



Computational Fluid Dynamics

(AE 706)

A report on

**Assignment-3: Computation of
Streamline Patterns for Steady
Inviscid Incompressible Fluid Flows**

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1. Governing Equation and discretization

Here the given 2-dimensional inviscid incompressible equation is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \dots\dots\dots (1)$$

Using second derivative approximation for equation (1)

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} \dots\dots\dots (2)$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} \dots\dots\dots (3)$$

Substituting equations (2) and (3) in equation (1)

Here the given 2-dimensional inviscid incompressible equation is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} = 0 \dots\dots\dots (4)$$

Substituting $\Delta x = \Delta y = 0.1$ in equation (4) and simplifying it will result as:

$$\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1} - 4\psi_{i,j} = 0 \dots\dots\dots (5)$$

Rearranging for $\psi_{i,j}$, we get Point Jacobi formula as:

$$\psi_{i,j}^{n+1} = \frac{1}{4}(\psi_{i+1,j}^n + \psi_{i-1,j}^n + \psi_{i,j+1}^n + \psi_{i,j-1}^n) \dots\dots\dots (6)$$

Here, $\psi_{i,j}^{n+1}$ is the updated value of $\psi_{i,j}$ at the $(n+1)^{\text{th}}$ iteration level.

2. Boundary Conditions

The chamber walls are the streamlines, so ψ is constant along the walls.

Left boundary ($x=0$) = ψ_3

Right boundary ($x=3$) = ψ_3

Top boundary ($y=4$) = ψ_3

Bottom boundary

I. Left section ($x=0$ to $x=1.1$) = ψ_3

II. Inlet ($x=1.1$ to $x=2.0$) = ψ_1

III. Right section ($x=2$ to $x=3$) = ψ_3

Internal boundary

I. Lower part ($x=1.5$, $y=1.1$) = ψ_1

II. Middle part with slot ($x=1.5$, $y=1.1$ to $y=2.0$) = ψ_2

III. Upper part ($x=1.5$, $y=2.0$ to $y=4.0$) = ψ_3

Sets of Boundary conditions

	ψ_1	ψ_2	ψ_3
Test 1	100	150	300
Test 2	100	200	300
Test 3	100	250	300

3. Convergence Check

The iteration stops when the solution converges. The convergence condition is based on L2 norm of the error.

$$\text{ERROR} = \frac{\|\psi_{i,j}^{n+1} - \psi_{i,j}^n\|_2}{\|\psi_{i,j}^{n+1}\|_2} < 10^{-4}$$

Where L2 norm is defined as: $\|\psi_{i,j}\|_2 = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-1} |\psi_{i,j}|^2}$

Here, IM and JM are the maximum values indices i, j. L2 norm for the difference in the solution at a point at two consecutive solutions are denoted as $\|\psi_{i,j}^{n+1} - \psi_{i,j}^n\|_2$

$$\|\psi_{i,j}^{n+1} - \psi_{i,j}^n\|_2 = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-1} |\psi_{i,j}^{n+1} - \psi_{i,j}^n|^2}$$

4.Results:

(i) Numerical results

1. Test 1: $\psi_1=100$, $\psi_2=150$ and $\psi_3=300$

a. Initial guess: $\psi = 100$

Table:1

x-location	Y-values
0.0	[300. 300.]
1.0	[300. 232.93388617 211.41010627 201.89392408 196.45918828 193.01166067 190.93024067 190.01913236 190.22001637 191.50188539 193.78769413 196.89401845 200.56131738 204.67440289 209.29382758 214.59422305 220.79142182 228.04719548 236.33990935 245.30202848 254.19001535 262.26175869 269.14516973 274.79436123 279.34063761 282.97120879 285.87300451 288.20480386 290.09690771 291.64963101 292.9424097 294.03473861 294.97406446 295.79557876 296.52795179 297.1923786 297.80661889 298.38396424 298.93598058 299.47184054 300.]
2.0	[300. 232.93388617 211.41010627 201.89392408 196.45918828 193.01166067 190.93024067 190.01913236 190.22001637 191.50188539 193.78769413 196.89401845 200.56131738 204.67440289 209.29382758 214.59422305 220.79142182 228.04719548 236.33990935 245.30202848 254.19001535 262.26175869 269.14516973 274.79436123 279.34063761 282.97120879 285.87300451 288.20480386 290.09690771 291.64963101 292.9424097 294.03473861 294.97406446 295.79557876 296.52795179 297.1923786 297.80661889 298.38396424 298.93598058 299.47184054 300.]
3.0	[300. 300.]

b. Initial guess: $\psi = 150$

Table:2

x-locations	y-values
0.0	[300. 300.]
1.0	[300. 232.9473083 211.43661614 201.93312295 196.51008215 193.07349462 191.00139076 190.09857122 190.30554617 191.5923281 193.88036748 196.98774766 200.65306386 204.76317144 209.37638963 214.66994702 220.85708411 228.10269725 236.38223445 245.33173868 254.20450576 262.26237044 269.12994864 274.76547747 279.29689905 282.9155405 285.80499954 288.12799209 290.01160783 291.55975074 292.848848 293.94148926 294.88238999 295.70922915 296.44829445 297.12258752 297.74790411 298.33862865 298.90485184 299.45612932 300.]
2.0	[300. 232.9473083 211.43661614 201.93312295 196.51008215 193.07349462 191.00139076 190.09857122 190.30554617 191.5923281 193.88036748 196.98774766 200.65306386 204.76317144 209.37638963 214.66994702 220.85708411 228.10269725 236.38223445 245.33173868 254.20450576 262.26237044 269.12994864 274.76547747 279.29689905 282.9155405 285.80499954 288.12799209 290.01160783 291.55975074 292.848848 293.94148926 294.88238999 295.70922915 296.44829445 297.12258752 297.74790411 298.33862865 298.90485184 299.45612932 300]
3.0	[300. 300.]

c. Initial guess: $\psi = 200$

Table:3

x-locations	y-values
0.0	[300. 300.]
1.0	[300. 232.99150278 211.52428808 202.06214817 196.6783891 193.27693013 191.23676132 190.35988963 190.58886496 191.89012657 194.18841912 197.2974062 200.96036708 205.05885398 209.6573171 214.92681257 221.08810317 228.29911274 236.54400928 245.45120125 254.28363496 262.29522678 269.12087089 274.71075386 279.20310906 282.78129185 285.63876256 287.93057599 289.79284437 291.32230173 292.60319855 293.69158606 294.63853222 295.47592422 296.23476523 296.93311855 297.58979857 298.21521941 298.82082744 299.41330906 300]
2.0	[300. 232.99150278 211.52428808 202.06214817 196.6783891 193.27693013 191.23676132 190.35988963 190.58886496 191.89012657 194.18841912 197.2974062 200.96036708 205.05885398 209.6573171 214.92681257 221.08810317 228.29911274 236.54400928 245.45120125 254.28363496 262.29522678 269.12087089 274.71075386 279.20310906 282.78129185 285.63876256 287.93057599 289.79284437 291.32230173 292.60319855 293.69158606 294.63853222 295.47592422 296.23476523 296.93311855 297.58979857 298.21521941 298.82082744 299.41330906 300.]
3.0	[300. 300.]

a. Initial guess: $\psi = 100$

[illegible]

b. Initial guess: $\psi = 150$

Table:5

[illegible]

c. Initial guess: $\psi = 200$

Table:6

[illegible]

3. Test 2: $\psi_1=100$, $\psi_2=250$ and $\psi_3=300$

a. Initial guess: $\psi = 100$

Table: 7

[illegible]

b. Initial guess: $\psi = 150$

Table:8

x-locations	y-values
0.0	[300. 300.]
1.0	[300. 234.13121719 213.87412393 205.76758768 201.96977799 200.48223516 200.79581509 202.83020291 206.62651905 212.17134003 219.22878015 227.16373567 235.03398556 242.23807745 248.61055974 254.24917668 259.36743349 264.17505705 268.81260787 273.27668671 277.41810183 281.05622259 284.12086676 286.63484924 288.67570486 290.32923117 291.67925745 292.79282787 293.72694812 294.52337107 295.21701592 295.83241047 296.39043877 296.90523695 297.38916215 297.85022291 298.29560275 298.72977195 299.15683993 299.57942198 300.]
2.0	[300. 234.13121719 213.87412393 205.76758768 201.96977799 200.48223516 200.79581509 202.83020291 206.62651905 212.17134003 219.22878015 227.16373567 235.03398556 242.23807745 248.61055974 254.24917668 259.36743349 264.17505705 268.81260787 273.27668671 277.41810183 281.05622259 284.12086676 286.63484924 288.67570486 290.32923117 291.67925745 292.79282787 293.72694812 294.52337107 295.21701592 295.83241047 296.39043877 296.90523695 297.38916215 297.85022291 298.29560275 298.72977195 299.15683993 299.57942198 300.]
3.0	[300. 300.]

c. Initial guess: $\psi = 200$

Table: 9

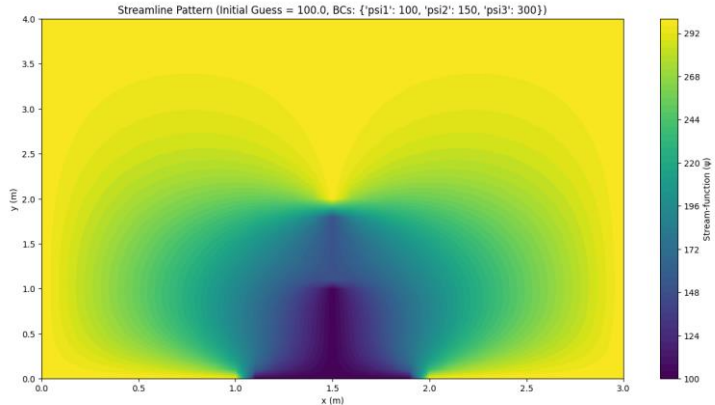
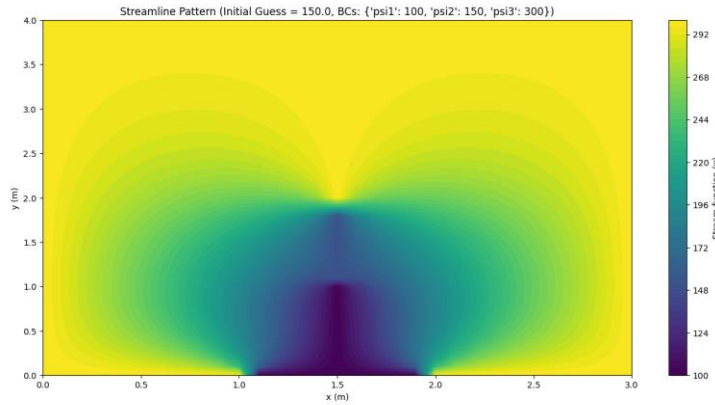
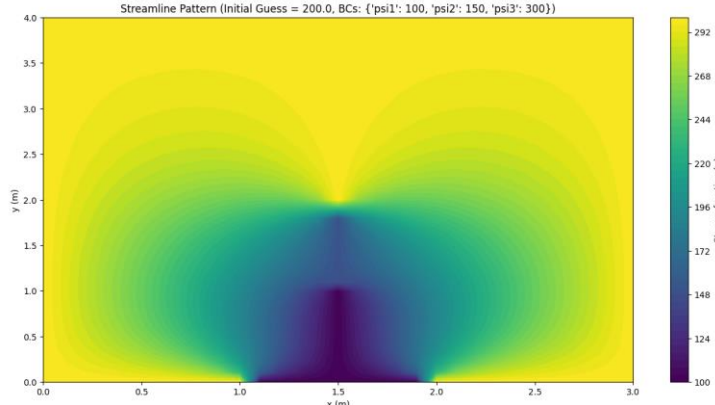
[illegible]

(ii). Effect of Initial Guess value

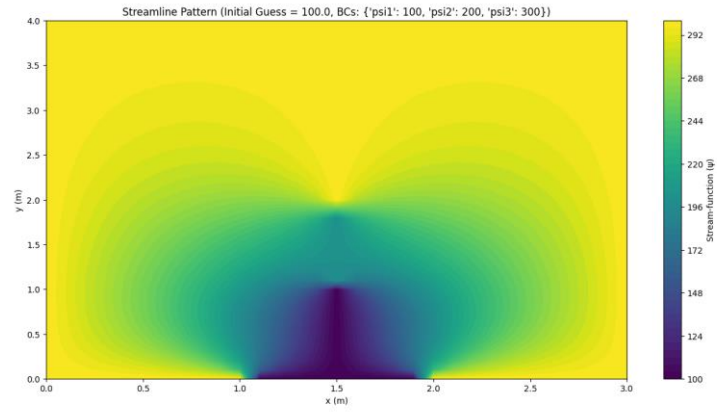
Table 10: Effect of initial guess on results

Case	No. of iteration	Error
$\Psi=100$, Test 1	334	9.962030803418954e-05
$\Psi=150$, Test1	305	9.969060838894907e-05
$\Psi=200$, Test1	260	9.962307952851038e-05
$\Psi=100$, Test2	336	9.932550071931199e-05
$\Psi=150$, Test2	308	9.973546016651792e-05
$\Psi=200$, Test2	266	9.943795075938226e-05
$\Psi=100$, Test3	338	9.88267089370632e-05
$\Psi=150$, Test3	311	9.94601551411883e-05
$\Psi=200$, Test3	271	9.990508704439758e-05

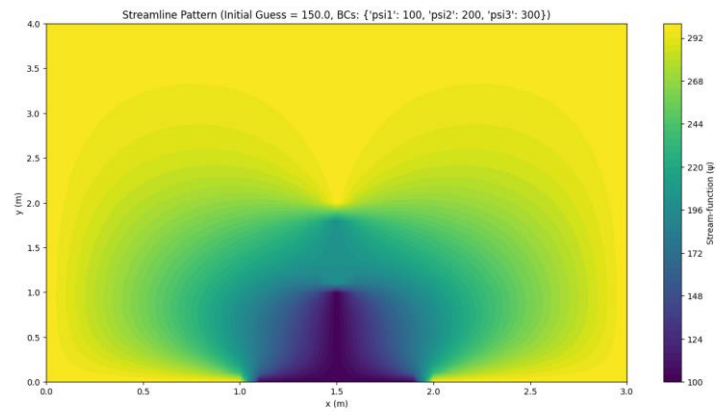
(iii) Contour Plots

Case	Contour
$\Psi=100$, Test 1	
$\Psi=150$, Test1	
$\Psi=200$, Test1	

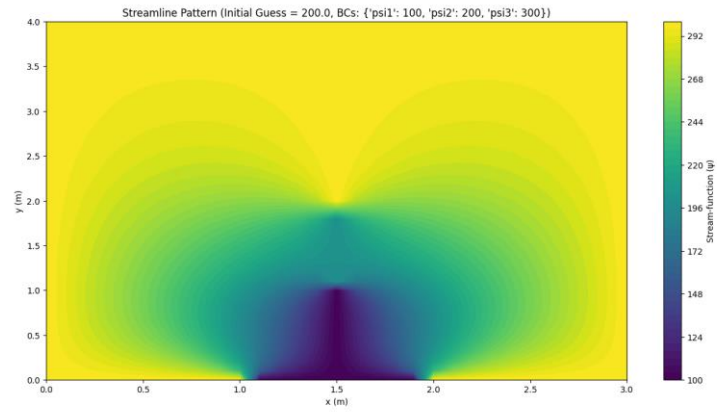
$\Psi=100$, Test2



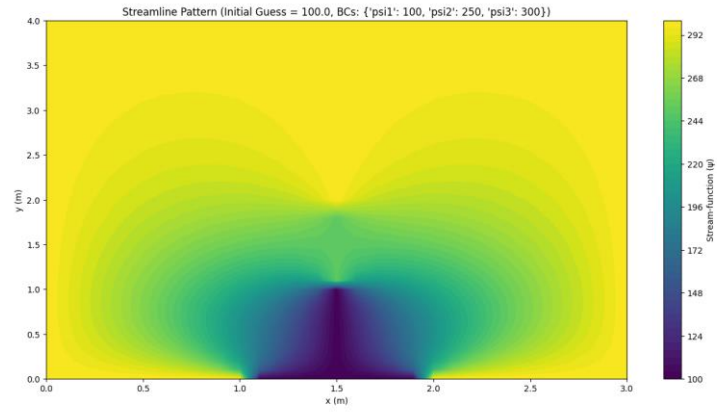
$\Psi=150$, Test2



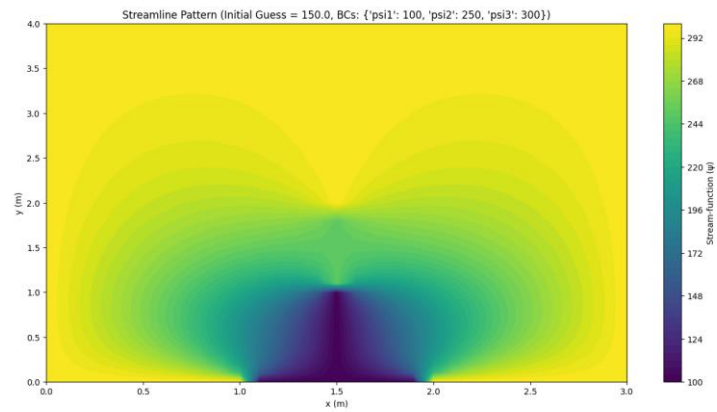
$\Psi=200$, Test2



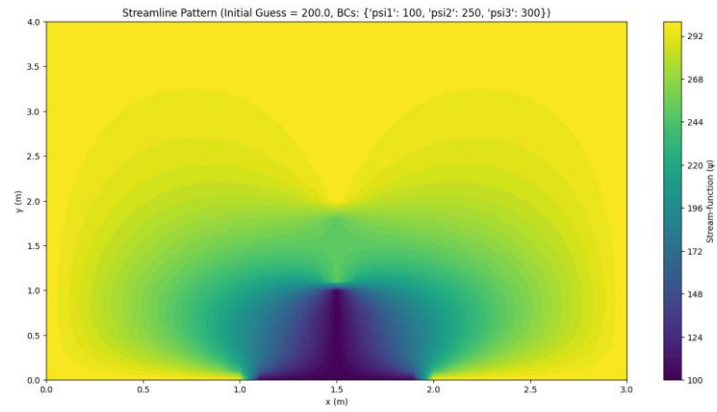
$\Psi=100$, Test3



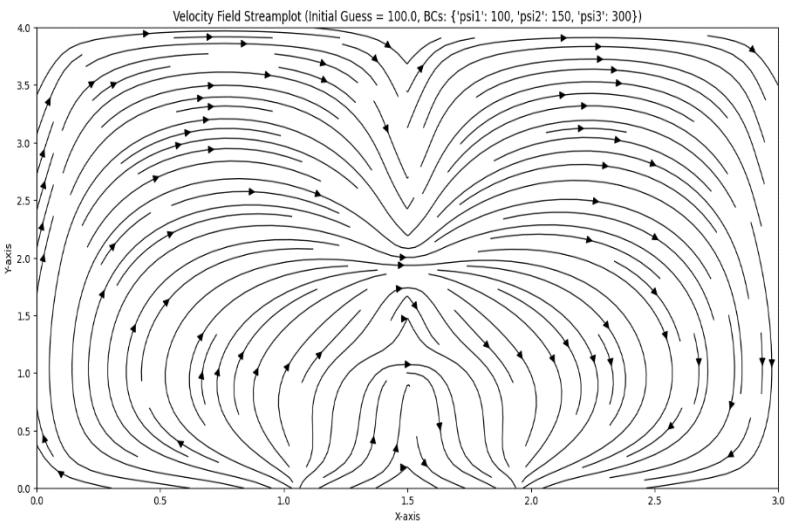
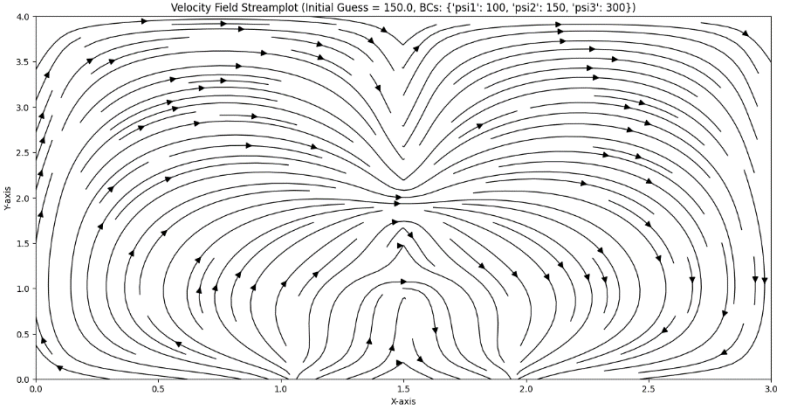
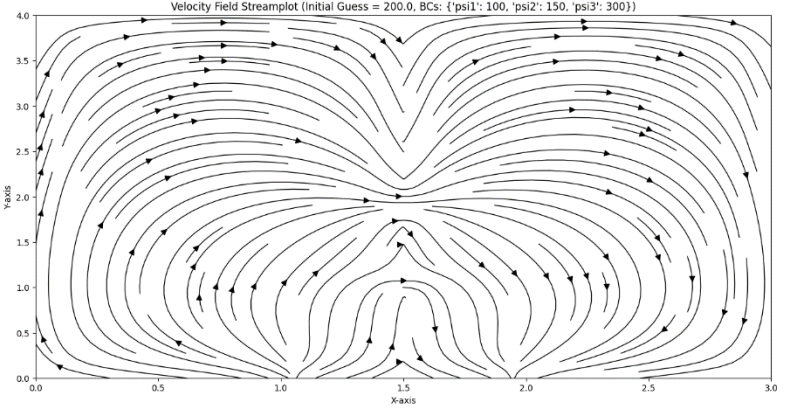
$\Psi=150$, Test3



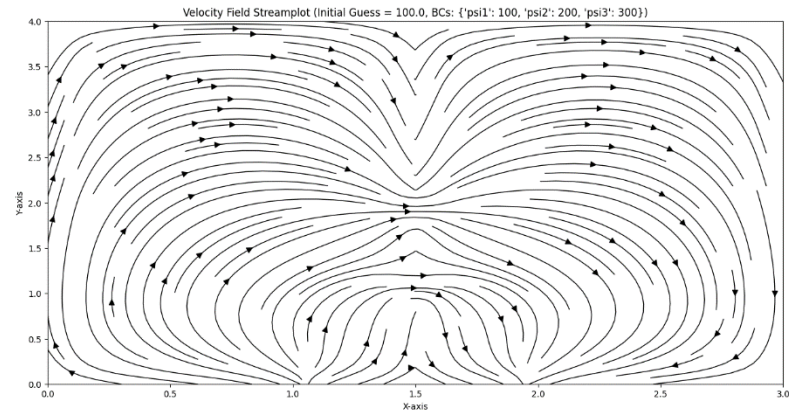
$\Psi=200$, Test3



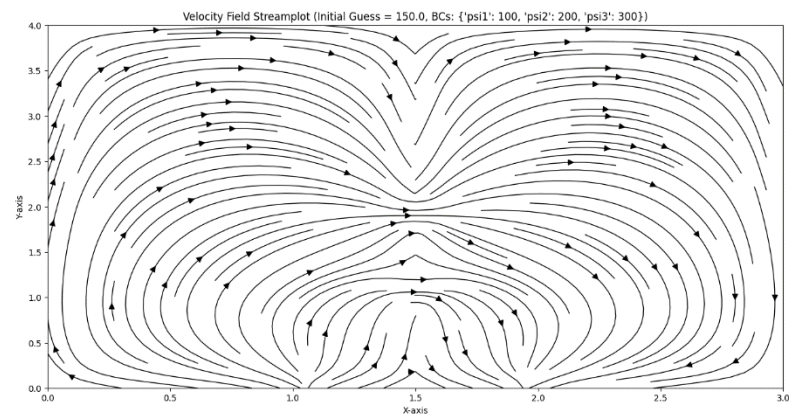
(iii) Velocity Vector Stream plots

Case	Plot
$\Psi=100$, Test 1	
$\Psi=150$, Test 1	
$\Psi=200$, Test 1	

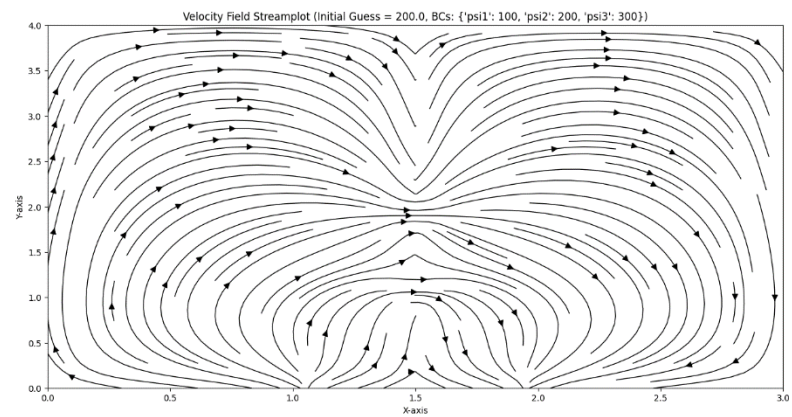
$\Psi=100$, Test 2



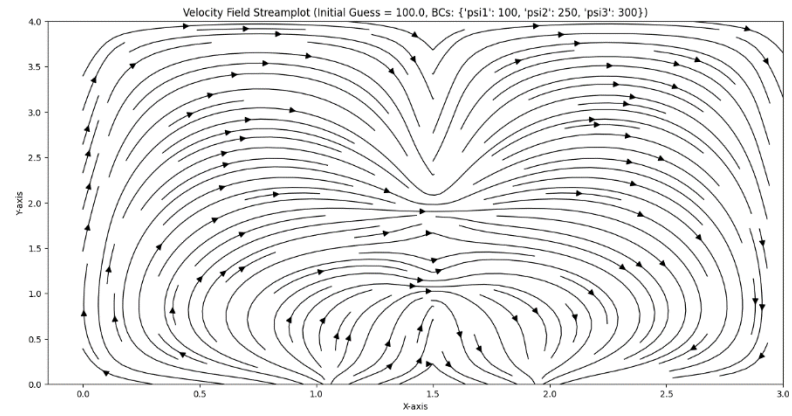
$\Psi=150$, Test 2



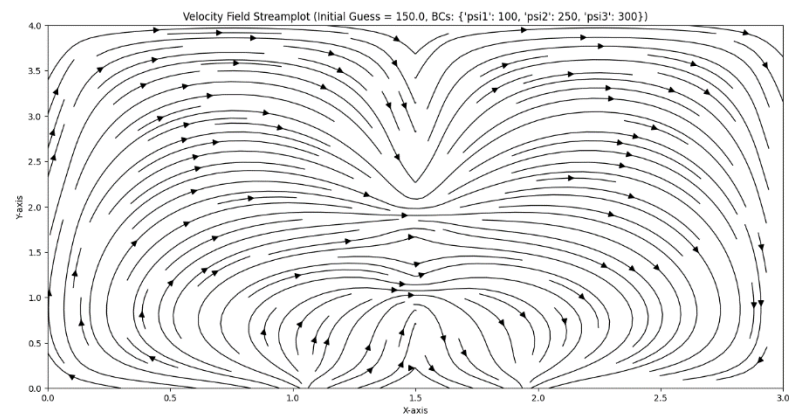
$\Psi=200$, Test 2



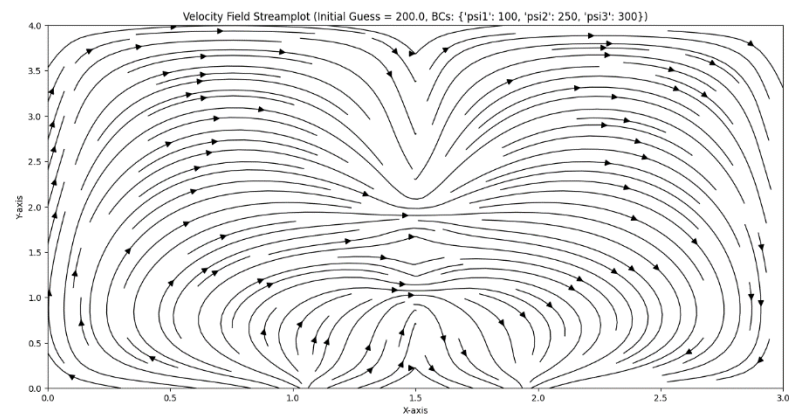
$\Psi=100$, Test 3



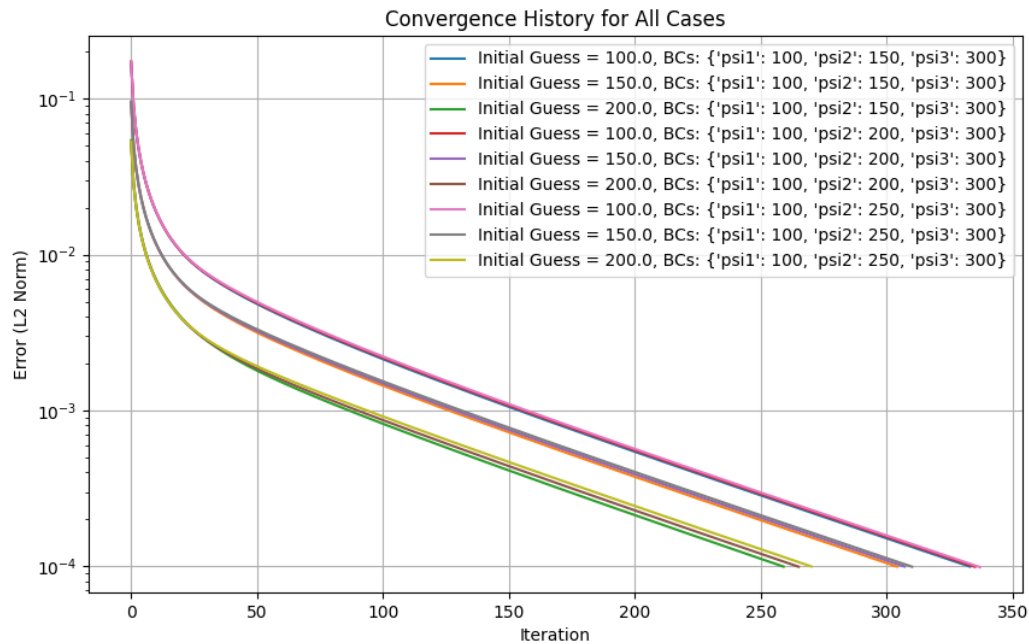
$\Psi=150$, Test 3



$\Psi=200$, Test 3



(iv) Convergence results



5. Comments and conclusions

- With the increase of initial guess the number of iterations decreases and it converges faster. The pattern is visible in all three test cases. For first test condition the no. of iteration decreases from 334 to 260 as initial guess increases from 100 to 200. The same pattern is observed in other test cases as well.
- The order of error for each initial guess and test cases is found to be same.
- The reason behind faster convergence for initial guess of 200 is that the stream function inside the chamber are closer to the guess value 200 as observed in the tabulated results and thus take a smaller number of iteration while 100 and 150 as initial guess take more iteration as they are far from the value of actual solutions of stream function inside the chamber. But there no effect on final solutions even if different initial guess has been used.
- The streamline pattern is consistent with the boundary condition provided in the problem statement. The walls are with constant stream function.
- Velocity filed stream plots are appearing to be originated from the inlet and appear to be terminated at the outlet.

6.Referemces

- [1] “Finite difference approximations” by Prof. JC Mandal
- [2] “Finite differencing on uniform cartesian mesh” by Prof. JC Mandal