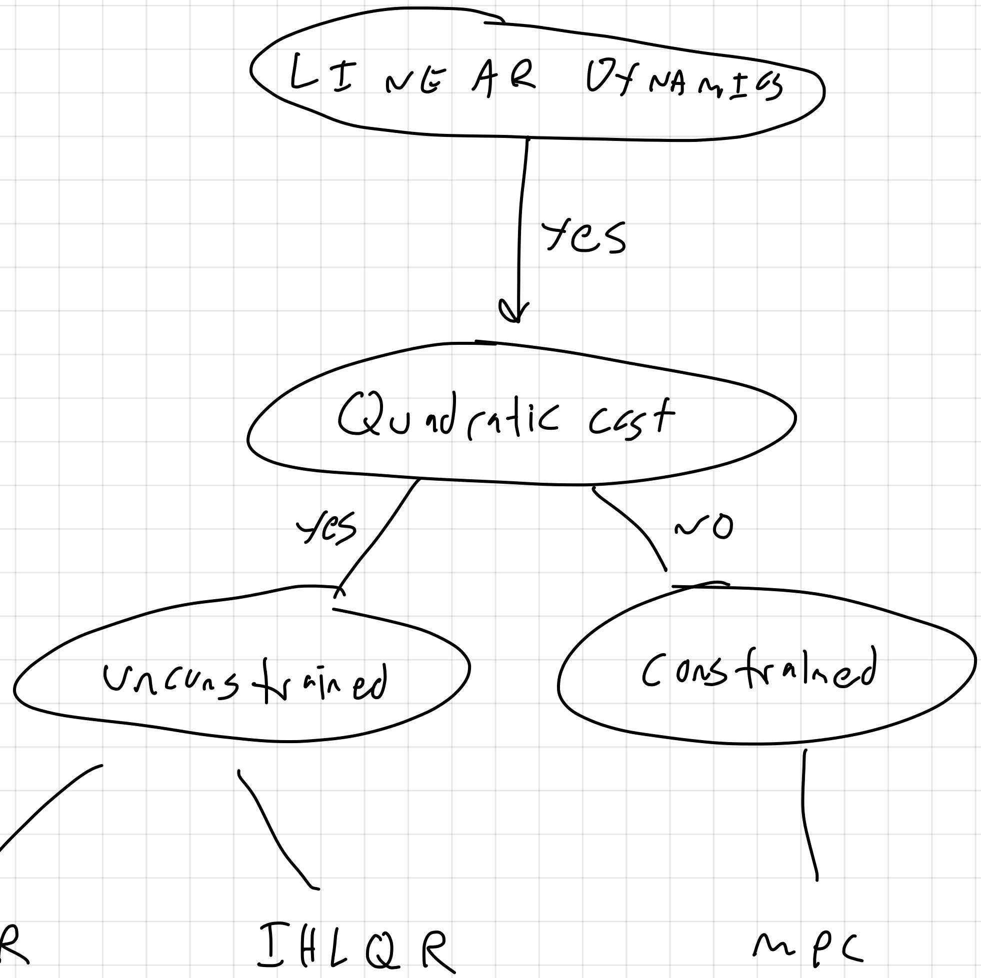
3/12 recitation



tracking, or any finite time trajectory

$t \to \infty$

TVLQR

$$\min \sum_{i=1}^{N} x_i^T Q x_i + u_i R u_i$$

- finite horizon
- can be time varying
- tracking

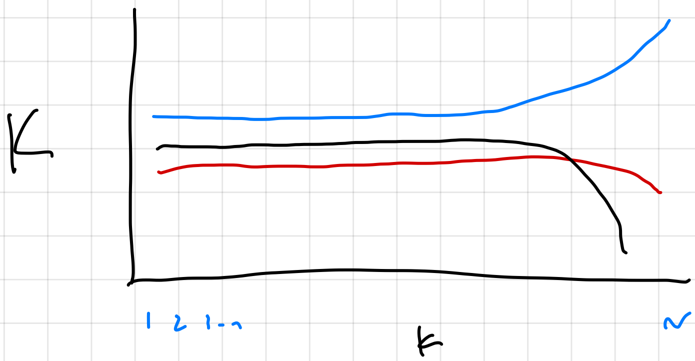
IHLQR

$$\min \sum_{i=1}^{\infty} x_i^T Q x_i + u_i R u_i$$

- Regulator

$$\min \sum_{i=1}^{100000} J(x, u)$$

$$\text{s.t.} \quad x_{k+1} = A x_k + B u_k$$

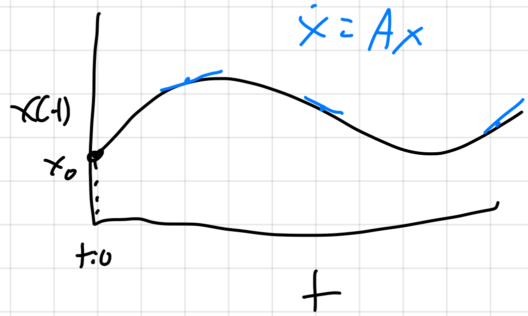


$$S = f(A, B, Q, R, S)$$

LTI ODE's

$$\dot{x} = Ax$$

$$x(t) = e^{At} \underbrace{x(0)}_{\text{I.C.}}$$

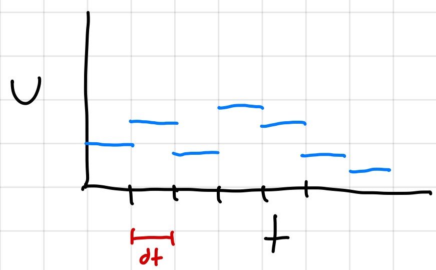


$$x_{k+1} = \underbrace{e^{Adt}}_{A_d} x_k$$

FOH on U

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{u} = 0 \end{cases}$$

integrate fm k to $k+1$ (dt)



$$\underbrace{\begin{bmatrix} \dot{x} \\ u \end{bmatrix}}_{\dot{\tilde{x}}} = \underbrace{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} x \\ u \end{bmatrix}}_{\tilde{x}}$$

$$\exp(A) = I + A + \frac{A^2}{2} + \frac{A^3}{6} \dots$$

$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \end{bmatrix} = \underbrace{e^{\tilde{A}dt}}_{\begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \quad u_{k+1} = u_k$$

$$e^{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} dt} = I + \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} dt + \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}^2 \frac{dt^2}{2} \dots$$

$$= \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}$$

$$\boxed{x_{k+1} = A_d x_k + B_d u_k}$$

$$\dot{x} = f(x, u)$$

$$x_{k+1} = RK4(x_k, u_k, dt)$$

$t = 0$ $\quad k_1 = dt\, f(x_k, \boxed{u_k})$

$t = \frac{dt}{2}$ $\quad k_2 = dt\, f(x_k + k_1/2, \boxed{u_k})$ $\qquad$ ZOH

$t = \frac{dt}{2}$ $\quad k_3 = dt\, f(x_k + k_2/2, \boxed{u_k})$

$t = dt$ $\quad k_4 = dt\, f(x_k + k_3, \boxed{u_k})$

$$x_{k+1} = x_k + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Could use RK4, Euler, midpoint, etc.
HW1Q1

$$x_{k+1} = f_d(x_k, u_k) \quad , \quad \text{Linearize at } \bar{x}_k, \bar{u}_k$$

$$x_{k+1} = f_d(x_k, u_k) \approx f_d(\bar{x}_k, \bar{u}_k) + \left[\frac{\partial f_d}{\partial x}\bigg|_{\bar{x}_k, \bar{u}_k}\right](x_k - \bar{x}_k) + \left[\frac{\partial f_d}{\partial u}\bigg|_{\bar{x}_k, \bar{u}_k}\right](u_k - \bar{u}_k)$$

$$x = \bar{x} + \Delta x \quad , \quad u = \bar{u} + \Delta u$$

$$\cancel{\bar{x}_{k+1}} + \Delta x_{k+1} = f_d(x_k, u_k) \approx \cancel{f_d(\bar{x}_k, \bar{u}_k)} + \overbrace{\left[\frac{\partial f_d}{\partial x}\bigg|_{\bar{x}_k, \bar{u}_k}\right]}^{A_k} \Delta x_k + \overbrace{\left[\frac{\partial f_d}{\partial u}\bigg|_{\bar{x}_k, \bar{u}_k}\right]}^{B_k} \Delta u_k$$

$(\bar{x}, \bar{u})$ dynamically feasible $= \left(\bar{x}_{k+1} = f_d(\bar{x}_k, \bar{u}_k)\right)$

$$\Delta x_{k+1} = A_k \Delta x_k + B_k \Delta u_k$$

ODE $\dot{x} = f(x, u)$

$$\dot{x} = f(x_k, u_k) \approx f(\bar{x}_k, \bar{u}_k) + \overbrace{\left[\frac{\partial f}{\partial x}\Big|_{\bar{x},\bar{u}}\right]}^{\bar{A}}(x_k - \bar{x}_k)$$

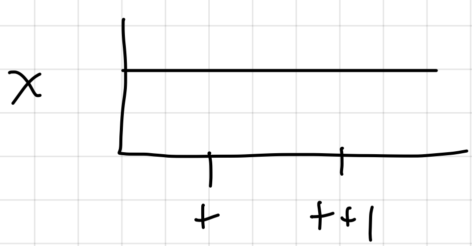
$$+ \overbrace{\left[\frac{\partial f}{\partial u}\Big|_{\bar{x},\bar{u}}\right]}^{\bar{B}}(u_k - \bar{u}_k)$$

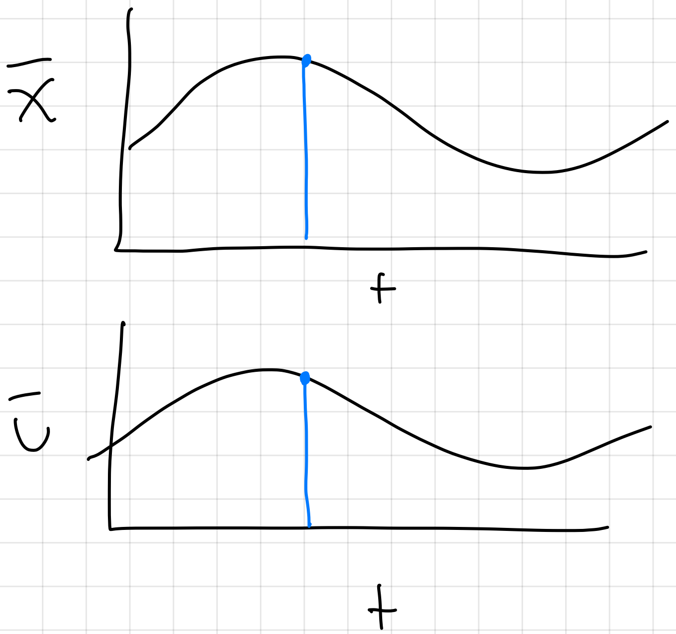
$$x = \bar{x} + \Delta x, \quad u = \bar{u} + \Delta u$$

$$\overbrace{\dot{\bar{x}}}^{0} + \Delta \dot{x} = \overbrace{f(\bar{x}_k, \bar{u}_k)}^{0} \quad \bar{A}\Delta x + \bar{B}\Delta u$$

$$\Delta \dot{x} = \bar{A}\Delta x + \bar{B}\Delta u$$

$$f(x_{eq}, u_{eq}) = \dot{x} = 0$$





$\bar{x}_k, \bar{u}_k$

$\frac{\partial \dot{x}}{\partial x}$ $\frac{\partial \dot{x}}{\partial u}$ $\frac{\partial x_{k+1}}{\partial x}$ $\frac{\partial x_{k+1}}{\partial u}$

$$\dot{x} = f(x_k) \xrightarrow{\text{discretize}} x_{k+1} = f_d(x_k)$$

$\bar{x}_k$ $\bar{x}_k$

$\downarrow$ discretize

$$\exp\left(\frac{\partial f}{\partial x}\Big|_{\bar{x}_k} \cdot dt\right) \qquad \frac{\partial f_d}{\partial x}\Big|_{\hat{x}_k}$$

BAD

Always discretize before linearizing if you're dealing with a trajectory. For an eq point, it doesn't really matter.