① current stock price (S) = \$38

strike price (K) = \$35

call option price (C) = \$4.20

$T = 4 \text{ months} = \frac{4}{12} = \frac{1}{3}$ year

$r = 6\% = 0.06 \quad , \quad q = 0$

(a) $c = S N(d_1) - K e^{-rT} N(d_2)$

where $d_1 = \dfrac{\ln(S/K) + (r + \sigma^2/2) T}{\sigma\sqrt{T}}$

$d_2 = d_1 - \sigma\sqrt{T}$

Let $\sigma = 0.2$

$$d_1 = \frac{\ln(38/35) + \left(0.06 + (0.2)^2/2\right) \times \frac{1}{3}}{0.2\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{0.1001}{0.1155} = 0.87$$

$d_2 = 0.87 - 0.2\sqrt{1/3} = 0.75$

$N(d_1) = 0.81 \qquad N(d_2) = 0.77$

$c = 38(0.81) - 35e^{-0.06/3} \cdot (0.77)$

$= 30.69 - 26.54 = 4.15 <$ market pric

4.20

Let $\sigma = 0.25$

$$d_1 = \frac{\ln(38/25) + \left(0.06 + (0.25)^2/2\right) \times \frac{1}{3}}{0.25 \times \frac{1}{\sqrt{3}}}$$

$= \dfrac{0.108}{0.1443} = 0.746$

$d_2 = 0.746 - \dfrac{0.25}{\sqrt{3}} = 0.6017$

$N(d_1) = 0.77, \quad N(d_2) = 0.73$

$C = 0.77(38) - 35e^{-0.06/3} \times 0.73 = \$4.39$

greater than market P $\$4.2$

Try $\sigma = 0.22$

$$d_1 = \frac{\ln(38/35) + \left(0.06 + \frac{0.22^2}{2}\right)\frac{1}{3}}{0.22\frac{1}{\sqrt{3}}} = 0.82$$

$d_2 = 0.82 - \dfrac{0.22}{\sqrt{3}} = 0.693$

$N(d_1) = 0.79, \quad N(d_2) = 0.76$

$C = 0.79(38) - 35e^{-0.06/3} \times 0.76 = \$4.21$

$\boxed{\text{implied volatility} \simeq 22\%}$

(b) for put option

$P = ke^{-rT}N(-d_2) - SN(-d_1)$

$$d_1 = \frac{\ln(38/35) + \left(0.06 + \frac{0.28^2}{2}\right)\frac{1}{3}}{0.28 \times \frac{1}{\sqrt{3}}}$$

$= \dfrac{0.1122}{0.1616} = \cancel{0.723}\ \cancel{0.69}\ 0.713$

$d_2 = \cancel{0.69}\ 0.713 - \dfrac{0.28}{\sqrt{3}} = \cancel{0.53}\ 0.55$

$N(-d_1) = \cancel{0.76}\ \cancel{0.25}\ 0.23 \quad N(-d_2) = \cancel{0.70}\ \cancel{0.30}\ 0.29$

$P = \overset{0.29}{(\cancel{0.2})}\, 35 e^{-0.06/3} - 38\,(\overset{0.23}{\cancel{0.38}})$

$= \cancel{0.97}\ \$0.83 \rightarrow \boxed{P = \$0.83}$

(c) $\sigma = 0.28$

$\Rightarrow d_1 = 0.713 \qquad d_2 = 0.55$

$N(d_1) = 0.78 \qquad N(d_2) = 0.71$

$C = 38(0.78) - 35e^{-0.02} \times 0.71$

$= \$\ 5.18$ million

Launching now gives = 38-35 = \$3 million

but waiting give \$ 5.18 million

(\$5.18 > \$3) million

$\therefore$ firm should wait

②

A) stock price starts at 100, strike 105, 10 days maturity. Each day, price moves ±\$1 with 50% probability.

a)

The option is in the money if $S_T > 105$

The terminal price

$$S_T = 100 + (\text{no of up moves}) - (\text{no of down moves}).$$

let n_u be up moves.

$$S_T = 100 + n_u - (10 - n_u) = 90 + 2n_u$$

For $S_T > 105 \Rightarrow 90 + 2n_u > 105 \Rightarrow n_u > 7.5$.

so n_u can be 8, 9 or 10.

$$P(n_u = 8) = \binom{10}{8} / 1024 = \frac{45}{1024}$$

$$P(n_u = 9) = \binom{10}{9} / 1024 = \frac{10}{1024}$$

$$P(n_u = 10) = \binom{10}{10} / 1024 = \frac{1}{1024}$$

$$\text{Total probability} = \frac{56}{1024} = \frac{7}{128}$$

b)

payoff for $n_u = 8$: $S_T = 106$, payoff $= 106 - 105 = 1$

payoff for $n_u = 9$: $S_T = 108$, payoff $= 108 - 105 = 3$

payoff for $n_u = 10$: $S_T = 110$, payoff $= 110 - 105 = 5$

$$\text{Expected payoff} = \left(1 \times \frac{45}{1024}\right) + \left(3 \times \frac{10}{1024}\right) + \left(5 \times \frac{1}{1024}\right)$$

$$= 5/64$$

c)

$$\text{Fair value} = \text{Expected payoff} = \frac{80}{1024} = \frac{5}{64}$$

b) Daily price move has mean 0.

$$E[|X|] = \sigma \cdot \sqrt{\frac{2}{\pi}} = 1$$

a) From $E[|X|] = 1 \Rightarrow \sigma \cdot \sqrt{\frac{2}{\pi}} = 1 \Rightarrow \sigma = \sqrt{\frac{\pi}{2}} \approx 1.2533$

for 10 days, $\sigma_T = \sigma \cdot \sqrt{10} = \sqrt{\frac{\pi}{2}} \times \sqrt{10} = \sqrt{5\pi} \approx 3.9633$

b) The terminal price S_T follows $N(100, \sigma_T^2)$

$$E[\max(S_T - K, 0)] = \int_{105}^{\infty} (s - 105) \frac{1}{\sigma_T \sqrt{2\pi}} \cdot e^{-\frac{(s-100)^2}{2\sigma_T^2}} \, ds$$

where $\sigma_T = \sqrt{5\pi}$

c) let $Y = S_T - K \Rightarrow Y \sim N(S_0 - K, \sigma_T^2) = N(-5, (3.9633)^2)$

The expected value of $\max(Y, 0)$ is

$$\mu \Phi(\mu/\sigma) + \sigma \phi(\mu/\sigma)$$

$\mu = -5, \ \sigma = 3.9633$

$$d = \frac{\mu}{\sigma} = \frac{-5}{3.9633} = -1.2616$$

$\Phi(-1.2616) = 0.1035, \quad \phi(-1.2616) = 0.2036$

Expected payoff $= (-5)(0.1035) + (3.9633)(0.2036)$

≈ 0.2894

c) Daily price move from uniform distribution $X \sim U(a, b)$ with $E[|X|] = 1$.

a) Assuming symmetry around 0, $a = -b$

$$E[|X|] = \int_{-b}^{b} |x| \cdot \frac{1}{2b} \, dx = \frac{b}{2} = 1$$

$\Rightarrow b = 2$

so, the daily price move is from $U[-2, 2]$.

b) Binomial - Discrete, Symmetric, bell-shaped, finite range ([90,110]).

Normal - Continuous, symmetric, bell-shaped, infinite range ({-∞,∞}).

Uniform - Continuous, symmetric, bell shaped, finite but wider range, [80,120]

c) Simulation model - Monte Carlo.

i) Repeat many times.

ii) For each run, simulate 10 daily price moves by drawing a random number from $U[-2,2]$ for each day.

iii) Sum these 10 daily moves and add to the initial stock price $S_0 = 100$ to get the terminal price S_T.

iv) Calculate the payoff: $\max(S_T - 105, 0)$.

v) Average all the calculated payoffs across all runs. This average will be the estimated fair value.

Work contribution

Q (1) ⟶ 2300053

Q (2) ⟶ 2300392

Q (3) ⟶ 2300527