1. a) Q → transition matrix

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix} \end{array}$$

b) states $\{1,2\}$ are only transient between them

states $\{3,4\}$ are transisent between them

→ There is no transisent states

c) $\{1,2\}$, $\{3,4\}$ are independent so they have diff stationary states

i) $P(X=1) = 0.5\,P(x=1) + 0.25\,P(x=2) \rightarrow (1)$

$P(X=2) = 0.5\,P(x=1) + 0.75\,P(x=2) \rightarrow (2)$

from (1) & (2) $\quad P(X=2) = 2P(x=1)$

$P(X=1) + P(X=2) = 1$

$\Rightarrow P(X=1) = \frac{1}{3}$ & $P(x=2) = \frac{2}{3}$

~~ii)~~ stationy distribution (1, $= (1/3, 2/3, 0, 0)$

ii) $P(x=3) = 0.25\,P(x=3) + 0.75\,P(x=4)$

$P(x=4) = 0.75\,P(x=3) + 0.25\,P(x=3)$ $\Big\} \Rightarrow P(x=3) = P(x=4)$

as $P(x=3) + P(x=4) = 1$

$\Rightarrow P(x=3) = P(x=4) = \frac{1}{2}$

stationary state 2 $\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$

2) b) $X_n \to$ Dinner together

$\therefore P(X_n) = 0.7\,P(W) + 0.2\,P(L)$

$= 0.7\,P(W) + 0.2\,[1 - P(W)]$

$= 0.2 + 0.5\,P(W)$

$= 0.2 + 0.5(0.6) = 0.2 + 0.3$

$= 0.5$

$\boxed{P(X_n) = 0.5}$

c) As $P(X_n) = 0.5$, the expected no. of games the team needs to play for dinner is "2" i.e $\frac{1}{0.5}$

6) a) $g \to h$

3 cases are possible.

i) $h = g$, ii) h is different from g by one pair swap

iii) h is different from g by more than one pair swap.

* From g total $^{26}C_2 = 325$ different permutations can be formed by one pair swap.

$$W_{hg}(1) = \begin{cases} \frac{1}{325} & ; \text{if differ by one pair swap} \\ 0 & ; \text{otherwise} \end{cases}$$

* In long run, probability of getting any permutation equal.

$\therefore$ Stationary distribution is uniform over all 26! permutations.

Probability $= \frac{1}{26!}$

(3)

a) Cat chain,

let $\pi_c = (\pi_{c_1}, \pi_{c_2})$ be the stationary distribution for the cat

π_{c_1} - prob. in room 1

π_{c_2} - prob. in room 2

∴ transition matrix $P_{cat} = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

∴ $\pi_c P_{cat} = \pi_c$; $\pi_{c_1} + \pi_{c_2} = 1$.

$(0.2\pi_{c_1} + 0.8\pi_{c_2}, 0.8\pi_{c_1} + 0.2\pi_{c_2}) = (\pi_{c_1}, \pi_{c_2})$

∴ $0.2\pi_{c_1} + 0.8\pi_{c_2} = \pi_{c_1}$ ⇒ $\pi_{c_1} = \pi_{c_2} = 0.5$.

∴ stationary distribution for cat chain is

$\pi_{cat} = (0.5, 0.5)$

Mouse chain,

let $\pi_M = (\pi_{M_1}, \pi_{M_2})$ - S.D for the mouse.

$P_{mouse} = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

$\pi_M P_{Mouse} = \pi_M$; $\pi_{M_1} + \pi_{M_2} = 1$

$(0.7\pi_{M_1} + 0.6\pi_{M_2}, 0.3M_{M_1} + 0.4\pi_{M_2}) = (\pi_{M_1}, \pi_{M_2})$

$0.7\pi_{M_1} + 0.6\pi_{M_2} = \pi_{M_1}$ ⇒ $M_1 = 2\pi_{M_2}$

⇒ $\pi_{M_1} = 2/3$; $\pi_{M_2} = 1/3$.

⇒ stationary distribution for the mouse chain is

$\pi_{Mouse} = (2/3, 1/3)$

b) $Z_n \in \{(1,1), (1,2), (2,1), (2,2)\}$

⇒ $(x,y) \to$ cat in room x, Mouse in room y.

$P(Z_{n+1} = z_{n+1} | Z_n = z_n, Z_{n-1} = z_{n-1} \ldots)$

$= P(C_{n+1} = c_{n+1} | C_n = c_n, C_{n-1} = c_n \ldots) \times$

$P(M_{n+1} = m_{n+1} | M_n = m_n, M_{n-1} = m_n \ldots)$

$= P(C_{n+1} = c_{n+1} | C_n = c_n) \times P(M_{n+1} = m_{n+1} | M_n = m_n)$

$= P(Z_{n+1} = z_{n+1} | Z_n = z_n)$ (This process is Markov chain)

(ii) Types of squares & No' of legal moves,

1) corner squares,

A king in a corner has 3 legal moves
And there are 4 corner squares.

2) Edge square.

legal moves - 5
No' of squares - 24

3) Inner squares

legal moves = 8
No' of squares = 64 - 28 = 36

→ The no' of legal moves determines the degree of each state in Markov chain.

stationary distribution (π_i) $\propto$ degree of each state i

$$\pi_i = \frac{d_i}{\sum_j d_j}$$

where di be the no' of legal moves from square i

→ Total no' of degrees of all squares,

corner square ⇒ $\underset{(sq.)}{4} \times \underset{(moves)}{3} = 12$ moves
Edge squares → $24 \times 5 = 120$ moves.
Inner squares ⇒ $36 \times 8 = 288$

Total moves = 420
Total sum of degrees $(\sum_j d_j) = 420$

→ Then stationary probability for

A corner square $= \frac{3}{420} = \frac{1}{140}$

A Edge square $= \frac{5}{420} = \frac{1}{84}$

A Inner square $= \frac{8}{420} = \frac{2}{105}$

(5) a) as stock value increases with 0.01 with probility 0.1 & decreases 0.01 with probability 0.05
since there is positive rise we can't have recurrent states

b) Same reason as part (a), we can't have stationary distribution

c) steps $= \frac{3 \text{ hours}}{5} = \frac{10800}{5} = 2160$

for stock price after time T 130 is almost zero

so, I simulate for ~~₹125~~ ~~₹122~~, ₹121 even though it makes no sense of exercising call option (loss)

6) b) Let $q(g,h)$ be the probability of going from g to h in one step.

$q(g,h)$ = (probability of proposing h from g) × (probability of accepting the proposal).

$$= \frac{1}{325} \cdot \left[P(g,h) + \frac{s(h)}{s(g)} \cdot (1 - P(g,h)) \right]$$

$P(g,h) \Rightarrow$ probability of $s(g) \leq s(h)$

$$q(h,g) = \frac{1}{325} \left[(1 - \underbrace{P(g,h)}_{P}) + \frac{s(g)}{s(h)} \cdot [1 - (1 - P(g,h))] \right]$$

$$= \frac{1}{325} \left[1 - P + \frac{s(g)}{s(h)} \cdot P \right]$$

$$s(h)\, q(h,g) = \frac{1}{325} \left[s(h) - P \cdot s(h) + P \cdot s(g) \right]$$

$$s(g) \cdot q(g,h) = \frac{1}{325} \left[s(g) \cdot P + s(h) - P \cdot s(h) \right]$$

$$\Rightarrow s(g) \cdot q(g,h) = s(h) \cdot q(h,g).$$

Define $\pi(g) = \dfrac{s(g)}{\sum_g s(g)}$; $\Rightarrow \sum_g \pi(g) = 1$

$\Rightarrow s(g) \cdot q(g,h) = s(h) \cdot q(h,g)$

$\Rightarrow \pi(g) \cdot q(g,h) = \pi(h) \cdot q(g,h)$

$\therefore$ This chain is reversible.

$\therefore$, $\pi(g)$ is stationary distribution.

$$\boxed{\pi(g) \propto s(g)}$$

WORK DISTRIBUTION

230053 $\longrightarrow$ 1, 5

230527 $\longrightarrow$ 2, 6

230392 $\longrightarrow$ 3, 4