Outline

QP-based Task Space Inverse Dynamics

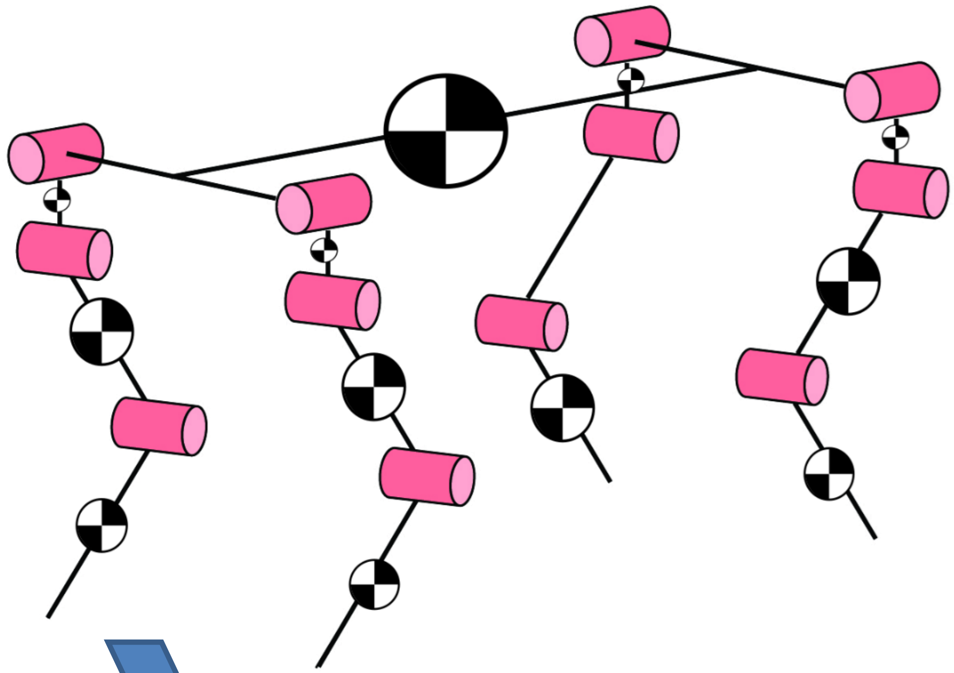
Feasibility Constraints

QP-based Task Space Inverse Dynamics in Contact

Formulation of the Reduced Problem

Model Based approach:

Model of the HyQ Robot



- Simplified representation
- Predict future behavior
- Guarantees constraints satisfaction
- Feedback controller corrects inaccuracies

Generalization

RECALL:

inverse dynamics: find τ such that ...

- q follows q^{des} JOINT SPACE ID
- x follows x^{des} TASK SPACE ID

JOINT SPACE ID | TASK SPACE ID

$$\tau = M\ddot{q}^{ref} + R$$

$$\ddot{q}^{ref} = \ddot{q}^{des} + K_q(q^{des} - q) + D_q(\dot{q}^{des} - \dot{q})$$

$q(t) \longrightarrow q^{des}(t)$

$$f^* = \Lambda \ddot{x}^{ref} + \mu$$

$$\ddot{x}^{ref} = \ddot{x}^{des} + K_x(x^{des} - x) + D_x(\dot{x}^{des} - \dot{x})$$

$$\tau = J^T f^*$$

$x(t) \longrightarrow x^{des}(t)$

$\ominus$ No feasibility constraints...

CONVEX OPTIMIZATION PROBLEMS TAXONOMY

A LP PROGRAMS:

- linear cost function $c^T x$
- linear equality / inequality

B QUADRATIC PROGRAMS

- convex quadratic cost function

  $x^T G x + g^T x \quad G \succ 0$
- linear equality/inequality constraints

C LS PROGRAMS (special case of QP)

- 2-norm of linear cost function $\|Ax - b\|^2$
- linear equality/inequality

+ QP, LP fast to be solved with off-the-shelf solvers

Let's cast as optimization problems:

$$\min_{y=\tau,\ddot{q}} \|\ddot{q} - \ddot{q}^{ref}\|^2$$

$$\text{s.t.} \quad M\ddot{q} + R = \tau$$

dynamics is enforced as an equality constraint.

ARE LEAST SQUARE PROBLEMS

we need to find the relationship WITH the decision variables $y = \tau, \ddot{q}$

$$\ddot{x} = J\ddot{q} + \dot{J}\dot{q}$$

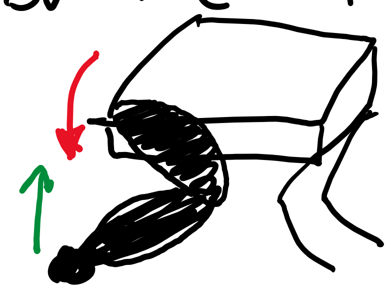
$$\ddot{x} - \ddot{x}^{ref} = \underbrace{J\ddot{q}}_{A} + \underbrace{\dot{J}\dot{q} - x^{ref}}_{b}$$

→ task error expressed as AFFINE function of $\ddot{q}$

$$\min_{\tau,\ddot{q}} \|J\ddot{q} + \dot{J}\dot{q} - x^{ref}\|^2$$

$$\text{st} \quad M\ddot{q} + R = \tau$$

(+) Differently from the quasi-static case we consider the full dynamics, so this controller takes into account inertial couplings and can compensate for the influence of heavy legs



(+) straightforward extension to floating-base (use FB dynamics instead of fixed-base)

$$M\ddot{q} + h = S^T \tau \quad \rightarrow +6 \text{ equations}$$

$$\ddot{q} \in \mathbb{R}^{n+6} \quad \rightarrow +6 \text{ decision variables}$$

(+) we can add additional constraints for feasibility

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FEASIBILITY CONSTRAINTS

We can add them as additional inequalities:

$$\min_{\ddot{q},\tau} \|\ddot{q} - \ddot{q}^{ref}\|^2$$

st

$$M\ddot{q} + h = \tau$$

$$\tau_{min} \leq \tau \leq \tau_{max}$$ TORQUE LIMITS

$$\ddot{q}_{min} \leq \ddot{q} \leq \ddot{q}_{max}$$ ACCELERATION LIMITS

VELOCITY LIMITS

$$\dot{q}_{min} - \dot{q} \leq dT\ddot{q} \leq \dot{q}_{max} - \dot{q}$$

assumes constant acceleration during dT

$$\dot{q}(t+dT) = \dot{q}(t) + dT\ddot{q}$$

POSITION LIMITS

$$q_{min} - dT\dot{q} - q \leq \frac{1}{2}dT^2\ddot{q} \leq q_{max} - dT\dot{q} - q$$

$$q(t+dT) = q(t) + dT\dot{q} + \frac{1}{2}dT^2\ddot{q}$$

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QP INVERSE DYNAMICS IN CONTACT

With contact forces we have k additional decision variables and contact constraints:

$$\min_{y=\ddot{q},\tau,f} \|\ddot{q}-\ddot{q}^{ref}\|^2 + f^T W f + \ddot{q}^T\ddot{q} + \tau^T\tau$$

→ BOUND SOLUTION

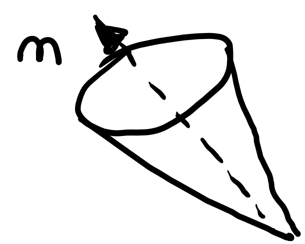
REGULARIZATION MATRIX

st

$$M\ddot{q} + h = S^T\tau + J^T f$$ DYNAMICS $(n+6)$

$$J\ddot{q} + \dot{J}\dot{q} = 0$$ CONTACT CONSTRAINTS (RIGID) (k)

- $n+6+k$ equations
- $\ddot{q}, \tau, f$ $\quad n+6+n+k$
- shaping the cost function we can have different solutions:
  - $W = I$: minimize 2-norm of force vector
  - $W = R^T R$: robustness
  - $W = J J^T$: minimize torques



COMPLIANT CONTACT CASE

add these additional constraints: ⓞ

- $f_z = \underset{\uparrow}{K_c}\underbrace{(P_0 - P_z)}_{\varepsilon} - \underset{\uparrow}{D_c}\underbrace{\dot{P}_z}_{-\dot{\varepsilon}}$ $\qquad P_z \leq 0$

Terrain stiffness

Terrain damping

↳ result of penetration of contact point below the surface. $(P_z \leq 0)$

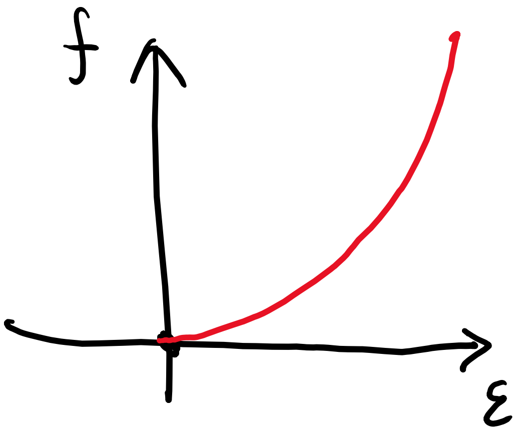
⊖ I need to measure the penetration (state estimation should be accurate)

- add penetration ε to decision variables
- link penetration to joint accelerations

$$\ddot{\varepsilon} = -\ddot{x}_f = -(J_c\ddot{q} + \dot{J}_c\dot{q}) \neq 0 \quad , \quad \varepsilon \geq 0$$

- f can be a generic function of ε, $\dot{\varepsilon}$ ⊕

$f = F(\varepsilon, \dot{\varepsilon})$ eg. non linear stiffness/foam



⊖ The problem becomes non-linear and is no longer a QP.

PROBLEM FORMULATION WITH COMPLIANT CONTACTS

since f should obey a specific model we no longer have f as a decision variable: $\ominus$

$$\min_{y = \ddot{q}, \tau, \varepsilon} \| A y - a \|^2$$

$$\text{st} \quad M \ddot{q} + h = \tau + J^T \left(K_c \varepsilon + D_c \dot{\varepsilon} \right)$$

$$\ddot{\varepsilon} = -\left(J_c \ddot{q} + \dot{J} \dot{q} \right)$$

$$\ddot{\varepsilon}_k = \frac{1}{dT} \left[\varepsilon_k - 2 \varepsilon_{k-1} + \varepsilon_{k-2} \right]$$

$$\varepsilon > 0$$

$\oplus$ No need to measure ε

[FAHMI 2019]

FORCE TASK / ACCELERATION TASK

$$\min_{\ddot{q},\tau,f} \quad \|Ay - a\|^2 + y^T W y$$

$\rightarrow \|A_q \ddot{q} - a_q\|^2$ acceleration Task

$\rightarrow \|A_f f - a_f\|^2$ force Task

st

$$M\ddot{q} + h = S^T \tau + J^T f$$

$$J\ddot{q} + \dot{J}\dot{q} = 0$$

$\ddot{q} \in \mathbb{R}^{n+6}$

$\tau \in \mathbb{R}^n$

$f \in \mathbb{R}^k \rightarrow 3c / 6c$

COMPUTATIONAL ASPECTS

- The computational cost in a QP is dominated by the HESSIAN
- cholesky decomposition is $O(n^3)$ with n = # decision variables
- exploiting the sparsity structure of the dynamics we can split actuated from underactuated part

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REDUCTION EXPLOITING THE SPARSITY OF THE PROBLEM

$$\min_{y=\ddot{q},\tau,f} \| A y - a \|^2$$

$$\text{s.t.} \quad \begin{bmatrix} \underbrace{[M_b \;\; M_{bJ}]}_{M_u} & -J_{c_b}^T & 0 \\ \underbrace{[M_{bJ}^T \;\; M_J]}_{M_a} & -J_{c_J}^T & -I \\ J & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f \\ \tau \end{bmatrix} = \begin{array}{c} \begin{bmatrix} -h_b \\ -h_J \\ -\dot{J}\dot{q} \end{bmatrix} \end{array} \begin{array}{c} 6 \\ m \\ k \end{array}$$

- express τ as affine function of $\ddot{q}$ and f

$$\tau = M_a \ddot{q} + h_J - J_{c_J}^T f$$

$$\underbrace{\begin{bmatrix} \ddot{q} \\ f \\ \tau \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} I & 0 \\ 0 & I \\ M_a & -J_{c_J}^T \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} \ddot{q} \\ f \end{bmatrix}}_{\bar{y}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ h_J \end{bmatrix}}_{d}$$

change of variables

REDUCED PROBLEM (removed τ and n equality)

$$y^* = \arg\min \| A\bar{y} - a \|^2$$

$$\bar{y} = (\ddot{q}, f)$$

st

$$\begin{bmatrix} M_u & -J_{cb}^T \\ J_c & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f \end{bmatrix} = \begin{bmatrix} -h_b \\ -\dot{J}_c \dot{q} \end{bmatrix} \begin{array}{l} 6 \\ K \end{array}$$

6 → NEWTON EULER EQUATIONS !

K → CONTACTS

$A_{f_c} f \leq 0$ → FRICTION CONES

$$\tau_{min} - h_J \leq [M_a \,\vdots\, -J_{cJ}^T]\, \bar{y} \leq \tau_{max} - h_J$$ → TORQUE LIMITS

Last step...

$$\tau^* = [M_a \quad -J_{cJ}^T]\, \bar{y}^* + h_J$$

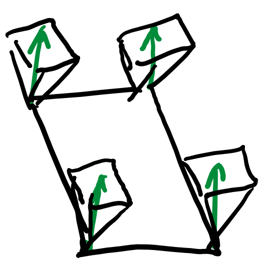
ELEMENTS OF COMPUTATION

- typical control frequency : 1 kHz → 1 ms
- dimension of the problem :
  - variables $\overbrace{n+6}^{\ddot{q}} + \overset{\tau}{n} + \overset{f}{k}$

    $(n+6+k)$ reduced
  - equality constraints $\overbrace{n+6}^{\text{dyn}} + \overset{\text{contact}}{k}$

    $(6+k)$ reduced
  - inequality constraints $\overset{\text{pos}}{n} + \overset{\text{vel}}{n} + \overset{\text{acc}}{n} + \overset{\text{cones}}{4c}$ → # LEGs in contact
- standard humanoid : $n = 30$ → 90 variables + 122 constraints in 1 ms !

  $k = 24$



4 3D forces for each foot

EXAMPLE : STATIC WALK ON QUADRUPED

6

Task 1 : Track CoM trajectory
Task 2 : move swinging foot

$\Rightarrow$ non conflicting Tasks, no need for PRIORITIES

WITH priorities we can exploit kinematic redundancy to achieve secondary Tasks

QUADRUPED 18 DoFs −
6 DoFs CoM Task
9 DoFs stance constraints (3 LEGS)
3 DoFs swing Task (1 LEG)

0 $\Rightarrow$ no kinematic redundancy

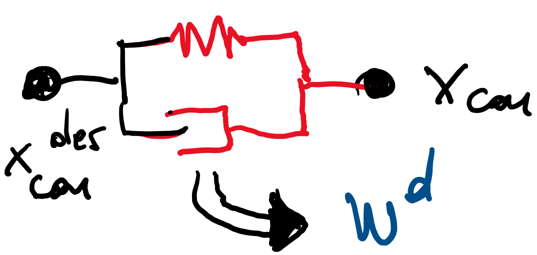
INGREDIENTS:

(A) decision variables:

$$y = [\ddot{q}, f] \quad \text{(reduced problem)}$$

(B) COM TASK : A_{com}, a_{com} ?

- Force level: $m\ddot{x}_{com} + mg = \sum f$

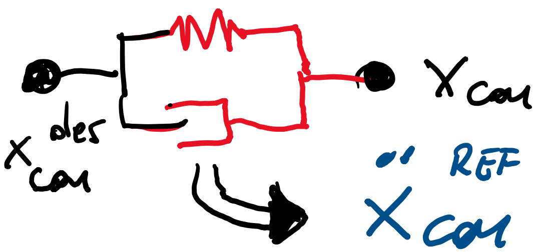


$$w_{com} = \underbrace{[I_{3\times3} \cdots I_{3\times3}]}_{A} f - mg$$

$$\| w - w^d \|^2_{W_{com}} = \| \underbrace{[0 \quad A]}_{A_{f\,com}} y - \underbrace{w^{des} - mg}_{a_{f\,com}} \|^2_{W_{com}}$$

- Acceleration level:

0



$$\ddot{x}_{com} = J_{com}\,\ddot{q} + \dot{J}_{com}\,\dot{q}$$

$$J_{com} = \left[I_{3\times3} \quad -[x_{b,com}]_{\times} \; J_{b,com}(q) \right] \ddot{q}$$

$$\| \ddot{x}_{com} - \ddot{x}_{com}^{REF} \|^2_{W_{com}} = \| \underbrace{J_{com}}_{A_{q_{com}}} \ddot{q} - \underbrace{x_{com}^{ref} + \dot{J}_{com}\,\dot{q}}_{b_{q_{com}}} \|^2_{W_{com}}$$

(c) SWING TASK: $\ddot{X}_{sw} = J_{sw}\ddot{q} + \dot{J}_{sw}\dot{q}$

$$\|\ddot{X}_{sw} - \ddot{X}_{sw}^{ref}\|^2_{W_{sw}} = \| \underbrace{J_{sw}}_{A_{q_{sw}}} \ddot{q} \underbrace{- \ddot{x}_{sw}^{ref} + \dot{J}_{sw}\dot{q}}_{a_{q_{sw}}} \|^2_{W_{sw}}$$

(D) STANCE CONSTRAINTS: $0 = \ddot{X}_c = J_c\ddot{q} + \dot{J}_c\dot{q}$

$$\min_{\ddot{q},f} \overbrace{\|\ddot{X}_{com} - \ddot{X}^{ref}\|_{W_{com}}}^{\text{COM TASK}} + \overbrace{\|\ddot{X}_{sw} - \ddot{X}_{sw}^{ref}\|_{W_{sw}}}^{\text{SWING TASK}}$$

s.t.

$$\begin{bmatrix} M_u & J_{c_b}^T \\ J_c & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f \end{bmatrix} = \begin{bmatrix} -h_b \\ -\dot{J}_c\dot{q} \end{bmatrix}$$

- W_{com}, W_{sw} are weights to tune precedence of task in case of constraint violation.