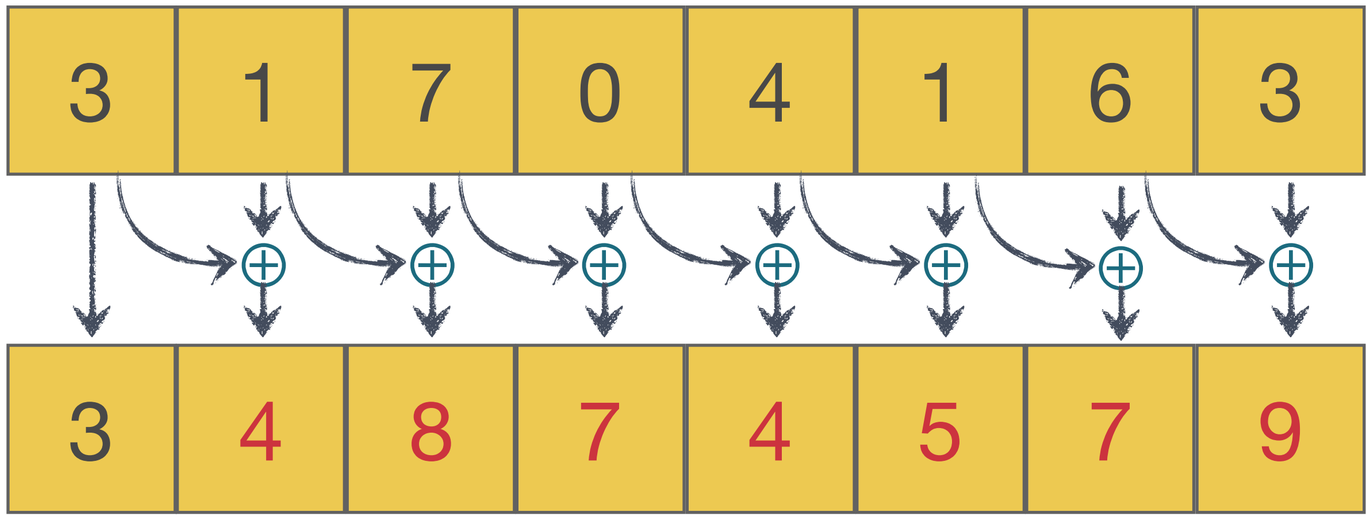

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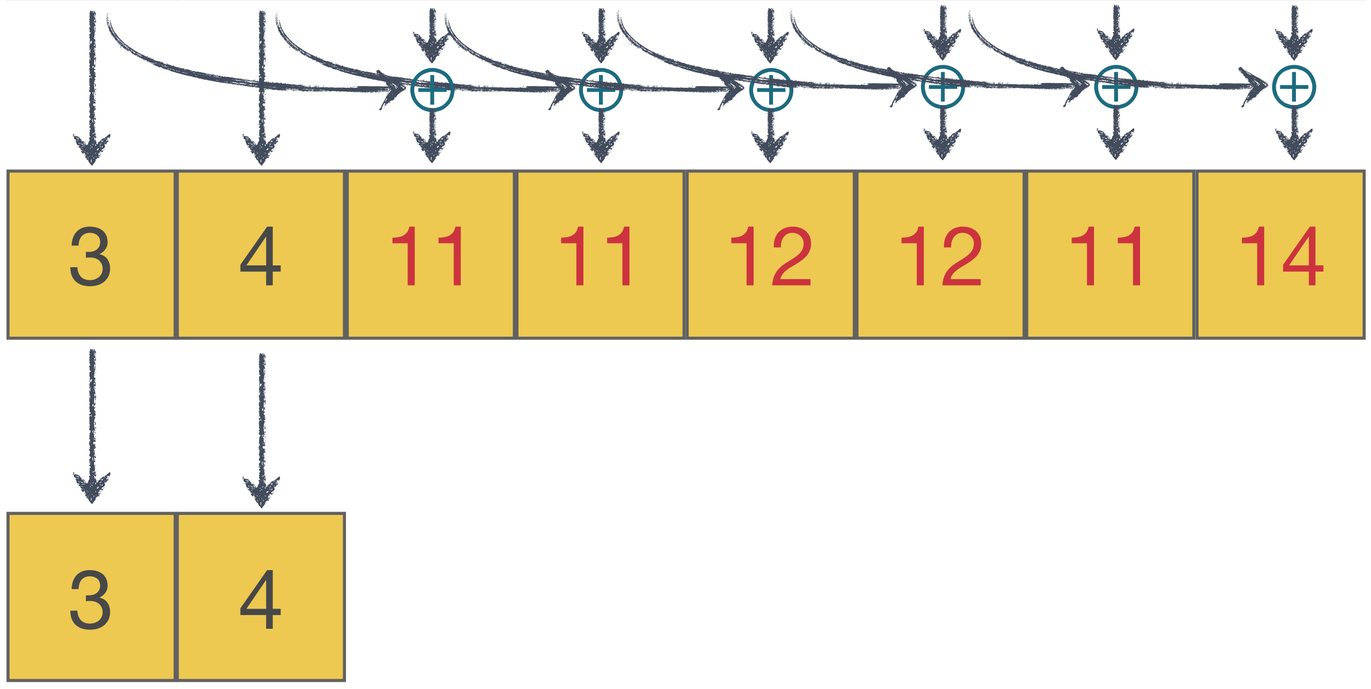

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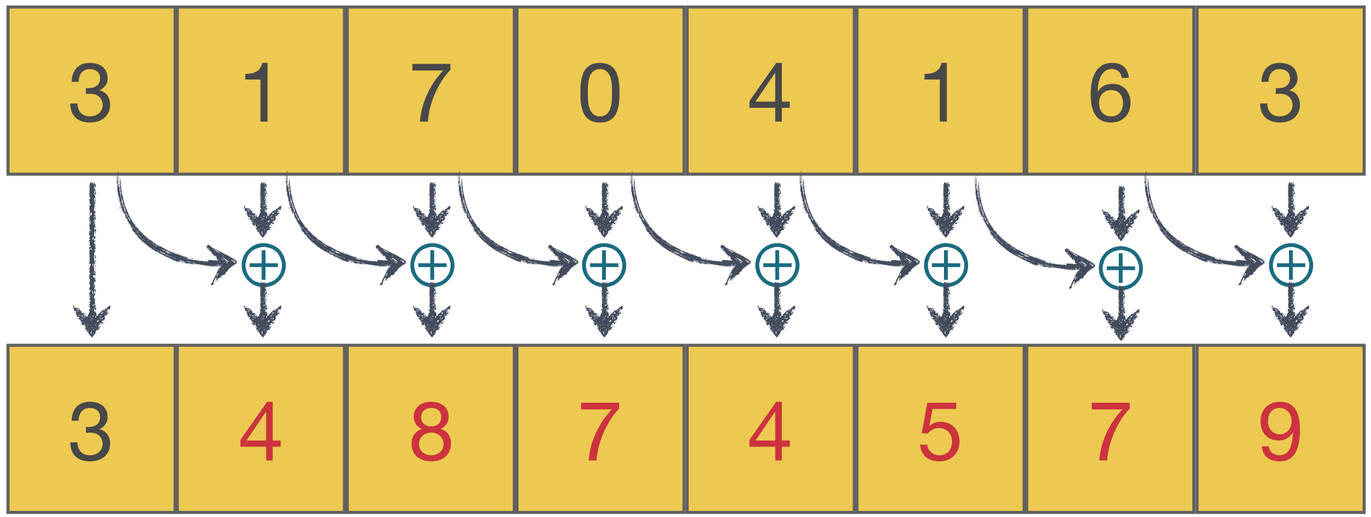

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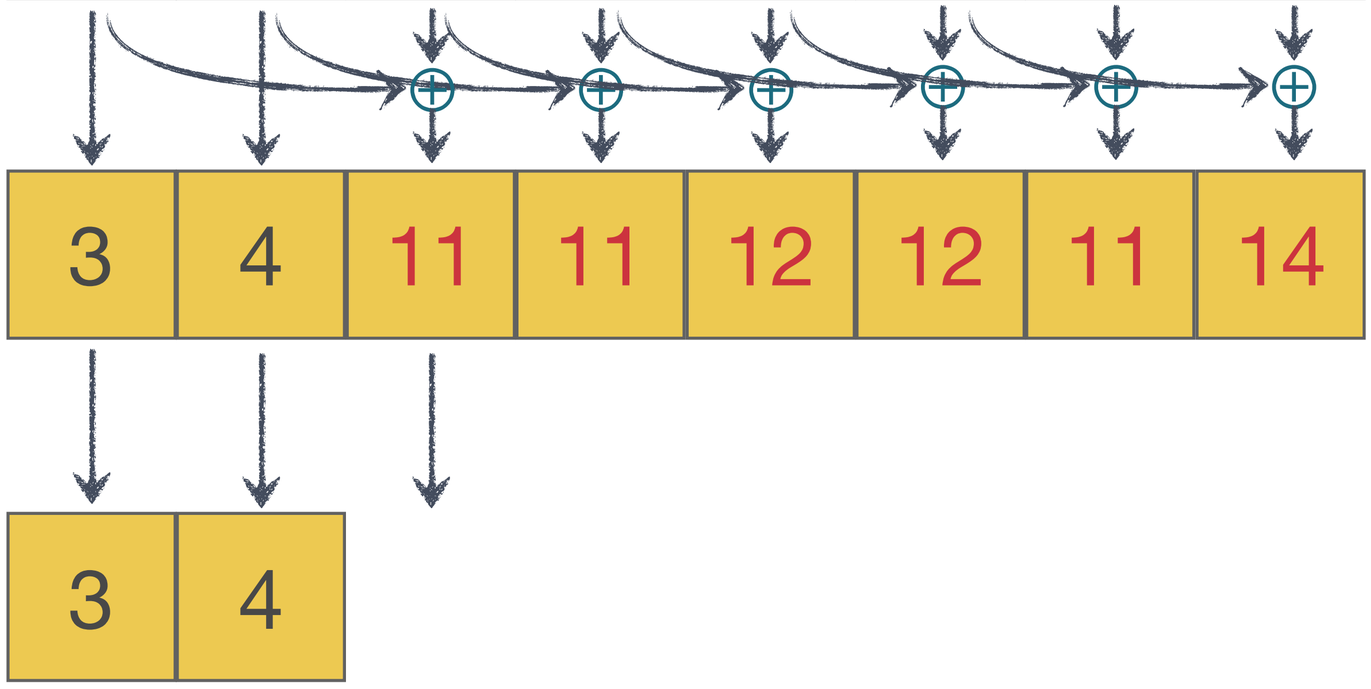

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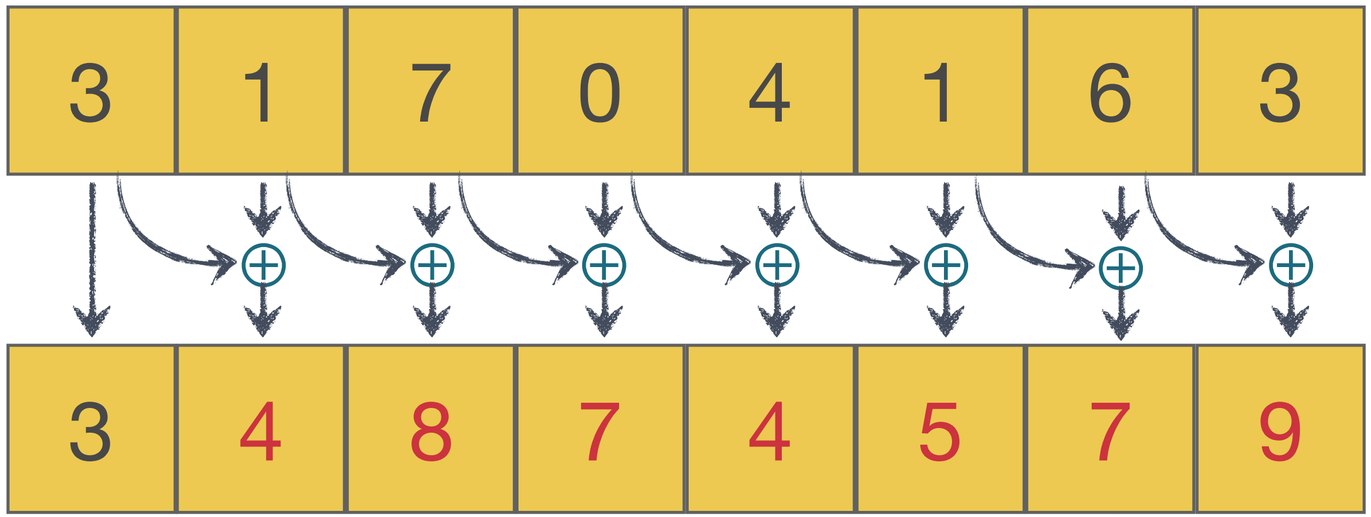

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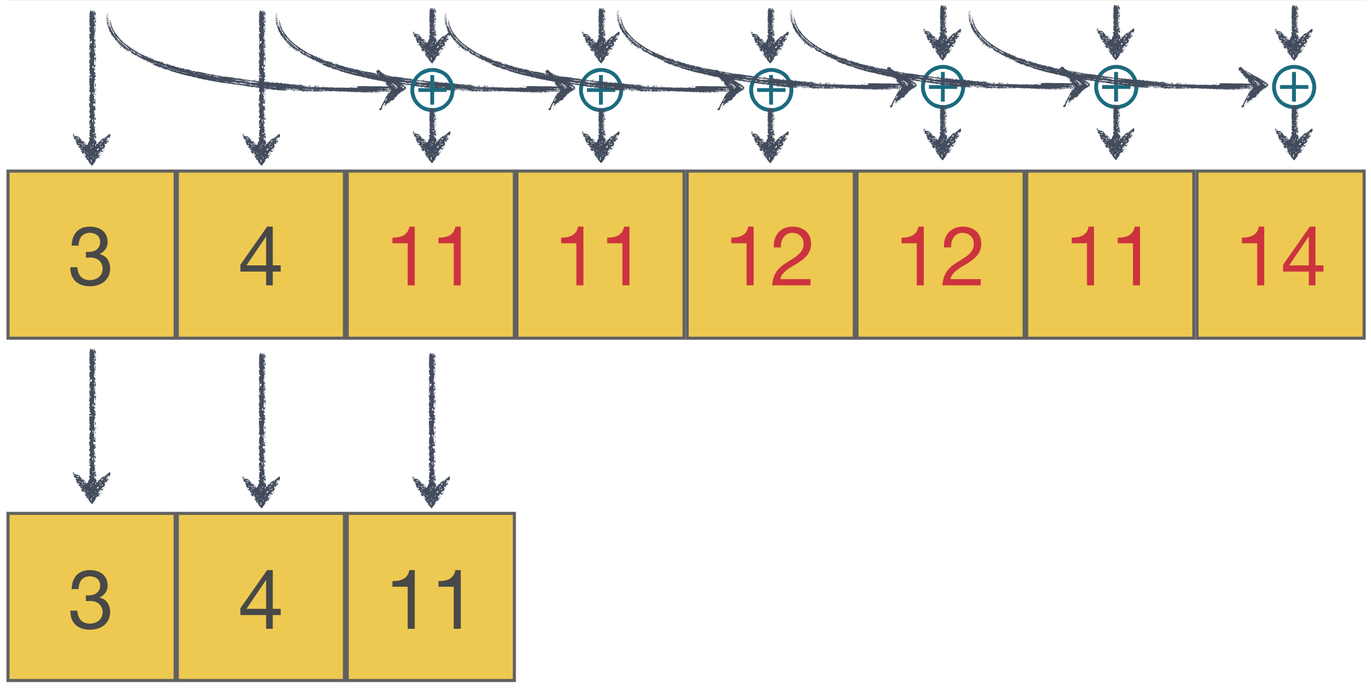

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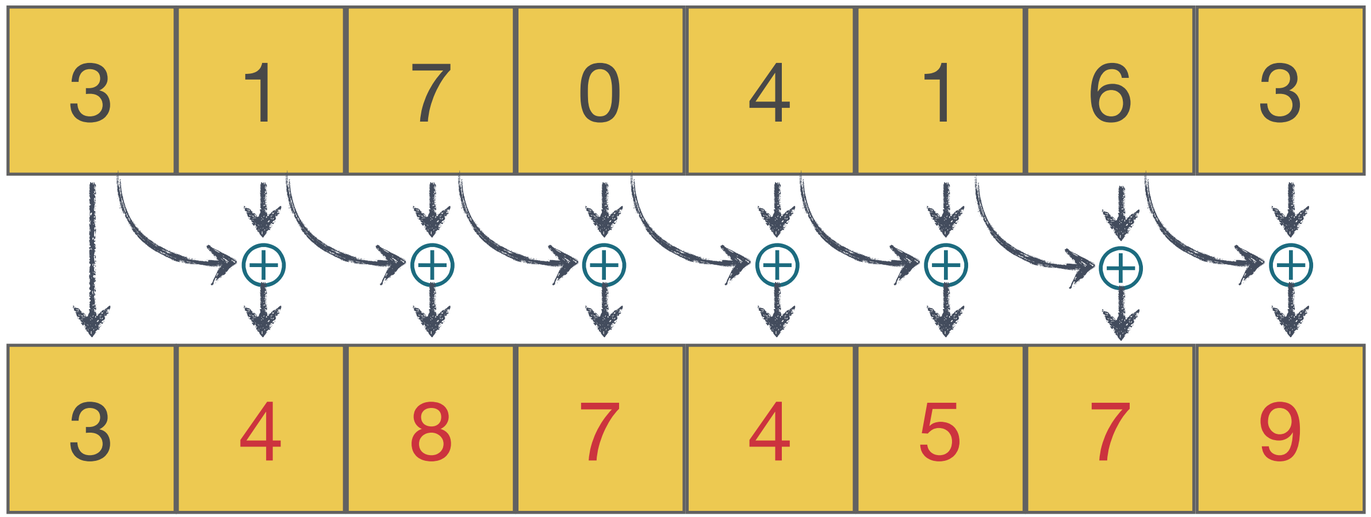

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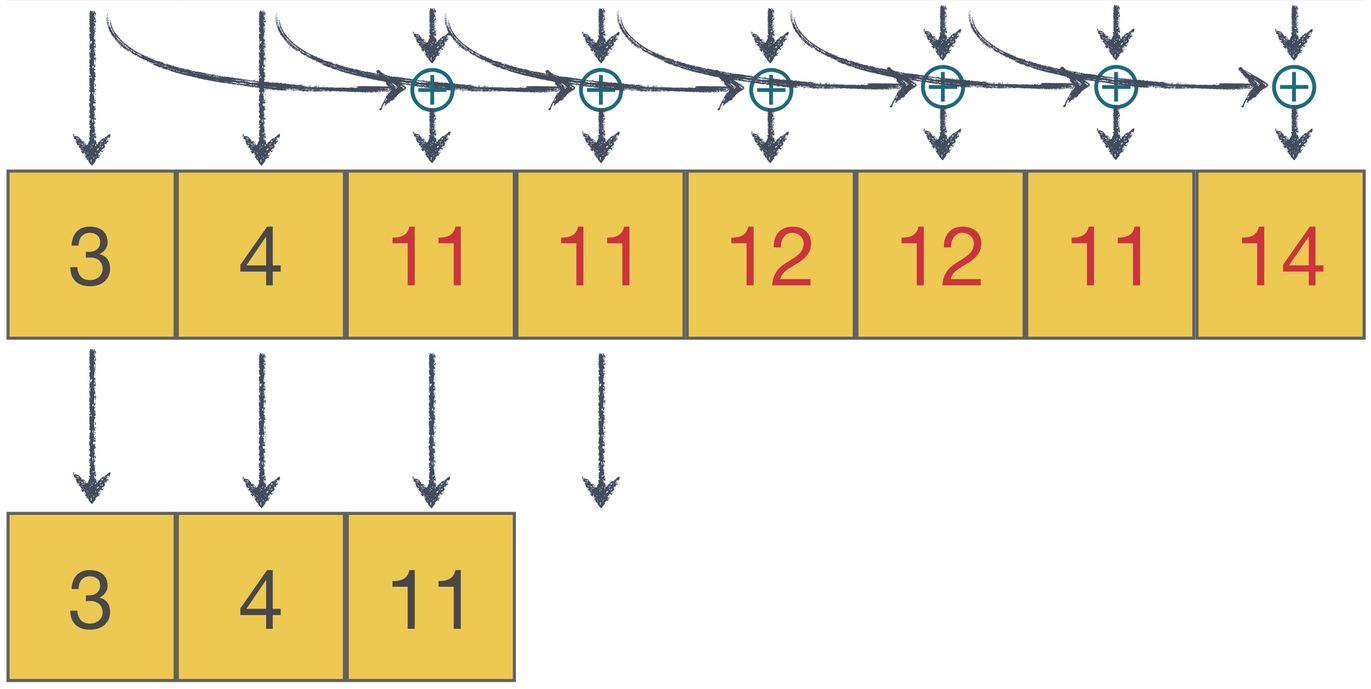

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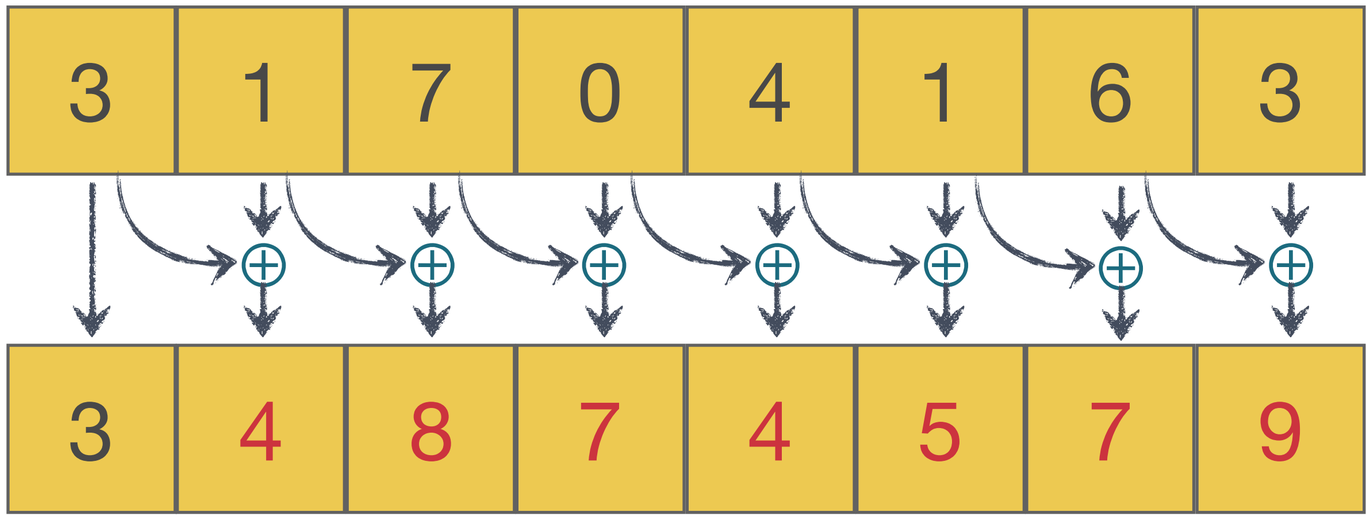

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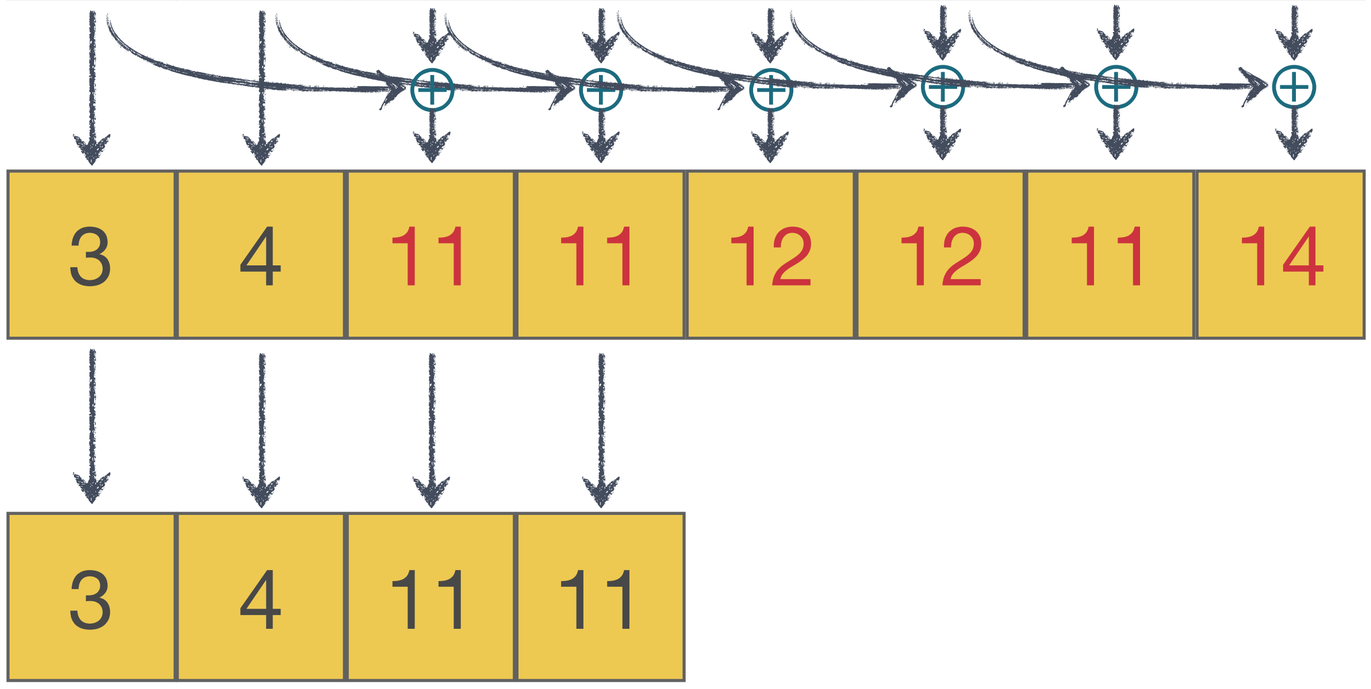

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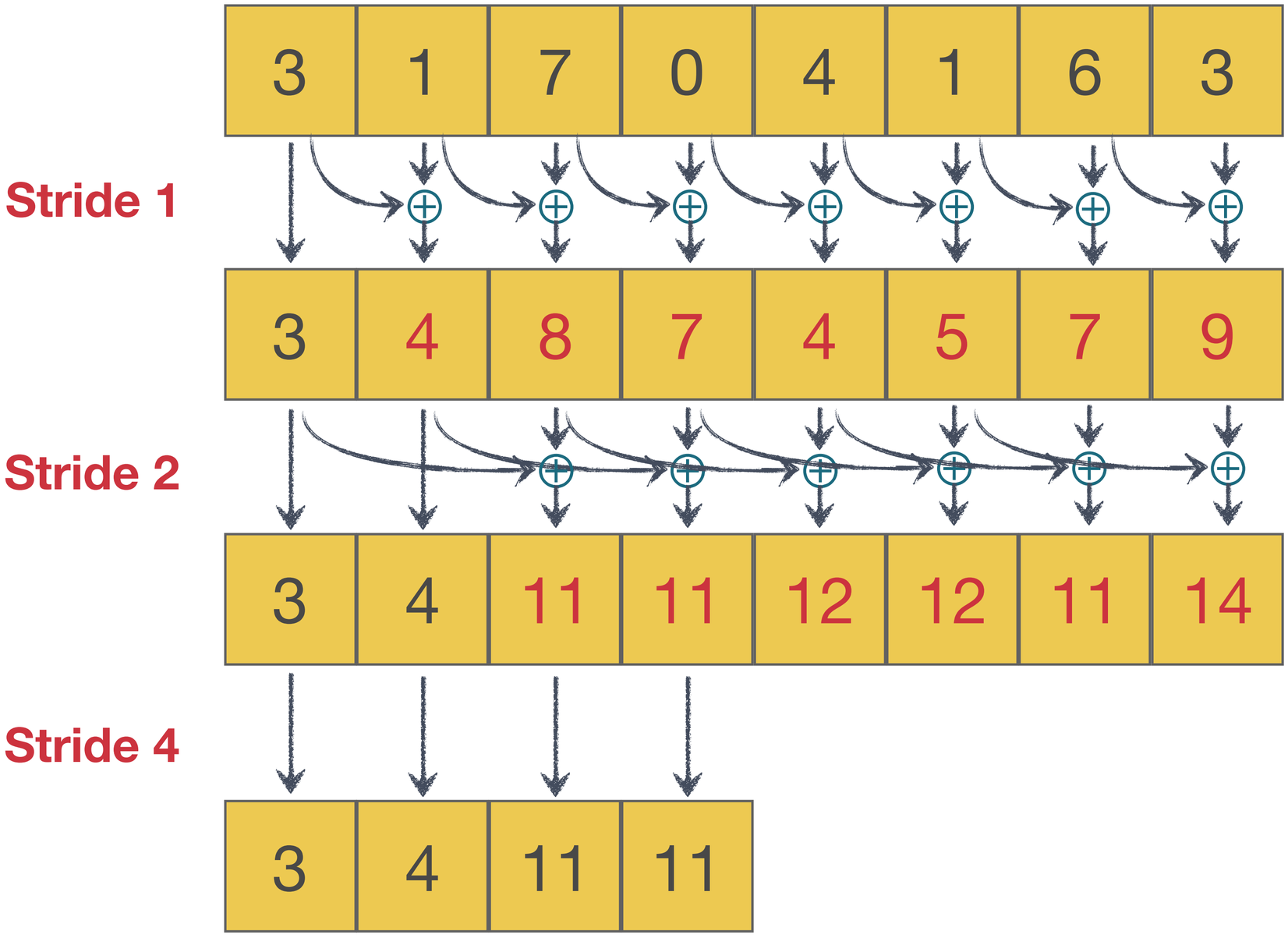

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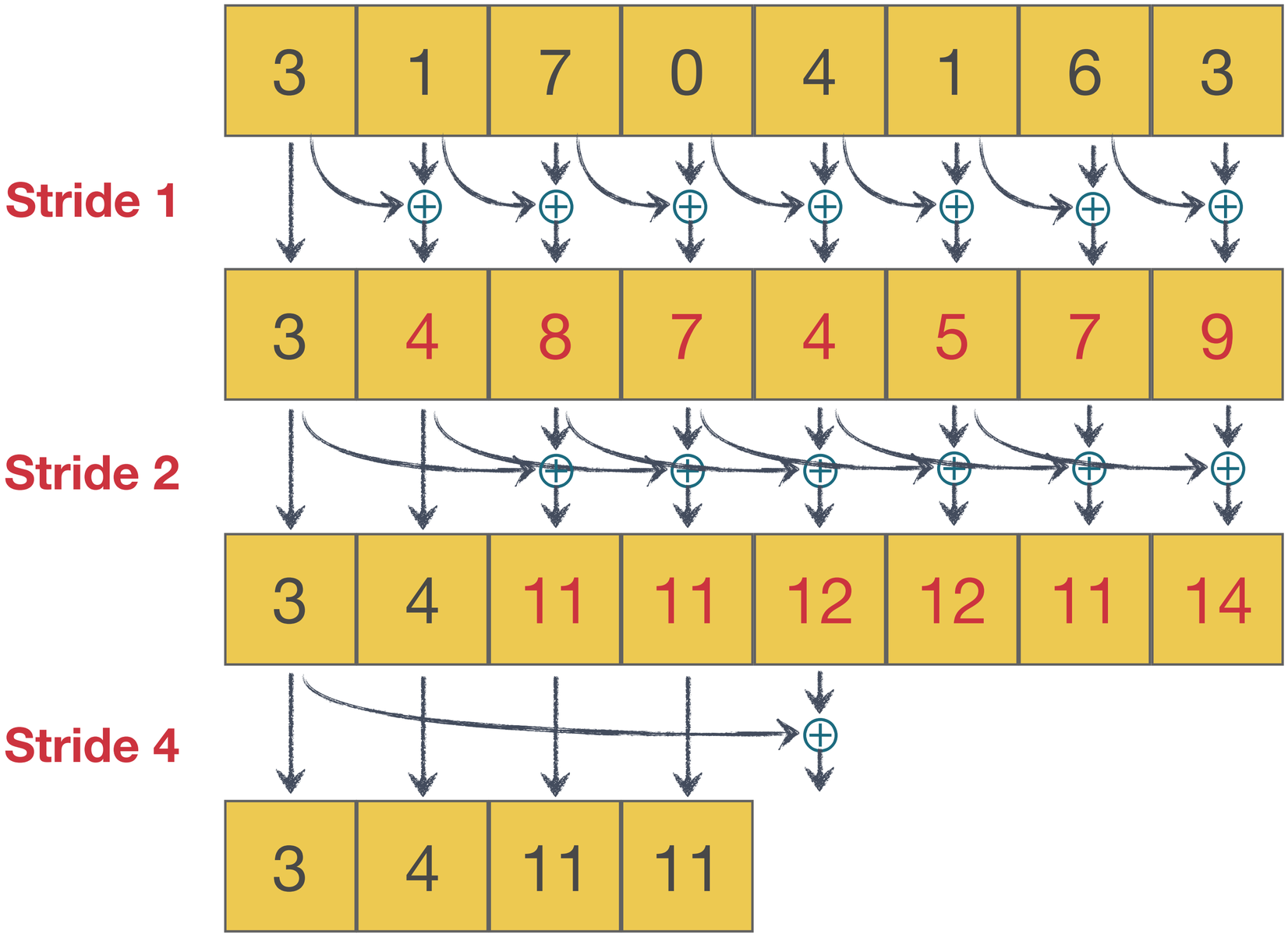

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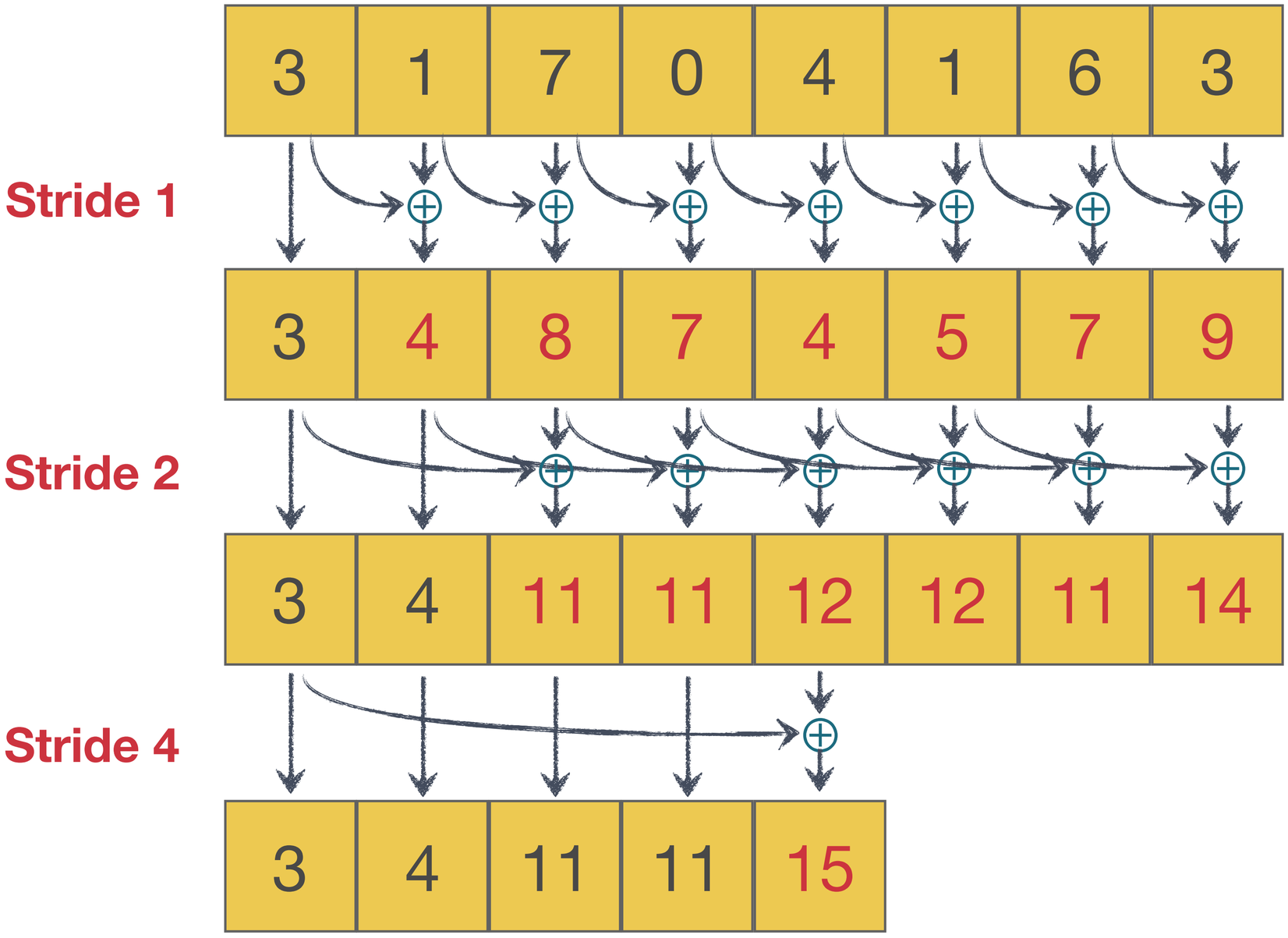

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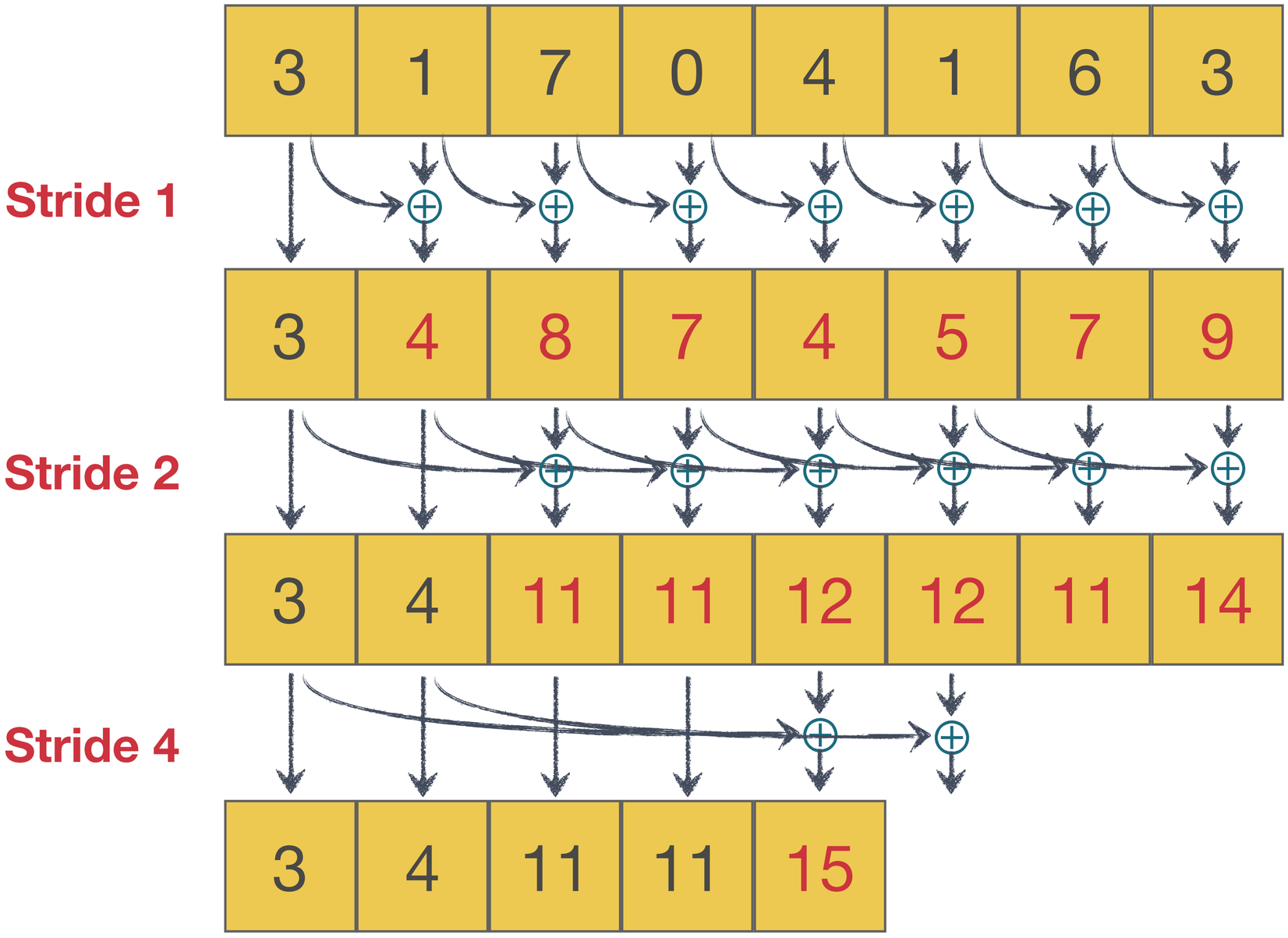

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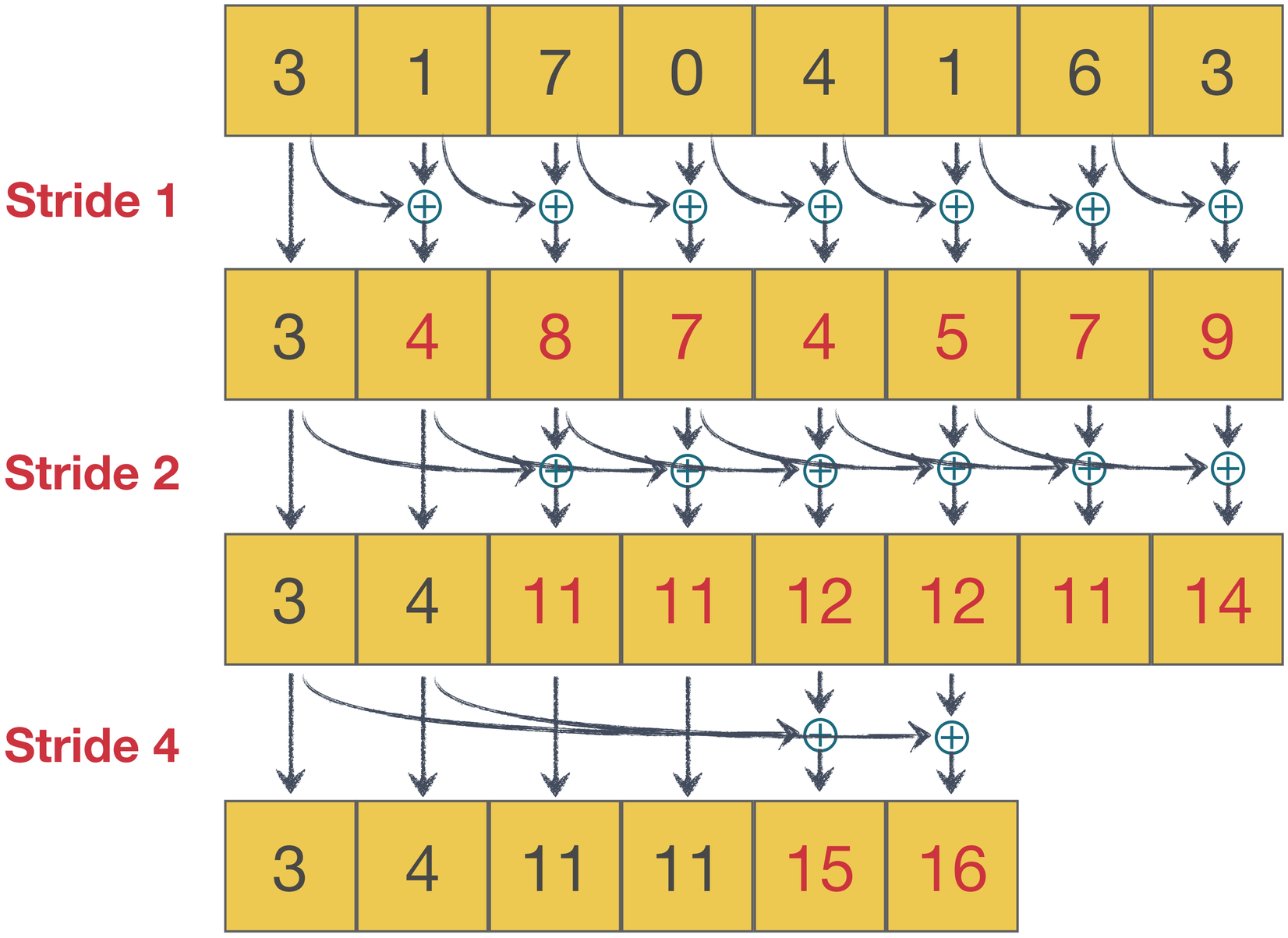

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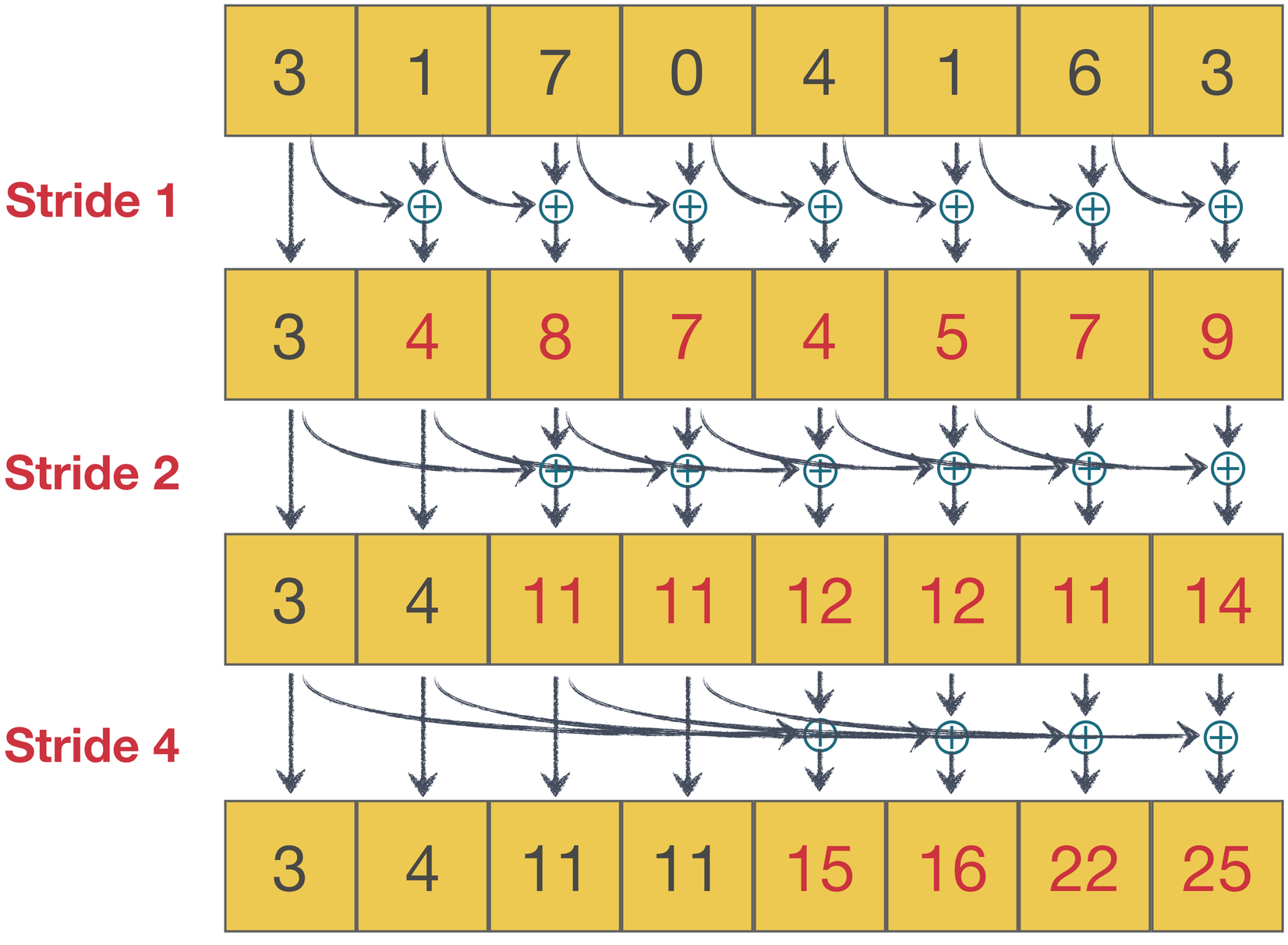

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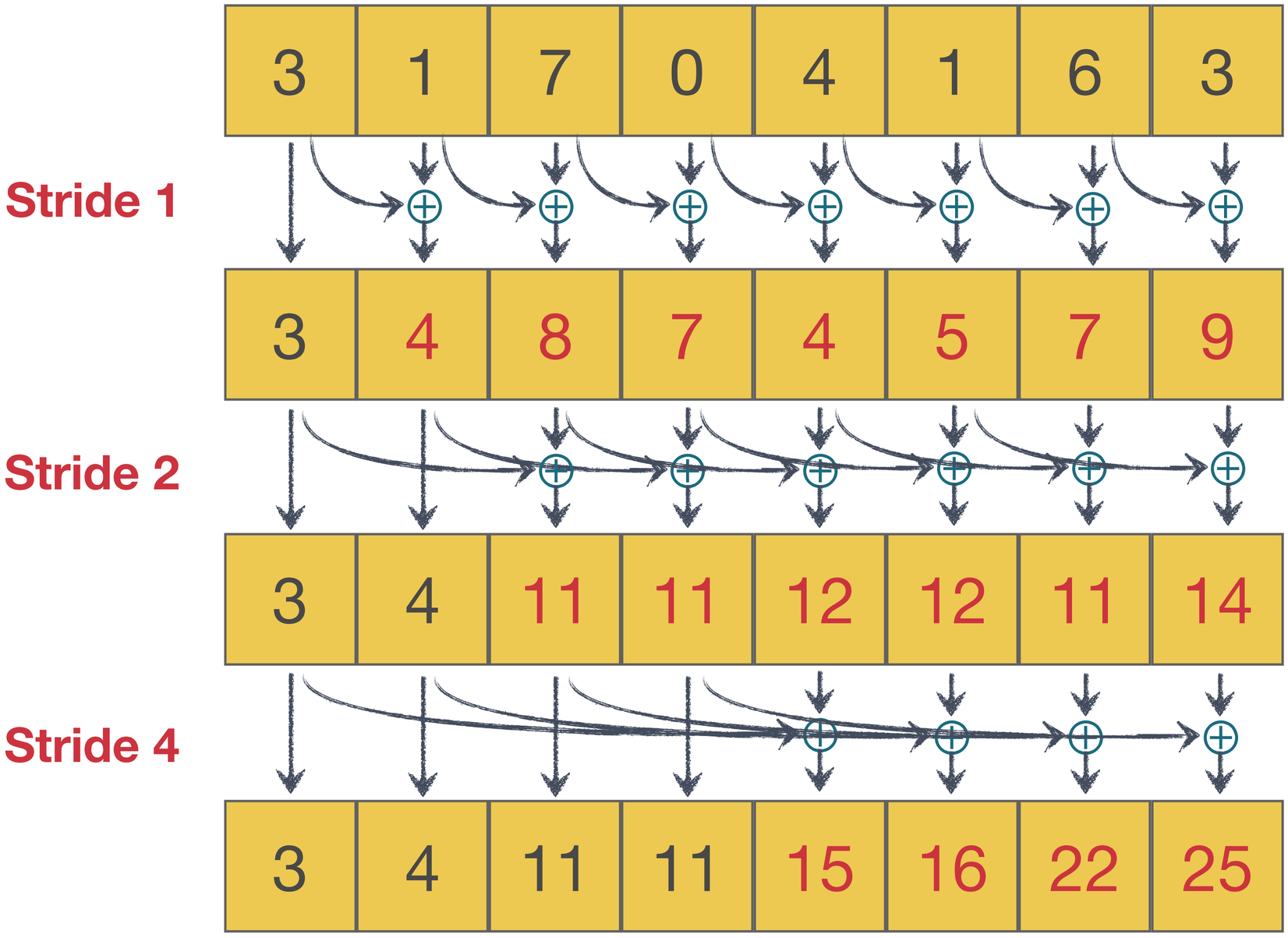
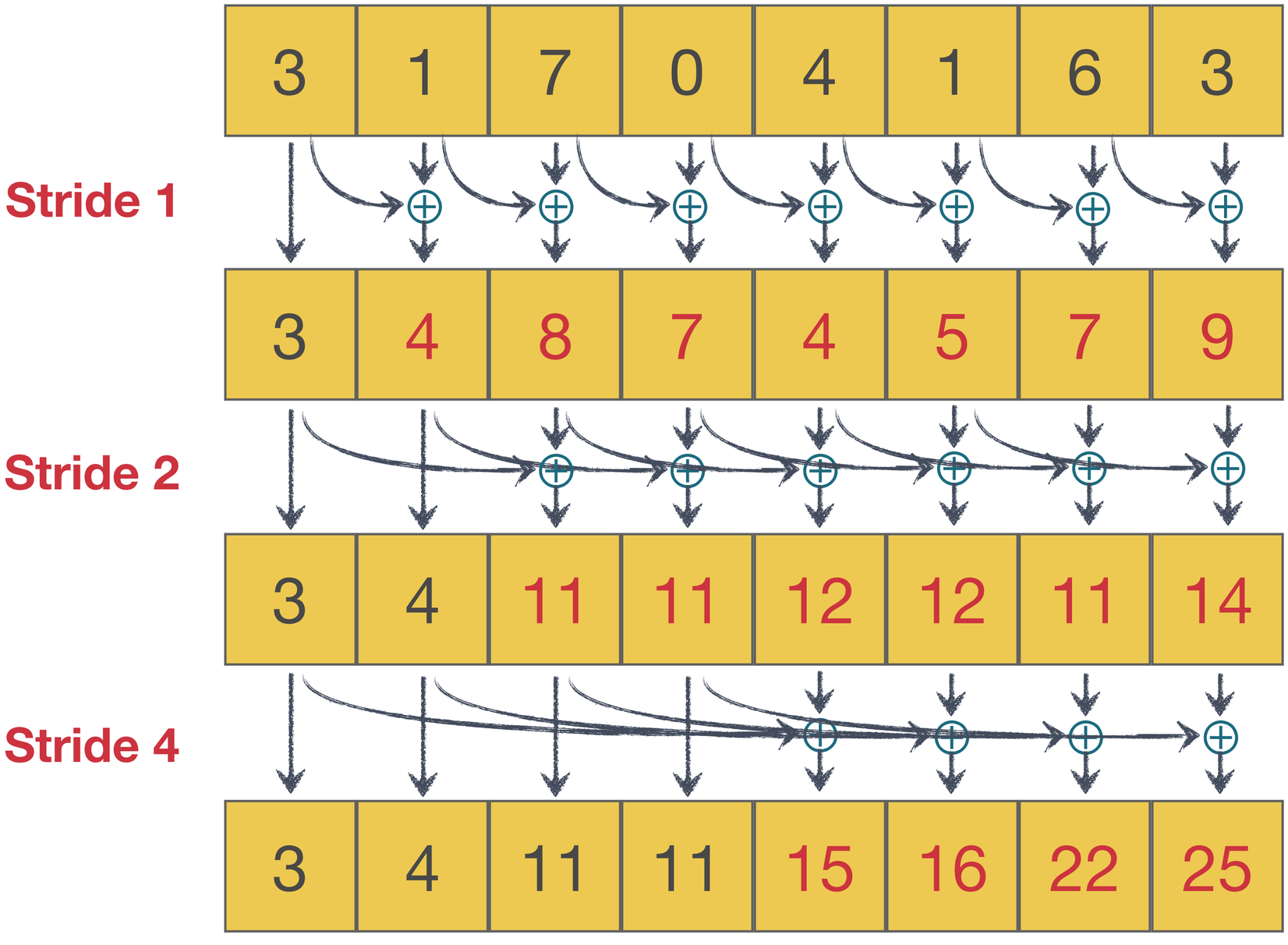


Iterate log(n) times adding pairs of elements *stride* elements apart.

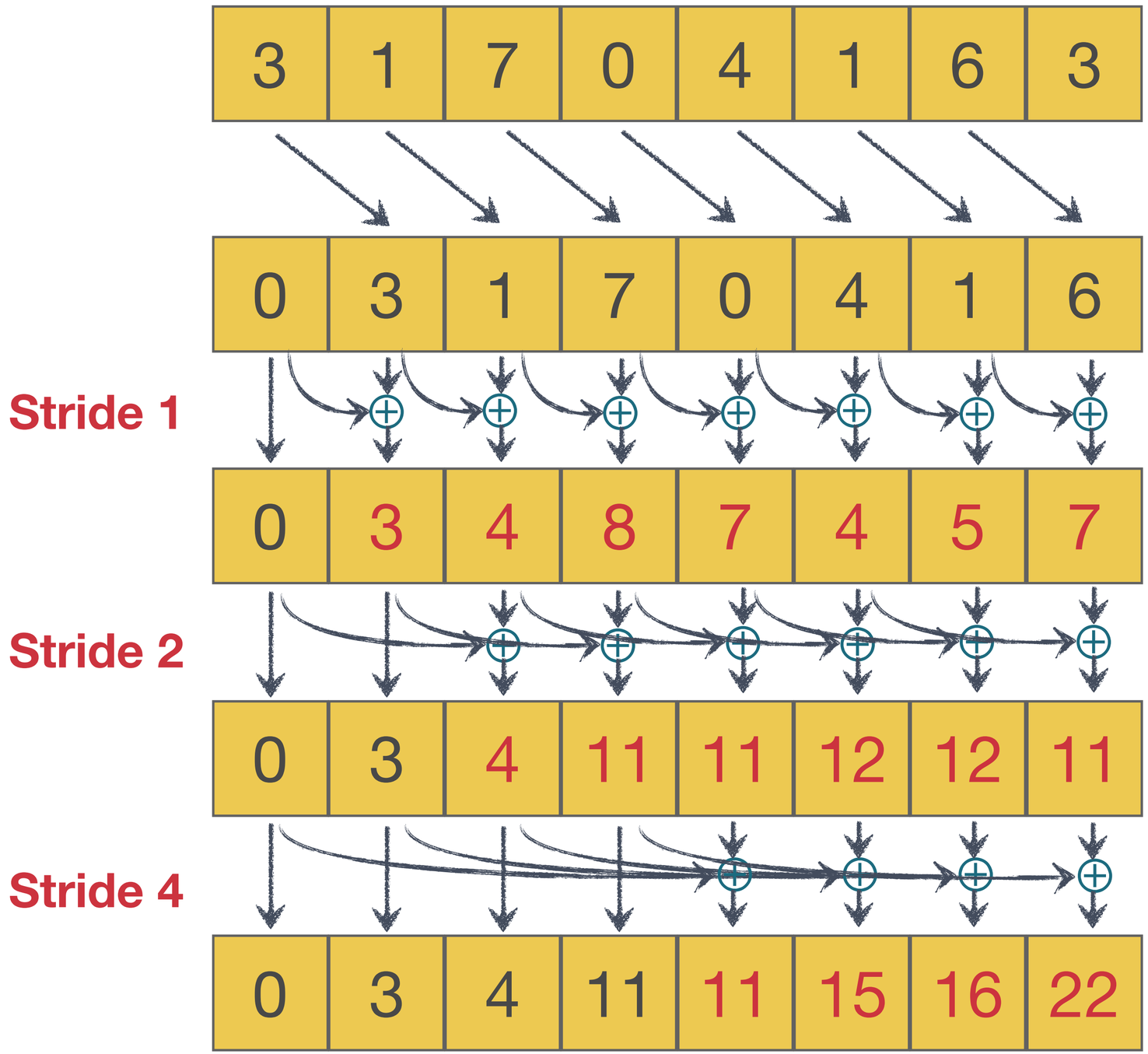
Inclusive Parallel Scan



Iterate log(n) times adding pairs of elements *stride* elements apart.

$$W(n) = \mathcal{O}(n \log n)$$
$$D(n) = \mathcal{O}(\log n)$$

Exclusive Parallel Scan



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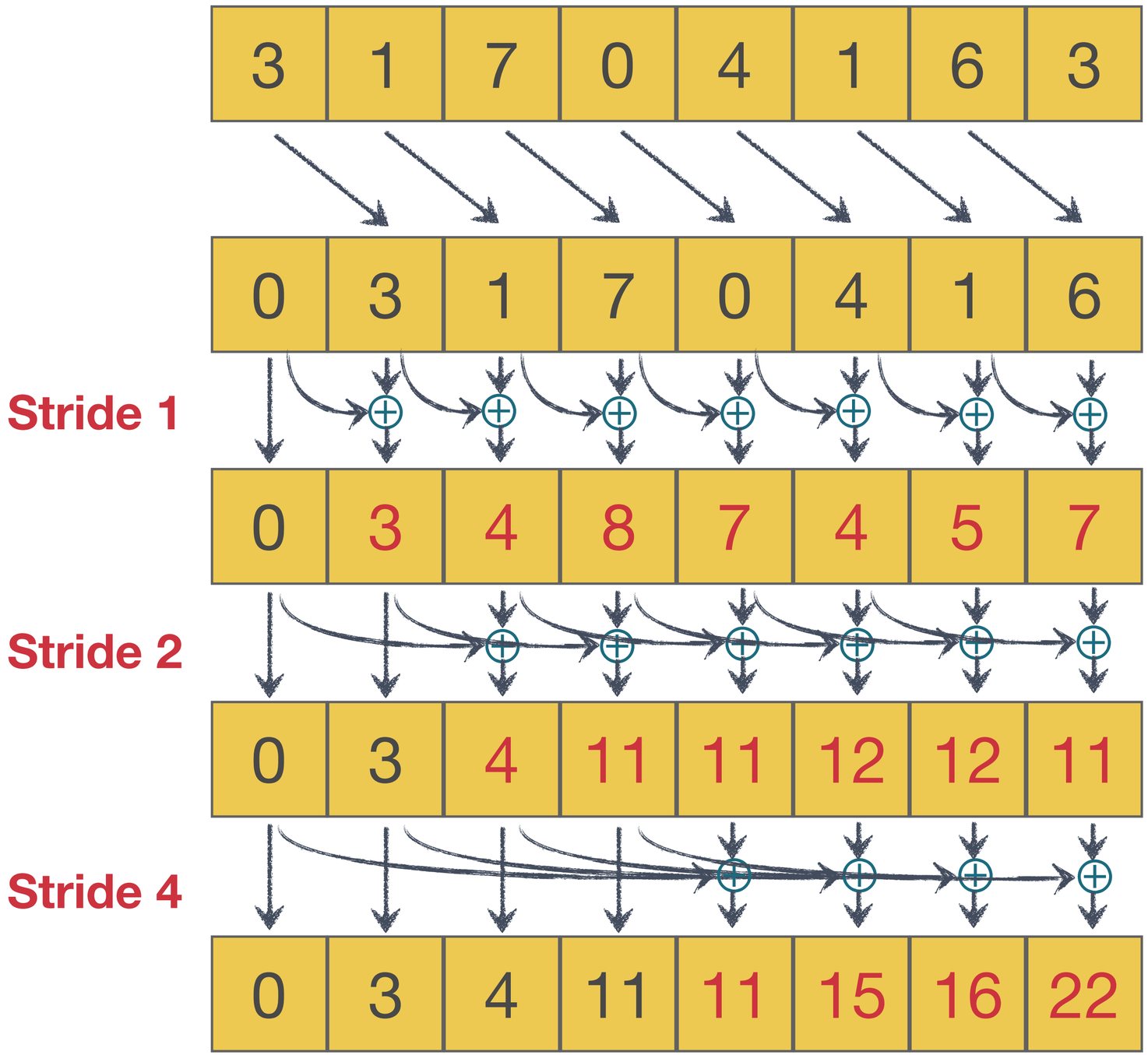

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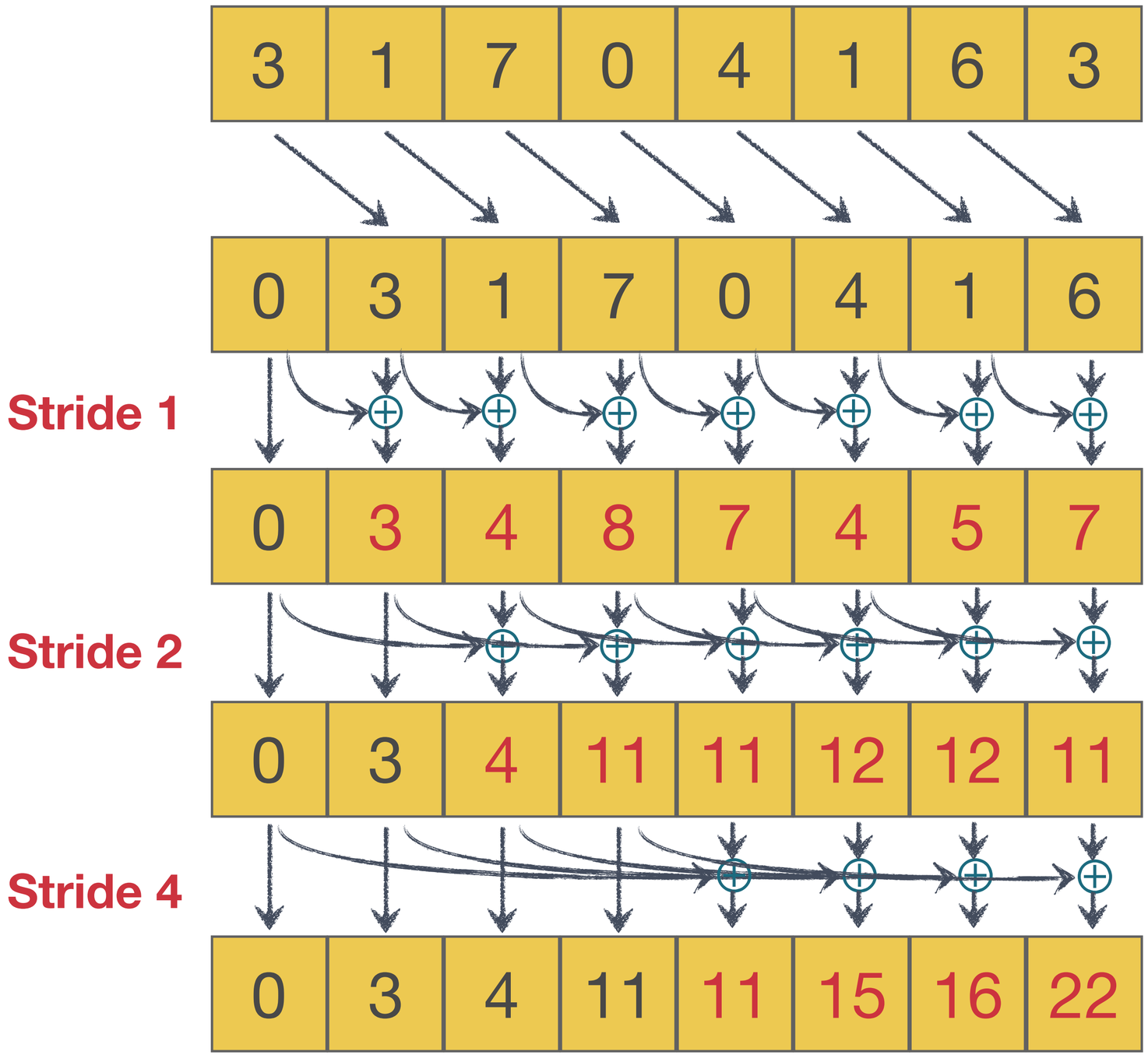

Iterate log(n) times adding pairs of elements *stride* elements apart.

$$W(n) = \mathcal{O}(n \log n)$$
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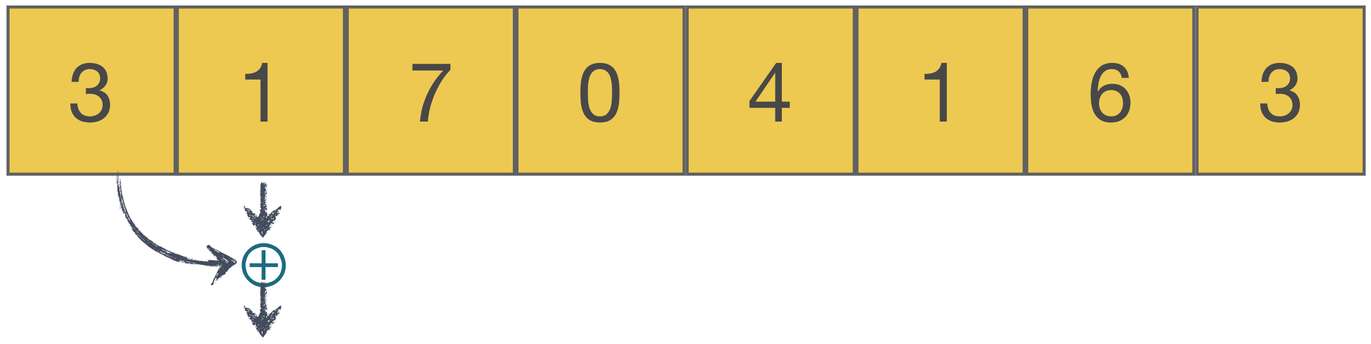
Can we do better?

Parallel Scan: Reduction

Parallel Scan: Reduction

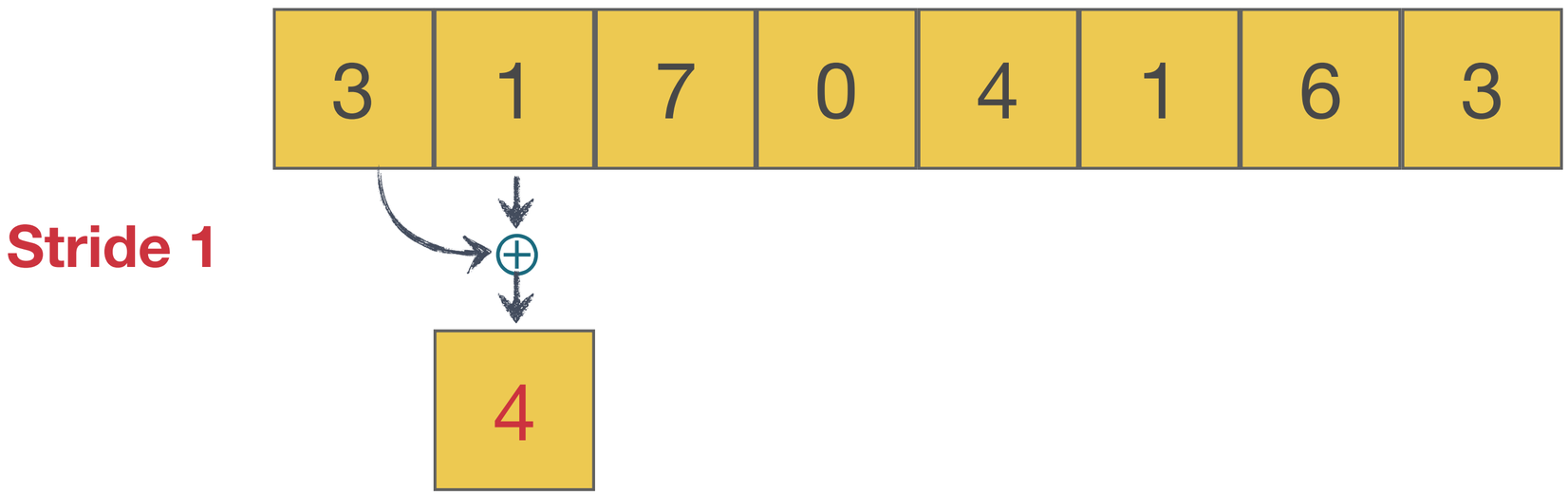
Stride 1

Parallel Scan: Reduction

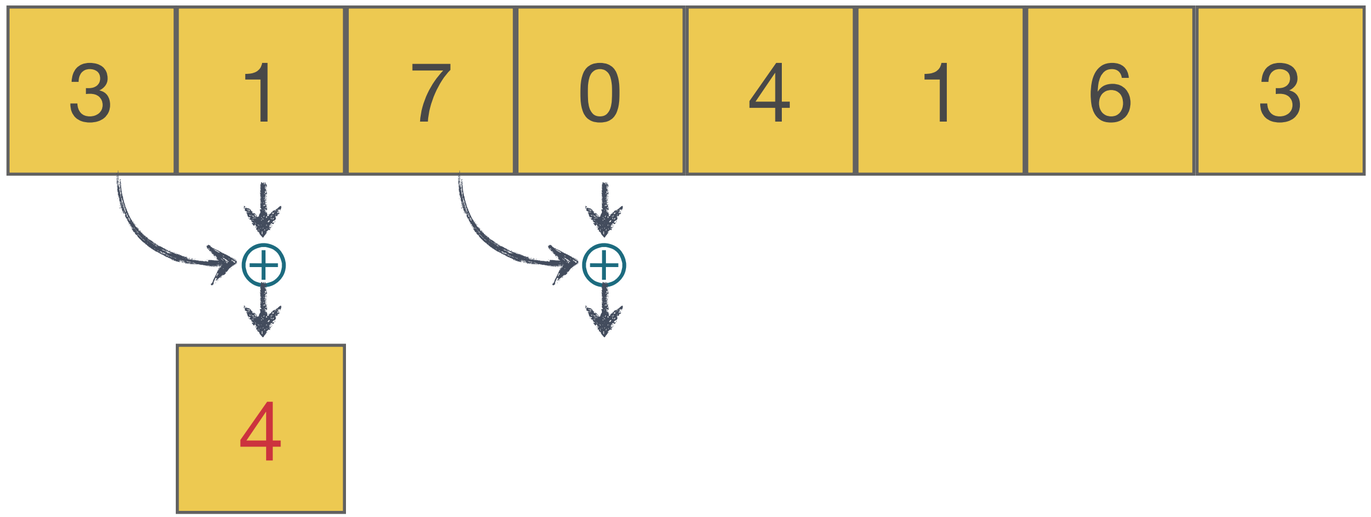


Stride 1

Parallel Scan: Reduction

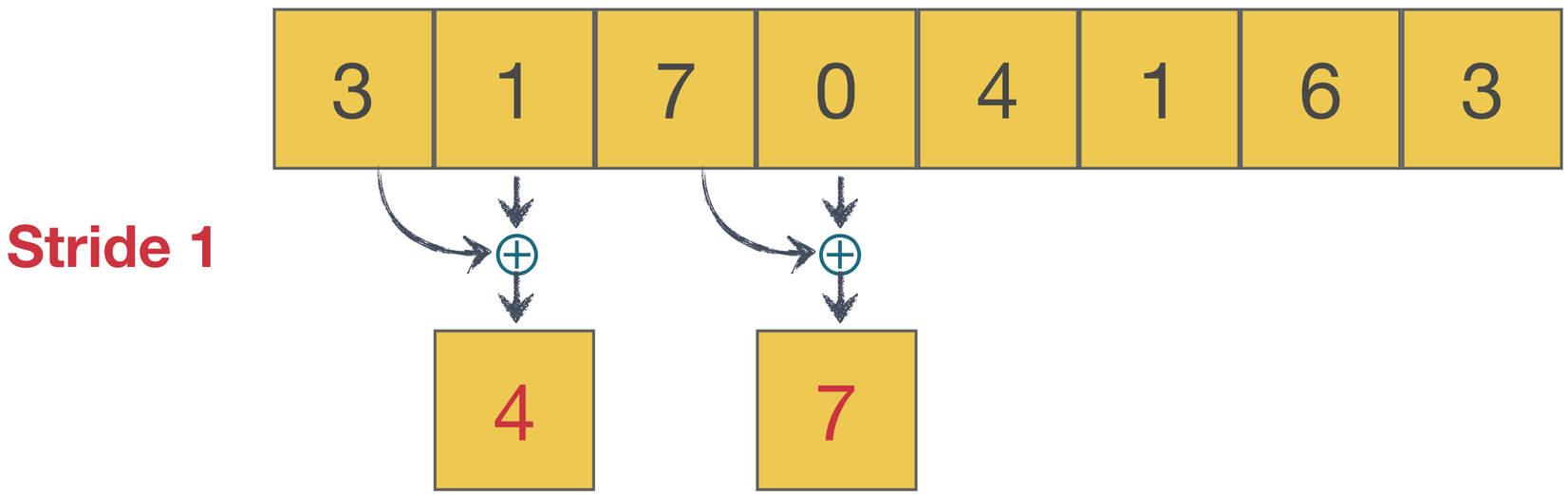


Parallel Scan: Reduction

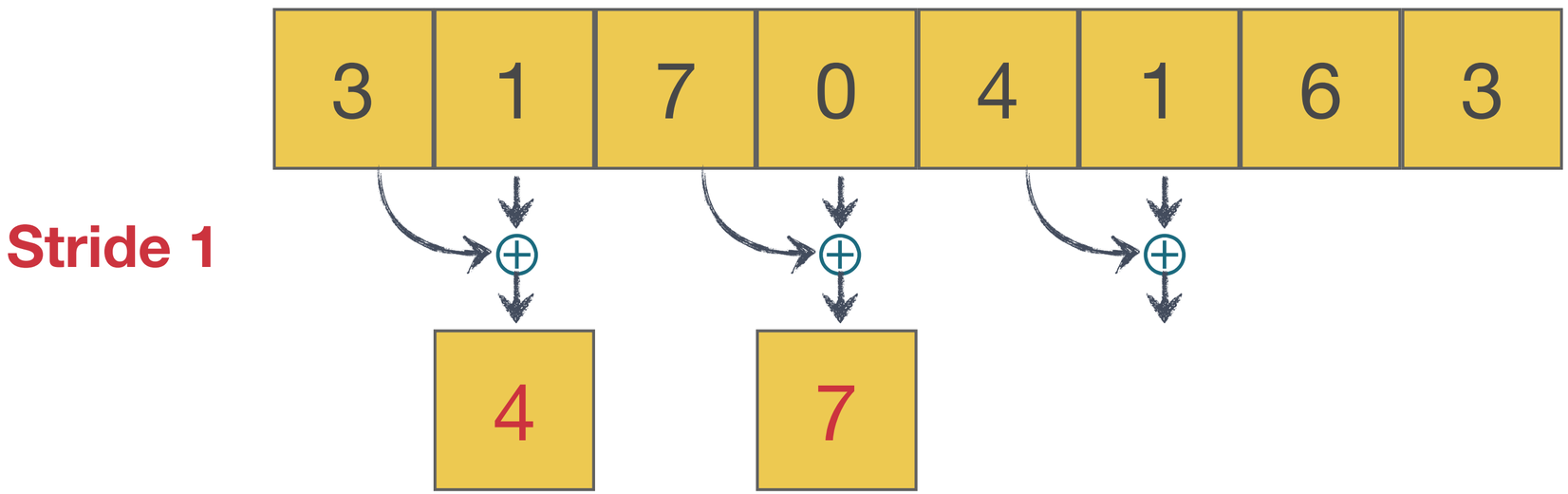


Stride 1

Parallel Scan: Reduction

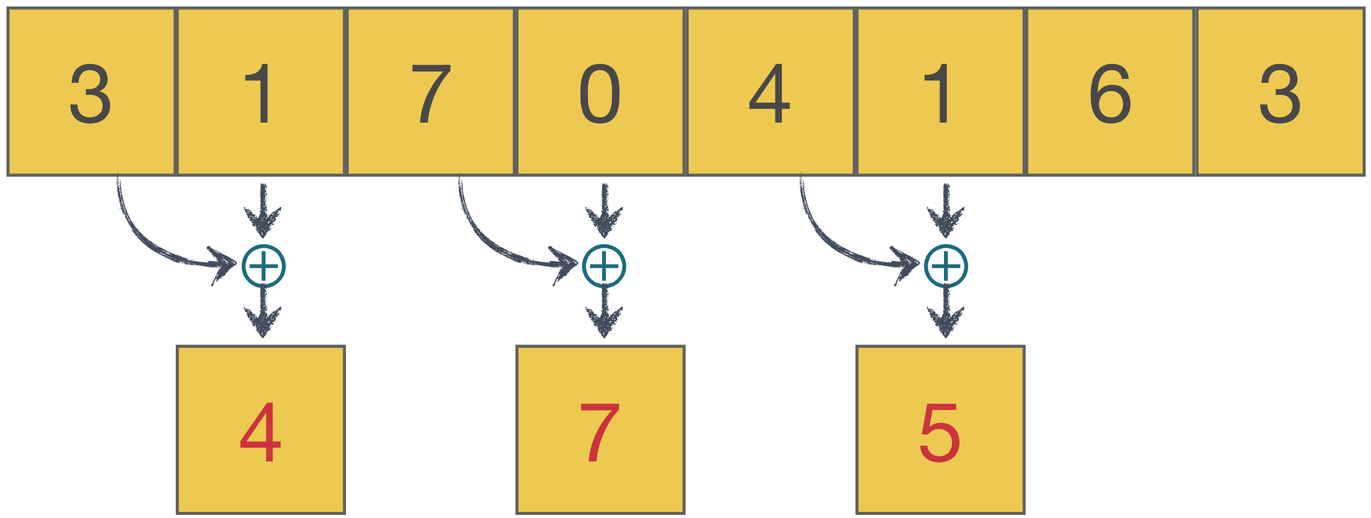


Parallel Scan: Reduction



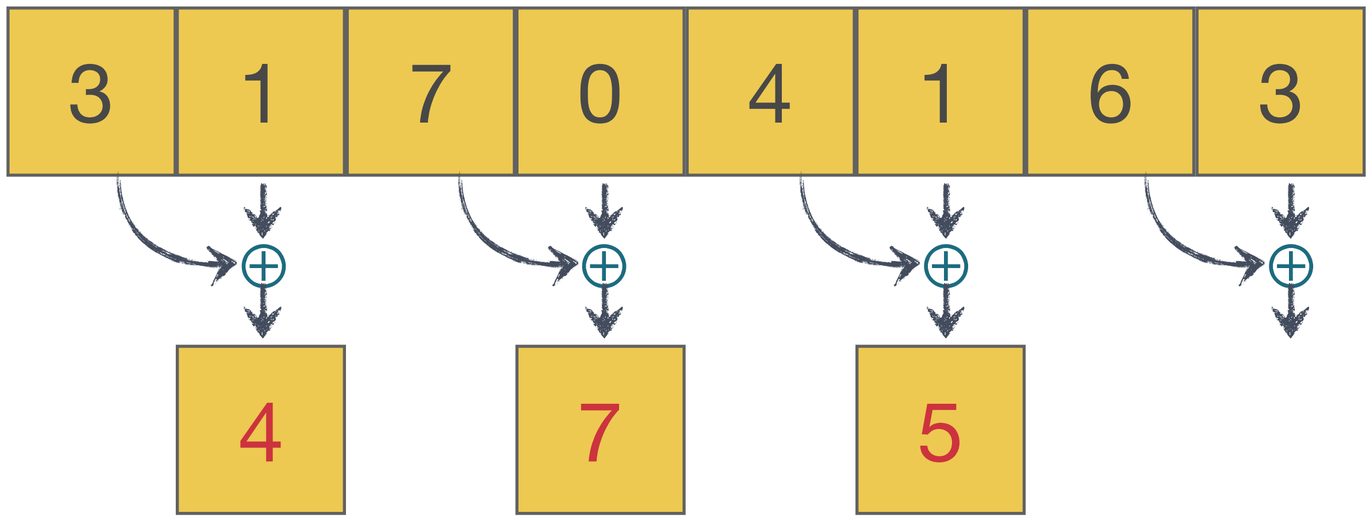
Parallel Scan: Reduction

Stride 1



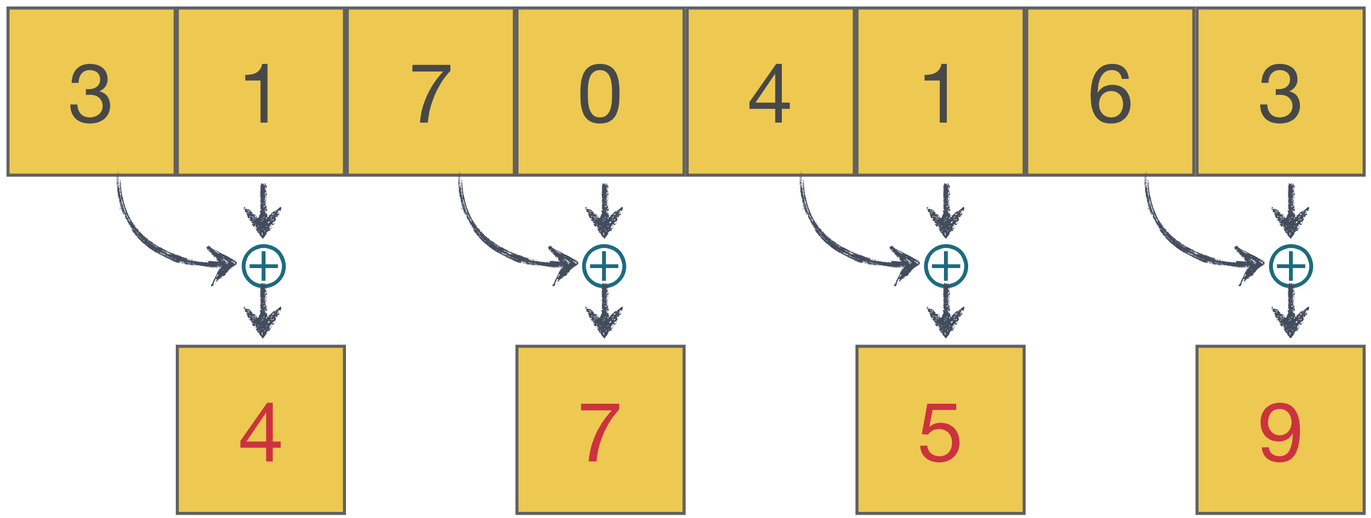
Parallel Scan: Reduction

Stride 1



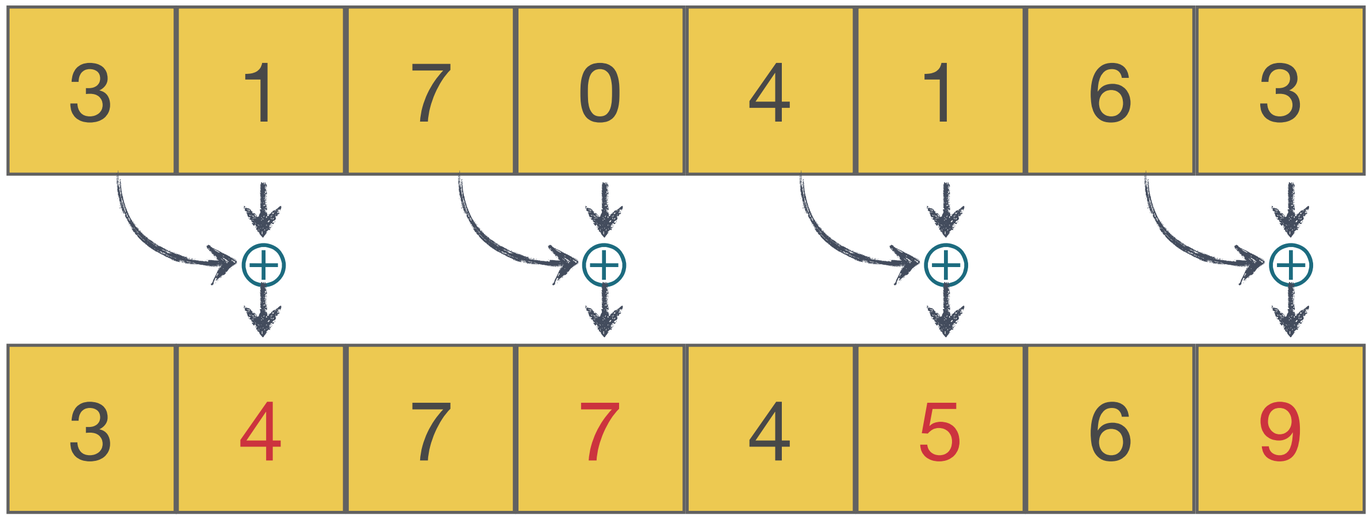
Parallel Scan: Reduction

Stride 1



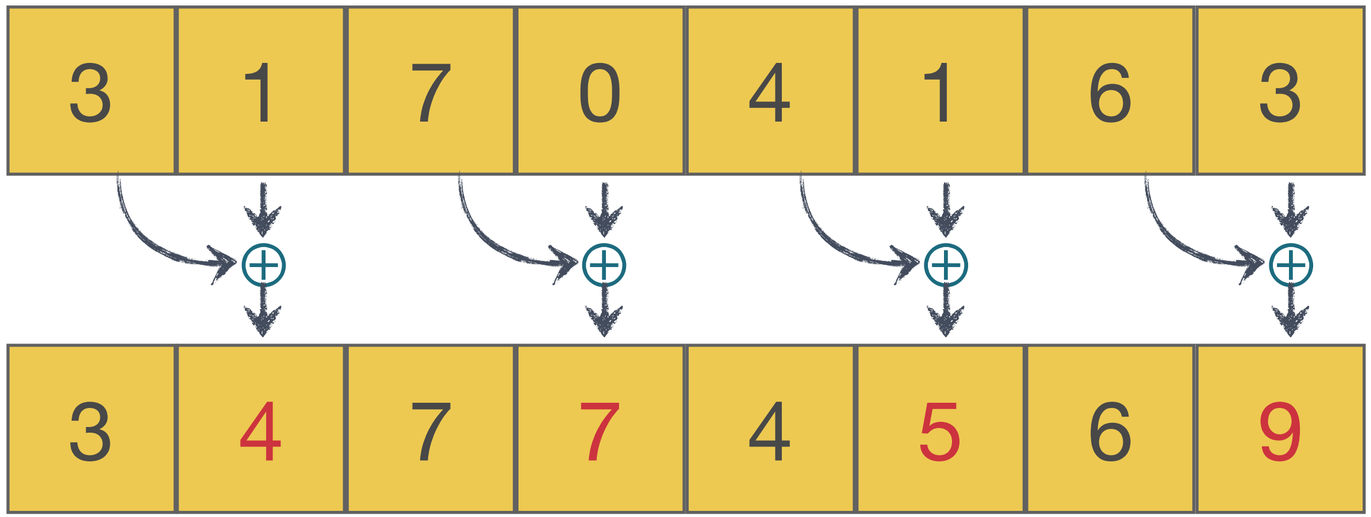
Parallel Scan: Reduction

Stride 1



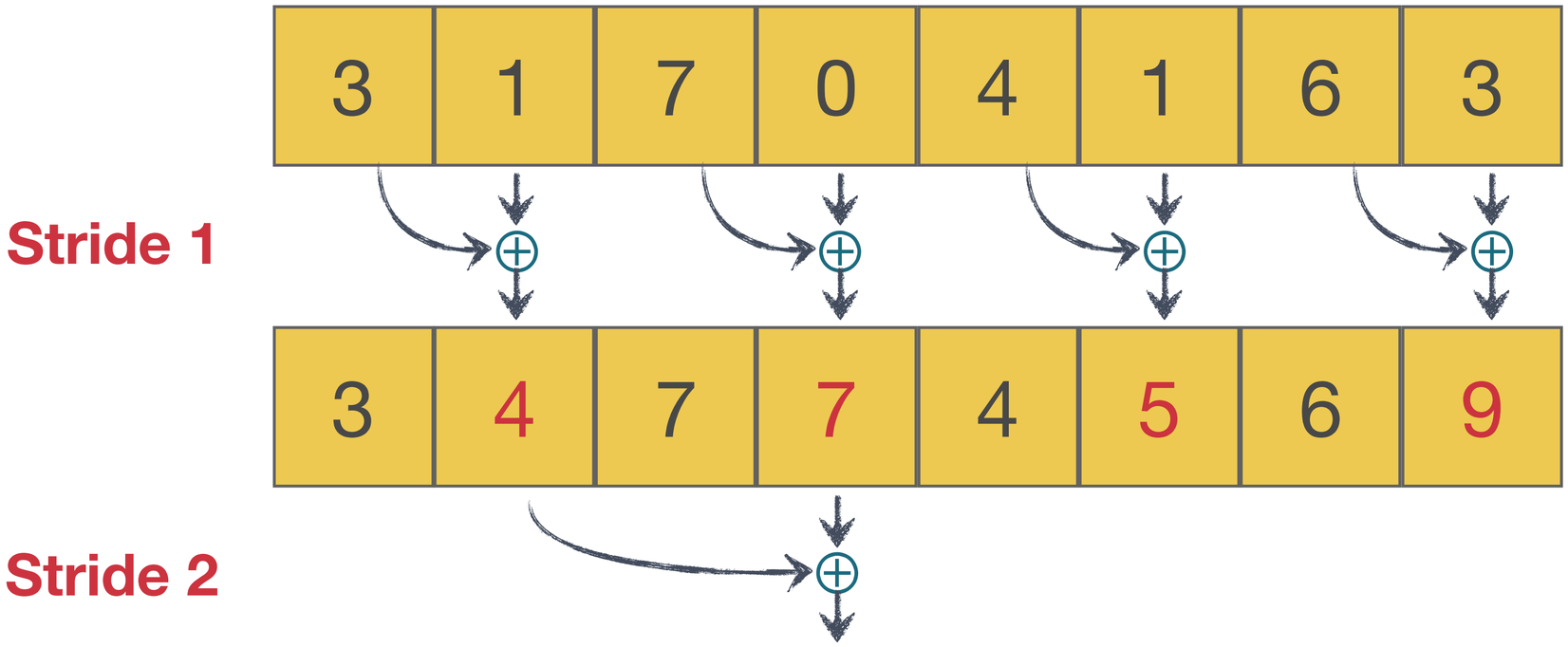
Parallel Scan: Reduction

Stride 1

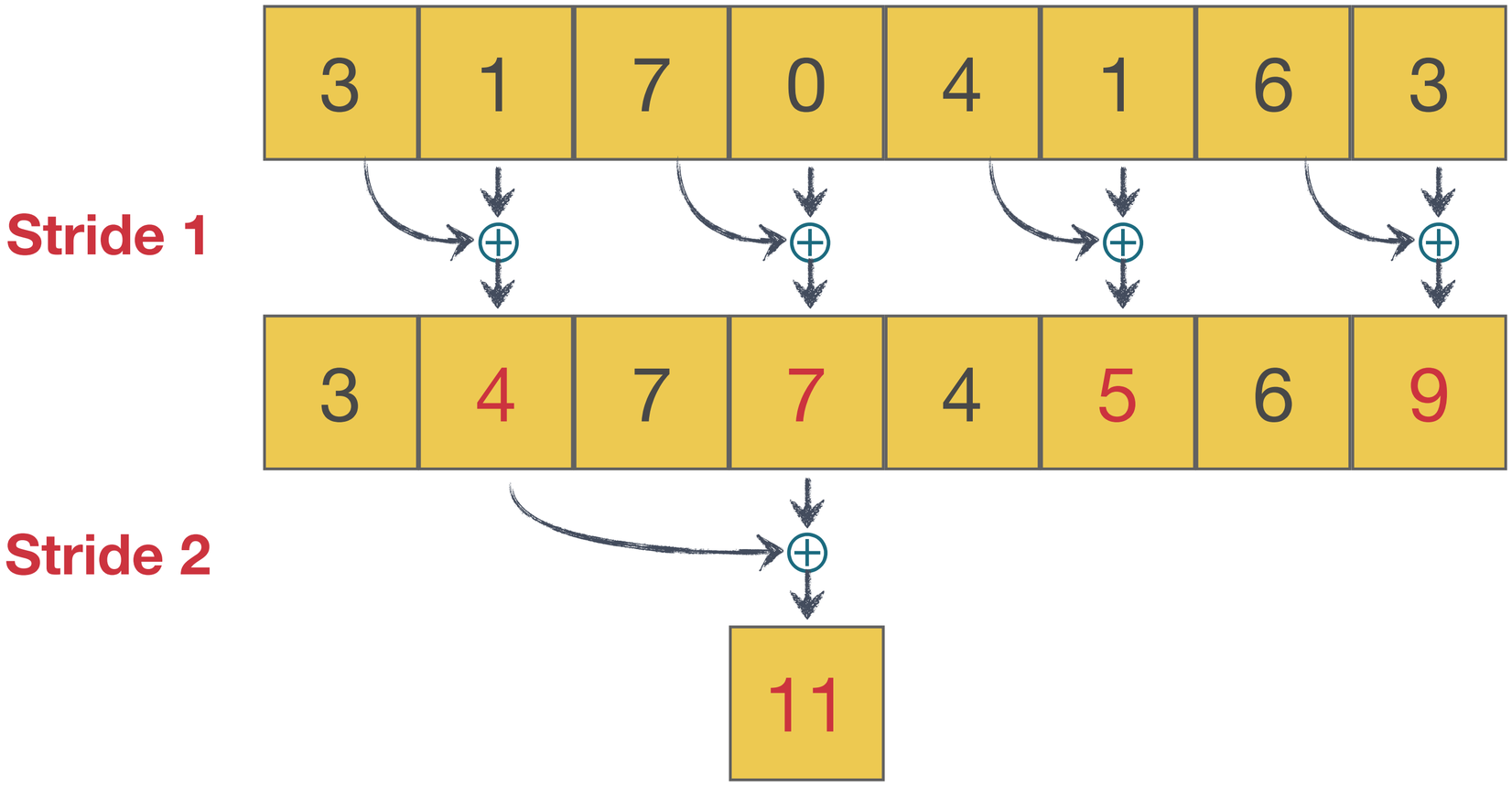


Stride 2

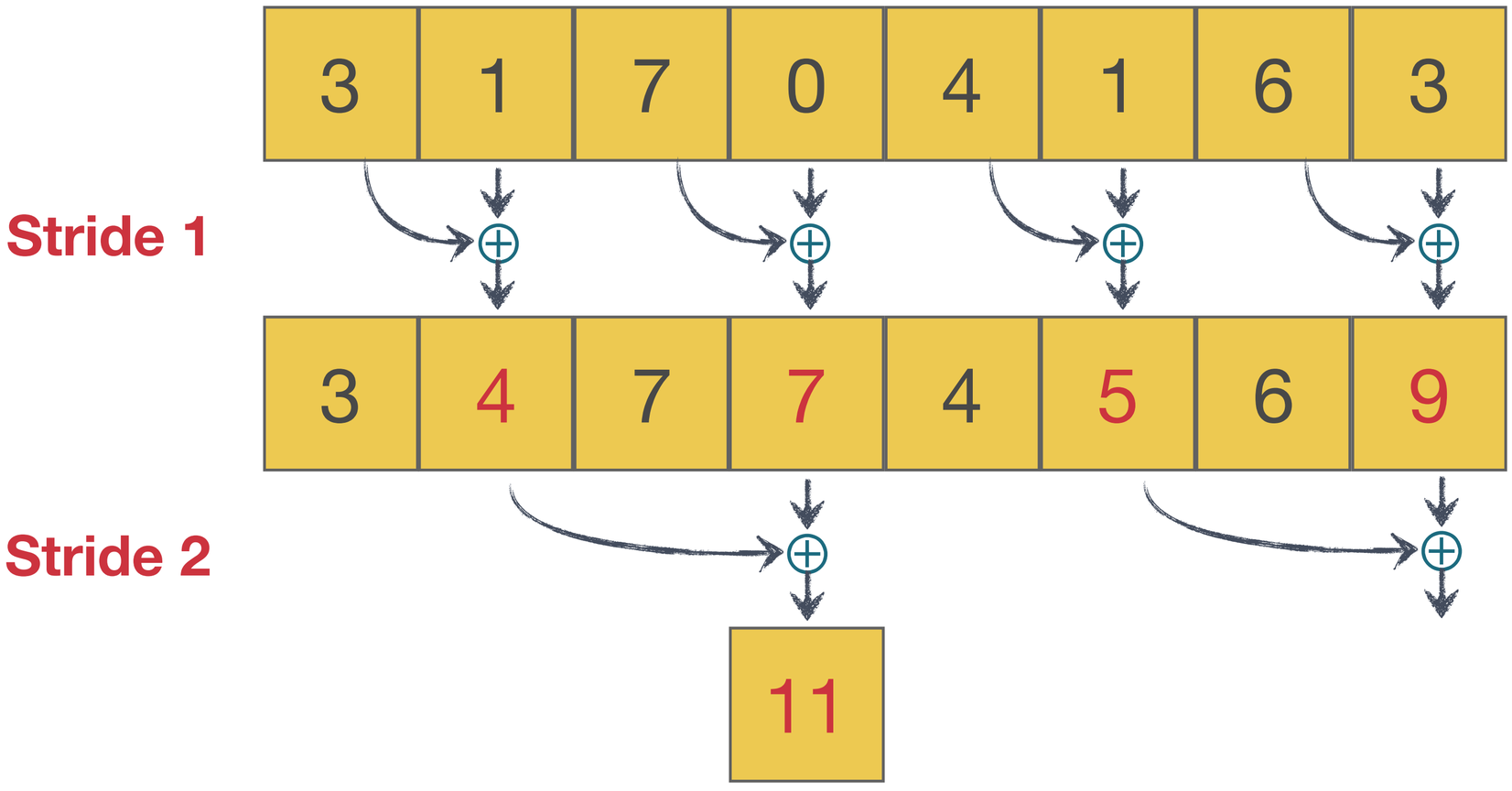
Parallel Scan: Reduction



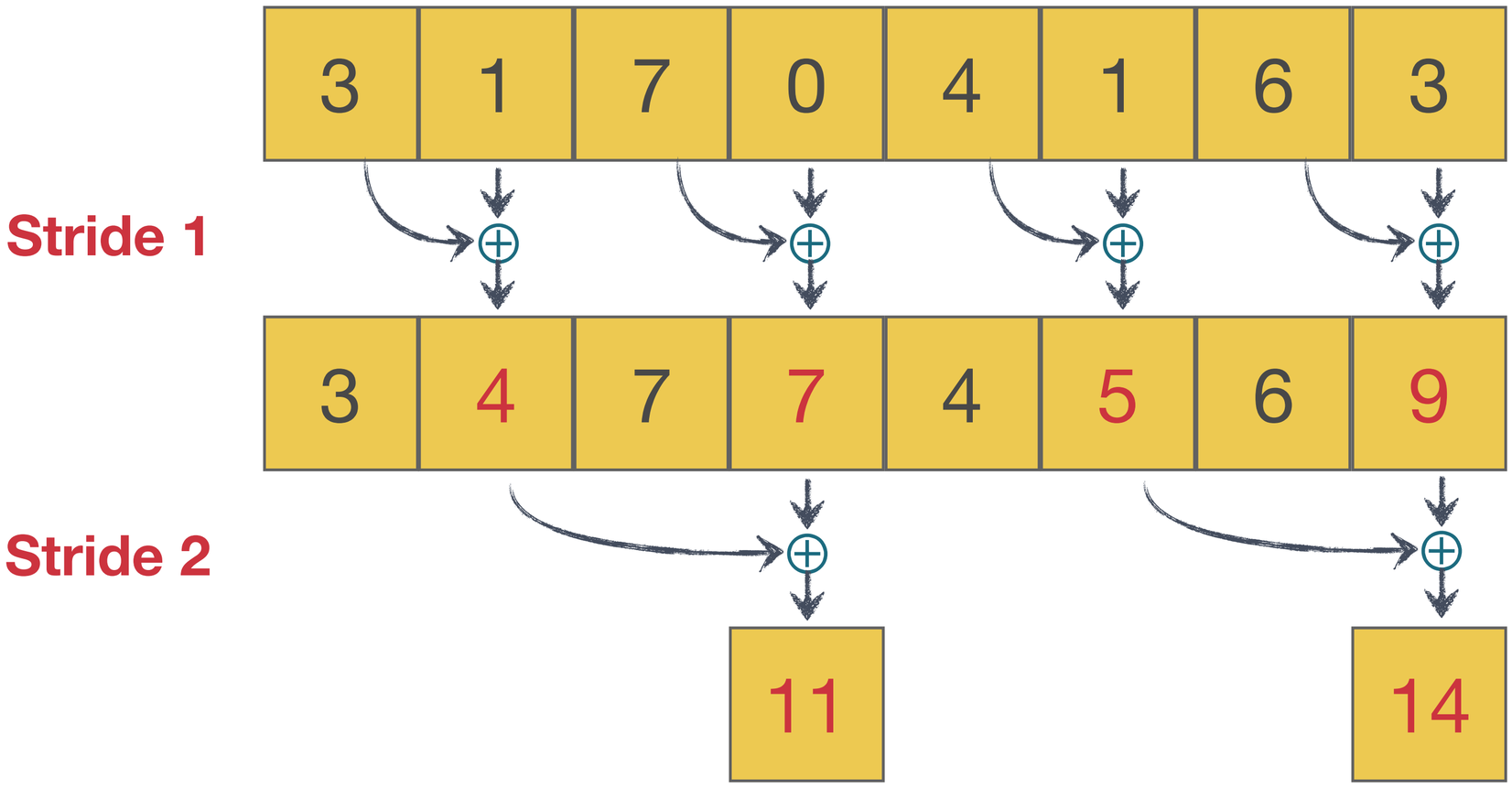
Parallel Scan: Reduction



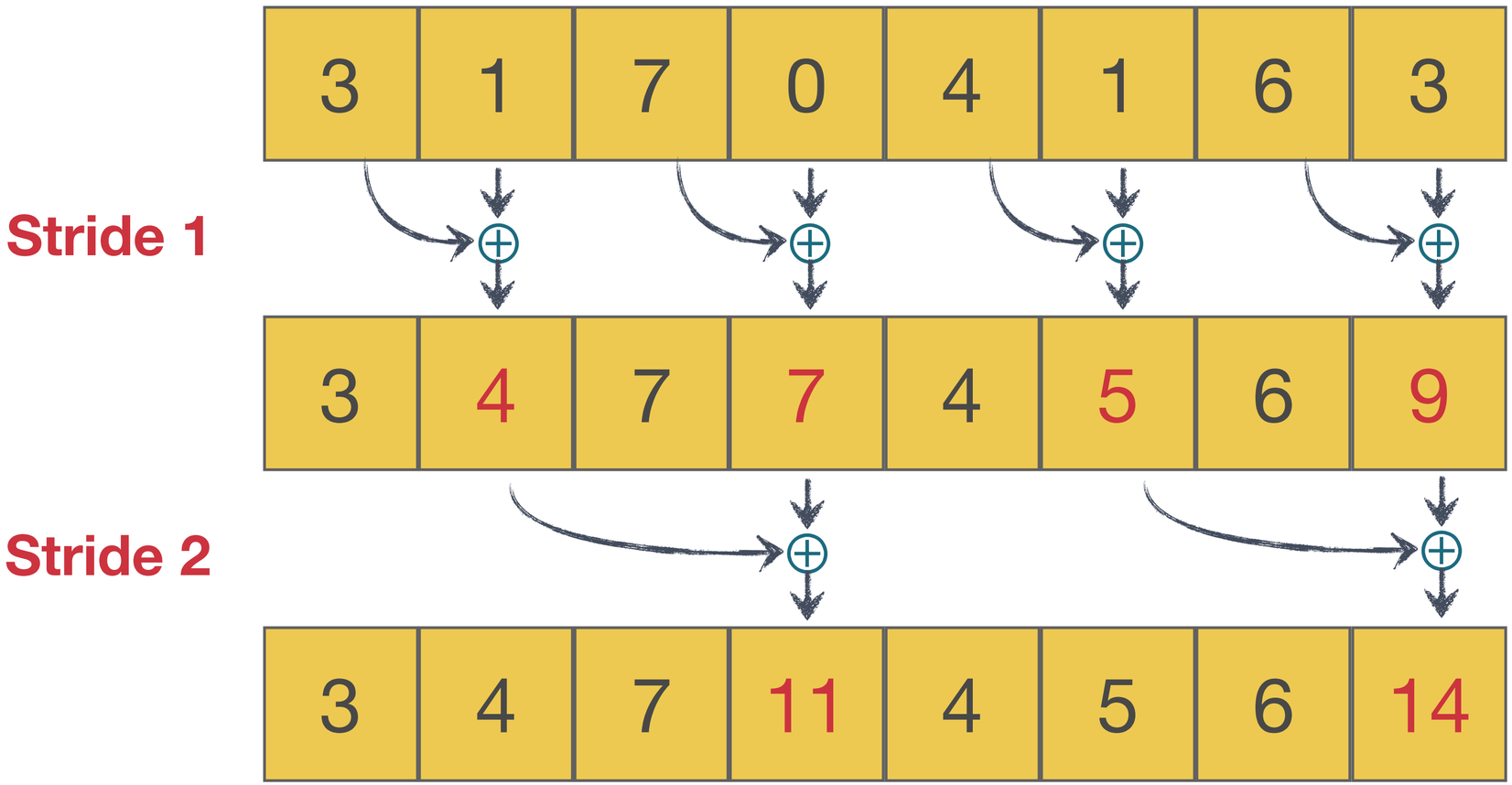
Parallel Scan: Reduction



Parallel Scan: Reduction

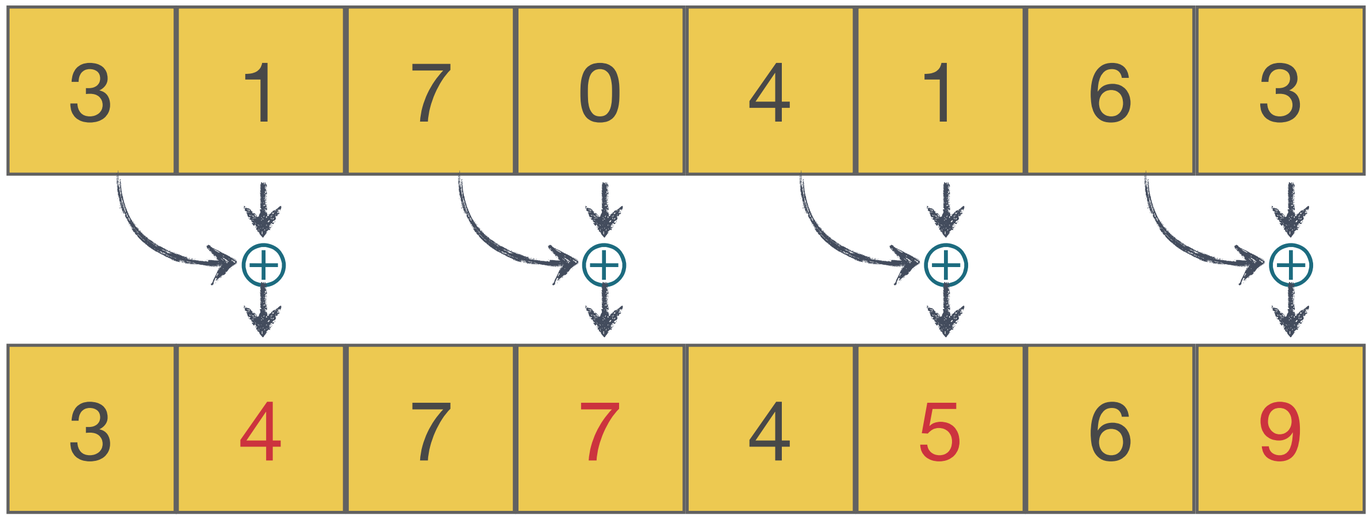


Parallel Scan: Reduction

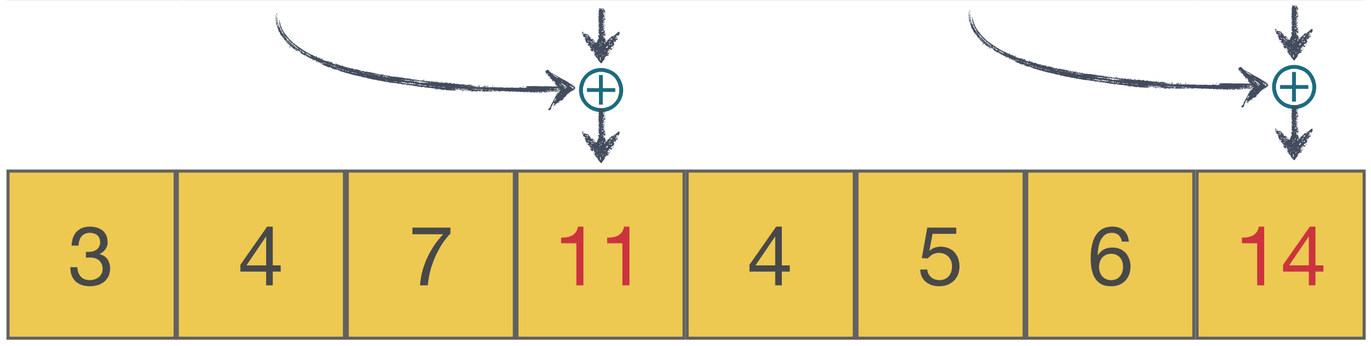


Parallel Scan: Reduction

Stride 1

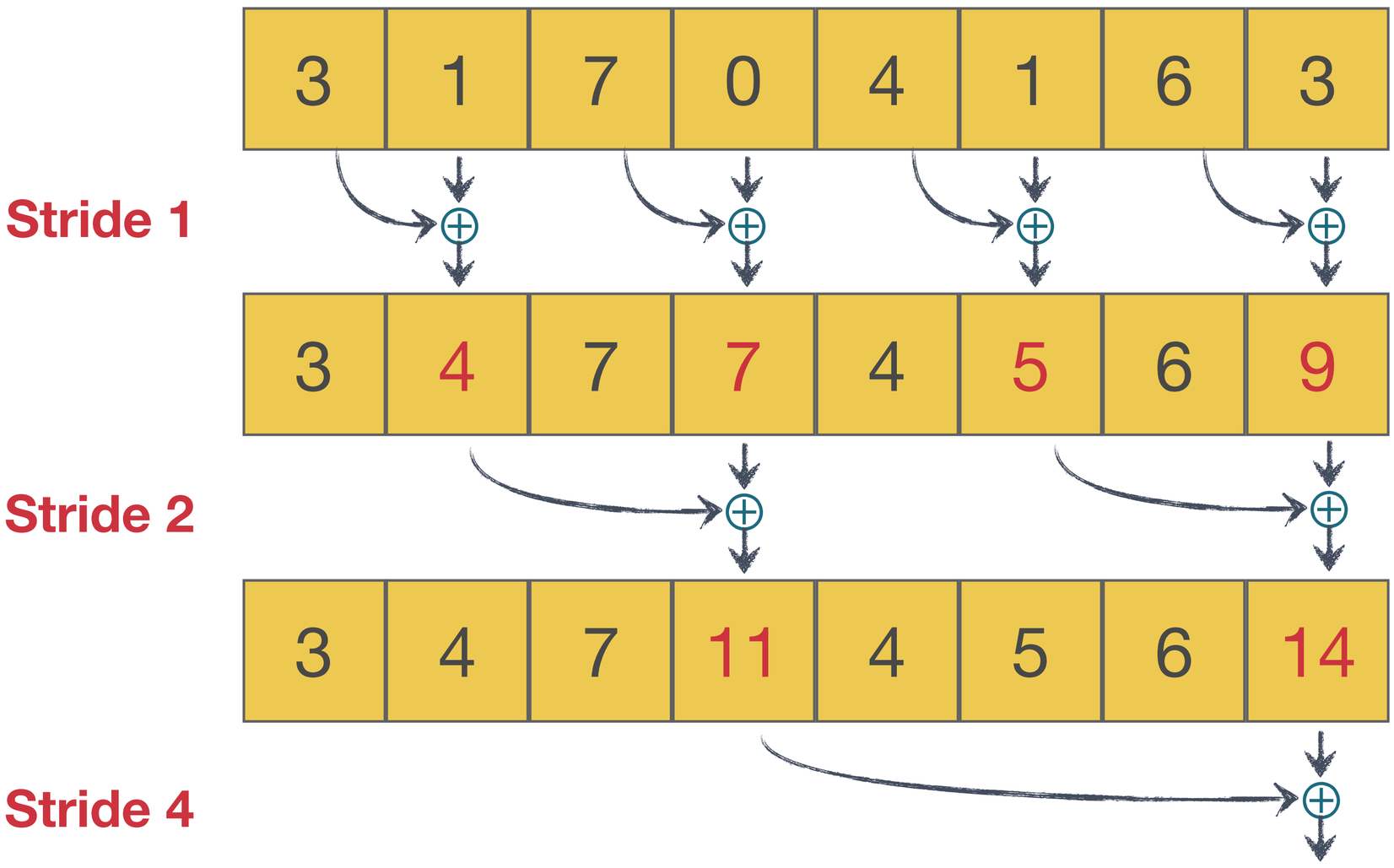


Stride 2

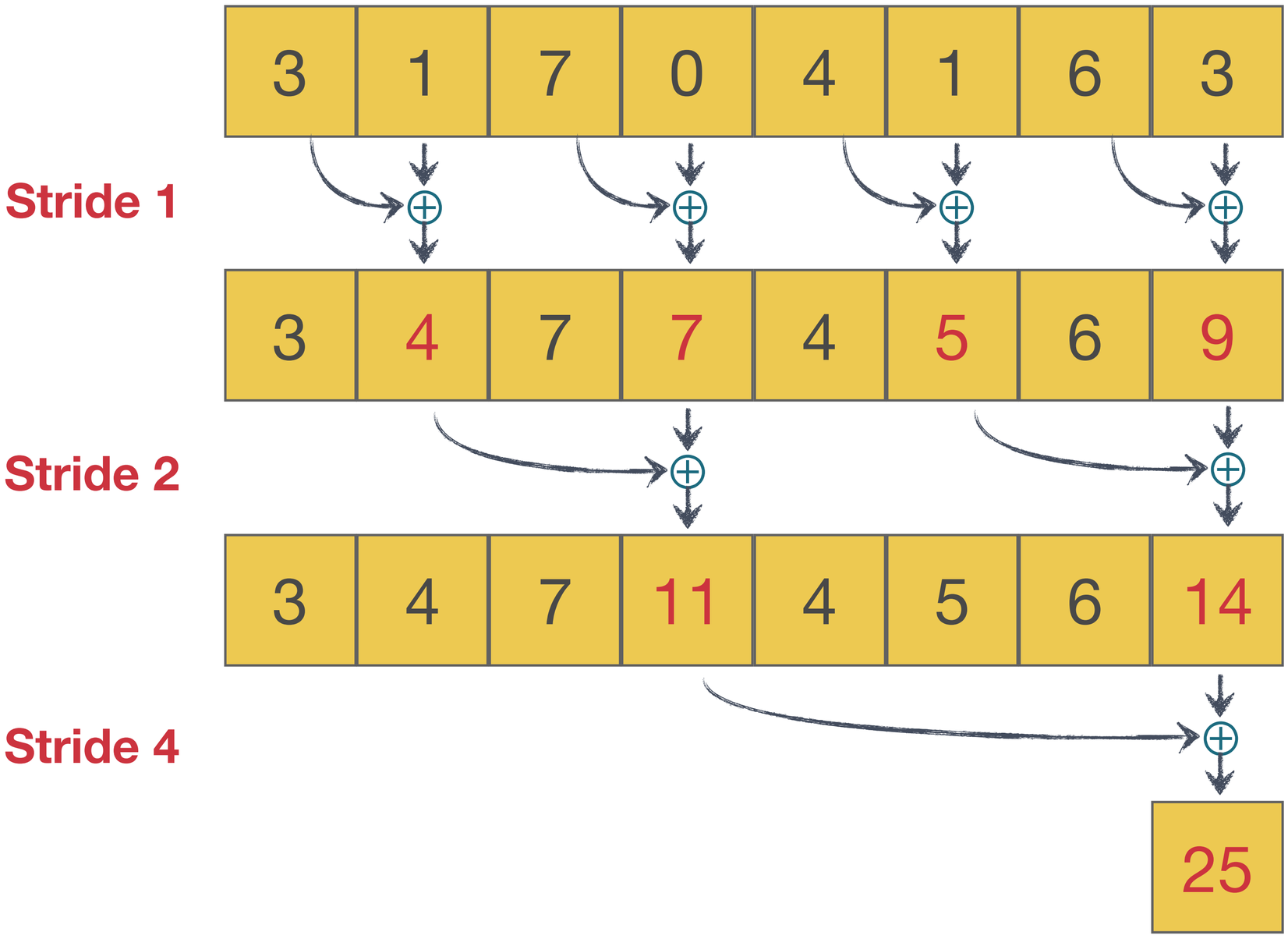


Stride 4

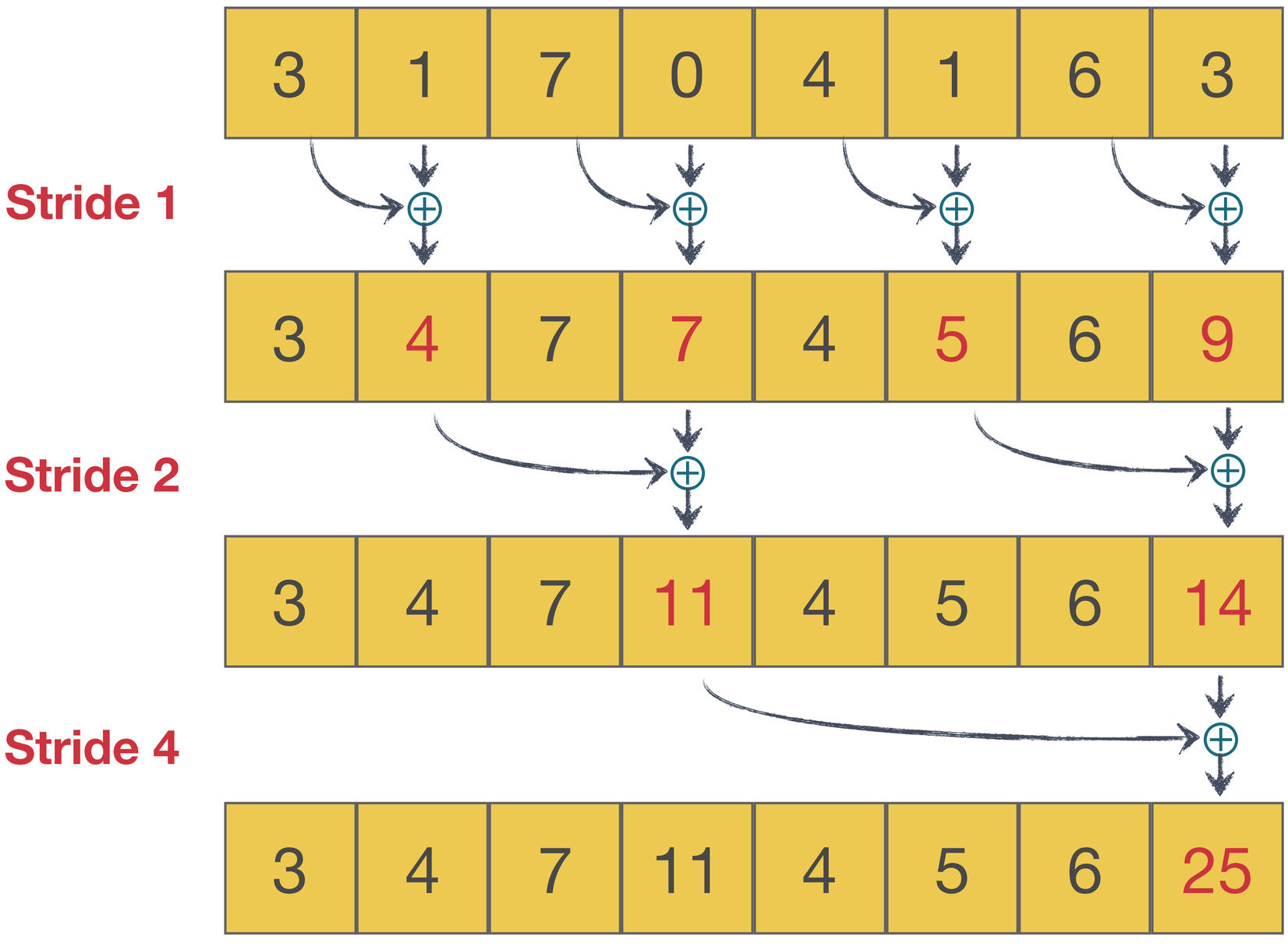
Parallel Scan: Reduction



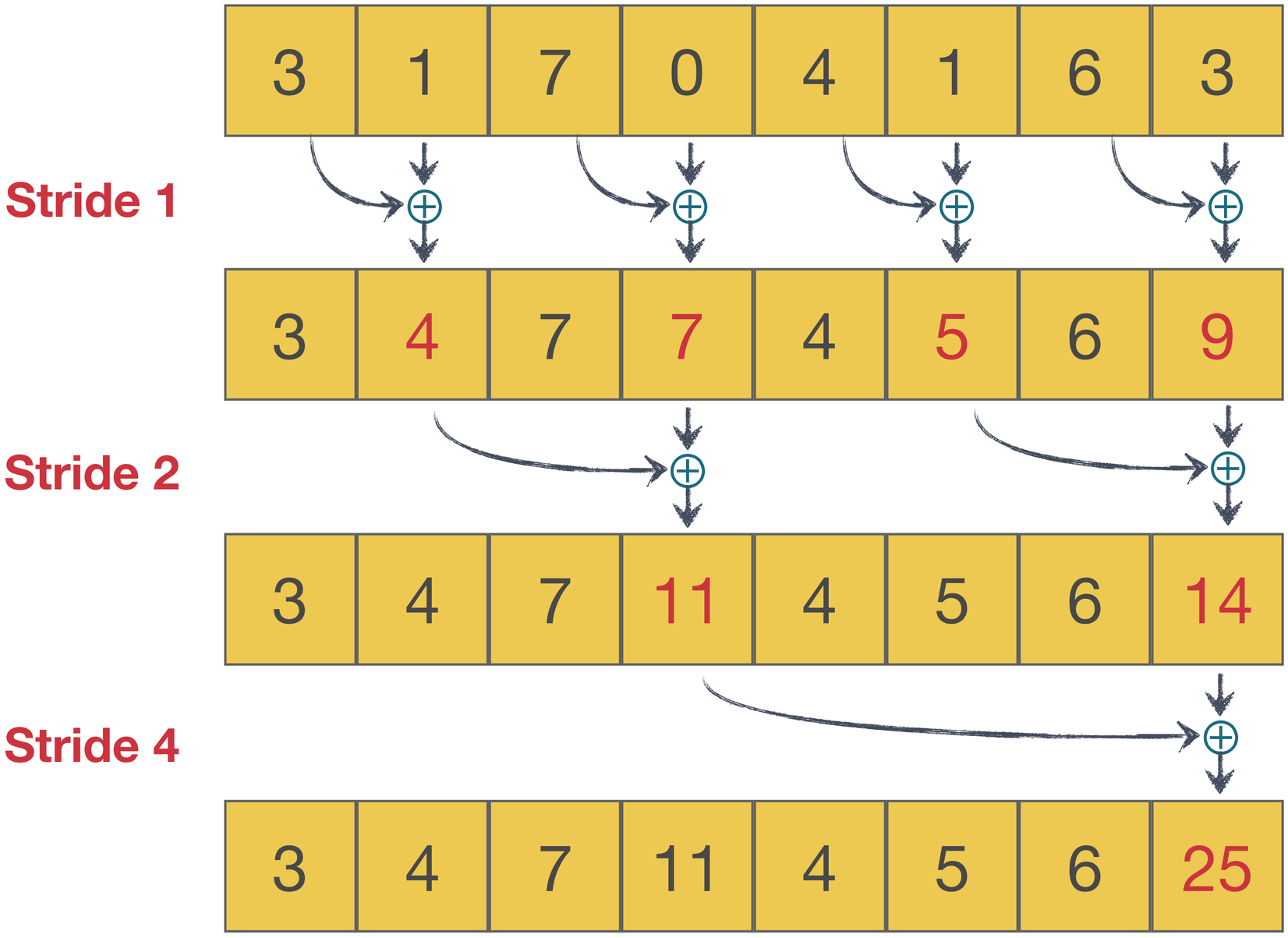
Parallel Scan: Reduction



Parallel Scan: Reduction

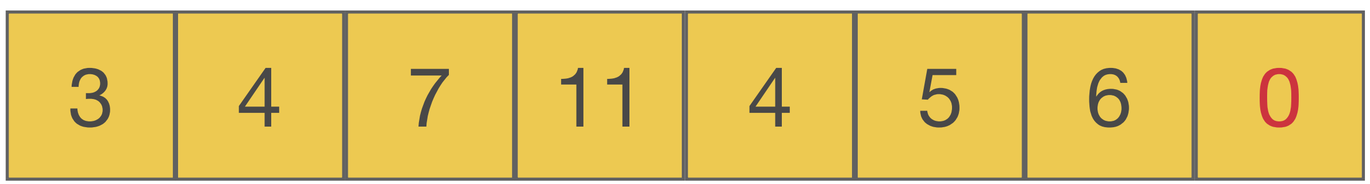


Parallel Scan: Reduction



Iterate log(n) times adding value *stride* elements away to its own value.

Parallel Scan: Zero the last element



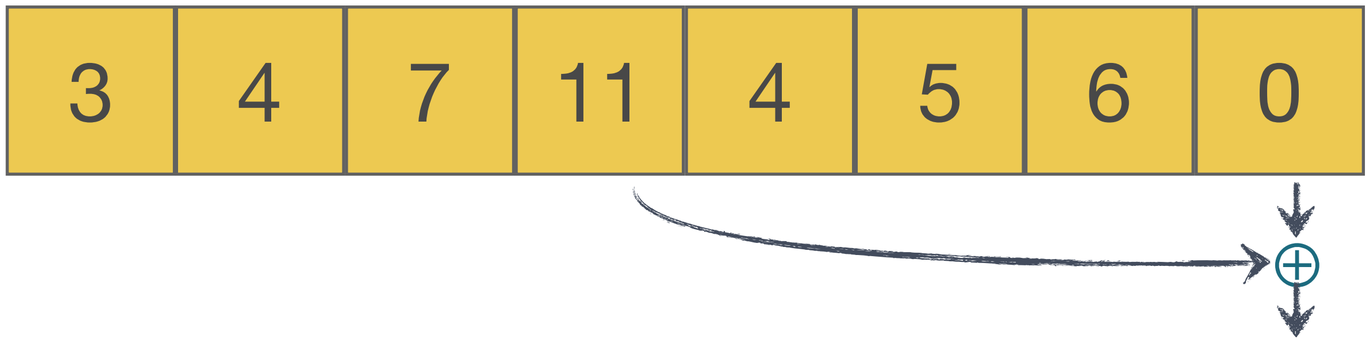
Since this is an exclusive scan,set the last element to zero. It will propagate back to the first element.

Parallel Scan: Post Scan

Parallel Scan: Post Scan

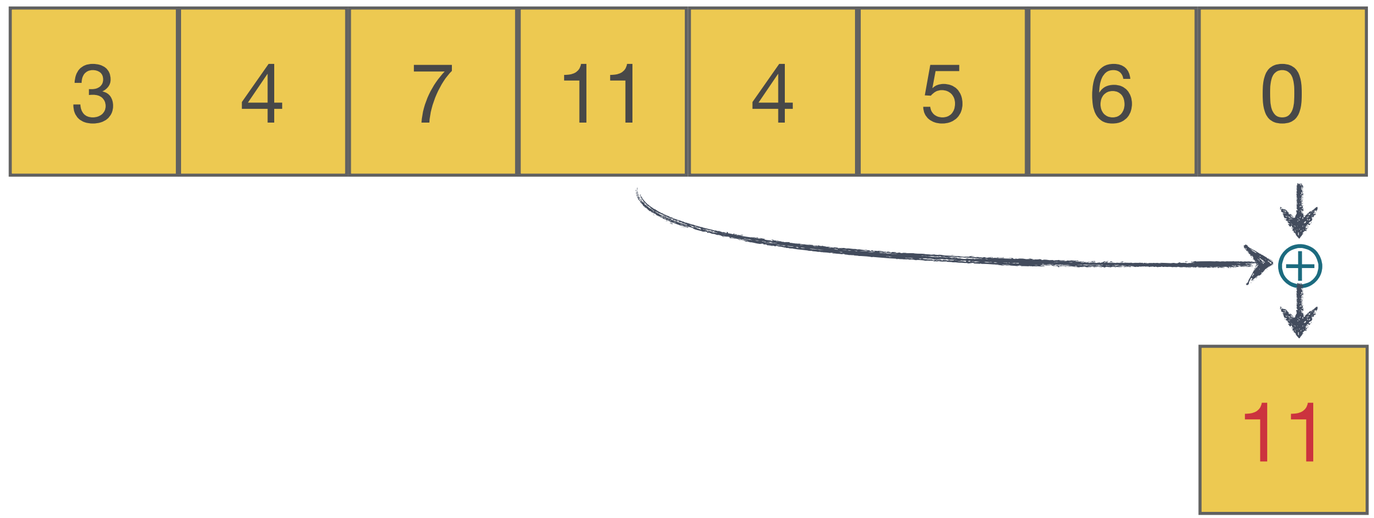
Stride 4

Parallel Scan: Post Scan



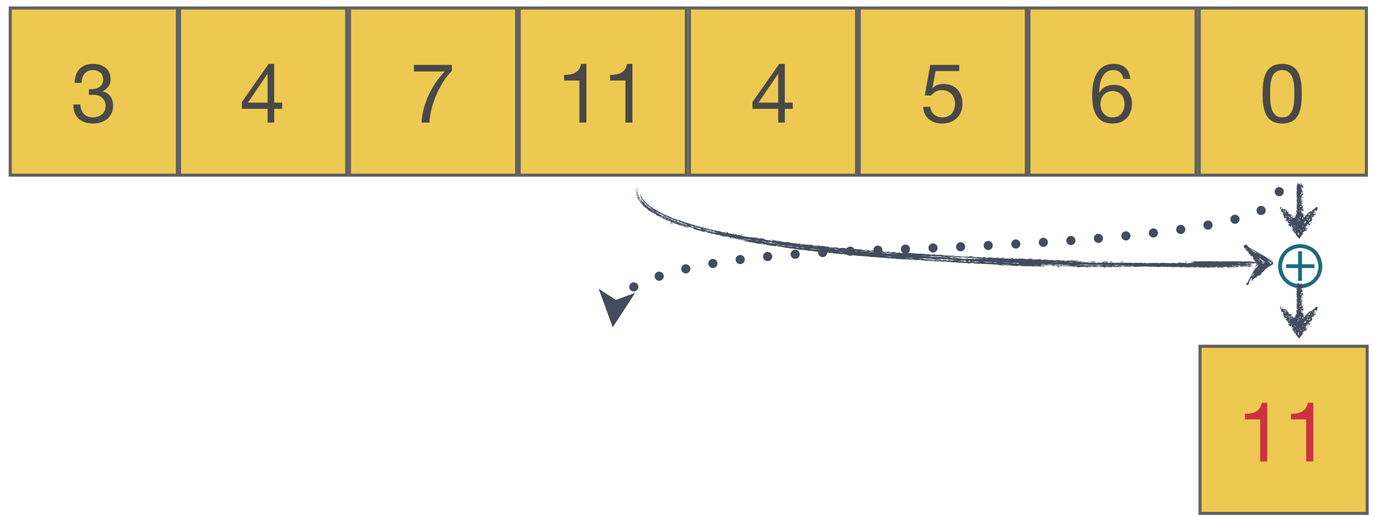
Stride 4

Parallel Scan: Post Scan



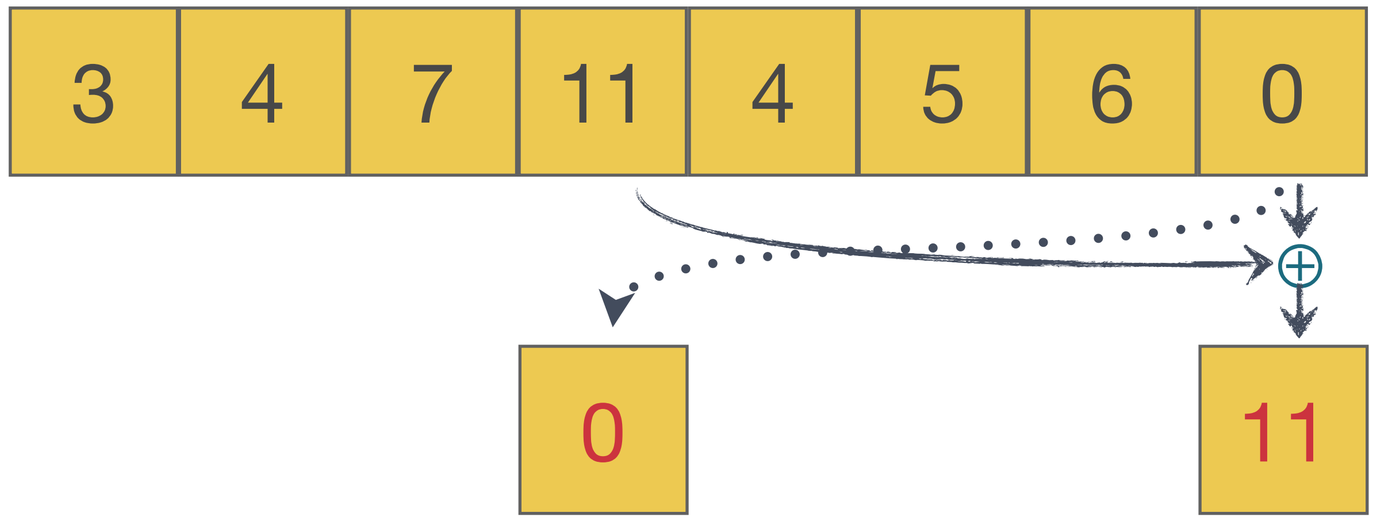
Stride 4

Parallel Scan: Post Scan



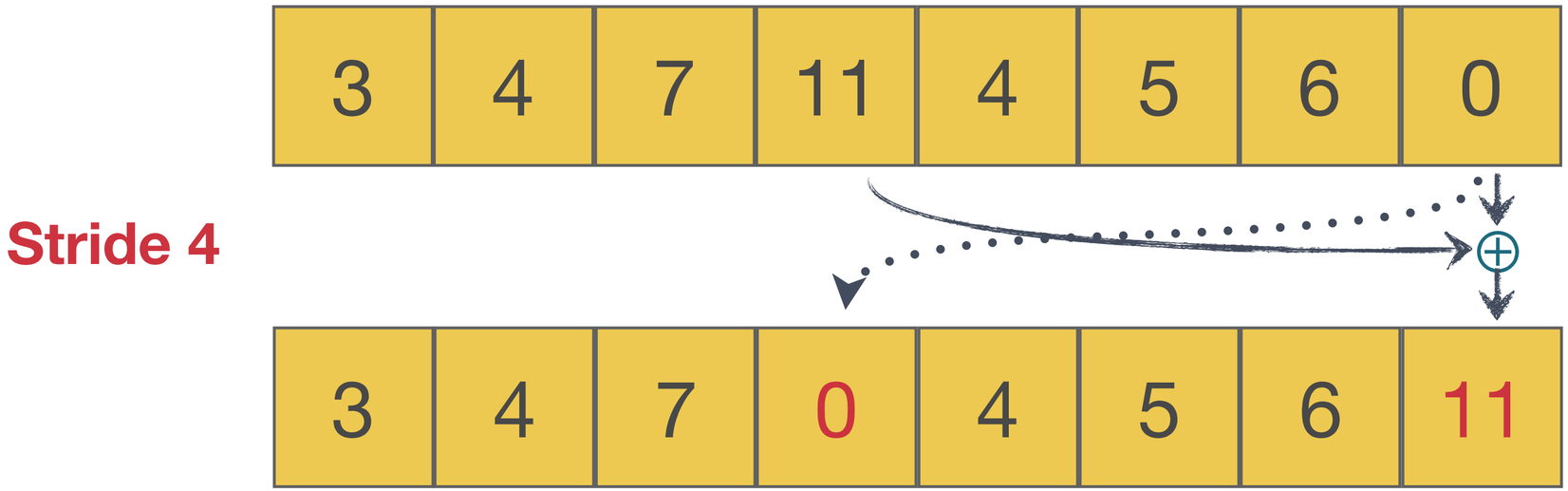
Stride 4

Parallel Scan: Post Scan

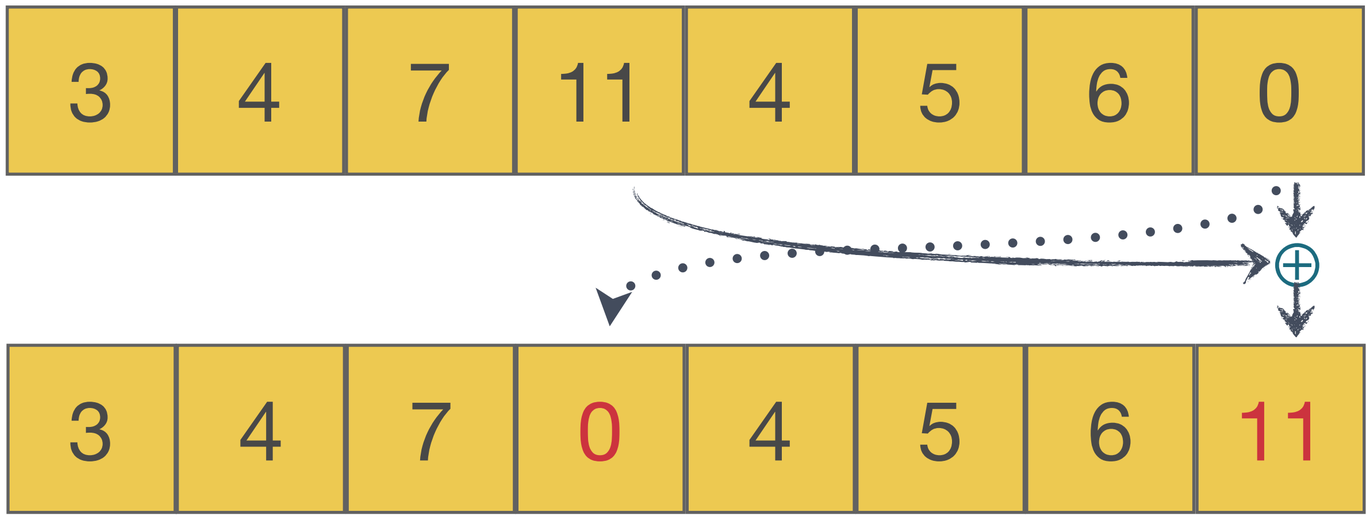


Stride 4

Parallel Scan: Post Scan

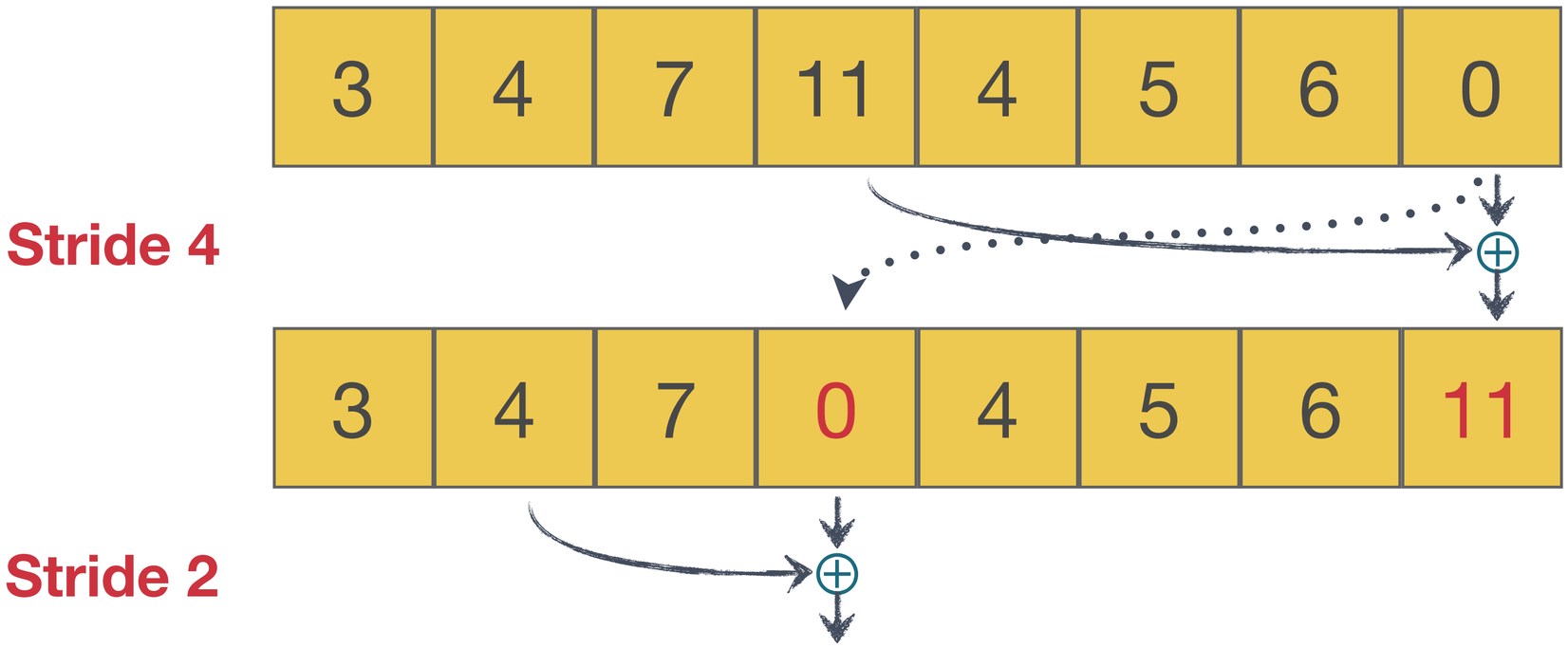


Parallel Scan: Post Scan

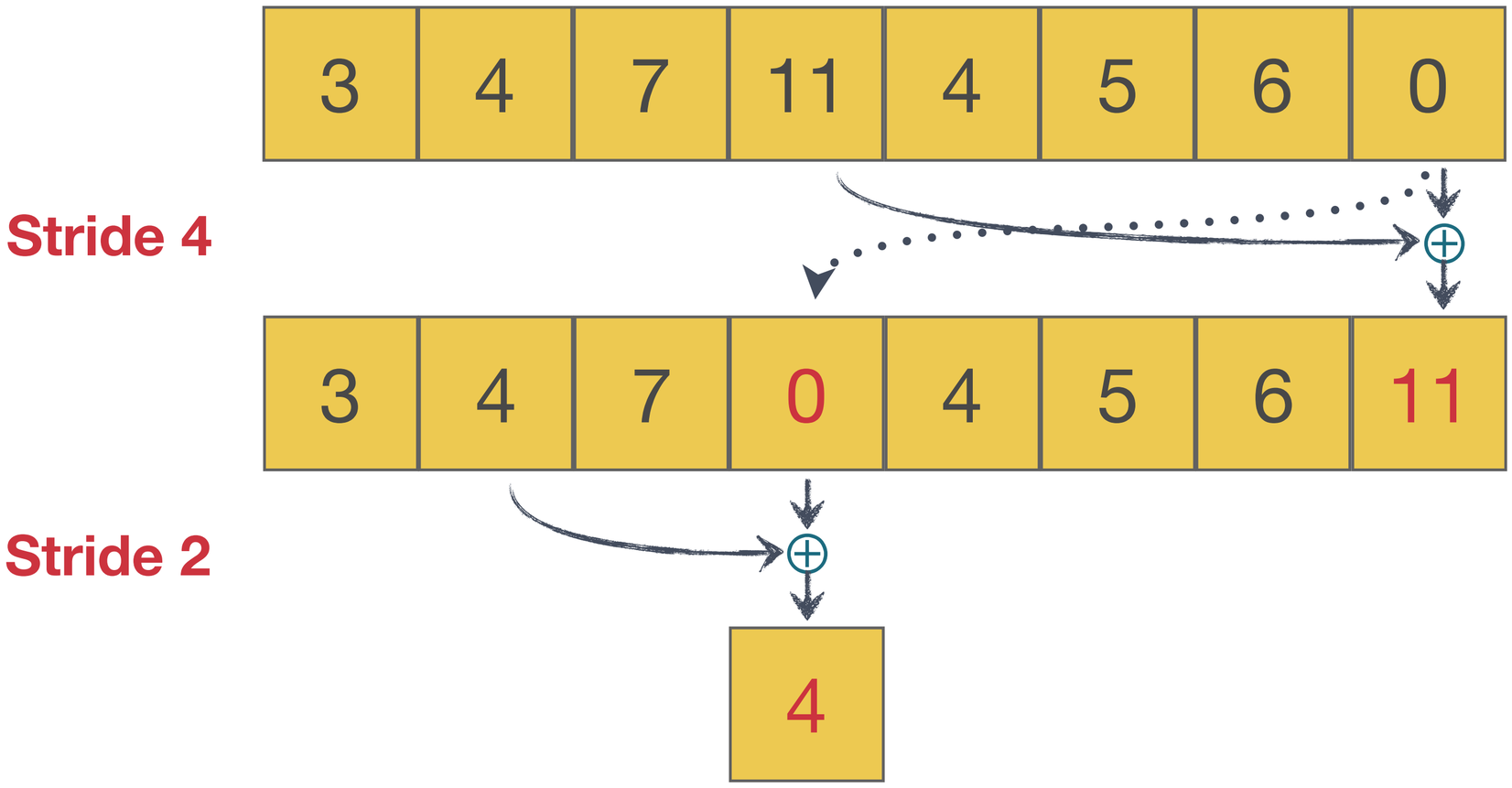


Stride 2

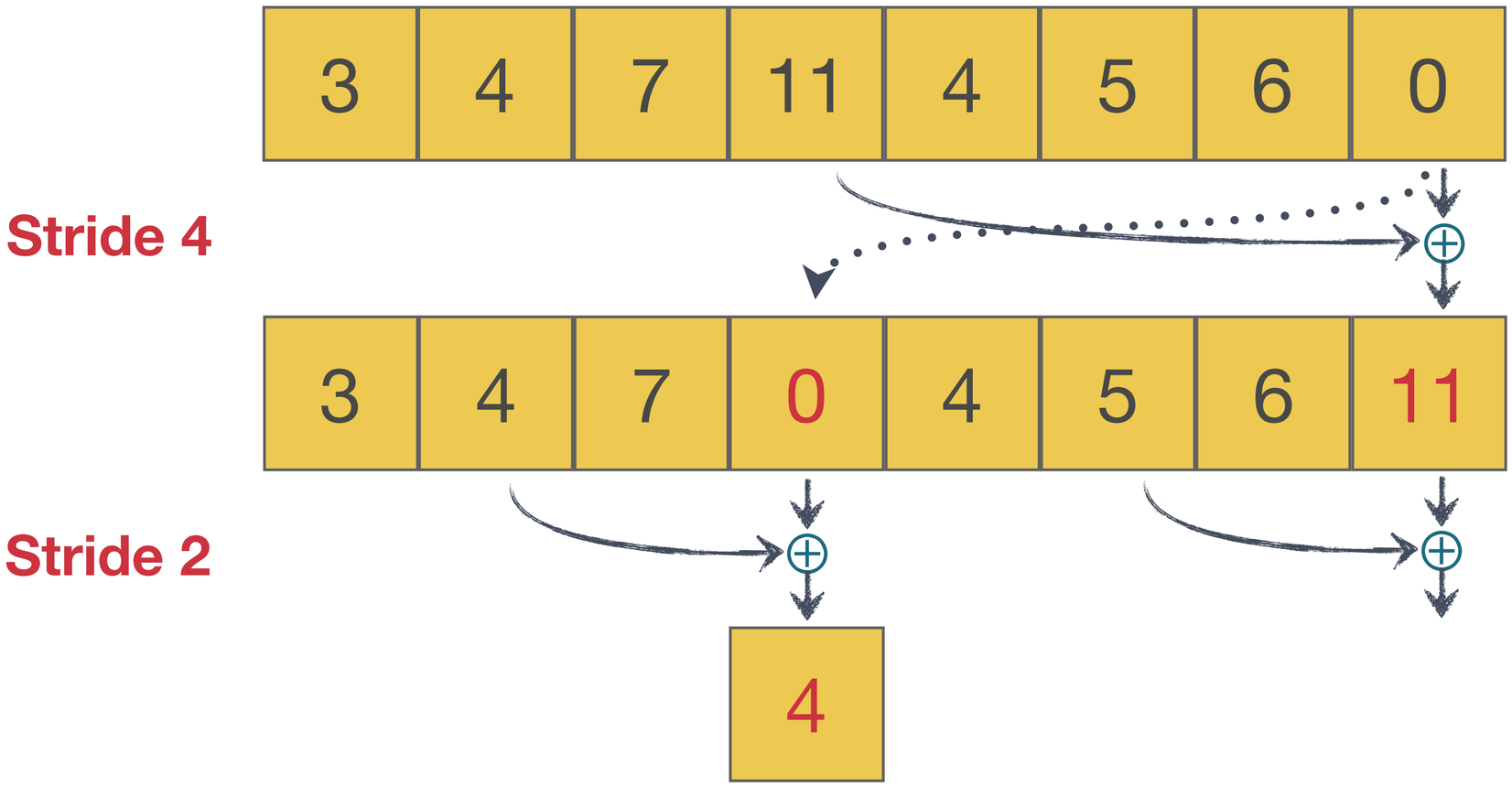
Parallel Scan: Post Scan



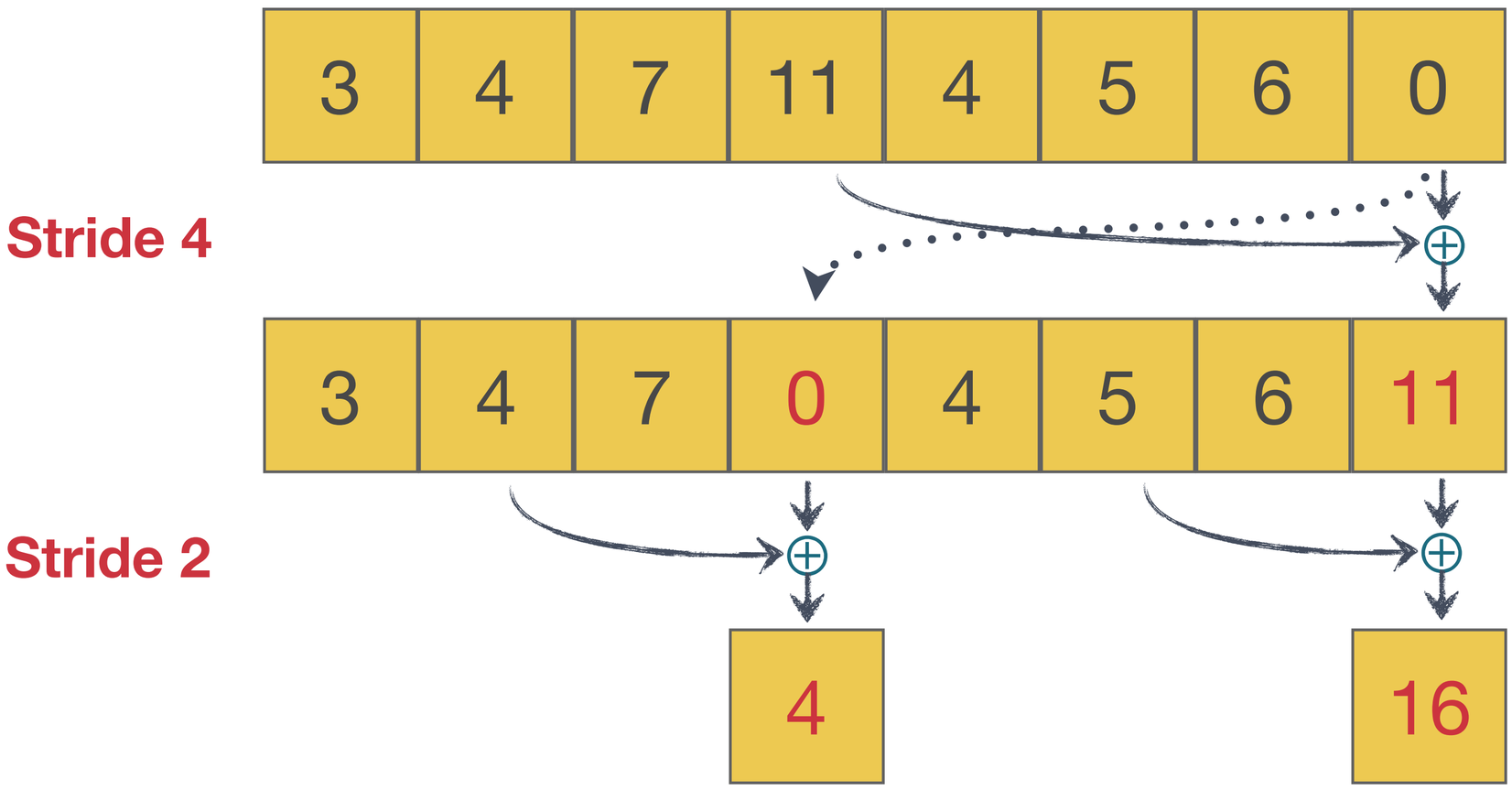
Parallel Scan: Post Scan



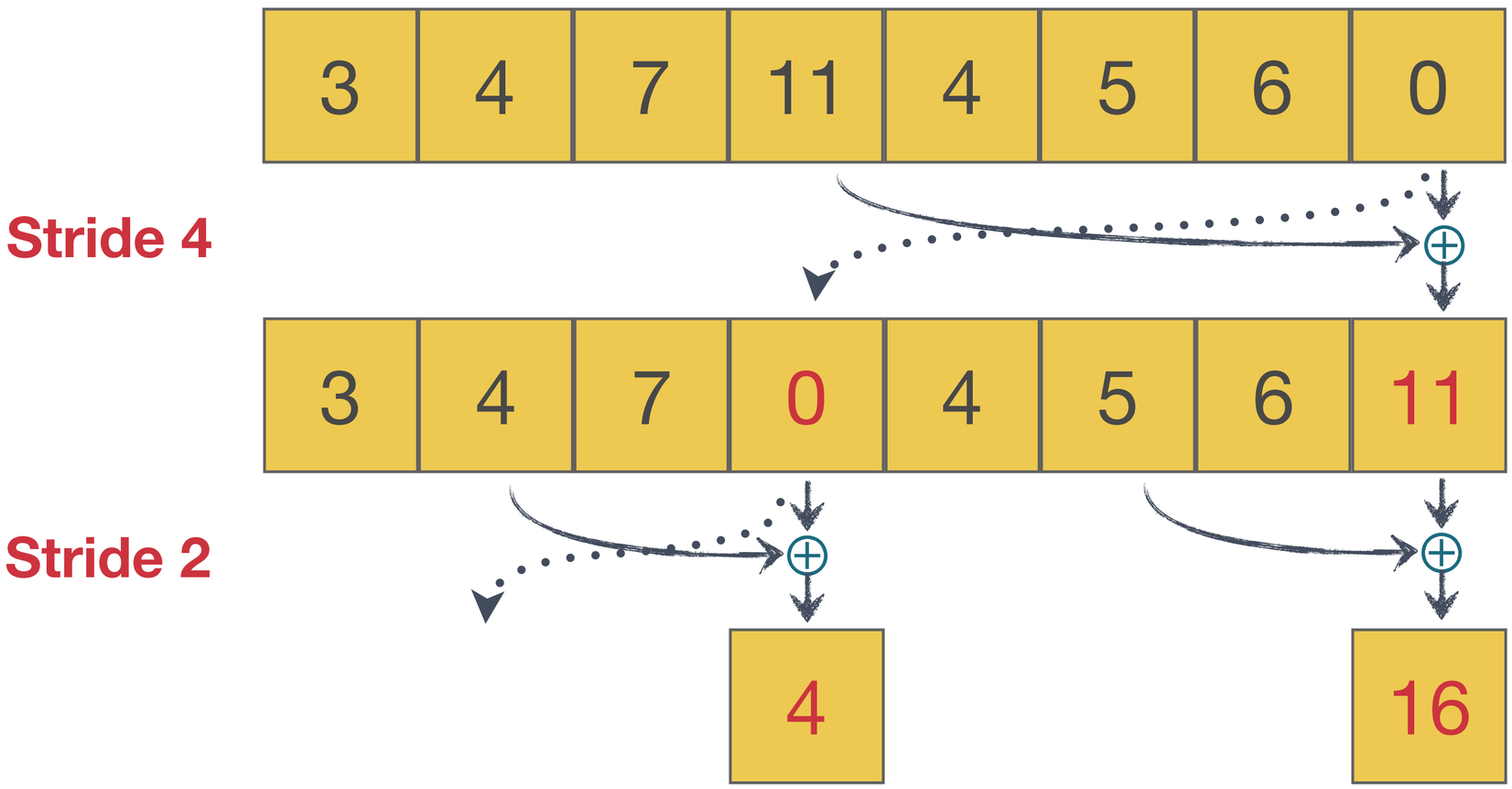
Parallel Scan: Post Scan



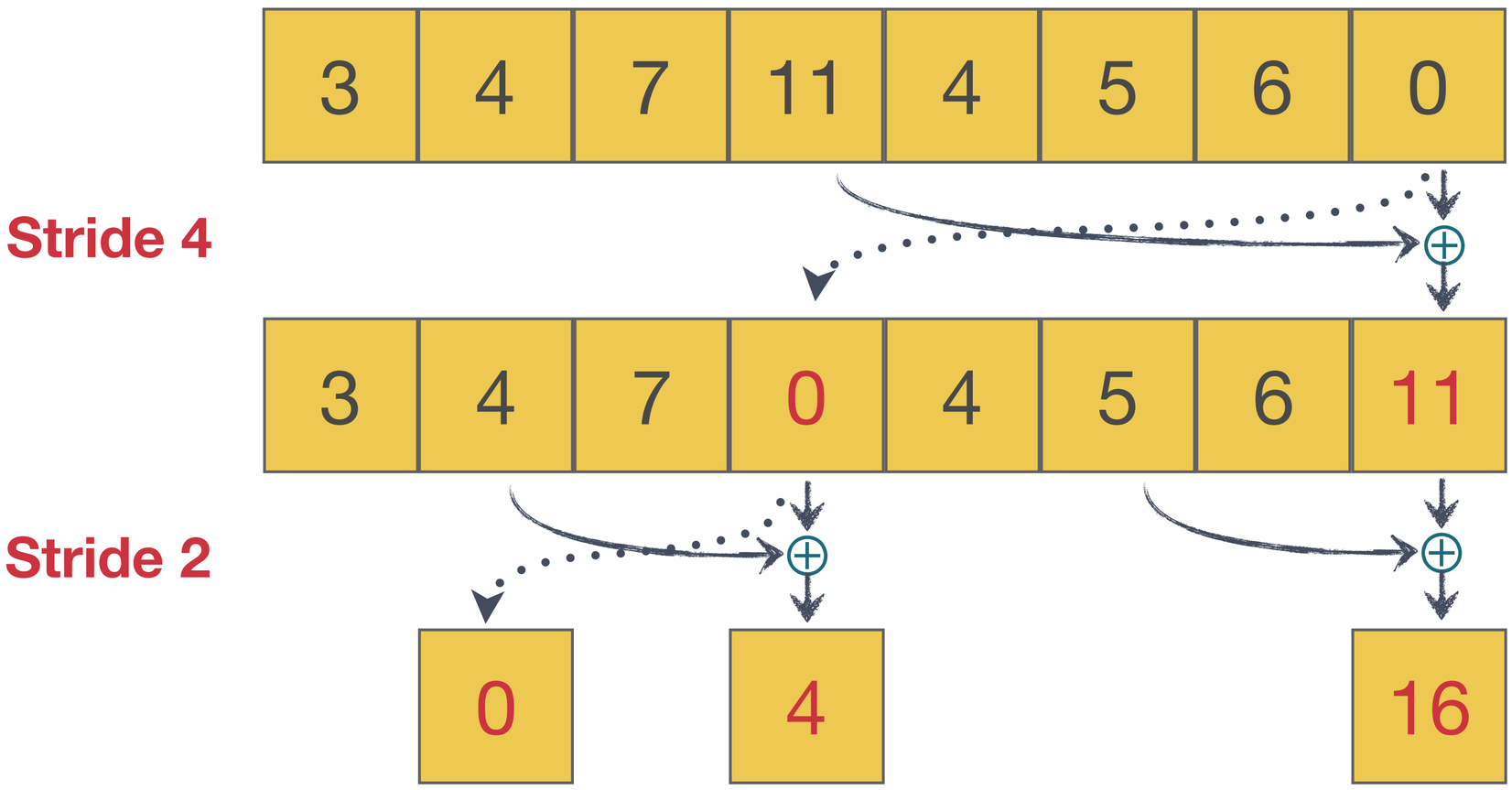
Parallel Scan: Post Scan



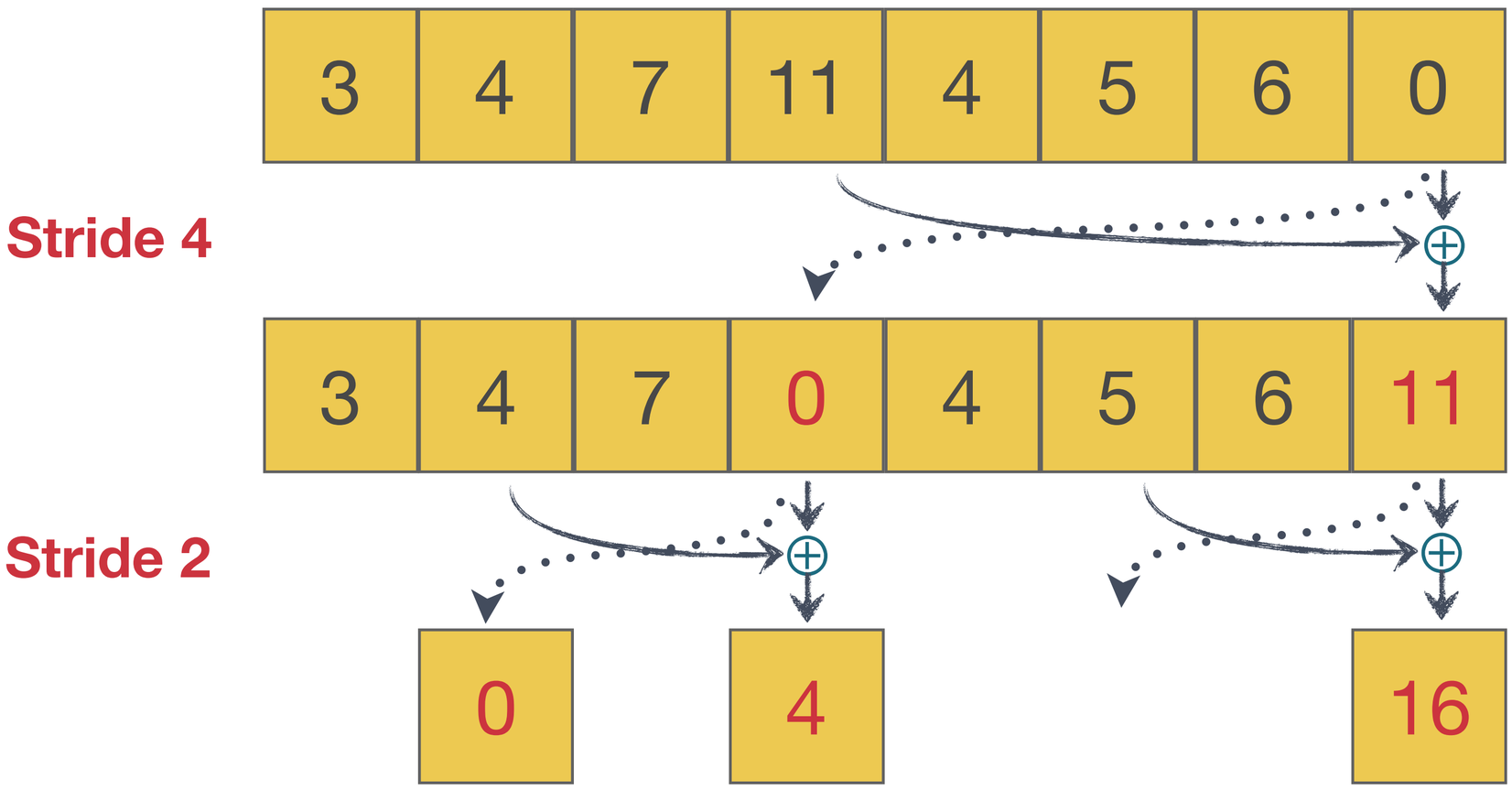
Parallel Scan: Post Scan



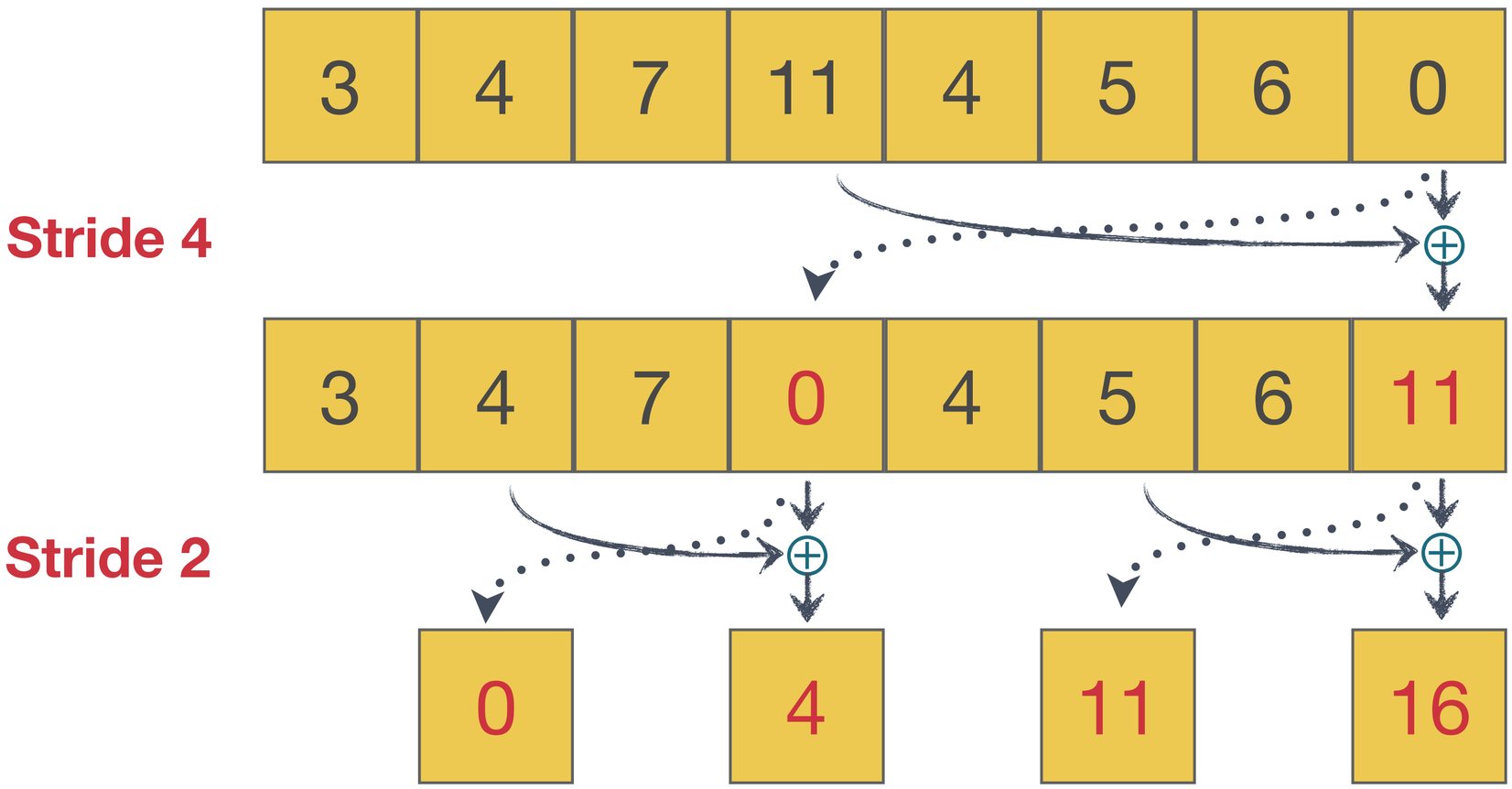
Parallel Scan: Post Scan



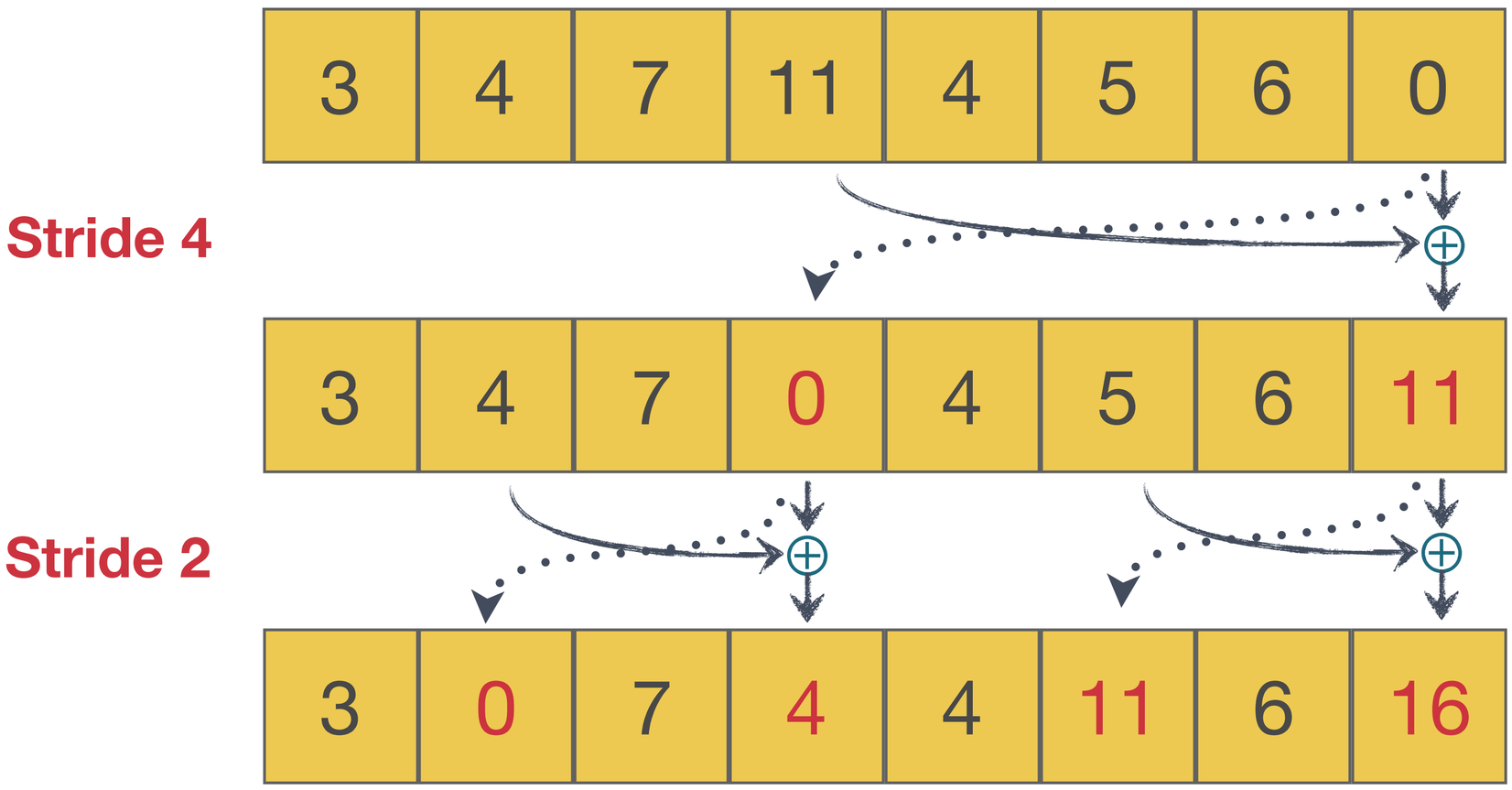
Parallel Scan: Post Scan



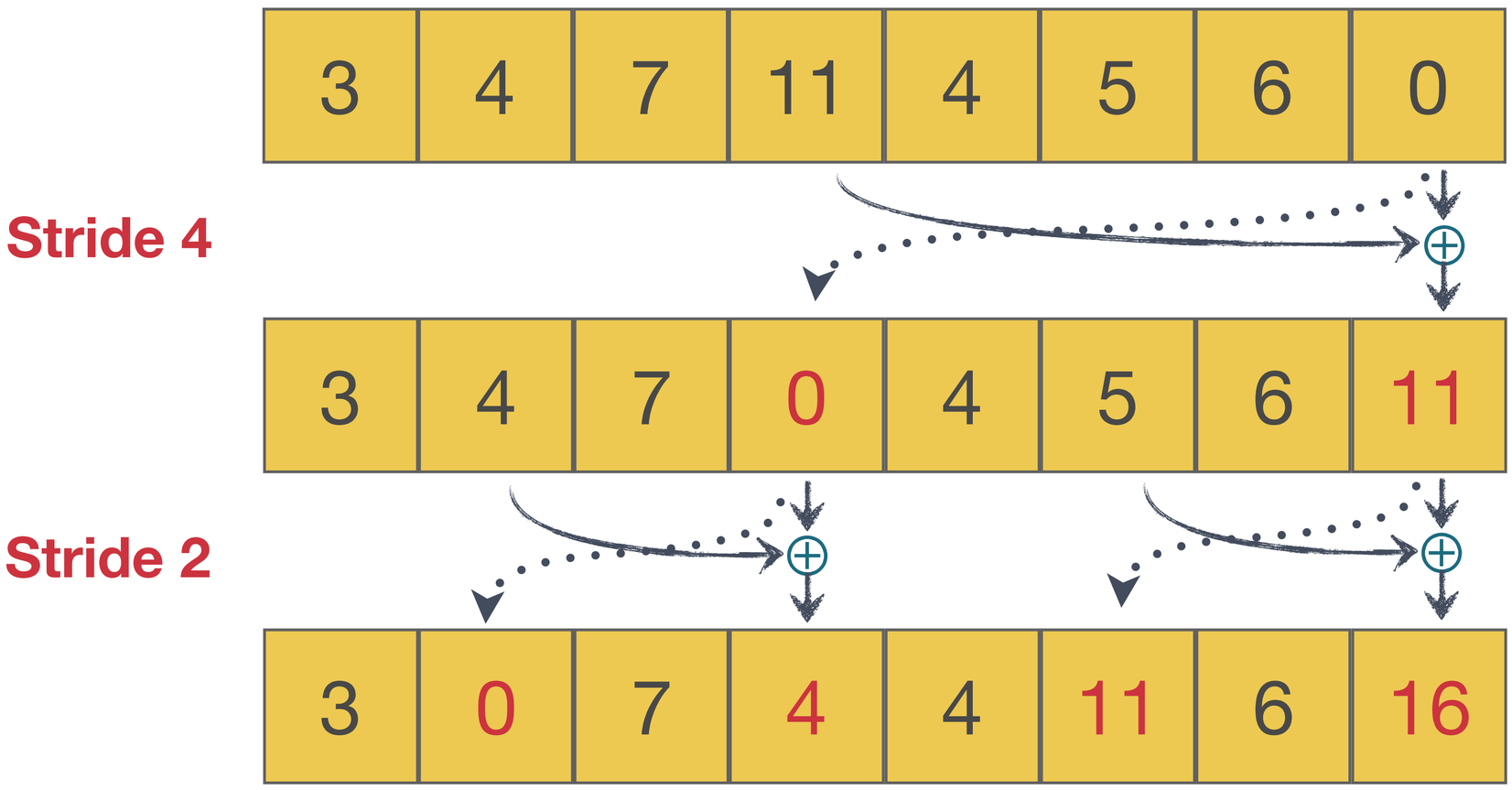
Parallel Scan: Post Scan



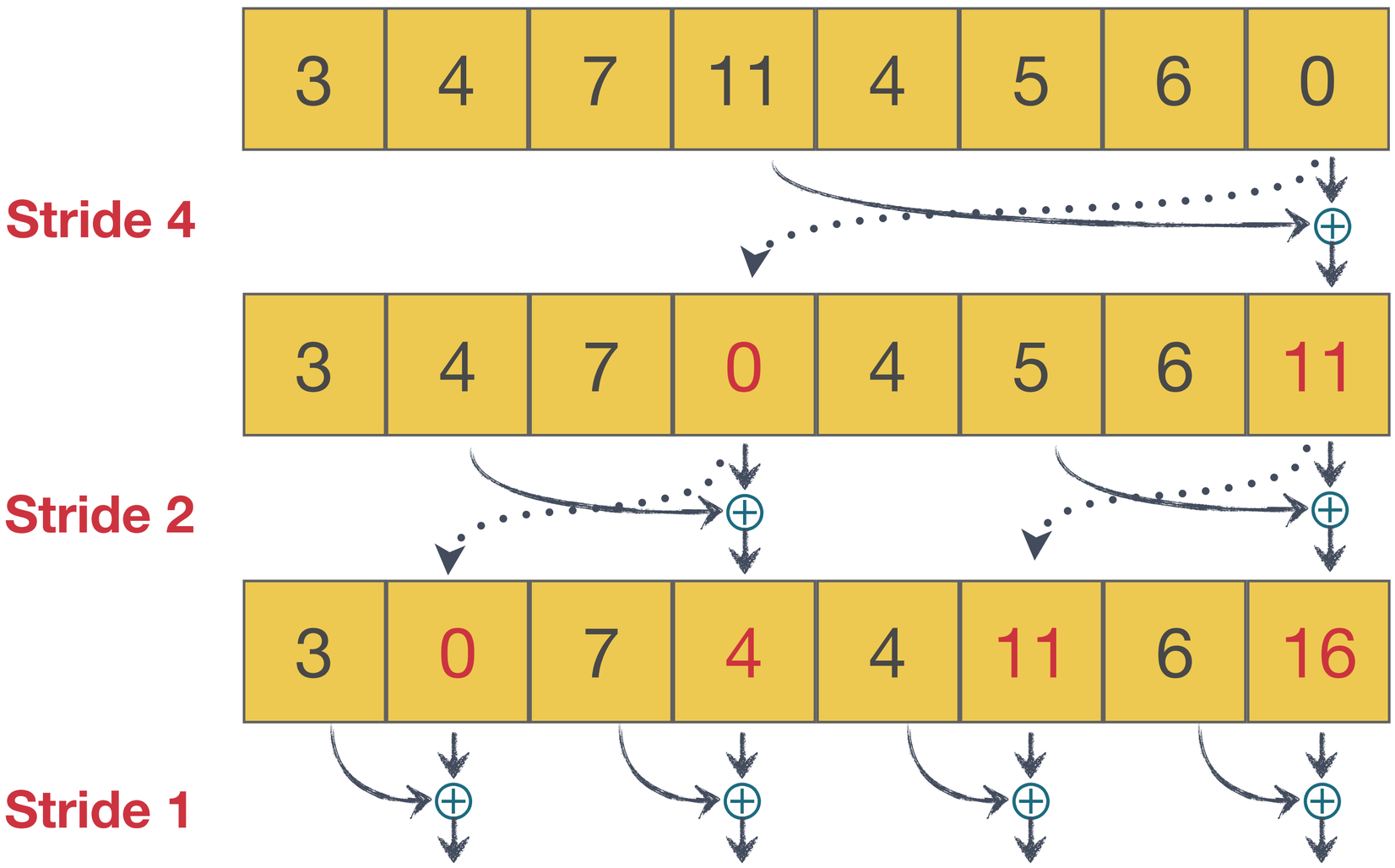
Parallel Scan: Post Scan



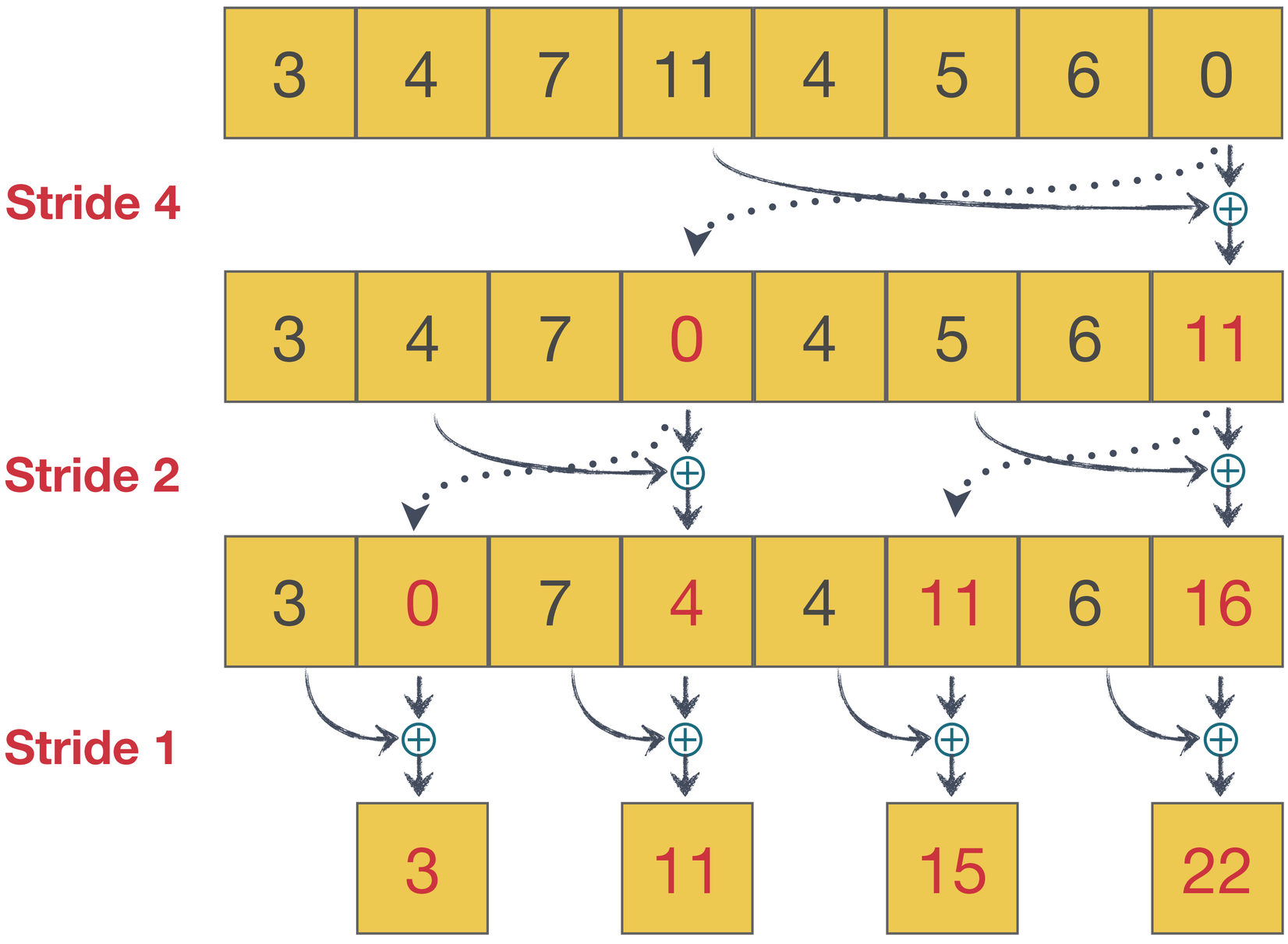
Parallel Scan: Post Scan



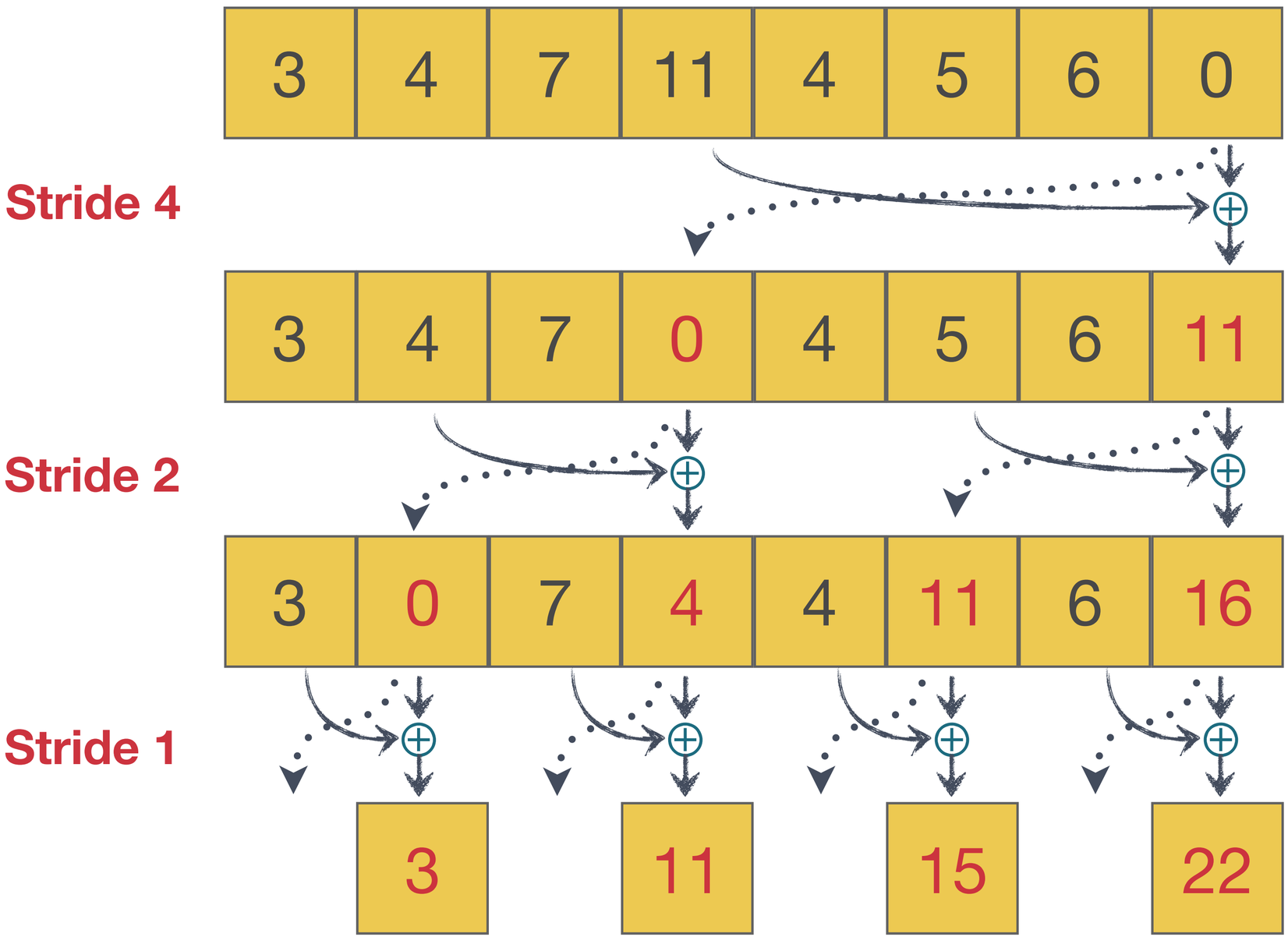
Parallel Scan: Post Scan



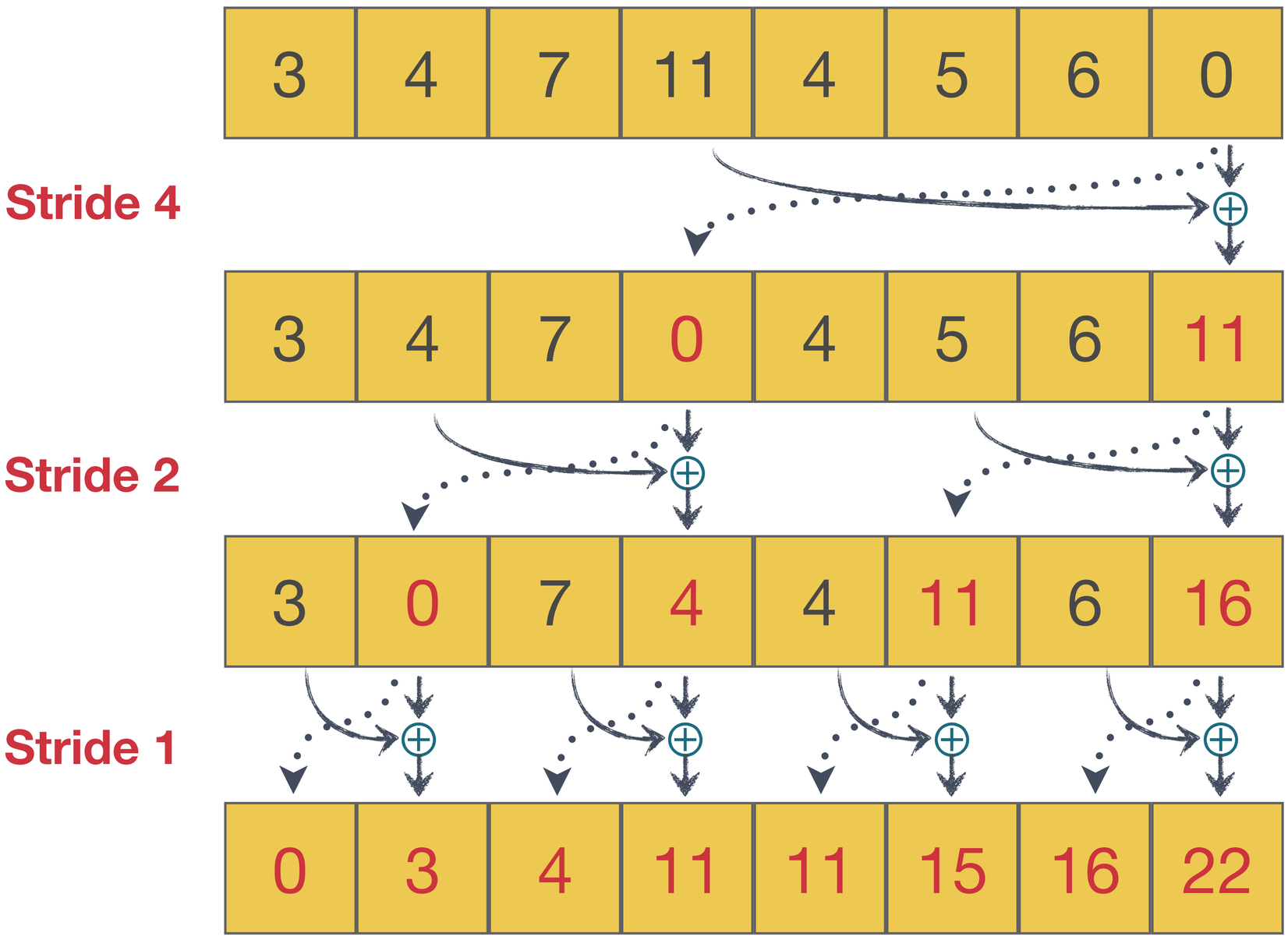
Parallel Scan: Post Scan



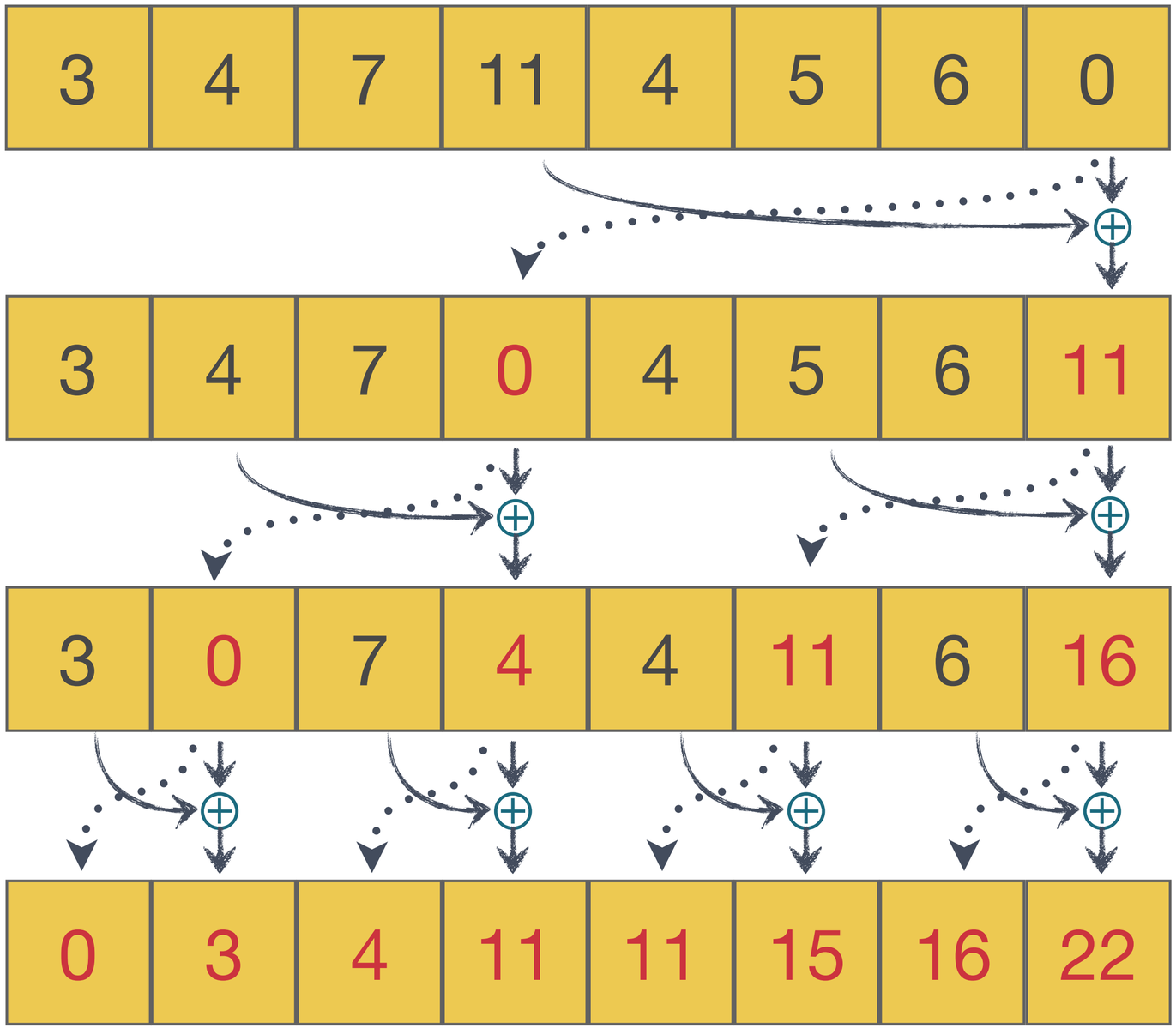
Parallel Scan: Post Scan



Parallel Scan: Post Scan



Parallel Scan: Post Scan



Iterate log(n) times adding value *stride* elements away to its own value, and set the value *stride* elements away to its previous value.

$$W_{scan}(n) = \Theta(n)$$

$$D_{scan}(n) = \Theta(\log n)$$

$$p \equiv \frac{W}{D} = \Theta\left(\frac{n}{\log n}\right)$$