CFD (AE 706)

Bonus assignment no.1

A report on

# Grid Generation over NACA 63-412 Airfoil

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# 1 Formulation and Numerical Implementation

## 1.1 TFI Formulation

The Transfinite Interpolation (TFI) method is used to generate structured grids between known boundaries. For an O-type grid around an airfoil, we interpolate between:

- The inner boundary (airfoil surface, closed loop)
- The outer boundary (far-field boundary, typically a large circle or rectangle)

**Step 1: Parametric Representation of Boundaries**

**1.1 Inner Boundary (Airfoil)** Given discrete points on the airfoil, we first fit a periodic cubic spline to ensure smoothness and closed-loop continuity.

The airfoil is resampled uniformly in parametric space $s \in [0, 1]$ to get $N$ points:

$$\mathbf{r}_{\text{inner}}(s) = \left(x_{\text{inner}}(s),\ y_{\text{inner}}(s)\right), \quad s = \frac{i}{N},\ i = 0, 1, \ldots, N-1$$

**1.2 Outer Boundary (Far Field)** The outer boundary is constructed as a large rectangle or circle at a distance $R = 10\,\text{m}$ from the leading edge.

$$\mathbf{r}_{\text{outer}}(s) = \left(x_{\text{outer}}(s),\ y_{\text{outer}}(s)\right)$$

For a circular boundary:

$$x_{\text{outer}}(s) = R\cos(2\pi s), \quad y_{\text{outer}}(s) = R\sin(2\pi s)$$

**Step 2: Transfinite Interpolation (TFI) Formulation**

**2.1 Linear TFI (Basic Form)** The simplest TFI is a linear interpolation between the inner and outer boundaries:

$$\mathbf{r}(t, s) = (1 - t)\,\mathbf{r}_{\text{inner}}(s) + t\,\mathbf{r}_{\text{outer}}(s)$$

Where:

- $t \in [0, 1]$ is the radial interpolation parameter
- $s \in [0, 1]$ is the circumferential parameter

**2.2 Non-Uniform Point Distribution (Improved TFI)** To cluster more grid points near the airfoil (where high resolution is needed), we apply a stretching function:

$$t' = t^k, \quad k > 1$$

With $k = 2$ for quadratic distribution.

The modified interpolation becomes:

$$\mathbf{r}(t, s) = (1 - t')\,\mathbf{r}_{\text{inner}}(s) + t'\,\mathbf{r}_{\text{outer}}(s)$$

**2.3 Mathematical Formulation** For each radial line $i$ and circumferential point $j$:

$$\begin{cases} x(i,j) = (1 - t_i)\,x_{\text{inner}}(j) + t_i\,x_{\text{outer}}(j) \\ y(i,j) = (1 - t_i)\,y_{\text{inner}}(j) + t_i\,y_{\text{outer}}(j) \end{cases}$$

Where:

$$t_i = \left(\frac{i}{M-1}\right)^2, \quad i = 0, 1, \ldots, M-1, \quad j = 0, 1, \ldots, N-1$$

## 1.2 Elliptic Grid Generation Method

### 1. Governing Equations

We solve the following Poisson equations in computational coordinates $(\xi, \eta)$:

$$\begin{cases} \alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0 \\ \alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0 \end{cases}$$

Where:

- $\xi$: Radial direction (from airfoil to far field)
- $\eta$: Circumferential direction (around the airfoil)
- $\alpha, \beta, \gamma$: Metric coefficients

### 2. Metric Coefficients

Using central finite differences:

$$\alpha = x_\eta^2 + y_\eta^2 + \varepsilon$$
$$\beta = x_\xi x_\eta + y_\xi y_\eta$$
$$\gamma = x_\xi^2 + y_\xi^2 + \varepsilon$$

With:

$$\varepsilon = 10^{-6} \quad \text{(small regularization term)}$$

Approximations:

$$x_\xi \approx \frac{x_{i+1,j} - x_{i-1,j}}{2}, \quad x_\eta \approx \frac{x_{i,j+1} - x_{i,j-1}}{2}$$

(Similarly for $y_\xi, y_\eta$)

**3. Discretized Update Rule**

The iterative update rule is:

$$x_{i,j}^{\text{new}} = \frac{\alpha(x_{i+1,j} + x_{i-1,j}) + \gamma(x_{i,j+1} + x_{i,j-1}) - \frac{\beta}{2}(x_{i+1,j+1} - x_{i+1,j-1} - x_{i-1,j+1} + x_{i-1,j-1})}{2(\alpha + \gamma)}$$

$$y_{i,j}^{\text{new}} = \frac{\alpha(y_{i+1,j} + y_{i-1,j}) + \gamma(y_{i,j+1} + y_{i,j-1}) - \frac{\beta}{2}(y_{i+1,j+1} - y_{i+1,j-1} - y_{i-1,j+1} + y_{i-1,j-1})}{2(\alpha + \gamma)}$$

**4. Boundary Conditions**

**Dirichlet Conditions (Fixed boundaries):**

- At the airfoil surface ($i = 0$):

$$x_{0,j} = x_{\text{airfoil}}(j), \quad y_{0,j} = y_{\text{airfoil}}(j)$$

- At the far-field boundary ($i = N - 1$):

$$x_{N-1,j} = x_{\text{outer}}(j), \quad y_{N-1,j} = y_{\text{outer}}(j)$$

**Periodic Conditions (Circumferential direction):**

$$j + 1 \to (j + 1) \bmod N_\eta, \quad j - 1 \to (j - 1) \bmod N_\eta$$

## 2 Plots

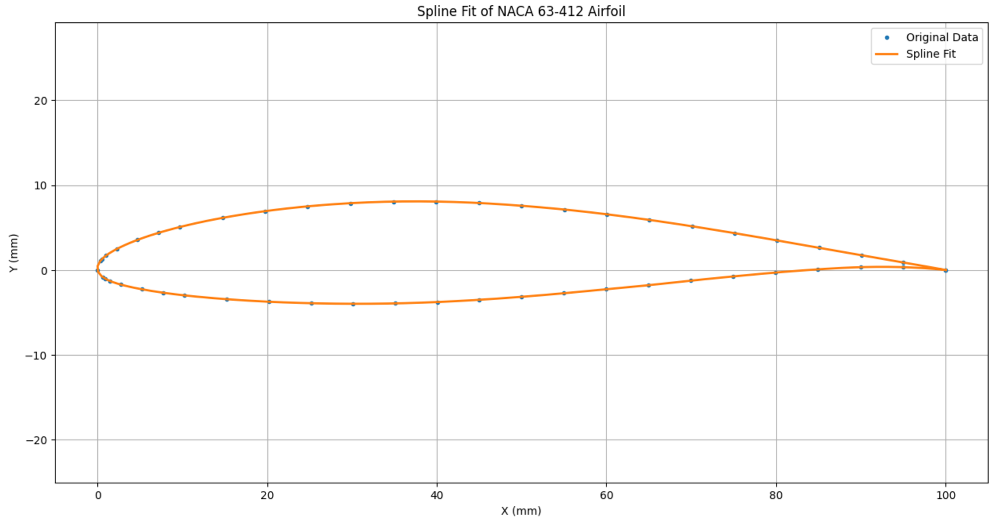


Figure 1: Spline fitting

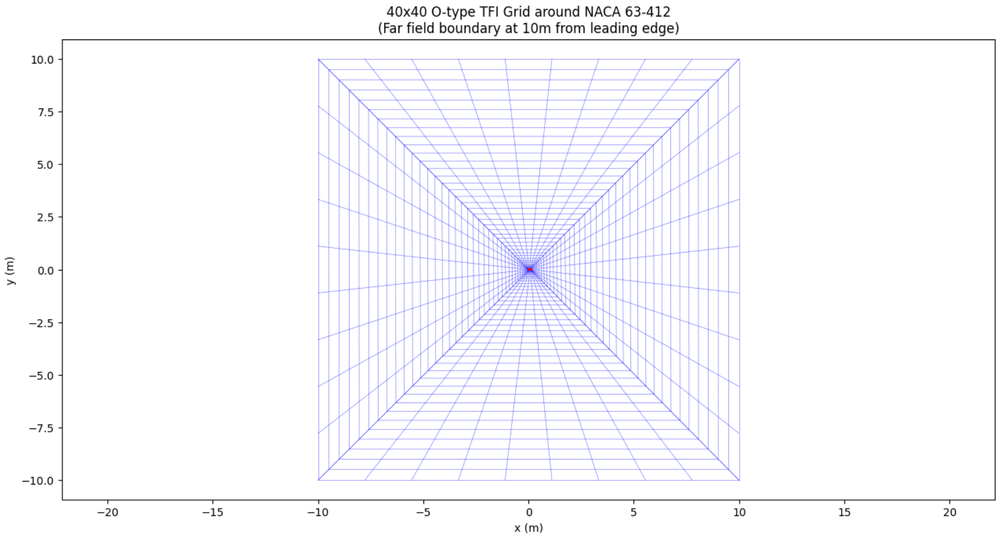


Figure 2: 40x40 TFI grid generation

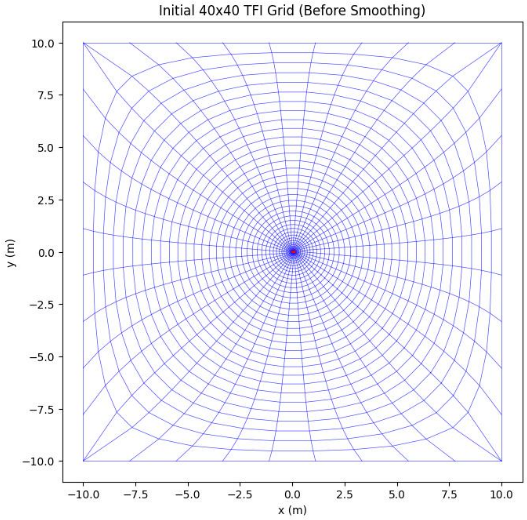


Figure 3: Grid before smoothing

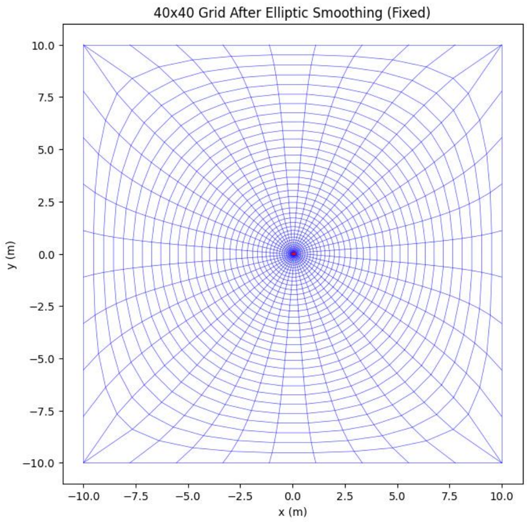


Figure 4: Grid after smoothing

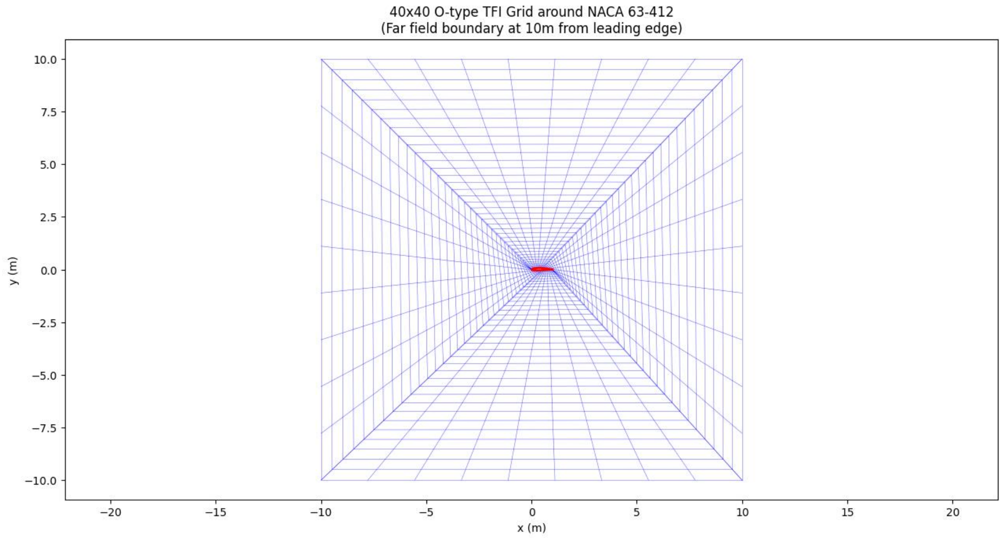


Figure 5: TFI grid with airfoil chord 1m

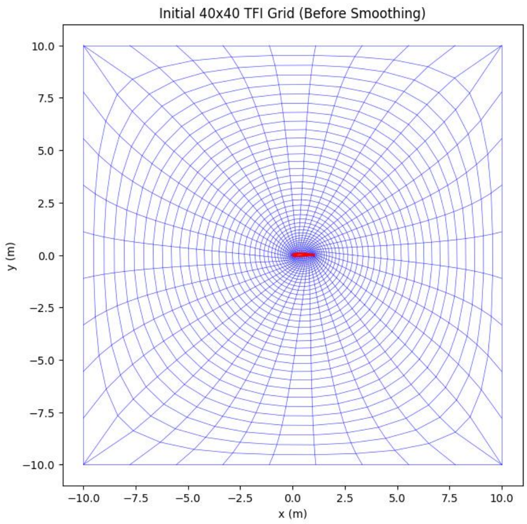


Figure 6: Elliptic grid with chord 1m (before smoothing)

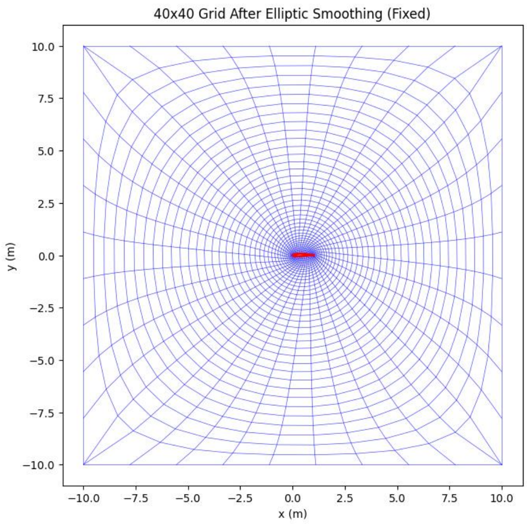


Figure 7: Elliptic grid with 1m chord (after smoothing)