3)

a) $P(A \wedge B|C) = P(A|B \wedge C) \cdot P(B|C)$

sol: $P(A \wedge B|C) = \dfrac{P(A \wedge B \wedge C)}{P(C)}$

$P(A|B \wedge C) = \dfrac{P(A \wedge B \wedge C)}{P(B \wedge C)}$, $P(B|C) = \dfrac{P(B \wedge C)}{P(C)}$

$\Rightarrow P(A \wedge B|C) = P(A|B \wedge C) \cdot P(B|C)$

$\boxed{\text{True}}$

b) $P(A \wedge B|C) = P(A|C) \cdot P(B|C)$ [A, B → independent]

sol: $P(A \wedge B|C) = \dfrac{P(A \wedge B \wedge C)}{P(C)}$ → L.H.S

$P(A|C) = \dfrac{P(A \wedge C)}{P(C)}$; $P(B|C) = \dfrac{P(B \wedge C)}{P(C)}$

$\dfrac{P(A \wedge B \wedge C)}{P(C)} \neq P(A|C) \cdot P(B|C)$

unless A, B are conditionally independent given C

$\boxed{\text{False}}$

2) Presents P_1, P_2, P_3

Probability of getting price (total prob).

$$= \underbrace{\frac{2}{3} \cdot \frac{1}{2}}_{\text{choose wrong \& change option}} + \underbrace{\frac{1}{3} \cdot \frac{1}{2}}_{\text{choose correct \& doesnot change}}$$

Probability of getting price if option changed

$$= \frac{\frac{2}{3}\left(\frac{1}{2}\right)}{\frac{2}{3}\cdot\left(\frac{1}{2}\right)+\frac{1}{3}\left(\frac{1}{2}\right)} = 2/3$$

expected winning $= 2/3 \times 1000$

$= 666.67$

$\therefore$ Switching is better option.

Bonus Question:

$$P_n = C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$C_n = \frac{1}{n+1} \cdot \frac{(2n)!}{(n!)(n!)}$$

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$ [stirling's approximation for large n]

$$(2n!) \approx \sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}$$

$$(n!)^2 \approx 2\pi n\left(\frac{n}{e}\right)^{2n}$$

$$C_n \approx \frac{1}{n+1} \cdot \frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{2\pi n\left(\frac{n}{e}\right)^{2n}}$$

$$\approx \frac{1}{n+1} \cdot \frac{4^n}{\sqrt{\pi n}}$$

$$C_n \approx \frac{4^n}{(n+1)\sqrt{\pi n}}$$

for large n ; $n+1 \approx n$

$$\boxed{P_n = C_n \approx \frac{4^n}{\sqrt{\pi}\, n^{3/2}}}$$