Statistical Analysis of Temperature Data of Dutch Cities from 1910 to 2020

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Table of Contents

1. Introduction

Climate change is a modern global issue. Global warming generally refers to the increase in the Earth's temperature caused by the presence of greenhouse gases in the atmosphere, which causes changes in climate patterns across the globe (Riebeek, 2010). According to the European Commission (n.d), some of the consequences of global warming are rising sea levels, more extreme hurricanes, faster glacial retraction, and more extreme weather patterns on the whole. Even though these consequences all sound very alarming and urgent, there are still movements of climate change denialism.

This paper aims to find evidence for the main consequence of climate change, namely temperature rise. With the help of statistical methods, there will be an analysis of three cities in the Netherlands: Maastricht, de Bilt and Eelde. This paper will primarily focus on the former with some comparisons to the other two. The provided data for this analysis consists of daily, monthly and annual temperatures of the aforementioned cities over a time period of 114 successive years, from 1907 up until 2020.

The main research question of this paper states: "Is there statistical evidence that there is a linear upward temperature trend over the last 114 years in the Netherlands?" In order to answer the main research question, there are some sub-research questions.

- Do we find similar results if we apply the bootstrap method?
- Do we find similar results if we use smoothed monthly data instead of annual data?
- Is there a statistical difference in temperature rise between the three cities given in our data set?

2. Comparison of Average Temperatures

To start, we were given four different temperature data sets with annual data, smoothed monthly data, monthly data and daily data. In our analysis, we will focus primarily on the annual data, as it includes the average temperature over a year and, therefore, seasonal patterns that play a role in the other data sets can safely be ignored. The monthly data, as well as the daily data, are not going to be considered in this paper, as they are both influenced by seasonal fluctuations. On the contrary, the smoothed monthly data also prevents this seasonal bias by assigning to every month the average temperature of the six months before and after the respective one. Hence, we will also extend the analysis to the smoothed monthly data, to check whether the results are in line with both data sets.

In order to investigate a possible upward trend of the temperature in the Netherlands, the annual data was divided into intervals of different lengths. The first four years of the data are ignored in this analysis so that the division starts from 1911 onwards. Firstly, the data was divided into eleven ten-year intervals and, secondly, into two 55-year intervals. Then, we calculated the average temperatures and also the standard deviation over each respective time interval. Next, we used these to construct 95% confidence intervals of each time partition. Those intervals show us for which values of the mean we are 95% confident that it ranges between the calculated lower and upper bound of the confidence interval.

We came to the result that overall, there is more than a two-degree increase from the average of the first ten-year interval (1911-1920) to the last 10-year interval (2011-2020). As it can be seen in the graph below, the temperature has risen constantly, except for one drop in the interval from 1961 to 1970. Afterward, the average temperature started rising again and seemingly at an even faster rate.

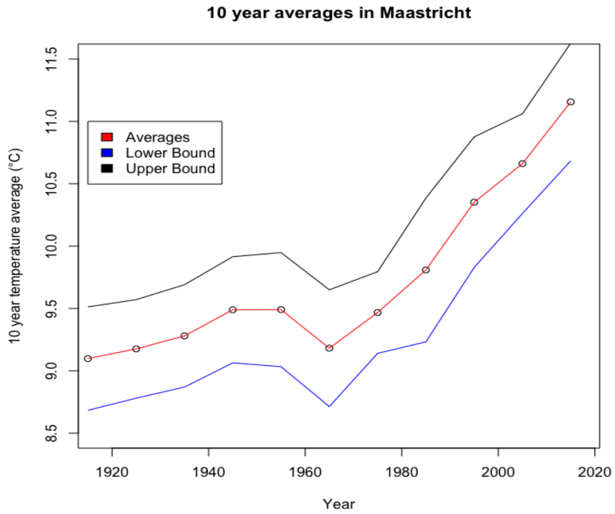

Figure 1: 10-year interval average temperatures of Maastricht

The graph below also shows, besides the ten-year averages in Maastricht on the red line, the lower bound of the confidence interval as the blue line and the upper bound of the confidence interval colored in black. Similar to the average temperature, the lower and upper bounds of the confidence intervals also increased over the years. This rise in the average temperatures happened to such an extent that the first confidence interval and the one of the last period do not overlap at all, which is another strong indication of an upward trend in the average temperatures over the 110 years.

The ten-year averages of Eelde and De Bilt show a similar development like the ones of Maastricht, even though we can see that the temperatures in Eelde are significantly lower compared to the other two cities. Moreover, next to the drop between 1961 to 1970, there is another decrease in these two data sets in the period from 1921 to 1930. All in all, we can say that the increase in temperature averages is not a unique phenomenon in Maastricht, but it also holds for Eelde and De Bilt. This increases our confidence that there is an upward trend in temperatures, not only locally in Maastricht, but also on a larger scale.

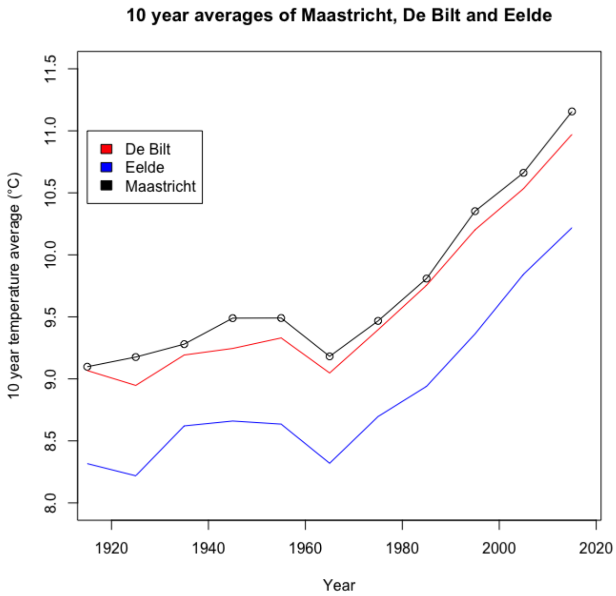

Figure 2: 10-year interval average temperatures of Maastricht, De Bilt and Eelde

The second interval division partitions the annual data from 1911 to 1965 and 1966 to 2020. When calculating the average temperature in Maastricht of these two-time intervals, it was found that the average temperature rose from 9.27°C in the first interval to 10.21°C in the second interval. This still indicates a rise in the average temperature, but the increase is only approximately 1°C compared to the 2°C increase that was found for the ten-year intervals. This is likely due to more observations evening out the volatility and the increase in temperatures over time.

Furthermore, we also extended our analysis to overlapping intervals, to check whether the changes in temperatures between the time intervals are stable or influenced by possible outliers. For that purpose, we introduced ten-year time intervals with a five-year overlap to the previous partition. From the graph below, which compares the non-overlapping temperature averages in Maastricht to the overlapping temperature averages, we can see that the increase in temperature is not as steady as suspected before but indicates more fluctuations than we expected. Nevertheless, our confidence in an overall upward trend in temperatures in Maastricht is not weakened by this finding, as it aligns with what we found before.

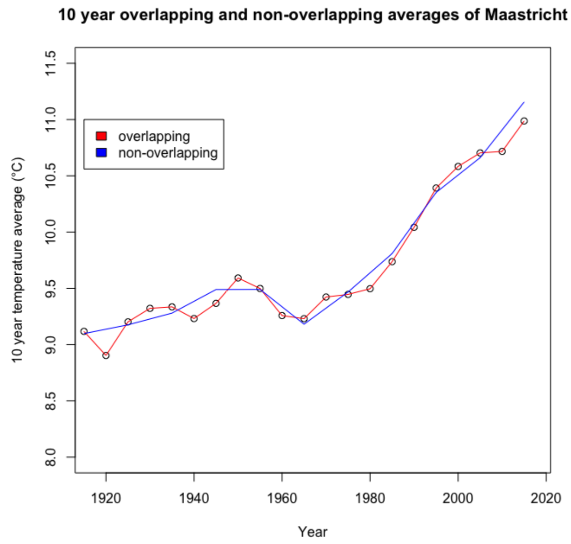

Figure 3: 10-year interval with 5-year overlap average temperatures of Maastricht

Also, when comparing the overlapping temperature averages from Maastricht with the ones from Eelde and De Bilt, it is clear that the net developments from one time period to another were quite similar for all three cities. This again strengthens our argument of an upward trend in temperatures independent of the city considered.

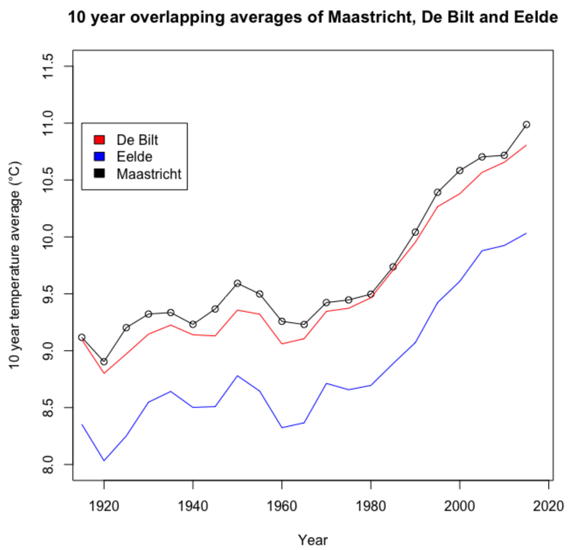

Figure 4: 10-year overlapping and non-overlapping interval average temperatures of Maastricht

3. Testing Differences in Means

In order to test whether there is a difference in both parts of the 55-year sample, a paired sample t-test was conducted since the sample size is the same and the true variances of each 55-year sample will be different. The following hypothesis test was carried out:

$$H_0: \bar{X} - \bar{Y} = 0 \text{ vs. } H_1: \bar{X} - \bar{Y} \neq 0$$

Carrying out the following test in R, found that the test statistic $|T(X)| = 7.119 > 2.004$, for a 5% significance level and therefore, the null hypothesis can be rejected. This implies that there is a difference between the means of each 55-year sample. Now, this test only indicates a difference in the means but does not tell us anything about which 55-year sample has a greater mean. Hence, the following right-tailed two sample t-test was conducted:

$$H_0: \bar{X} - \bar{Y} \leq 0 \text{ vs. } H_1: \bar{X} - \bar{Y} > 0$$

The results of the right-tailed test found that $T(X) = 7.119 > -1.674$, for a 5% significance level and therefore, the null hypothesis can be rejected. In the case of a right-tailed test, this implied that the average temperature in the last 55 years was larger than that of the first 55 years, which is also an indication of a linear upward trend. Considering the sharp rise in global temperatures in the 21st century this rejection does make sense.

4. Testing Differences in Means (Bootstrap)

We also wished to investigate the aforementioned 55-year hypotheses with respect to the bootstrap. While the exact implementation will be dealt with later in the assignment, we did not wish to assume any sort of distribution on our observations and thus we chose to perform a nonparametric bootstrap. Through this, we could gain insight into the accuracy of our assumptions of normality on the observations. However, we still required the independence assumption which indeed may not be entirely accurate to assume, as temperatures of the previous years likely influence the current temperature.

The regular t-test did indeed find a rejection for a two-tailed test; however, the bootstrap found a critical value of 8.94 which is larger than the aforementioned t-statistic of 7.119. It could similarly not be rejected in the case of the right-tailed test. While this is certainly not solid proof of an absence of a difference in means, it is important to evaluate this result. This may indeed mean that our normality assumption on the original observations may not have been completely warranted as we saw that the regular paired sample t-test very easily shows that there was an indication of a positive difference between the two means.

It is important to note here that the null hypothesis was not accepted, but simply failed to be rejected and thus, no conclusions can be directly made from the analysis itself. Another issue may have been the test itself that was used, the paired sample t-test. This test was designed to test measure similar observations two times, rather than comparing two different sets of information. The other option, a two-sample t-test, could also not be directly applied to this case as it requires the true variances of the two samples to be the same. The paired sample t-test makes no such requirement, however. Finally, the implementation of the test may have been flawed. The test subtracts "pairs" of data, X_i and Y_i, which in this case, would be the respective years in the 55-year samples (e.g. year 5 and year 60). There is, however, no real relation between these two pairs which may have skewed the results even further.

5. Linear Regression on Annual Data

To be able to investigate the presence of a linear upward trend, a linear regression was used to investigate the relationship between time (years) and temperatures in Maastricht. A linear regression model is represented by the following equation:

$$Y_i = \alpha_i + \beta * X_i + \varepsilon_i$$

The dependent variables Y_1, …, Y_n represent the temperatures in Maastricht and the independent variables X_1, …, X_n represent the years 1907 – 2020. Through the regression, we want to be able to predict the value of β and α, however, for the purpose of investigating a linear upward trend the value of β is of more importance. The value of βrepresents the average change in temperature in Maastricht every year whereas α is the intercept of the line of best fit for the data. The values ε_i are the error terms, which indicate the difference in the line of best fit and the actual data for each data point from 1, …, n.

Applying the linear regression in the program R yields the following results for the value of β and its 95% confidence interval:

$$\beta = 0.00495$$

$$C(X) = [0.000448, 0.00944]$$

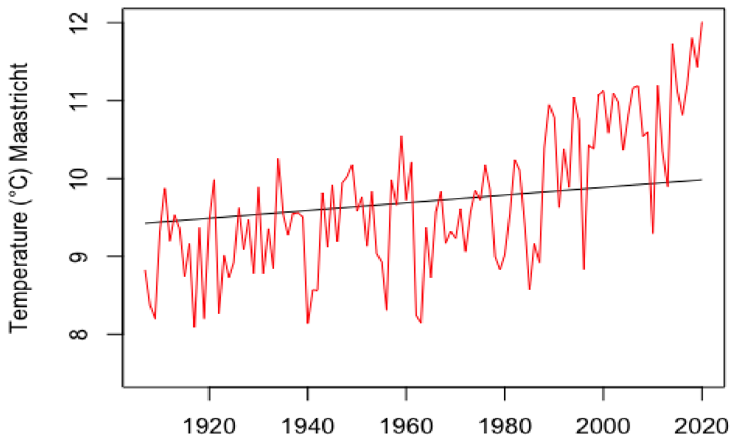

Figure 5: Linear regression on annual data (Maastricht)

The predicted value of β implies that on average between the years 1907-2020 the temperature rose by 0.00495 degrees celsius yearly. The value of β might not truly reflect reality due to the possibility of estimation errors. Therefore, in order to be sure of the presence of a linear upward trend a right-tailed hypothesis test was conducted using the t-test. The reason for the choice of a right-tailed t-test was because the variance is unknown, and the sample variance is used instead. The test is as follows:

$$H_0: \ \beta \leq 0 \text{ vs. } H_1: \beta > 0$$

Conducting this test in R, it was found that the test statistic, T(X) = 2.1787 > 1.6584 and, hence, we reject the null hypothesis at a 5% significance level. Thus, there is evidence of a linear upward trend.

However, it is important to note that throughout the regression process we assumed normality, but this assumption might not truly reflect reality. Using asymptotic theory, the Central Limit Theorem implies that regardless of the original distribution of the parameter of interest, for large values of n (sample size) it converges to a standard normal distribution. It could be the case that temperatures in Maastricht are not normally distributed and finding the true distribution might be difficult. This is where the bootstrap plays an important role, as it allows us to make inferences about our parameter of interest without having to know the distribution of the data.

6. Bootstrap Linear Regression on Annual Data

The bootstrap treats the sample as the population and samples from this original sample with replacement in order to simulate different samples. This ensures that the confidence interval will remain close to the wanted 1-ɑ confidence interval (in this case 95%) instead of using the Central Limit Theorem which may only slowly converge to this confidence. Here, 1000 different samples were simulated from the original sample in order to find new critical values for the beta hat confidence interval. Specifically, a residual bootstrap was done as it imposes a linear model which will help to compare it to the original regression analysis which also approximates the unknown distribution of the original observations using the Empirical Distribution Function (EDF) (Smeekens, 2020). Furthermore, the other option of a pairs bootstrap may not be applicable as it cannot be used for a fixed X which we have in this case since X is simply the year the data was taken. Doing this we find a new confidence interval of:

$$C(X) = [0.00489, 0.00503]$$

This is significantly smaller than the original confidence interval for beta while preserving the approximate 95% confidence level. However, we again find that βeasily falls outside the range of being lower than zero, so we can indeed conclude at a size of 5% that there is an upward trend in the data.

It is important to keep in mind however that while effective, this method is not without scrutiny. Firstly, bootstrapping requires independence of the original observations, which, given that temperatures affect one another, may not be accurate to assume. This is similar to the rest of the analysis where this was also assumed. Furthermore, bootstrapping depends on the original sample and thus a very different confidence interval may have arisen if one had chosen a different sample from the true population. Despite this, it was not necessary to assume anything about the distribution of the original observations which is in strong contrast to the rest of the analysis where we imposed normality on the observations.

7. Regression for De Bilt and Eelde

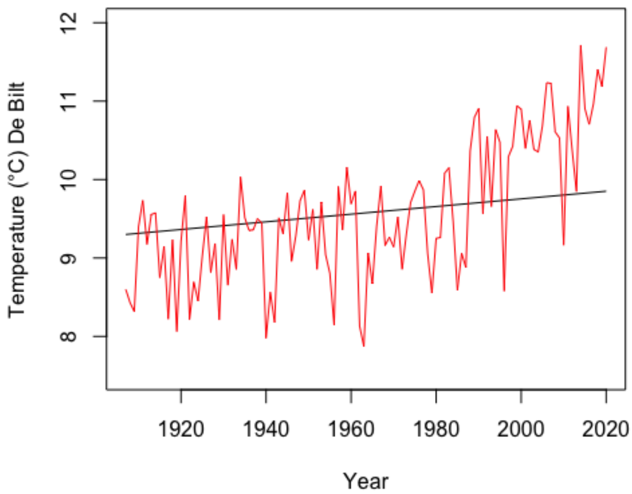

Figure 6: Linear regression on annual data (De Bilt)

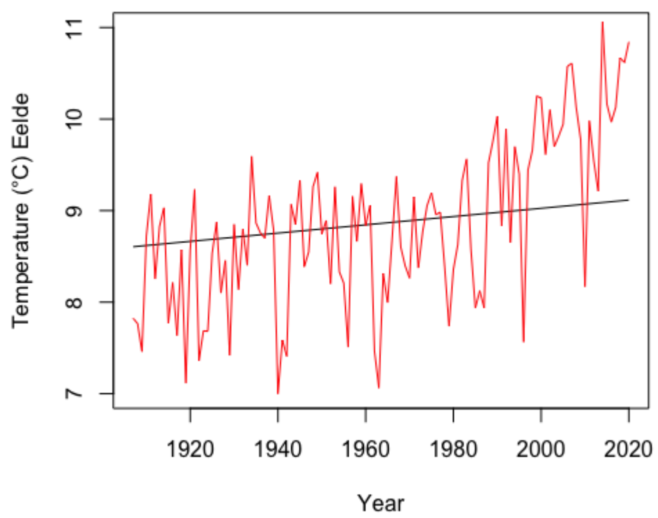

Figure 7: Linear regression on annual data (Eelde)

Similar regression analysis was done for the other two cities, De Bilt and Eelde. Just as for Maastricht, a positive beta was for De Bilt and for Eelde, 0.0049 and 0.0045 respectively. Furthermore we find beta confidence intervals for De Bilt and Eelde of [0.000480, 0.00928] and [0, 0.00912] respectively. This indeed once again is significant statistical evidence of warming in both cities with both confidence intervals being very positive. Therefore, we can indeed reason that there is some sort of warming occurring over the past century in various parts of the Netherlands.

8. Investigation of the presence of a linear upward trend in parts of the sample

In order to investigate the claim that temperatures only started increasing significantly from the seventies on, we divide the sample into two parts, one up to 1969 and the other one from 1970 onwards. Coincidentally, this is also the year when the first Earth Day was organized, which was supposed to underline the importance of environmental protection and gain more support for this topic (Earth Day, n.d.).

As we can see in our previous graphs showing the ten-year intervals average temperatures, the increase in temperatures seemed to slow down in the 1950s and 1960s. This was possibly caused by natural variability and cooling effects of some aerosols that were produced during that time (NASA Earth Observatory, n.d.). This effect was so strong that some scientists even warned of a coming ice age, but they did not consider in this prediction that the aerosols only stay in the atmosphere for a short amount of time compared to CO_2 (Le Treut, 2007).
Therefore, after considering the graphs showing a slowdown in average temperature increase before the seventies and also the cooling effect caused by the aerosols, we decided to compare the increase in temperatures from the year 1970 onwards to the rise in temperatures before that year.

To investigate the presence of a linear upward trend in parts of the sample, the respective β values for each subsample were determined. The β values in 1910 - 1969 and 1970 - 2020 were $\beta_2 = 0.00479$ and $\beta_3 = 0.00516$ respectively. Although $\beta_3 > \beta_2$, it is not sufficient to conclude from the respective β values that there is a presence of a linear upward trend from 1970 onwards just because $\beta_3 > \beta_2$. In order to explore this more formally, a test for structural breaks is required. A structural break can be defined as a sudden change in a time series at a certain point in time. The time series, in this case, is the change in temperatures over the years from 1910 - 2020. A test for a structural break can be conducted using the Chow test and since the investigation focuses on the presence of a linear upward trend, the following hypothesis test was carried out:

$$H_0: \beta_3 - \beta_2 = 0 \text{ vs. } H_1: \beta_3 - \beta_2 \neq 0$$

The test rejects the null hypothesis if the test statistic, $T(X) > F_{k,\, n1 + n2 - 2k}$, where k represents the number of parameters and n_1 and n_2 represent the populations of each sub-sample respectively. In our case, the test statistic $T(X) = 17.33 > 0.0513 = F_{2,\, 50 + 60 - 2*(3)} = F_{2,104}$ and therefore, we can reject the null hypothesis for 5% significance level. So, we have that there is a difference between the two regression coefficients for each sample and since, $\beta_3 > \beta_2$, the sample from 1970 onwards has a greater average increase in temperature than the sample before 1970. The average difference in the yearly increase in temperature of the two samples is β_3 - β_2 = 0.00516 - 0.00479 = 0.00037 degrees Celsius. Since there is evidence of a structural break from 1970 onwards, there is evidence of a presence of a linear upward trend.

Another structural break test was conducted on the sub-sample from 1910-1969. This sub-sample was split into two more sub-samples from 1910 - 1939 and 1940 - 1969. The β values were β_4 = 0.00477 and β_5 = 0.0048 respectively. Another similar hypothesis test was carried out and looks as follows:

$$H_0: \beta_5 - \beta_4 = 0 \text{ vs. } H_1: \beta_5 - \beta_4 \neq 0$$

The test statistic, $T(X) = 0.05 \leq 0.0513$ and therefore, we fail to reject the null hypothesis of a structural break. The difference between β_5 and β_4 is very minimal so, there is no concrete evidence of a structural break before the 1970s, which also supports our choice of splitting the sample from 1910 - 1969 and 1970 - 2020.

9. Extension to smoothed monthly data

As mentioned earlier, we extended our analysis to the smoothened monthly data. This data set prevents the seasonal bias because it takes weighted averages of the month itself, and 6 months before and after the respective month. Therefore, seasonal fluctuations or also called noise have been taken away.

We want to verify whether we get similar results if we use smoothened monthly data instead of annual data. We apply the same method as with the annual data, so first we split the sample into eleven subsets, of each 120 observations, this is equivalent to ten years. We chose to remove the first 36 months, and start our analysis in July 1910, this because our data set runs until June 2020. In this way it is possible to plot the graph, as shown below. The overall trend of the graph is upward, however, there are a few data points where we see a decrease in the 10-year average temperatures.

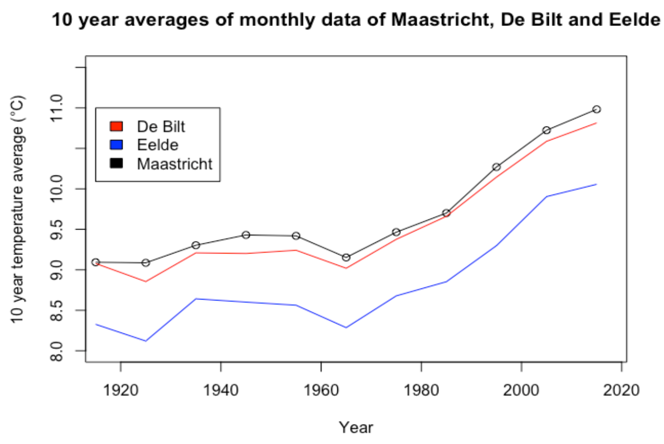

Figure 8: 10-year interval temperature averages using smoothed monthly data

In order to statistically confirm that the eleven different means indeed have an upward trend over the years, we perform another linear regression of Maastricht. Under the same assumptions and implementation as for the annual data, we observe that the slope of the regression is = 0.00494. This implies that there is a 0.0000493 degrees celsius increase of temperatures on a monthly basis. This beta sees a confidence interval of [0.000236, 0.00965] so it is indeed in the positive range for the given confidence. In the graph of our linear regression model, we see significantly higher peaks compared to the annual data possibly caused by more fluctuations in monthly data as the annual data would have eased some of the volatility by averaging out over more months.

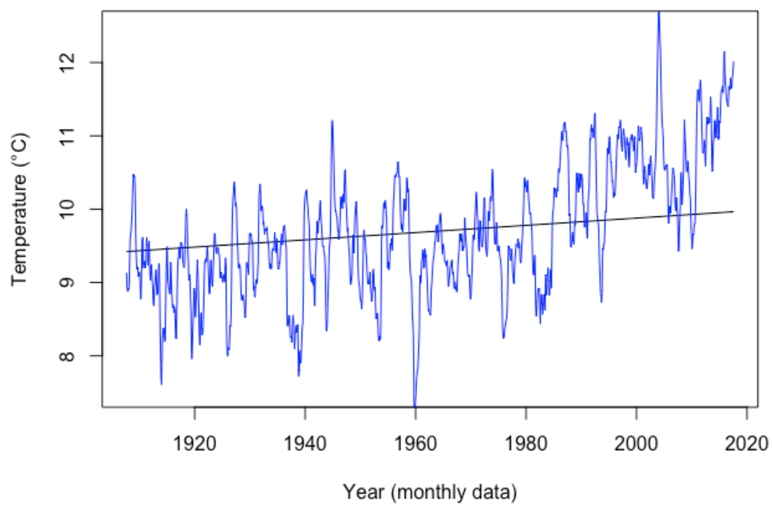

Figure 9: Linear regression on smoothed monthly data (Maastricht)

As our data sample of the smoothened monthly data contains more observations, it is more likely that the distribution lies closer to the standard normal distribution compared to the annual data. Therefore, we can say that the statistical results of the smoothened monthly data are likely stronger than the annual data.

As our data sample consists of temperatures over the last 114 years, there is no guarantee that one temperature on one day, month or year does not affect another temperature. Therefore, we cannot ensure independence of the observations. Furthermore, weather patterns may have temporarily changed the distribution of a handful of observations and thus there is also no way to maintain the identically distributed assumption. That being said, given the large sample sizes, much of the volatility will have been reduced and so while not having perfect assumptions, they may be somewhat reasonable.

10. Conclusion

In conclusion, through the statistical analysis there seems to be rather strong evidence in favor of an upward trend in the temperatures in the various cities. Firstly, through the application of a simple t-test it was indeed possible to show that the means trended upward in the ten-year and 55-year subsamples. Not only this, when attempting to mitigate possible outliers through the use of overlapping data, the same conclusions could be drawn from these overlapping subsample hypothesis tests. While it is important to stress that this is by no means definitive proof of warming, it does imply strong statistical evidence of such warming. There is no guarantee that the data is indeed normal and independent. Moreover, by implementing a non-parametric bootstrap to simulate the hypothesis test it could be seen that at the chosen confidence level, no statistical conclusions could be drawn from the performed tests. Therefore, it is important to further evaluate the data with possibly other tests and other hypotheses.

Secondly, various linear regressions were performed in order to investigate the specific trend in the data. Here too strong evidence was found in favor of an upward trend in the form of positive betas. In the yearly data as a whole, a positive beta was found with a confidence interval that was indeed entirely positive. Furthermore, splitting the data and using a Chow

Test to test how the two betas differed also allowed us to accept that there was an upward effect on the beta given the chosen confidence level. Additionally, the bootstrap was also implemented on the beta confidence interval, finding a smaller interval with a similar confidence level and indeed providing more evidence for a warming. Therefore, one can be confident in the existence of some sort of increase in average temperatures over the past century.

Thirdly, we extended the data analysis to monthly data and performed linear regression on this extended data. Once again, an upward trend was noticed in the averages and a positive beta for the linear regression was seen. The extension here was mainly implemented to alleviate seasonal biases that may indeed skew the yearly data as summer is undoubtedly warmer on average than winter. With more observations, the data converges more toward the asymptotic distribution and thus via the central limit theorem, there was less need to assume some sort of distribution on the observations. While strong evidence does indeed seem to exist, it is again important to note that we must take into consideration the assumptions made with ordinary least squares regressions such as strict exogeneity. Furthermore, while no direct distribution assumptions needed to be made for least squares regression, that is not to say that no assumptions needed to be made about the dependent and independent variables. The assumption here is that there is a linear relation between the two which may obscure several abnormalities with the data such as abnormally warm or cool periods. Despite this, the sample size was sufficiently large in order to mitigate these issues. Finally, the chosen alpha level of 5% is rather arbitrarily chosen and can indeed hide points of interests in the analysis, such as tests where 5% is not appropriate given the large number of assumptions made.

While several sacrifices were made in the data analysis, it is nonetheless important to properly evaluate the results that were found given the assumptions. The results at a glance provide strong evidence for warming and it is important to take these results and act upon them. While the results do not specify the causes of the warming, both consumers and firms need to remain mindful of their effect on the climate and take steps to mitigate their carbon footprints in order to avoid a possible climate disaster. These results are steppingstones to more internationally oriented data analyses to find the global effect on the climate over the past century and can give potential researchers reasons to seek out how humans interacting with the world affects the climate, in order to find exact culprits not only in the Netherlands, but also globally.

11. References

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12. List of tables and figures

Figures:

13. Appendix

R Code:

```
# Reading csv files
annual.data <- read.csv("AnnualTemp.csv", sep = ";", dec = ",")
annual.temp <- ts(annual.data[, 2:4], start = c(1907))
smoothed.monthly.data <- read.csv("SmoothedMonthlyTemp.csv", sep = ";",
dec = ",")
smoothed.monthly.temp <- ts(smoothed.monthly.data[, 2:4], start =
c(1907, 7), frequency = 12)
monthly.data <- read.csv("MonthlyTemp.csv", sep = ";", dec = ",")
monthly.temp <- ts(monthly.data[, 2:4], start = c(1907, 1), frequency =
12)

# We choose Maastricht as our sample of main interest
t <- nrow(annual.data)
averages <- colMeans(AnnualTemp[,-1])
t_10Y <- floor(t/10)
averages_10Y <- matrix(NA, t_10Y, 5)
sd_10Y <- matrix(NA, t_10Y, 5)
for(i in 1:t_10Y){
```

```
  index_i <- ((i-1)*10+1):(i*10)+4
  temp_i <- annual.data[index_i,]
  averages_10Y[i,1] <- (1900 + i*10)
  averages_10Y[i,2] <- (1910 + i*10)
  averages_10Y[i,3:5] <- colMeans(temp_i[,-1])
  sd_10Y[i,1] <- (1900 + i*10)
  sd_10Y[i,2] <- (1910 + i*10)
  sd_10Y[i,3:5] <- apply(temp_i[,2:4], 2, sd)
}
names <- c("start", "end", "De.Bilt", "Eelde", "Maastricht")
colnames(averages_10Y) <- names
colnames(sd_10Y) <- names

tConfidenceLower <- averages_10Y[,3:5]  - (qt(0.975, df=9) *
(sd_10Y[,3:5] / sqrt(10)))
tConfidenceUpper <- averages_10Y[,3:5] + (qt(0.975, df=9) *
(sd_10Y[,3:5] / sqrt(10)))

# Confidence intervals for 10 year averages
tConfidenceIntervalDe.Bilt <- cbind(averages_10Y[,1:2],
tConfidenceLower[,1], tConfidenceUpper[,1])
tConfidenceIntervalEelde<- cbind(averages_10Y[,1:2],
tConfidenceLower[,2], tConfidenceUpper[,2])
tConfidenceIntervalMaastricht <- cbind(averages_10Y[,1:2],
tConfidenceLower[,3], tConfidenceUpper[,3])

#plot of maastricht 10 year averages
plot(averages_10Y[,1] + 5, averages_10Y[,5], main = "10 year averages in
Maastricht", xlab = "Year", ylab = "10 year temperature average (°C)",
xlim = c(1917, 2017), ylim =  c(8.5,11.5))
lines(averages_10Y[,1] + 5, averages_10Y[,5], col="red")
lines(averages_10Y[,1] + 5, tConfidenceIntervalMaastricht[,3],
col="blue")
lines(averages_10Y[,1] + 5, tConfidenceIntervalMaastricht[,4], col
="black")
legend(1915,11, legend = c("Averages", "Lower Bound", "Upper Bound"),
fill = c("red", "blue", "black")  ,col = c("red", "blue", "black"))

#plot 10 year averages of all three cities
plot(averages_10Y[,1] + 5, averages_10Y[,5], main = "10 year averages of
Maastricht, De Bilt and Eelde", xlab = "Year", ylab = "10 year
temperature average (°C)", xlim = c(1917, 2017), ylim =  c(8,11.5))
lines(averages_10Y[,1] + 5, averages_10Y[,3], col="red")
lines(averages_10Y[,1] + 5, averages_10Y[,4], col="blue")
lines(averages_10Y[,1] + 5, averages_10Y[,5], col= "black")
```

```
legend(1915,11, legend = c("De Bilt", "Eelde", "Maastricht"), fill =
c("red", "blue", "black")  ,col = c("red", "blue", "black"))


# 5 year overlapping sample for 10 year averages

t_5overlapping10Y <- floor(t/5 - 1)
av_5overlapping10Y <- matrix(NA, t_5overlapping10Y, 5)
sd_5overlapping10Y <- matrix(NA, t_5overlapping10Y, 5)
for(i in 1:t_5overlapping10Y){
  index_i <- ((i-1)*5+4):((i-1)*5+14)
  temp_i <- annual.data[index_i,]
  av_5overlapping10Y[i,1] <- ((i-1)*5+1910)
  av_5overlapping10Y[i,2] <- ((i-1)*5+1920)
  av_5overlapping10Y[i, 3:5] <- colMeans(temp_i[,2:4])
  sd_5overlapping10Y[i,1] <- ((i-1)*5+1910)
  sd_5overlapping10Y[i,2] <- ((i-1)*5+1920)
  sd_5overlapping10Y[i, 3:5] <- apply(temp_i[,2:4], 2, sd)
}

plot(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,5], main = "10 year
overlapping averages of Maastricht, De Bilt and Eelde", xlab = "Year",
ylab = "10 year temperature average (°C)", xlim = c(1917, 2017), ylim =
c(8,11.5))
lines(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,3], col="red")
lines(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,4], col="blue")
lines(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,5], col= "black")
legend(1915,11, legend = c("De Bilt", "Eelde", "Maastricht"), fill =
c("red", "blue", "black")  ,col = c("red", "blue", "black"))

plot(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,5], main = "10 year
overlapping and non-overlapping averages of Maastricht", xlab = "Year",
ylab = "10 year temperature average (°C)", xlim = c(1917, 2017), ylim =
c(8,11.5))
lines(av_5overlapping10Y[,1] + 5, av_5overlapping10Y[,5], col="red")
lines(averages_10Y[,1] + 5, averages_10Y[,5], col="blue")
legend(1915,11, legend = c("overlapping", "non-overlapping"), fill =
c("red", "blue")  ,col = c("red", "blue"))

# Paired sample t-test for testing differences in mean (two-tailed test)
X.matrix <- matrix(annual.data[,1], t, 1)
Y.matrix <- matrix(annual.data[,4], t, 1)

first55Temp <- Y.matrix[5:59,]
last55Temp <- Y.matrix[60:114,]
```

```
subtractTemp <- last55Temp - first55Temp
subtractTemp.bar <- mean(subtractTemp)
subtractTemp.sd <- sd(subtractTemp)

t.statistic_twoSample <- (subtractTemp.bar)/(subtractTemp.sd/sqrt(55))
twoSample.criticalVal <- qt(p=0.05/2, 54, lower.tail = FALSE)
if (abs(t.statistic_twoSample) > twoSample.criticalVal) {
  print("We can reject the null hypothesis and therefore, the two means
are indeed different")
} else {
  print("We fail to reject the null hypothesis")
}

# Paired sample t-test for testing differences in mean (right-tailed
test)
t.statistic_twoSampleRightTail <-
(subtractTemp.bar)/(subtractTemp.sd/sqrt(55))
twoSample.criticalValRightTail <- qt(p=0.05, 54, lower.tail = FALSE)
if (t.statistic_twoSample > twoSample.criticalValRightTail) {
  print("We can reject the null hypothesis and therefore, so the mean of
the last 55 years is larger than that of the first 55")
} else {
  print("We fail to reject the null hypothesis and so, the mean of the
last 55 years is less than that of the first 55")
}

#Bootstrap paired sample test (55Y, 2 tailed)
B = 1000
Q.star <- rep(NA, B)
n.55Y = 55

for (b in 1:B) {

  J <- sample.int(n.55Y, size = n.55Y, replace = TRUE)
  Yfirst.star <- first55Temp[J]
  Ylast.star <- last55Temp[J]
  Yfirst.bar.star <- mean(Yfirst.star)
  Ylast.bar.star <- mean(Ylast.star)
  Yfirst.var.star <- sd(Yfirst.star)^2
  Ylast.var.star <- sd(Ylast.star)^2
  S.star <- (Yfirst.var.star + Ylast.var.star) / 2

  Q.star[b] <- (Yfirst.bar.star - Ylast.bar.star) / sqrt((2*S.star) /
n.55Y)
}
```

```r
c.alpha.star <- quantile(Q.star, probs = 1 - (alpha/2))
c.oneminusalpha.star <- quantile(Q.star, probs = alpha/2)

if (t.statistic_twoSample > c.alpha.star || t.statistic_twoSample <
c.oneminusalpha.star) {
  print("We can reject the null hypothesis and therefore, the two means
are indeed different")
} else {
  print("We fail to reject the null hypothesis")
}
rm(J, Yfirst.star, Ylast.star, Yfirst.bar.star, Ylast.bar.star,
Yfirst.var.star, Ylast.var.star, S.star, Q.star, B)

#Bootstrap paired sample t test (55Y, right tailed)
B = 1000
Q.star <- rep(NA, B)
n.55Y <- 55

for (b in 1:B) {

  J <- sample.int(n.55Y, size = n.55Y, replace = TRUE)
  subtractTemp.star <- subtractTemp[J]
  subtractTemp.bar.star <- mean(subtractTemp.star)
  subtractTemp.sd.star <- sd(subtractTemp.star)
  Q.star[b] <- (subtractTemp.bar.star) / (subtractTemp.sd.star /
sqrt(55))
}
c.alpha.star <- quantile(Q.star, probs = 1 - alpha)
if (t.statistic_twoSample > c.alpha.star) {
  print("We can reject the null hypothesis and therefore, so the mean of
the last 55 years is larger than that of the first 55")
} else {
  print("We fail to reject the null hypothesis and so, the mean of the
last 55 years is less than that of the first 55")
}
rm(J, Yfirst.star, Ylast.star, Yfirst.bar.star, Ylast.bar.star,
Yfirst.var.star, Ylast.var.star, S.star, Q.star, B)


#Linear Regression
X.matrix <- matrix(annual.data[,1], t, 1)
Y.matrix <- matrix(annual.data[,4], t, 1)

beta.hat <-
drop(solve(crossprod(X.matrix),crossprod(X.matrix,Y.matrix))) #drop
```

```
converts 1x1 matrix to scalar
alpha.hat <- colMeans(Y.matrix) - (beta.hat * colMeans(X.matrix))

plot(X.matrix, (beta.hat*X.matrix) + alpha.hat, type = "l",
ylim=c(7.5,12), xlab = "Year", ylab = "Temperature (°C)")
lines(X.matrix , Y.matrix, col="red")

# Confidence intervals for beta
residual <- Y.matrix - alpha.hat - beta.hat*(X.matrix)
sSquared <- sum((residual)^2)/112
sXX <- sum((X.matrix - colMeans(X.matrix))^2)
beta.criticalVal <- qt(p=alpha/2,112,lower.tail = FALSE)
betaLowerConfidence <- beta.hat -
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
betaUpperConfidence <- beta.hat +
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
print("Beta CI")
print(c(betaLowerConfidence,betaUpperConfidence))

# Confidence intervals for alpha
alpha.criticalVal <- qt(p=alpha/2,112,lower.tail = FALSE)
a <- sSquared/(114*sXX)
b <- sum((X.matrix)^2)
alphaLowerConfidence <- alpha.hat - alpha.criticalVal*sqrt(a*b)
alphaUpperConfidence <- alpha.hat + alpha.criticalVal*sqrt(a*b)
print("Alpha CI")
print(c(alphaLowerConfidence,alphaUpperConfidence))



#Linear Regression (De Bilt)

YDB.matrix <- matrix(annual.data[,2], t, 1)
betaDB.hat <-
drop(solve(crossprod(X.matrix),crossprod(X.matrix,YDB.matrix)))
alphaDB.hat <- colMeans(YDB.matrix) - (betaDB.hat * colMeans(X.matrix))

plot(X.matrix, (betaDB.hat*X.matrix) + alphaDB.hat, type = "l",
ylim=c(7.5,12), xlab = "Year", ylab = "Temperature (°C) De Bilt")
lines(X.matrix , YDB.matrix, col="red")

#Linear Regression (Eelde)

YE.matrix <- matrix(annual.data[,3], t, 1)
betaE.hat <-
```

```
drop(solve(crossprod(X.matrix),crossprod(X.matrix,YE.matrix)))
alphaE.hat <- colMeans(YE.matrix) - (betaE.hat * colMeans(X.matrix))

plot(X.matrix, (betaE.hat*X.matrix) + alphaE.hat, type = "l",
ylim=c(7,11), xlab = "Year", ylab = "Temperature (°C) Eelde")
lines(X.matrix , YE.matrix, col="red")

#Confidence intervals for beta (De Bilt)

residualDB <- YDB.matrix - alphaDB.hat - betaDB.hat*(X.matrix)
sSquared <- sum((residualDB)^2)/112
sXX <- sum((X.matrix - colMeans(X.matrix))^2)
beta.criticalVal <- qt(p=alpha/2,112,lower.tail = FALSE)
betaDBLowerConfidence <- betaDB.hat -
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
betaDBUpperConfidence <- betaDB.hat +
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
print("Beta CI (De Bilt)")
print(c(betaDBLowerConfidence,betaDBUpperConfidence))

# Confidence intervals for beta (Eelde)

residualE <- YE.matrix - alphaE.hat - betaE.hat*(X.matrix)
sSquared <- sum((residualE)^2)/112
sXX <- sum((X.matrix - colMeans(X.matrix))^2)
beta.criticalVal <- qt(p=alpha/2,112,lower.tail = FALSE)
betaELowerConfidence <- betaE.hat -
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
betaEUpperConfidence <- betaE.hat +
beta.criticalVal*(sqrt(sSquared)/sqrt(sXX))
print("Beta CI (Eelde)")
print(c(betaELowerConfidence,betaEUpperConfidence))



# Then we carry out the following hypothesis test: H0: beta <= 0 Vs H1:
beta > 0 (right-tailed test)
t.statistic <- (beta.hat - 0)/(sqrt((sSquared/sXX)))
criticalVal <- qt(p=alpha,113,lower.tail = FALSE)
if (t.statistic > criticalVal){
  print("We can reject the null hypothesis and therefore, there is
evidence of an upward trend in the data")
} else {
  print("We fail to reject the null hypothesis")
}
```

```
#Residual Bootstrap implementation
error.hat <- Y.matrix - alpha.hat - (beta.hat * X.matrix)
n <- nrow(X.matrix)
B <- n
Y.matrix.star <- rep(NA, B)
Q.star <- rep(NA, B)
beta.hat.star <- rep(NA, B)
alpha.hat.star <- rep(NA, B)

for (b in 1:B) {

  J <- sample.int(n, size = n, replace = TRUE)
  error.star <- error.hat[J]
  for (i in 1:n){
    Y.matrix.star[i] = alpha.hat + (beta.hat*X.matrix[i]) +
error.star[i]
  }
  Y.matrix.star <- matrix(Y.matrix.star)
  beta.hat.star[b] <-
drop(solve(crossprod(X.matrix),crossprod(X.matrix,Y.matrix.star)))
  alpha.hat.star[b] <- colMeans(Y.matrix.star) - (beta.hat *
colMeans(X.matrix))
  sSquared.star <- sum((error.star)^2)/112
  sXX <- sum((X.matrix - colMeans(X.matrix))^2)
  Q.star[b] <- (beta.hat.star[b] - beta.hat) / sqrt(sSquared.star / sXX)
}
c.alpha.star <- quantile(Q.star, probs = 1 - (alpha/2))
c.oneminusalpha.star <- quantile(Q.star, probs = alpha/2)
beta.hat.star.ci <- rep(NA, 2)
beta.hat.star.ci[1] <- beta.hat - (c.alpha.star * sqrt(sSquared / sXX))
beta.hat.star.ci[2] <- beta.hat - (c.oneminusalpha.star * sqrt(sSquared
/ sXX))


rm(n, J,  error.star, Y.matrix.star, beta.hat.star, alpha.hat.star,
sSquared.star, c.alpha.star, c.oneminusalpha.star, Q.star, B)

#Splting data at 1970
X2.matrix <- matrix(X.matrix[4:64,])
Y2.matrix <- matrix(Y.matrix[4:64,])

beta2.hat <-
drop(solve(crossprod(X2.matrix),crossprod(X2.matrix,Y2.matrix))) #drop
converts 1x1 matrix to scalar
```

```
alpha2.hat <- mean(Y2.matrix) - (beta2.hat * mean(X2.matrix))

X3.matrix <- X.matrix[65:114,]
Y3.matrix <- Y.matrix[65:114,]

beta3.hat <-
drop(solve(crossprod(X3.matrix),crossprod(X3.matrix,Y3.matrix)))
alpha3.hat <- mean(Y3.matrix) - (beta3.hat * mean(X3.matrix))

residualA <- Y2.matrix - alpha2.hat - beta2.hat*(X2.matrix)
residualSumA <- sum((residualA)^2)

residualB <- Y3.matrix - alpha3.hat - beta3.hat*(X3.matrix)
residualSumB <- sum((residualB)^2)

residualSumWholeSample <- sum((residual)^2)

# Structural Break & Chow Test for the samples from 1910-1969 &
1970-2020
chow_Num <- (residualSumWholeSample - residualSumA - residualSumB)/2
chow_Den <- (residualSumA + residualSumB)/(60 + 50 - 6)
chow.stat <- chow_Num/chow_Den
f.criticalVal <- qf(p=0.05, 2, 110 - 6)

if (chow.stat > f.criticalVal) {
  print("We can reject the null hypothesis and therefore, there is
evidence of a structural break from 1970 onwards")
} else {
  print("We fail to reject the null hypothesis and therefore, there is
no evidence of a structural break from 1970 onwards")
}


# Structural Break before 1970
X4.matrix <- matrix(X.matrix[4:34,])
Y4.matrix <- matrix(Y.matrix[4:34,])

beta4.hat <-
drop(solve(crossprod(X4.matrix),crossprod(X4.matrix,Y4.matrix))) #drop
converts 1x1 matrix to scalar
alpha4.hat <- mean(Y4.matrix) - (beta4.hat * mean(X4.matrix))

X5.matrix <- X.matrix[35:64,]
Y5.matrix <- Y.matrix[35:64,]
```

```
beta5.hat <-
drop(solve(crossprod(X5.matrix),crossprod(X5.matrix,Y5.matrix)))
alpha5.hat <- mean(Y5.matrix) - (beta5.hat * mean(X5.matrix))

residualC <- Y4.matrix - alpha4.hat - beta4.hat*(X4.matrix)
residualSumC <- sum((residualC)^2)

residualD <- Y5.matrix - alpha5.hat - beta5.hat*(X5.matrix)
residualSumD <- sum((residualD)^2)

# Chow Test before 1970
chow_NumA <- (residualSumA - residualSumC - residualSumD)/2
chow_DenA <- (residualSumC + residualSumD)/(30 + 29 - 6)
chow.statA <- chow_NumA/chow_DenA
f.criticalValA <- qf(p=0.05, 2, 59 - 6)


# We choose Maastricht as our sample of main interest for the smoothened
monthly data
m <- nrow(smoothed.monthly.data)
averages <- colMeans(SmoothedMonthlyTemp[,-1])
m_10Y <- floor(m/120)
averagesm_10Y <- matrix(NA, m_10Y, 5)
sdm_10Y <- matrix(NA, m_10Y, 5)
for(i in 1:m_10Y){
  index_i2 <- ((i-1)*120+1):(i*120)+36
  temp_i2 <- smoothed.monthly.data[index_i2,]
  averagesm_10Y[i,1] <- (1900 + i*10)
  averagesm_10Y[i,2] <- (1910 + i*10)
  averagesm_10Y[i,3:5] <- colMeans(temp_i2[,-1])
  sdm_10Y[i,1] <- (1900 + i*10)
  sdm_10Y[i,2] <- (1910 + i*10)
  sdm_10Y[i,3:5] <- apply(temp_i[,2:4], 2, sd)
}

names <- c("start", "end", "De.Bilt", "Eelde", "Maastricht")
colnames(averagesm_10Y) <- names
colnames(sdm_10Y) <- names

#Linear regression for smoothed monthly data
Xm.matrix <- matrix(smoothed.monthly.data[,1], m, 1)
Ym.matrix <- matrix(smoothed.monthly.data[,4], m, 1)
Xm2.matrix <- matrix(rep(NA, 1320))
for (i in 1:1320){
  Xm2.matrix[i] <- (1907 + (7/12)) + (i/12)
}
Ym2.matrix <- matrix(Ym.matrix[37:1356])
```

```
betaM2.hat <-
drop(solve(crossprod(Xm2.matrix),crossprod(Xm2.matrix,Ym2.matrix)))
alphaM2.hat <- colMeans(Ym2.matrix) - (betaM2.hat * colMeans(Xm2.matrix))

plot(Xm2.matrix, (betaM2.hat*Xm2.matrix) + alphaM2.hat, type = "l",
ylim=c(7.5,12.5), xlab = "Year (monthly data)", ylab = "Temperature (°C)")
lines(Xm2.matrix , Ym2.matrix, col="blue")

# Confidence intervals for beta
residual <- Ym2.matrix - alphaM2.hat - betaM2.hat*(Xm2.matrix)
sSquared <- sum((residual)^2)/112
sXX <- sum((Xm2.matrix - colMeans(Xm2.matrix))^2)
betaM2.criticalVal <- qt(p=alpha/2,112,lower.tail = FALSE)
betaM2LowerConfidence <- betaM2.hat -
betaM2.criticalVal*(sqrt(sSquared)/sqrt(sXX))
betaM2UpperConfidence <- betaM2.hat +
betaM2.criticalVal*(sqrt(sSquared)/sqrt(sXX))
print("Beta Monthly CI")
print(c(betaM2LowerConfidence,betaM2UpperConfidence))
```