CFD (AE 706)

Assignment no.4

A report on

Numerical Solution of Flow through CD Nozzle using van Leer Flux Vector Splitting Method

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1. Introduction

$$A = 1.0 + 2.0(x-1)^2, \quad 0 \le x \le 2$$

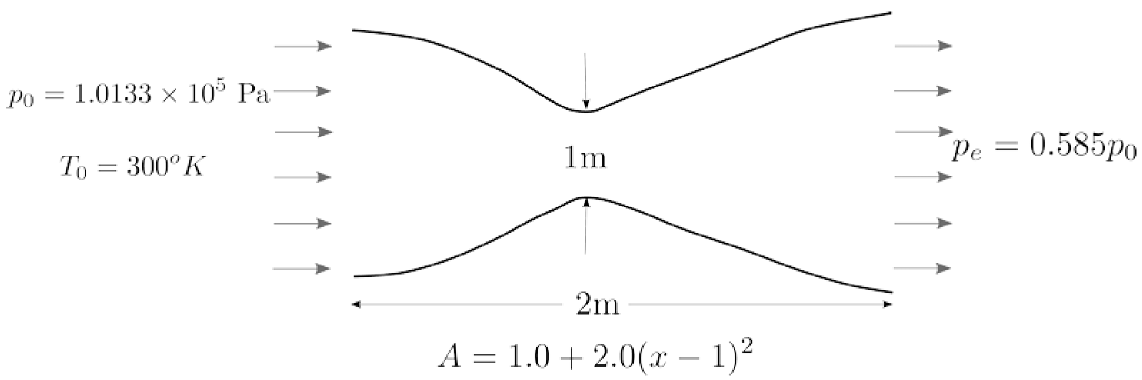

Figure 1: Flow through nozzle

Flow through nozzle problem is solved numerical using van Leer Flux Vector Splitting method. At the same time the flow through the nozzle is solved using relations of gas dynamics to get the exact solutions. 1-D Euler equation was solved numerically to demonstrate flow through nozzle. The discretised governing equations is:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{F}^{+}{}_i^n - \mathbf{F}^{+}{}_{i-1}^n\right) - \frac{\Delta t}{\Delta x}\left(\mathbf{F}^{-}{}_{i+1}^n - \mathbf{F}^{-}{}_i^n\right) + \Delta t \mathbf{S}_i$$

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Split Fluxes for van Leer Method

1. For $M \le -1$,

$$\mathbf{F}_{VL}^{+} = 0 \quad \text{and} \quad \mathbf{F}_{VL}^{-} = \mathbf{F}$$

2. For $-1 < M < 1$,

$$\mathbf{F}_{VL}^{+} = \frac{1}{4}\rho a (M+1)^2 A \begin{bmatrix} 1 \\ \dfrac{2a}{\gamma}\left(1 + \dfrac{\gamma-1}{2}M\right) \\ \dfrac{2a^2}{\gamma^2-1}\left(1 + \dfrac{\gamma-1}{2}M\right)^2 \end{bmatrix} \qquad \mathbf{F}_{VL}^{-} = \mathbf{F}_{VL} - \mathbf{F}_{VL}^{+}$$

3. For $M \ge 1$,

$$\mathbf{F}_{VL}^{+} = \mathbf{F} \quad \text{and} \quad \mathbf{F}_{VL}^{-} = 0$$

2.Results

2.1 Plots

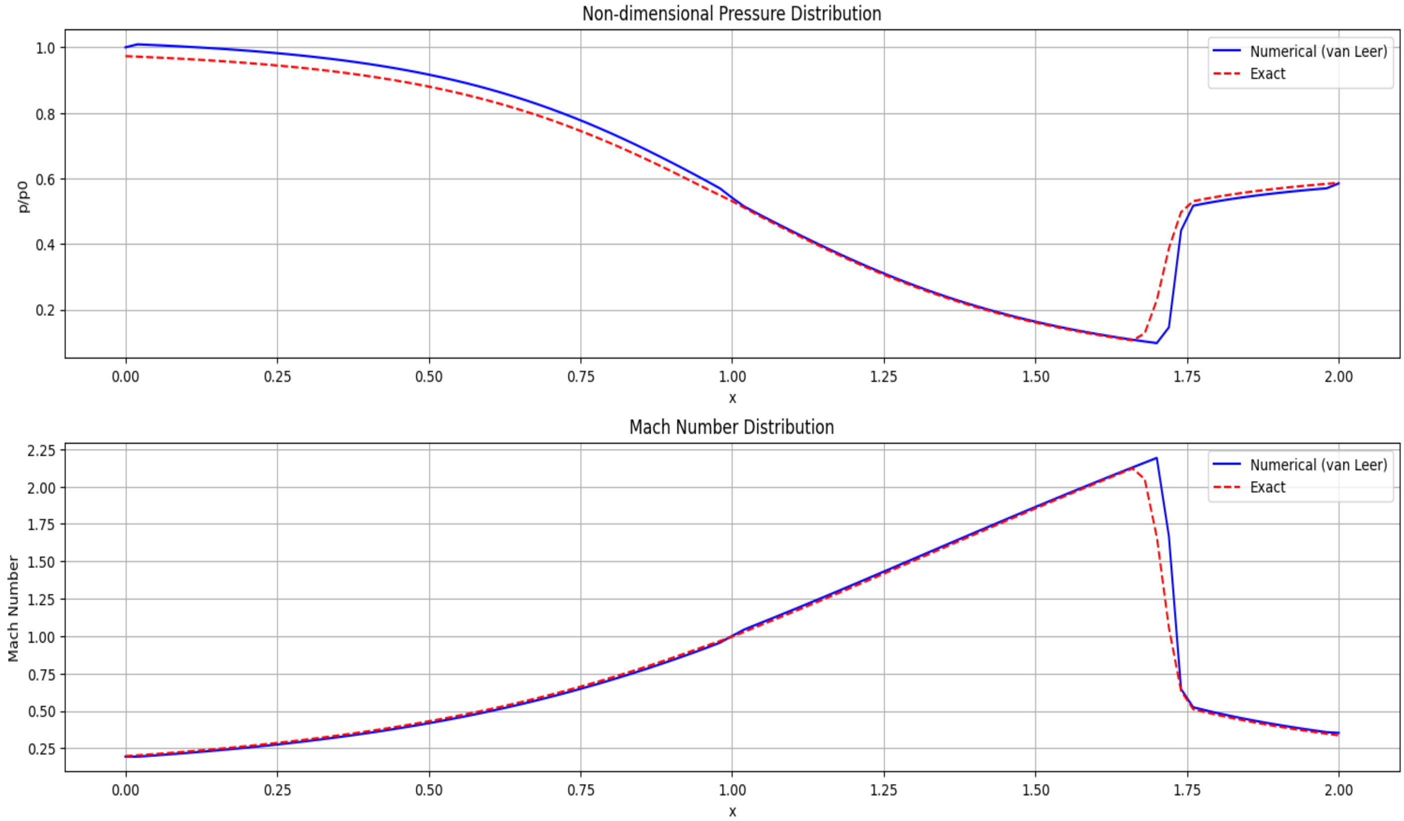

Figure 2: Pressure ratio and Mach number distribution along the length of nozzle

2.1.1 Pressure distribution (p/p0)

(i) Pre-shock Region (x<1.7)

The numerical solution (solid line) and the exact solution (dashed line) show good agreement in the converging section (x < 1.0 m) and part of the diverging section (x ≈ 1.0–1.7 m). Both exhibit a gradual decrease in pressure as the flow accelerates, consistent with isentropic expansion in a converging-diverging nozzle. Minor deviations are observed, likely due to numerical diffusion or the finite grid resolution.

(ii) Shock region (x=1.7)

The shock is represented by the numerical solution as a smeared transition instead of a crisp discontinuity, which is typical of the upwind nature of the Van Leer FVS scheme. Starting at about x = 1.7 m, this smearing is seen as a slow pressure increase. There is a noticeable jump at the shock position (about x = 1.7m) in the exact solution, which is followed by a quick adjustment to the post-shock pressure. The exact solution and the numerical shock location roughly match, suggesting that the exit pressure ratio (pe/p0 = 0.585) was used to accurately estimate the shock position.

(ii) Post Shock region (x>1.7m)

In contrast to the perfect solution, the numerical pressure shows oscillations or a somewhat wider profile after the shock, but it increases to match the enforced exit pressure. This implies that there may be instability or numerical dissipation in the FVS system close to the shock.The specified exit Mach number (M = 0.337) and boundary condition are met by the exact solution, which rapidly stabilizes to a constant pressure ratio (about 0.5866).

2.1.2 Mach Number distribution

(i) Pre-shock Region (x<1.7)

The two solutions have a comparable pattern. In the diverging section, the Mach number rises farther into the supersonic zone after reaching unity at the throat (x = 1.0 m) from a subsonic value at the inlet. The isentropic acceleration predicted by both methods is validated by the tight alignment of the precise and numerical curves.

(ii) Shock region (x=1.7)

Because of the diffusive character of the Van Leer FVS scheme, the numerical solution shows the shock as a slow decrease in Mach number from a supersonic value (about 2.15), to a subsonic value (approximately 0.5). The ideal normal shock relations are reflected in the exact solution, which exhibits a dramatic drop at the shock from roughly 2.15 to 0.5. The accuracy of the numerical method in predicting the shock position is confirmed by the good match between the two.

(ii) Post Shock region (x>1.7m)

Following the shock, the numerical Mach number steadily drops toward the exit value (about 0.3532), with a few slight variations that could be caused by numerical artefacts or the implementation of boundary conditions. The exact solution exhibits a more stable subsonic deceleration as it smoothly approaches the designated exit Mach number (0.3370). The little variation in the numerical solution indicates that time-stepping or the implementation of the boundary conditions may generate tiny inaccuracies.

2.2 Numerical results and flowchart

Table 1: Comparison of van Leer and exact solutions at various locations

Location (x)	Area (A(x))	p/p0 (num) van Leer	p/p0 (exact solution)	M(num) van Leer	M (exact solution)
0.00	3.0000	1.0000	0.9732	0.1945	0.1974
0.50	1.5000	0.9168	0.8803	0.4180	0.4305
1.00	1.0000	0.5410	0.5306	1.0009	0.9965
1.50	1.5000	0.1631	0.1604	1.8632	1.8540
2.00	3.0000	0.5850	0.5866	0.3532	0.3370

Table 2: Comparison of property at throat and shock location

Method	Throat location	Throat area	Mach number	p/p0	Shock location
Van Leer	1.0	1.0	1.0009	0.5410	1.7000
Exact solution	1.0	1.0	0.9965	0.5306	1.7000

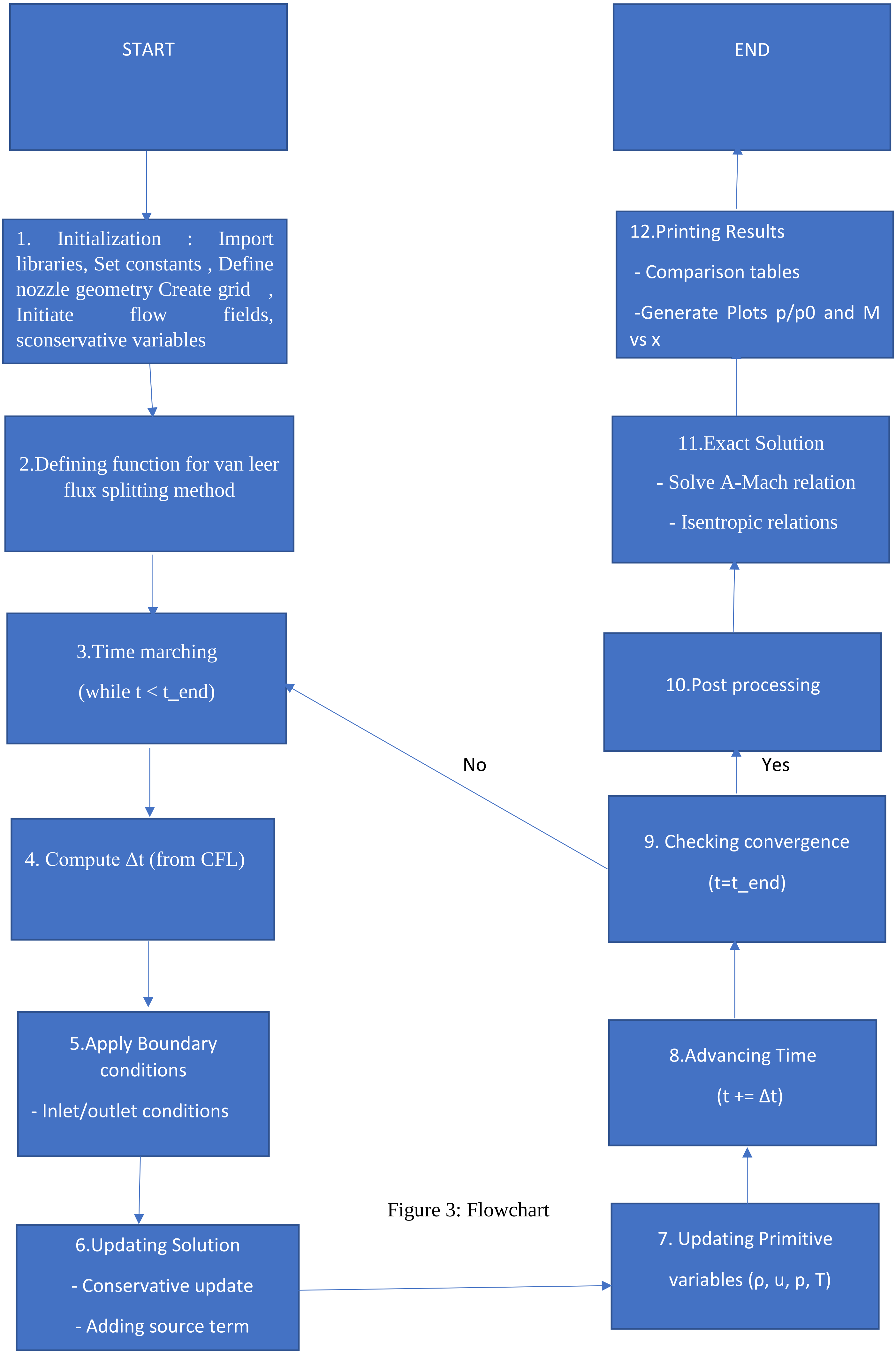

Figure 3: Flowchart

3. Conclusions

The Van Leer FVS numerical solution precisely predicts the location of the shock and general flow patterns, offering a decent approximation of the exact solution for the nozzle flow with a shock. However, smeared shocks are caused by the scheme's inherent diffusion and slight post-shock oscillations, which contrast with the precise solution's steadiness and abrupt transitions. The numerical results could be closer to the exact solution with improvements in grid resolution and shock-capturing methods, increasing their usefulness for engineering applications.

4. References

[1] Charles Hirsch, Numerical Computation of Internal and External Flows, (Section 20.2.3 pages 420-425) Volume II, Wiley, 1990