

Computational Fluid Dynamics (AE 706)

A report on

Assignment-3: Computation of Streamline Patterns for Steady Inviscid Incompressible Fluid Flows

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1. Governing Equation and discretization

Here the given 2-dimensional inviscid incompressible equation is:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0 \dots (1)$$

Using second derivative approximation for equation (1)

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\Psi_{i+1,j} - 2\Psi_{i,j} + \Psi_{i-1,j}}{(\Delta x)^2} \dots \dots \dots \dots \dots (2)$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} \dots (3)$$

Substituting equations (2) and (3) in equation (1)

Here the given 2-dimensional inviscid incompressible equation is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} = 0 \dots (4)$$

Substituting $\Delta x = \Delta y = 0.1$ in equation (4) and simplifying it will result as:

Rearranging for $\psi_{i,j}$, we get Point Jacobi formula as:

$$\psi_{i,j}^{n+1} = \frac{1}{4} (\psi_{i+1,j}^n + \psi_{i-1,j}^n + \psi_{i,j+1}^n + \psi_{i,j-1}^n).....(6)$$

Here, $\psi_{i,j}^{n+1}$ is the updated value of $\psi_{i,j}$ at the (n+1) th iteration level.

2.Boundary Conditions

The chamber walls are the streamlines, so ψ is constant along the walls.

Left boundary(x=0) = ψ_3

Right boundary(x=3) = ψ_3

Top boundary(y=4) = ψ_3

Bottom boundary

- I. Left section (x=0 to x=1.1) = ψ_3
- II. Inlet (x=1.1 to x=2.0) = ψ_1
- III. Right section (x=2 to x=3) = ψ_3

Internal boundary

- I. Lower part (x=1.5, y=1.1) = ψ_1
- II. Middle part with slot (x=1.5, y=1.1 to y=2.0) = ψ_2
- III. Upper part (x = 1.5, y=2.0 to y = 4.0) = ψ_3

Sets of Boundary conditions

	ψ_1	ψ_2	ψ3
Test 1	100	150	300
Test 2	100	200	300
Test 3	100	250	300

3. Convergence Check

The iteration stops when the solution converges. The convergence condition is based on L2 norm of the error.

ERROR =
$$\frac{\left\|\psi_{i,j}^{n+1} - \psi_{i,j}^{n}\right\|_{2}}{\left\|\psi_{i,j}^{n+1}\right\|_{2}} < 10^{-4}$$

Where L2 norm is defined as:
$$\|\psi_{i,j}\|_2 = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-1} |\psi_{i,j}|^2}$$

Here, IM and JM are the maximum values indices i, j. L2 norm for the difference in the solution at a point at two consecutive solutions are denoted as $\left\|\psi_{i,j}^{n+1}-\psi_{i,j}^{n}\right\|_{2}$

$$\left\| \psi_{i,j}^{n+1} - \psi_{i,j}^{n} \right\|_{2} = \sqrt{\sum_{i=2}^{IM-1} \sum_{j=2}^{JM-2} \left| \psi_{i,j}^{n+1} - \psi_{i,j}^{n} \right|^{2}}$$

4.Results:

(i) Numerical results

1. Test 1: $\psi_1 = 100$, $\psi_2 = 150$ and $\psi_3 = 300$

a. Initial guess: $\psi = 100$

x-location	Y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300 232.93388617 211.41010627 201.89392408 196.45918828
	193.01166067 190.93024067 190.01913236 190.22001637 191.50188539
	193.78769413 196.89401845 200.56131738 204.67440289 209.29382758
	214.59422305 220.79142182 228.04719548 236.33990935 245.30202848
	254.19001535 262.26175869 269.14516973 274.79436123 279.34063761
	282.97120879 285.87300451 288.20480386 290.09690771 291.64963101
	292.9424097 294.03473861 294.97406446 295.79557876 296.52795179
	297.1923786 297.80661889 298.38396424 298.93598058 299.47184054
	300]
2.0	[300. 232.93388617 211.41010627 201.89392408 196.45918828
	193.01166067 190.93024067 190.01913236 190.22001637 191.50188539
	193.78769413 196.89401845 200.56131738 204.67440289 209.29382758
	214.59422305 220.79142182 228.04719548 236.33990935 245.30202848
	254.19001535 262.26175869 269.14516973 274.79436123 279.34063761
	282.97120879 285.87300451 288.20480386 290.09690771 291.64963101
	292.9424097 294.03473861 294.97406446 295.79557876 296.52795179
	297.1923786 297.80661889 298.38396424 298.93598058 299.47184054
	300.]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

b. Initial guess: $\psi = 150$

x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 232.9473083 211.43661614 201.93312295 196.51008215
	193.07349462 191.00139076 190.09857122 190.30554617 191.5923281
	193.88036748 196.98774766 200.65306386 204.76317144 209.37638963
	214.66994702 220.85708411 228.10269725 236.38223445 245.33173868
	254.20450576 262.26237044 269.12994864 274.76547747 279.29689905
	282.9155405 285.80499954 288.12799209 290.01160783 291.55975074
	292.848848 293.94148926 294.88238999 295.70922915 296.44829445
	297.12258752 297.74790411 298.33862865 298.90485184 299.45612932
	300.]
2.0	[300. 232.9473083 211.43661614 201.93312295 196.51008215
	193.07349462 191.00139076 190.09857122 190.30554617 191.5923281
	193.88036748 196.98774766 200.65306386 204.76317144 209.37638963
	214.66994702 220.85708411 228.10269725 236.38223445 245.33173868
	254.20450576 262.26237044 269.12994864 274.76547747 279.29689905
	282.9155405 285.80499954 288.12799209 290.01160783 291.55975074
	292.848848 293.94148926 294.88238999 295.70922915 296.44829445
	297.12258752 297.74790411 298.33862865 298.90485184 299.45612932
	300]
3.0	[300, 300, 300, 300, 300, 300, 300, 300,
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

c. Initial guess: $\psi = 200$

Table:3

x-locations	y-values						
0.0	[300. 300.	300. 300.	300. 300.	300. 300	. 300. 300.	. 300. 300	0. 300. 300.
	300. 300. 3	300. 300.	300. 300.	300. 300	. 300. 300.	300. 300	0. 300. 300.
	300. 300. 3	300. 300.	300. 300.	300. 300	. 300. 300.	300. 300	0. 300.]
1.0	[300.	232.9915	0278 211.5	52428808	3 202.0621	4817 196	6.6783891
	193.27693	8013	191.23676	132	190.35988	8963	190.58886496
	191.89012	657					
	194.18841	912 197.	2974062 2	00.96036	5708 205.0	5885398	209.6573171
	214.92681	257	221.08810	317	228.2991	1274	236.54400928
	245.45120	125					
	254.28363	3496	262.29522	678	269.12087	7089	274.71075386
	279.203109	906					
	282.78129	185	285.63876	256	287.9305	7599	289.79284437
	291.32230	173					
	292.60319	9855	293.69158	606	294.63853	3222	295.47592422
	296.23476	523					
	296.93311	.855	297.58979	857	298.2152	1941	298.82082744
	299.41330	906					
	300]						
2.0	[300.	232.9915	0278 211.5	52428808	3 202.0621	4817 196	6.6783891
	193.27693	8013	191.23676	132	190.35988	8963	190.58886496
	191.89012	657					
	194.18841	912 197.	2974062 2	00.96036	5708 205.0	5885398	209.6573171
	214.92681	257	221.08810	317	228.2991	1274	236.54400928
	245.45120	125					
	254.28363	3496	262.29522	678	269.1208	7089	274.71075386
	279.203109	906					
	282.78129	185	285.63876	256	287.9305	7599	289.79284437
	291.32230	173					
	292.60319	855	293.69158	606	294.63853	3222	295.47592422
	296.23476	523					
	296.93311	855	297.58979	857	298.2152	1941	298.82082744
	299.41330	906					
	300.]						
3.0	[300. 300.	300. 300.	300. 300.	300. 300	. 300. 300.	. 300. 300	0. 300. 300.
	300. 300. 3	300. 300.	300. 300.	300. 300	. 300. 300.	300. 300	0. 300. 300.
	300. 300. 3	300. 300.	300. 300.	<u>300. 30</u> 0	. 300. 300.	300. 300	0. 300.]

2. Test 2: $\psi_1 = 100$, $\psi_2 = 200$ and $\psi_3 = 300$

a. Initial guess: $\psi = 100$

x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 233.52883631 212.63461518 203.81995013 199.2001667
	196.73005219 195.84321955 196.40327971 198.39984776
	201.81282153
	206.48351146 212.00511238 217.77417779 223.43508046
	228.93256902
	234.40564611 240.06604857 246.10235637 252.57107256 259.2894826
	265.80833091 271.66883794 276.64713646 280.7340582 284.03152467
	286.67814952 288.80709875 290.53312441 291.94798352 293.1242851
	294.11764507 294.97139791 295.71848573 296.38466017
	296.98954202
	297.54860001 298.07372609 298.57445534 299.05833601
	299.53170273
	300]
2.0	[300. 233.52883631 212.63461518 203.81995013 199.2001667
	196.73005219 195.84321955 196.40327971 198.39984776
	201.81282153
	206.48351146 212.00511238 217.77417779 223.43508046
	228.93256902
	234.40564611 240.06604857 246.10235637 252.57107256 259.2894826
	265.80833091 271.66883794 276.64713646 280.7340582 284.03152467
	286.67814952 288.80709875 290.53312441 291.94798352 293.1242851
	294.11764507 294.97139791 295.71848573 296.38466017
	296.98954202
	297.54860001 298.07372609 298.57445534 299.05833601
	299.53170273
	300]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

b. Initial guess: $\psi = 150$

Table:5

x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 233.53869068 212.65411315 203.84866298 199.23747692
	196.7751231 195.89507144 196.46071127 198.46159134 201.87737988
	206.54943196 212.07069778 217.83791895 223.49520253
	228.98765364
	234.45398407 240.10646697 246.13336387 252.59188717
	259.29897583
	265.8062354 271.65452753 276.62096336 280.69599483 283.98256727
	286.6188872 288.73912536 290.45759551 291.86697469 293.03939755
	294.03125478 294.88536839 295.63526241 296.30614298
	296.91799096
	297.48571188 298.02132826 298.53382154 299.03065449
	299.51765593
	300.]
2.0	[300. 233.53869068 212.65411315 203.84866298 199.23747692
	196.7751231 195.89507144 196.46071127 198.46159134 201.87737988
	206.54943196 212.07069778 217.83791895 223.49520253
	228.98765364
	234.45398407 240.10646697 246.13336387 252.59188717
	259.29897583
	265.8062354 271.65452753 276.62096336 280.69599483 283.98256727
	286.6188872 288.73912536 290.45759551 291.86697469 293.03939755
	294.03125478 294.88536839 295.63526241 296.30614298
	296.91799096
	297.48571188 298.02132826 298.53382154 299.03065449
	299.51765593
	300.]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

c. Initial guess: $\psi = 200$

x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 233.57270651 212.72143359 203.94791525 199.36658959
	196.93144113 196.07534529 196.66118225 198.67812052
	202.10536594
	206.78422057 212.30717798 218.07135215 223.72035589
	229.20015757
	234.64889907 240.28022332 246.28171261 252.7124075 259.38843769
	265.86363403 271.67803787 276.61133168 280.65306665
	283.90882901
	286.51582826 288.61078305 290.30693191 291.69924698
	292.85871117
	293.8436116 294.69554013 295.44940928 296.12912412 296.7554868
	297.34205855 297.9011117 298.44029519 298.96679755 299.4852111
	300]
2.0	[300. 233.57270651 212.72143359 203.94791525 199.36658959
	196.93144113 196.07534529 196.66118225 198.67812052
	202.10536594
	206.78422057 212.30717798 218.07135215 223.72035589
	229.20015757
	234.64889907 240.28022332 246.28171261 252.7124075 259.38843769
	265.86363403 271.67803787 276.61133168 280.65306665
	283.90882901
	286.51582826 288.61078305 290.30693191 291.69924698
	292.85871117
	293.8436116 294.69554013 295.44940928 296.12912412 296.7554868
	297.34205855 297.9011117 298.44029519 298.96679755 299.4852111
	300.]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

3. Test 2: $\psi_1 = 100$, $\psi_2 = 250$ and $\psi_3 = 300$

a. Initial guess: $\psi = 100$

x-locations	y-values			
0.0	[300. 300. 300.	300. 300. 300. 300.	300. 300. 300. 300.	. 300. 300. 300.
	300. 300. 300.	300. 300. 300. 300.	300. 300. 300. 300.	300. 300. 300.
	300. 300. 300.	300. 300. 300. 300.	300. 300. 300. 300.	300. 300.]
1.0	[300. 234.]	12432928 213.86020	0778 205.74758102	2 201.94326542
	200.45104084	200.75926352	202.79090416	206.58355717
	212.12796577			
	219.18385504	227.12096814	234.99202453	242.2008785
	248.5765567			
	254.22234718	259.34598088	264.16275615	268.80740758
	273.28194841			
	277.43150091	281.08053079	284.1534913	286.67785089
	288.72624091			
	290.38860135	291.74441916	292.86434907	293.8016794
	294.60125126			
	295.29492467	295.90982063	296.46442875	296.97501675
	297.4521957			
	297.90567314	298.34150074	298.76543155	299.18101177
	299.59172206			
	300]			
2.0	L	12432928 213.86020		
	200.45104084	200.75926352	202.79090416	206.58355717
	212.12796577			
	219.18385504	227.12096814	234.99202453	242.2008785
	248.5765567			
	254.22234718	259.34598088	264.16275615	268.80740758
	273.28194841			
	277.43150091	281.08053079	284.1534913	286.67785089
	288.72624091			
	290.38860135	291.74441916	292.86434907	293.8016794
	294.60125126			
	295.29492467	295.90982063	296.46442875	296.97501675
	297.4521957			•••••••
	297.90567314	298.34150074	298.76543155	299.18101177
	299.59172206			
	300.]	200 200 200 200	200 200 200 200	
3.0	-	300. 300. 300. 300.		
		300. 300. 300. 300.		
	300. 300. 300.	300. 300. 300. 300.	300. 300. 300. 300.	300. 300.]

b. Initial guess: $\psi = 150$

Table:8

x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 234.13121719 213.87412393 205.76758768 201.96977799
	200.48223516 200.79581509 202.83020291 206.62651905 212.17134003
	219.22878015 227.16373567 235.03398556 242.23807745 248.61055974
	254.24917668 259.36743349 264.17505705 268.81260787 273.27668671
	277.41810183 281.05622259 284.12086676 286.63484924 288.67570486
	290.32923117 291.67925745 292.79282787 293.72694812 294.52337107
	295.21701592 295.83241047 296.39043877 296.90523695 297.38916215
	297.85022291 298.29560275 298.72977195 299.15683993 299.57942198
	300.]
2.0	[300. 234.13121719 213.87412393 205.76758768 201.96977799
	200.48223516 200.79581509 202.83020291 206.62651905 212.17134003
	219.22878015 227.16373567 235.03398556 242.23807745 248.61055974
	254.24917668 259.36743349 264.17505705 268.81260787 273.27668671
	277.41810183 281.05622259 284.12086676 286.63484924 288.67570486
	290.32923117 291.67925745 292.79282787 293.72694812 294.52337107
	295.21701592 295.83241047 296.39043877 296.90523695 297.38916215
	297.85022291 298.29560275 298.72977195 299.15683993 299.57942198
	300.]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.

c. Initial guess: $\psi = 200$

Table: 9

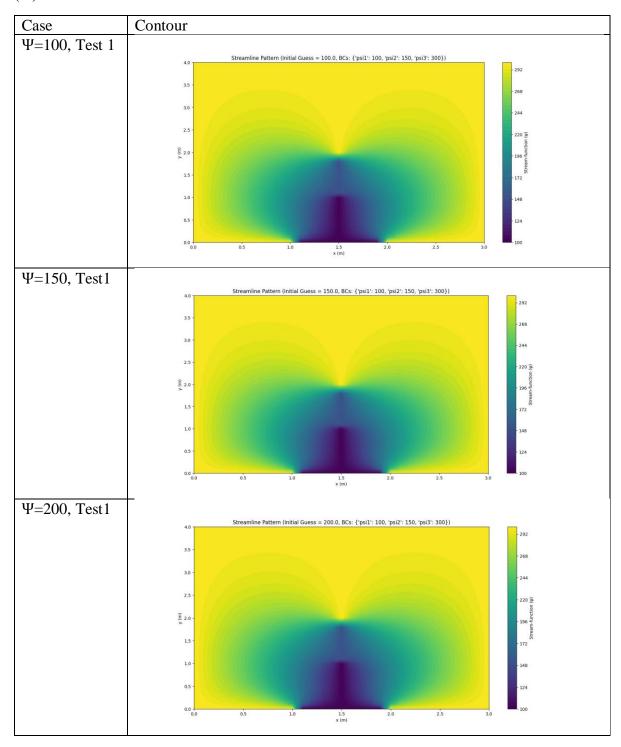
x-locations	y-values
0.0	[300. 300. 300. 300. 300. 300. 300. 300.
0.0	
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
1.0	[300. 234.1553647 213.92195168 205.83787051 202.06115064
	200.59235406 200.92252445 202.97024514 206.77712013 212.32861312
	219.38956016 227.32384793 235.19014219 242.38618624 248.74750136
	254.37130927 259.47209685 264.2593892 268.87466482 273.31464317
	277.43094495 281.04339395 284.08250964 286.57179268 288.58928867
	290.22160585 291.5528755 292.65097406 293.57298888 294.3614173
	295.05103315 295.66692243 296.22960198 296.75350717 297.25042884
	297.72837888 298.19381992 298.6509328 299.10301826 299.55215059
	300]
2.0	[300. 234.1553647 213.92195168 205.83787051 202.06115064
2.0	200.59235406 200.92252445 202.97024514 206.77712013 212.32861312
	219.38956016 227.32384793 235.19014219 242.38618624 248.74750136
	254.37130927 259.47209685 264.2593892 268.87466482 273.31464317
	277.43094495 281.04339395 284.08250964 286.57179268 288.58928867
	290.22160585 291.5528755 292.65097406 293.57298888 294.3614173
	295.05103315 295.66692243 296.22960198 296.75350717 297.25042884
	297.72837888 298.19381992 298.6509328 299.10301826 299.55215059
	300.]
3.0	[300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	300. 300. 300. 300. 300. 300. 300. 300.
	1

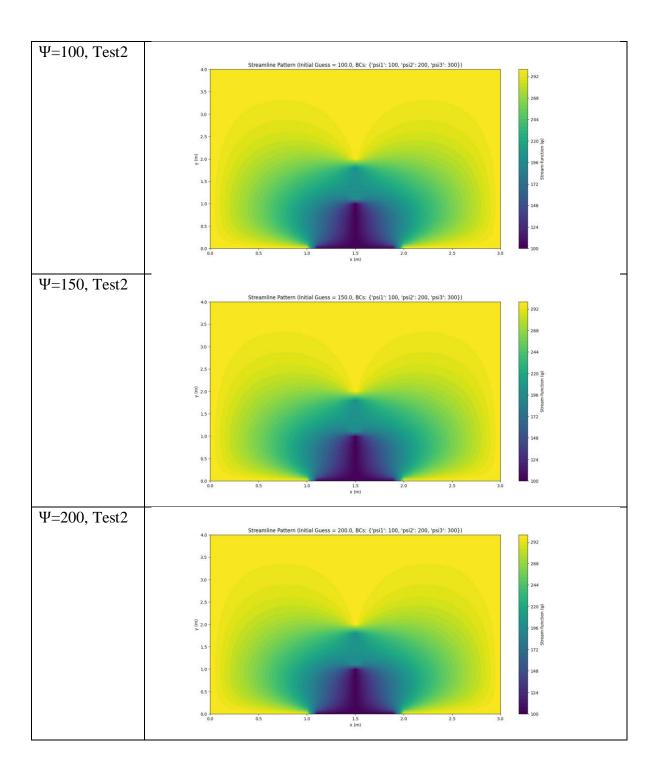
(ii). Effect of Initial Guess value

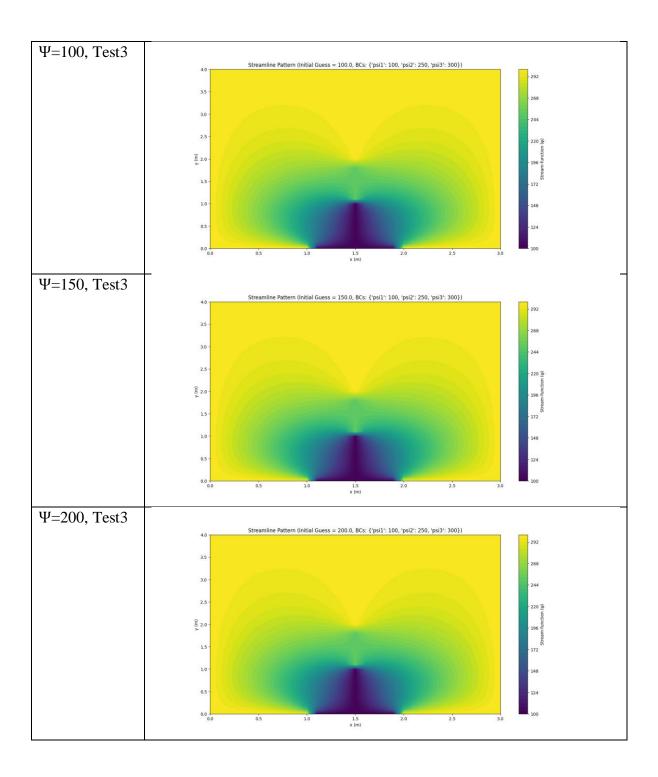
Table 10: Effect of initial guess on results

Case	No. of iteration	Error
Ψ=100, Test 1	334	9.962030803418954e-05
Ψ=150, Test1	305	9.969060838894907e-05
Ψ=200, Test1	260	9.962307952851038e-05
Ψ=100, Test2	336	9.932550071931199e-05
Ψ=150, Test2	308	9.973546016651792e-05
Ψ=200, Test2	266	9.943795075938226e-05
Ψ=100, Test3	338	9.88267089370632e-05
Ψ=150, Test3	311	9.94601551411883e-05
Ψ=200, Test3	271	9.990508704439758e-05

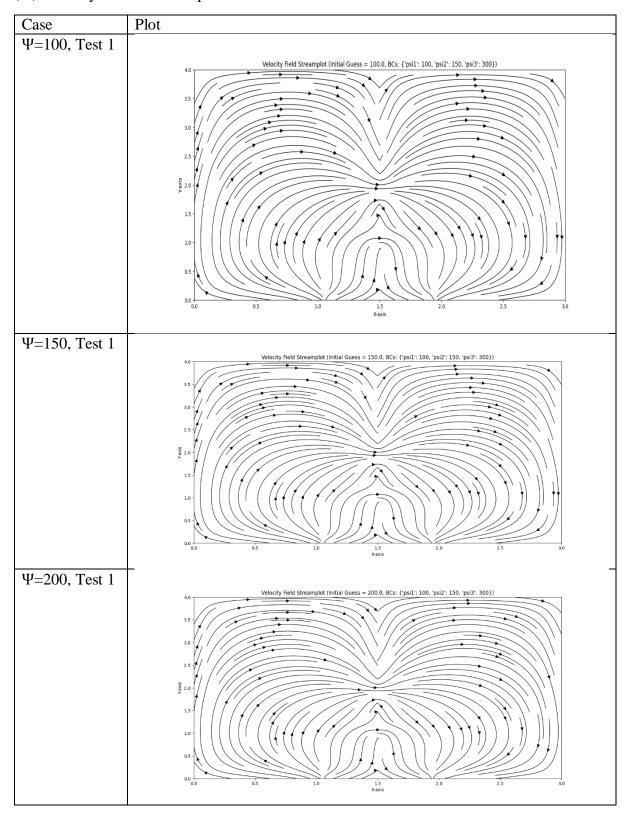
(iii) Contour Plots

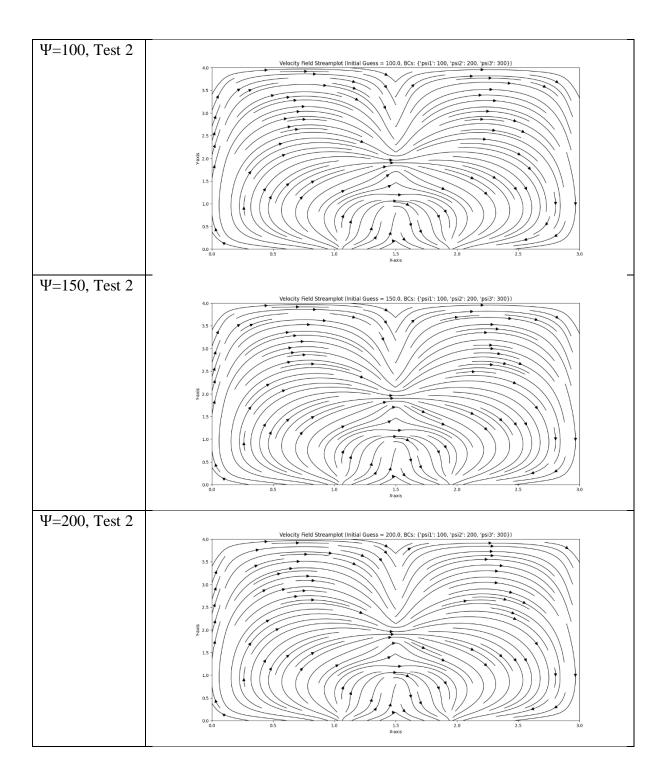


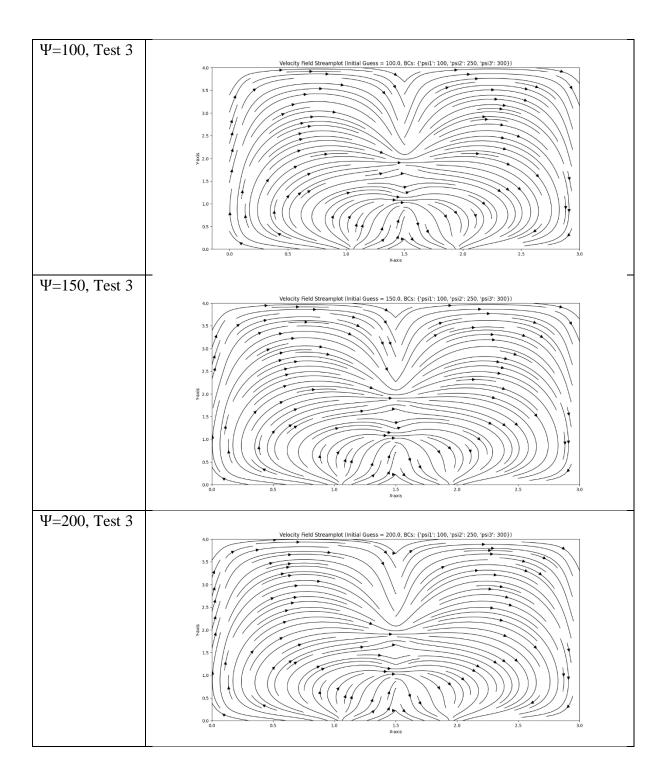




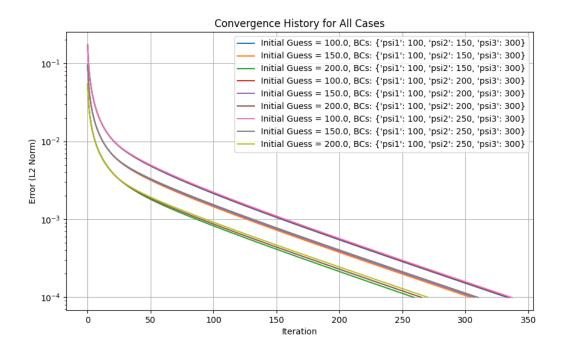
(iii) Velocity Vector Stream plots







(iv) Convergence results



5. Comments and conclusions

- a. With the increase of initial guess the number of iterations decreases and it converges faster. The pattern is visible in all three test cases. For first test condition the no. of iteration decreases from 334 to 260 as initial guess increases from 100 to 200. The same pattern is observed in other test cases as well.
- b. The order of error for each initial guess and test cases is found to be same.
- c. The reason behind faster convergence for initial guess of 200 is that the stream function inside the chamber are closer to the guess value 200 as observed in the tabulated results and thus take a smaller number of iteration while 100 and 150 as initial guess take more iteration as they are far from the value of actual solutions of stream function inside the chamber. But there no effect on final solutions even if different initial guess has been used.
- d. The streamline pattern is consistent with the boundary condition provided in the problem statement. The walls are with constant stream function.
- e. Velocity filed stream plots are appearing to be originated from the inlet and appear to be terminated at the outlet.

6.References

- [1] "Finite difference approximations" by Prof. JC Mandal
- [2] "Finite differencing on uniform cartesian mesh" by Prof. JC Mandal