# Nonlinear optics of photonic hyper-crystals: optical limiting and hyper-computing

Photonic Project: Group 8

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PH 421 Course Project

Laboratory of Optics of Quantum Materials

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# Presentation Overview

# Abstract

- Goal: to achieve enhanced photon-photon interaction for application in optical computing
- Photonic hyper-crystals combine the most interesting features of hyperbolic metamaterials and photonic crystals.
- Since the dispersion law of extraordinary photons in hyperbolic metamaterials does not exhibit the usual diffraction limit, photonic hyper-crystals exhibit light localization on deep subwavelength scales, leading to considerable enhancement of nonlinear photon–photon interaction.
- Nonlinear optics of photonic hyper-crystalsis such that one of the spatial coordinates would play a role of effective time in a 2+1-dimensional “optical space-time”
- Mapping the conventional optical computing onto nonlinear optics of photonic hyper-crystals results in a “hyper-computing” scheme, which may considerably accelerate computation time.

# Motivation

Hyperbolic Metamaterials and Photonic Crystals
Foundations of Photon Control

- Photonic Crystals: Periodic dielectric nanostructures with changing refractive index. Period : order of the optical wavelength.
- Hyperbolic Metamaterials: Artificial materials with positive dielectric constant in one direction and negative in another, leading to a hyperbolic dispersion relation

Mitigating Loss in Hyperbolic Metamaterials

- Hyperbolic metamaterials experience losses.
- Solution: Integrate gain media as a dielectric component.
- Result: Lossless and potentially active hyperbolic materials.

# Motivation

- Photonic Crystals: Periodic dielectric nanostructures with changing refractive index.
  Period : order of the optical wavelength.

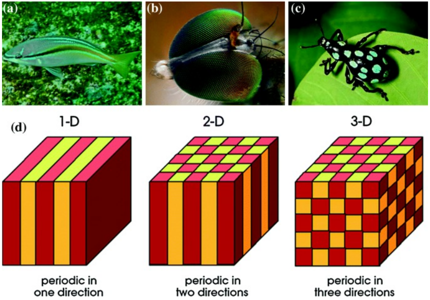



Figure: Photonic Crystals

# Motivation

- Hyperbolic Metamaterials: The metamaterials with hyperbolic dispersion relation

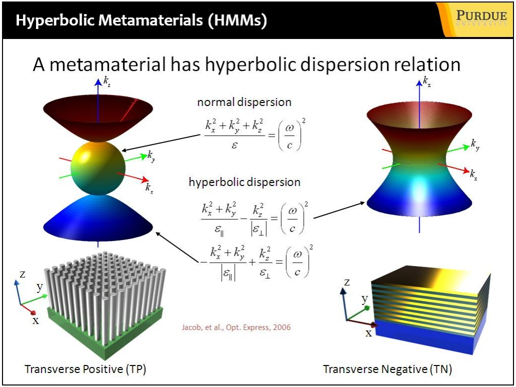



Figure: Hyperbolic metamaterials (HMMs)

- Analogy to Gravity: The z-axis can be treated as a time-like axis, 2+1-dimensional “optical space-time”

# Motivation

- Quantum computers require qubits or quantum bits
- Single photons: a promising candidate
  1. Can encode in: polarization, path, time bin
  2. Free of noise

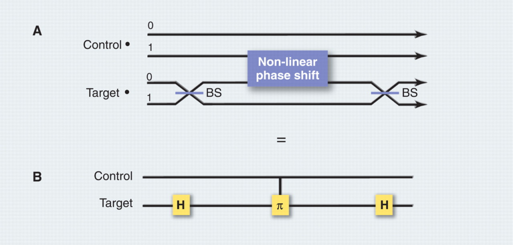



Figure: CNOT gate

- Control = 0 $\implies$ No non-linear phase shift $\implies$ $2^{nd}$ BS undoes $1^{st}$ $\implies$ No mixing of target states
- Control = 1 $\implies$ Non-linear phase shift is activated $\implies$ $2^{nd}$ BS can't undo $1^{st}$ $\implies$ Mixing of target states

# Motivation

- Challenge: Strong enough non-linearity for this conditional shift for single photons not available.
- Reason: The Kerr Effect!!
  The non-linearity of materials is proportional to intensity.
  e.g. $n = n_0 + n_2 I$. Single photons $\implies$ Less I $\implies$ No non-linearity
- A way out: Increasing I or photon interaction at single photon level. $I \propto \frac{1}{V}$. I $\uparrow$ if V $\downarrow$. So need small volumes.
- Recent research: Photonic-crystal-based ultra-small mode volumes used $\implies$ Non-linearities observed at single photon level.

# Motivation

- Why Photonic Hyper-crystals???
- Conventional diffraction limit: $d << \lambda$
- Our $d = L << \lambda$ indeed! So diffraction should occur
- Twist!! dispersion relation of HMMs for extraordinary photons $\implies$ ~~Conventional diffraction limit~~
- $\implies$ Bragg scattering of extraordinary photons $\implies$ Formation of a photonic band structure no matter how small L is.
- Therefore, photonic hyper-crystals exhibit light localization on deep subwavelength scales, which far exceeds the demonstrated mode volume reduction in photonic crystals.

# Motivation

- Goal: Proving that non-linearity time-like z-axis of hyper-crystal is relation is useful in:
- Optical limiting i.e. Limiting the intensity of light
- Optical computing

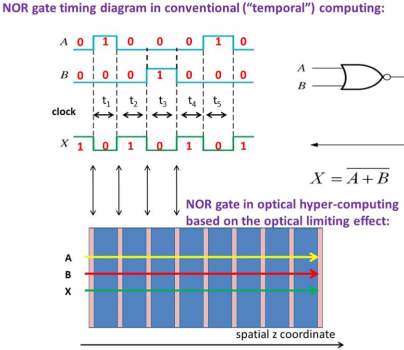



Figure: Mapping Temporal Computation to Optical

- Periodic resonator boundaries serve as a computer 'clock.'
- Optical 'hyper-computing' accelerates computation time.
- Optical limiting: material darkens with intense laser light to protect eyes and sensors.
- Optical computing replaces electrons with photons for faster, long-distance processing.

# Structure of our Photonic Hyper-crystal

- Artificial hyperbolic metamaterial is created using porous alumina filled with silver nanowires.
- Exhibits low-loss type I hyperbolic behavior in the visible range.
- Type I: $\varepsilon_x, \varepsilon_y > 0; \varepsilon_z < 0$ and Type II: $\varepsilon_x, \varepsilon_y < 0; \varepsilon_z > 0$
- Chosen material parameters for the hyperbolic metamaterial: $\varepsilon_x = \varepsilon_y = \varepsilon_1 = 5.28$ and $\varepsilon_z = \varepsilon_2 = -4.8$ (at 660nm).
- $\varepsilon_{xy} = \varepsilon_1$ throughout the crystal.
- $\sqrt{\varepsilon_1}$ = 2.3 nearly matches the refractive index of lithium niobate (LiNbO3), a thin dielectric layer between hyperbolic resonators.
- To counter losses in the silver-based hyperbolic metamaterial in the visible range, we consider using Rh800 dye-doped dielectric polymer as a gain medium
- One solution to mitigate losses is to use a 660 nm wavelength-saturated dye in porous alumina.
- Grating structures embedded within these layers for controlled nonlinear photon-photon interaction.

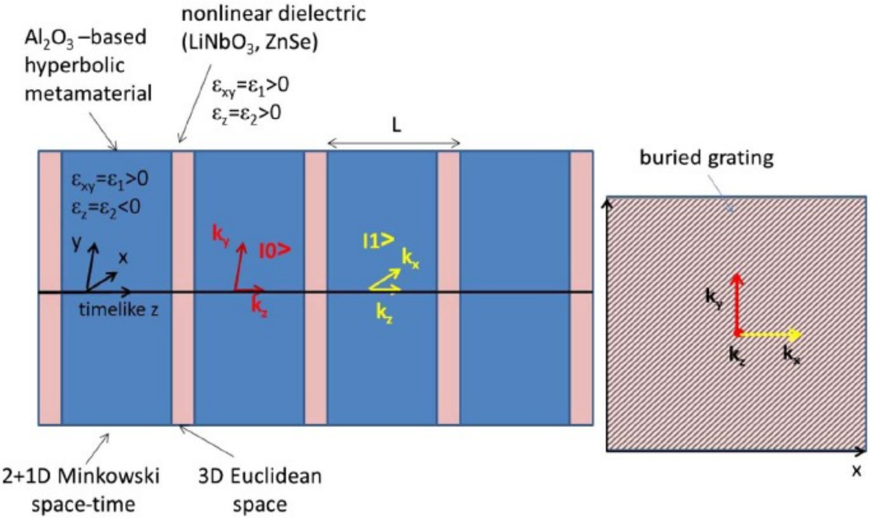


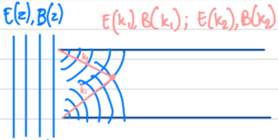

Figure: Photonic hyper-crystal with grating

# Suggested Improvement

- Natural hyperbolic materials, such as pure alumina, can be used in the long-wave infrared (LWIR) range.
- Pure alumina exhibits low-loss type I hyperbolic behavior in the 19.0–20.4 m and 23–25 m LWIR ranges.
- In the LWIR range, thin nonlinear dielectric layers can be made of ZnSe due to its high transparency.
- Mention of LWIR single-photon detectors suggests a specific application for this structure in the infrared range.

# TE and TM polarized modes

- Ordinary and extraordinary waves reduce to TE (Transverse Electric) and TM (Transverse Magnetic) polarized modes in a uniaxially symmetric medium.
- Ordinary waves have their Electric field perpendicular to the ray path while this is not the case for extraordinary waves.





$\vec{E}(k_1)+\vec{E}(k_2)$ add in the out-of-plane dirn.

$\vec{B}(k_1)+\vec{B}(k_2)$ add non-trivially and have a component along $\hat{k}$

$\boxed{\text{TE}}$ – ordinary

Viceversa $\boxed{\text{TM}}$ – extraordinary

- Macroscopic Maxwell Equation

$$\frac{\omega^2}{c^2}\vec{D}_\omega = \vec{\nabla}\times\vec{\nabla}\times\vec{E}_\omega$$

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(\mathbf{r}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ \frac{\partial}{\partial \mathrm{x}} & \frac{\partial}{\partial \mathrm{y}} & \frac{\partial}{\partial \mathrm{z}} \\ \frac{\partial}{\partial \mathrm{y}}E_z - \frac{\partial}{\partial \mathrm{z}}E_y & \frac{\partial}{\partial \mathrm{z}}E_x - \frac{\partial}{\partial \mathrm{x}}E_z & \frac{\partial}{\partial \mathrm{x}}E_y - \frac{\partial}{\partial \mathrm{y}}E_x \end{vmatrix}$$

Hence

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(\mathbf{r}) = {} & \left(\frac{\partial}{\partial \mathrm{y}}\left\{\frac{\partial}{\partial \mathrm{x}}E_y - \frac{\partial}{\partial \mathrm{y}}E_x\right\} - \frac{\partial}{\partial \mathrm{z}}\left\{\frac{\partial}{\partial \mathrm{z}}E_x - \frac{\partial}{\partial \mathrm{x}}E_z\right\}\right)\hat{\imath} \\ & - \left(\frac{\partial}{\partial \mathrm{x}}\left\{\frac{\partial}{\partial \mathrm{x}}E_y - \frac{\partial}{\partial \mathrm{y}}E_x\right\} - \frac{\partial}{\partial \mathrm{z}}\left\{\frac{\partial}{\partial \mathrm{y}}E_z - \frac{\partial}{\partial \mathrm{z}}E_y\right\}\right)\hat{\jmath} \\ & + \left(\frac{\partial}{\partial \mathrm{x}}\left\{\frac{\partial}{\partial \mathrm{z}}E_x - \frac{\partial}{\partial \mathrm{x}}E_z\right\} - \frac{\partial}{\partial \mathrm{y}}\left\{\frac{\partial}{\partial \mathrm{y}}E_z - \frac{\partial}{\partial \mathrm{z}}E_y\right\}\right)\hat{\mathrm{k}}\end{aligned}$$

So we have

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(\mathbf{r}) = {} & \left(\left\{\frac{\partial^2}{\partial \mathrm{yx}}E_y - \frac{\partial^2}{\partial \mathrm{yy}}E_x\right\} - \left\{\frac{\partial^2}{\partial \mathrm{zz}}E_x - \frac{\partial^2}{\partial \mathrm{zx}}E_z\right\}\right)\hat{\imath} \\ & - \left(\left\{\frac{\partial^2}{\partial \mathrm{xx}}E_y - \frac{\partial^2}{\partial \mathrm{xy}}E_x\right\} - \left\{\frac{\partial^2}{\partial \mathrm{zy}}E_z - \frac{\partial^2}{\partial \mathrm{zz}}E_y\right\}\right)\hat{\jmath} \\ & + \left(\left\{\frac{\partial^2}{\partial \mathrm{xz}}E_x - \frac{\partial^2}{\partial \mathrm{xx}}E_z\right\} - \left\{\frac{\partial^2}{\partial \mathrm{yy}}E_z - \frac{\partial^2}{\partial \mathrm{yz}}E_y\right\}\right)\hat{\mathrm{k}}\end{aligned}$$

# Linear Optical Properties of Photonic Hyper-crystals

- For extraordinary part: $E_x = 0$

  $\nabla \times \nabla \times \mathbf{E_w} =$

  $\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\mathbf{i} - \left(\frac{\partial^2 E_y}{\partial x^2} - \frac{\partial^2 E_z}{\partial y \partial z} + \frac{\partial^2 E_y}{\partial z^2}\right)\mathbf{j} + \left(-\frac{\partial^2 E_z}{\partial x^2} - \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_y}{\partial y \partial z}\right)\mathbf{k}$

- $\nabla \cdot \mathbf{D}_\omega = 0$

  $\mathbf{D}_\omega = \varepsilon_\omega \mathbf{E}_\omega$

  $\nabla \cdot \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \cdot \left(\varepsilon_1 E_y \mathbf{i} + \varepsilon_2 E_z \mathbf{k}\right) = 0$

  $\varepsilon_1 \frac{\partial E_y}{\partial y} + \varepsilon_2 \frac{\partial E_z}{\partial z} = 0$

  $\frac{\partial E_y}{\partial y} = -\frac{\varepsilon_2}{\varepsilon_1}\frac{\partial E_z}{\partial z}$

- Let's equate the z-component of $\nabla \times \nabla \times \mathbf{E_w} = \frac{\omega^2}{c^2} D_\omega$

$$-\frac{\partial^2 E_z}{\partial x^2} - \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_y}{\partial z \partial y} = \frac{\omega^2}{c^2} \varepsilon_2 E_z$$

- Substituting $\frac{\partial E_y}{\partial y} = -\frac{\varepsilon_2}{\varepsilon_1} \frac{\partial E_z}{\partial z}$

$$-\frac{\partial^2 E_z}{\partial x^2} - \frac{\partial^2 E_z}{\partial y^2} - \frac{\varepsilon_2}{\varepsilon_1} \frac{\partial^2 E_z}{\partial z^2} = \frac{\omega^2}{c^2} \varepsilon_2 E_z$$

$$E_z = \psi_\omega, \varepsilon_2 = -|\varepsilon_2|$$

$$-\frac{1}{\varepsilon_1} \frac{\partial^2 \psi_\omega}{\partial z^2} + \frac{1}{|\varepsilon_2|} \left( \frac{\partial^2 \psi_\omega}{\partial x^2} + \frac{\partial^2 \psi_\omega}{\partial y^2} \right) = \frac{\omega^2}{c^2} (\psi_\omega)$$

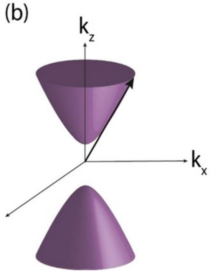
- Similar to 3D Klein-Gordon Equation describing a massive field propagating in a flat 2+1-dimensional Minkowski space-time

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2} \right) \Psi(t, x, y) = 0$$

# Linear Optical Properties of Photonic Hyper-crystals

- Dispersion law of extraordinary waves in uniaxial HMM

$$\frac{k_z^2}{\varepsilon_1} - \frac{k_x^2+k_y^2}{|\varepsilon_2|} = \frac{\omega^2}{c^2}$$




- Volume Between two such hyperboloids (corresponding to different values of frequency) is infinite: Infinite density of photonic states in the frequency bands of HMM

# Linear Optical Properties of Photonic Hyper-crystals

- Now we consider the field behavior near the $Al_2O_3/LiNbO_3$ interfaces
- $\varepsilon_{xy} = \varepsilon_1$ = 5.28 = constant

There is translational symmetry in x and y directions $\Longrightarrow$ ($k_x$, $k_y$) are good quantum number

- Propagating waves

$$E_\omega(\vec{r}) = E(z)\exp(ik_x x + ik_y y),$$

$$D_\omega(\vec{r}) = D(z)\exp(ik_x x + ik_y y),$$

$$B_\omega(\vec{r}) = B(z)\exp(ik_x x + ik_y y).$$

# Linear Optical Properties of Photonic Hyper-crystals

- z-component of the electric displacement of the TM wave
  $\psi(r) = D_z(r) = \varepsilon_z(z)E_z(r) = -\frac{c}{w}k_x B$
- For this case, $\nabla \cdot \mathbf{D} = 0$
  $\left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \cdot (\varepsilon_1 E_y \mathbf{j} + \varepsilon_z(z)E_z\mathbf{k}) = 0$
  $\varepsilon_1 \frac{\partial E_y}{\partial y} + \frac{\partial}{\partial z}(\varepsilon_z(z)E_z) = 0$
  $\frac{\partial E_y}{\partial y} = -\frac{1}{\varepsilon_1}\frac{\partial \psi}{\partial z}$
- Equating the z-component of $\nabla \times \nabla \times \mathbf{E_w} = \frac{\omega^2}{c^2}D_w$
  $-\frac{\partial^2 E_z}{\partial z^2} - \frac{\partial^2 E_z}{\partial y^2} - \frac{\partial^2 E_y}{\partial y \partial z} = \frac{\omega^2}{c^2}\varepsilon_z(z)E_z$
  $(k_x^2 + k_y^2)E_z - \frac{1}{\varepsilon_1}\frac{\partial^2 \psi}{\partial z^2} = \frac{\omega^2}{c^2}\psi$

# Linear Optical Properties of Photonic Hyper-crystals

$$-\frac{\partial^2 \psi}{\partial z^2} + \frac{\varepsilon_1}{\varepsilon_z(z)}(k_x^2 + k_y^2)\psi = \varepsilon_1 \frac{\omega^2}{c^2}\psi$$

- Let $k_x^2 + k_y^2 = k^2$
- Similar to Time Independent Schrodinger Equation, with a periodic potential $\frac{\varepsilon_1 k^2}{\varepsilon_z(z)}$
- A well-known problem in quantum mechanics which is solved using Bloch's Theorem

# Bloch's Theorem

## Theorem

*Bloch's Theorem tells us that for a differential equation*

$$-\frac{\partial^2\psi}{\partial z^2} + V(z)\psi = E\psi$$

*where* $V(z+L) = V(z)$ *, i.e. the potential is periodic, the solution can be given by*

$$\psi(z) = u(z)e^{ik_z z}$$

*where* $u(z)$ *is a periodic function with same period L, i.e.* $u(z+L) = u(z)$

- $\psi(z+L) = e^{ik_z z}\psi(z)$
- Born-von Karman Boundary Condition or continuous medium limit $\psi(z+NL) = \psi(z)$
  where N = total number of periods in the crystal

# Bloch's Theorem

- $u(z) = \sum_{m=0}^{\infty} \psi_m e^{i\frac{2\pi mz}{L}}$, a period function in terms of Fourier Series
- $\psi(z) = u(z)e^{ik_z z}$

  $\psi(z) = \sum_{m=0}^{\infty} \psi_m e^{i\frac{(k_z+2\pi m)z}{L}}$ where $k_z = \frac{2\pi n}{NL}$ for $n = 0, \pm 1, \ldots$
- $\psi(z+L) = e^{ik_z L}\psi(z)$ we need to solve only for region $0 \leq z \leq L$
- For large $|n|$, $|k_z z|$ will go beyond $\pi$ but we want $-\pi \leq k_z \leq \pi$ as solutions for $k_z$ will be redundant

  Since $0 \leq z \leq L \rightarrow -\frac{\pi}{L} \leq k_z \leq \frac{\pi}{L}$
  This region in the $k_z$ - space is called the first Brillouin zone
  $\psi(z) = \sum_{m=0}^{\infty} e^{i(kz+\frac{2\pi mz}{L})z}$ for $-\frac{\pi}{L} \leq k_z \leq \frac{\pi}{L}$

# Formation of Bandgaps

- Strong Bragg Scattering i.e. constructive interference will be observed for $k_z \approx \frac{\pi}{L}$
- This will lead to formation of photonics bandgaps in both wave number and the frequency domains

# Form of $\varepsilon_z(z)$

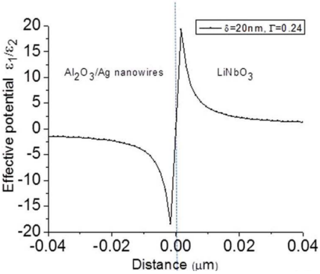
- We assume $\varepsilon_z(z)$ approach its extremes $\varepsilon_1$ and $\varepsilon_2$ at interfaces asymptotically
- Transition behavior of $\varepsilon_z(z)$ near one of the interfaces located at z=0 can be assumed as

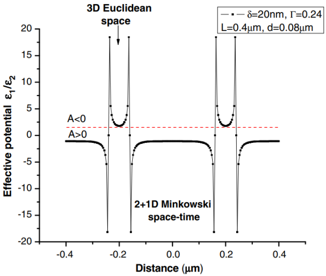
$$\varepsilon_Z(z) = \varepsilon_1\varepsilon_2 \frac{(1 - \exp(z/\delta))}{(\varepsilon_1 - \varepsilon_2 \exp(z/\delta))} + \frac{i\Gamma}{(1 - (\varepsilon_2/\varepsilon_1)\exp(z/\delta))},$$

# Form of $\varepsilon_z(z)$

$\Gamma = 0.24$, the imaginary part of $\varepsilon_2$

$\delta$ = transition layer length between $Al_2O_3$ and $LiNbO_3$




(a)

If we assume $\Gamma \ll 1$ and take only real part

$$\varepsilon_z = \frac{\varepsilon_1 \varepsilon_2 \left(1 - \exp\left(\frac{z}{\delta}\right)\right)}{\varepsilon_1 - \varepsilon_2 \exp\left(\frac{z}{\delta}\right)}$$

say $u = \exp\left(\frac{z}{\delta}\right)$

$$\frac{\varepsilon_1}{\varepsilon_z(u)} = \frac{\varepsilon_1 - \varepsilon_2 u}{\varepsilon_2(1-u)}$$

# Solving for $\psi$

- We had $-\frac{\partial^2\psi}{\partial z^2}+\frac{\varepsilon_1}{\varepsilon_z(z)}k^2\psi=\frac{\varepsilon_1\omega^2}{c^2}\psi$
- $u=e^{z/\delta}$ $\frac{\partial u}{\partial z}=\frac{u}{\delta}$
- $\frac{\partial^2\psi}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial\psi}{\partial z}\right)$

  $= \frac{\partial u}{\partial z}\frac{\partial}{\partial u}\left(\frac{\partial}{\partial z}u\frac{\partial}{\partial u}\psi\right)$

  $= \frac{u}{\delta}\frac{\partial}{\partial u}\left(\frac{u}{\delta}\frac{\partial}{\partial u}\psi\right)$

  $= \frac{u}{\delta^2}\left[u^2\frac{\partial^2\psi}{\partial u^2}+u\frac{\partial\psi}{\partial u}\right]$

  $\frac{\partial^2\psi}{\partial z^2}=\frac{1}{\delta^2}\left[u^2\frac{\partial^2\psi}{\partial u^2}+u\frac{\partial\psi}{\partial u}\right]$
- Now our differential equation becomes
  $u^2\frac{\partial^2\psi}{\partial u^2}+u\frac{\partial\psi}{\partial u}-\left[\frac{(\varepsilon_1-\varepsilon_2 u)k^2\delta^2}{\varepsilon_2(1-u)}-\varepsilon_1\frac{\omega^2}{c^2}\delta^2\right]\psi=0$
- Note: The equation given in the paper has a bit different form but we will proceed with ours as it will give results consistent with the paper later on. $u^2\frac{\partial^2\psi}{\partial u^2}+u\frac{\partial\psi}{\partial u}-\frac{Au+B}{u-1}\psi=0$

# Solving for $\psi$

where $A = (k^2 - \frac{\varepsilon_1 \omega^2}{c^2})\delta^2$ and $B = \left(\frac{\varepsilon_1 \omega^2}{c^2} - \frac{\varepsilon_1}{\varepsilon_2} k^2\right)\delta^2$

- This equation can be converted into the form $x(1-x)\frac{d^2y}{dx^2} + [c-(a+b+1)x]\frac{dy}{dx} - aby = 0$ with proper substitution
- Solutions: hypergeometric functions $y(x) = {}_2F_1(a,b,c,x)$
- Hypergeometric functions are a very general class of functions and many functions like trigonometric, exponential, Bessel, Jacobi, etc. functions can be expressed in terms of hypergeometric functions

$$\begin{aligned} {}_2F_1(a,b,c,x) &= 1 + \frac{ab}{1!c}x + \frac{a(a+1)b(b+1)}{2!c(c+1)}x^2 + \dots \\ &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{u^n}{n!} \end{aligned}$$

# Solving for $\psi$

- The solution of $\psi$ can be given as

$$\psi(u) = u^{i\sqrt{B}}{}_2F_1^*\left(-\sqrt{A}-i\sqrt{B}, \sqrt{A}-i\sqrt{B}, 1-2i\sqrt{B}, u\right) + ru^{-i\sqrt{B}}{}_2F_1^*\left(-\sqrt{A}+i\sqrt{B}, \sqrt{A}+i\sqrt{B}, 1+2i\sqrt{B}, u\right),$$

where r is the reflection coefficient

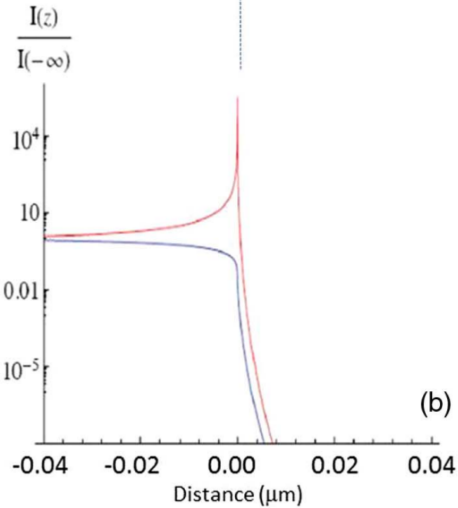
- At the interface i.e. z=0 or u=1, $\psi = 0$

$$r = -\frac{2F1^*(\sqrt{-A-i\sqrt{B}}, \sqrt{A-i\sqrt{B}}, 1-2i\sqrt{B}, 1)}{2F1^*(\sqrt{-A+i\sqrt{B}}, \sqrt{A+i\sqrt{B}}, 1+2i\sqrt{B}, 1)}$$

$$r = -\left[\frac{\Gamma(\sqrt{A}+i\sqrt{B})\Gamma(1+\sqrt{A}+i\sqrt{B})}{\Gamma(\sqrt{A}-i\sqrt{B})\Gamma(1+\sqrt{A}-i\sqrt{B})}\right]^* \frac{\Gamma\left(1+2i\sqrt{B}\right)^*}{\Gamma\left(1+2i\sqrt{B}\right)} \times \exp\left(-2\pi\sqrt{B}\right). \quad (13)$$

# Intensity at the interface $Al2O3/LiNbO3$

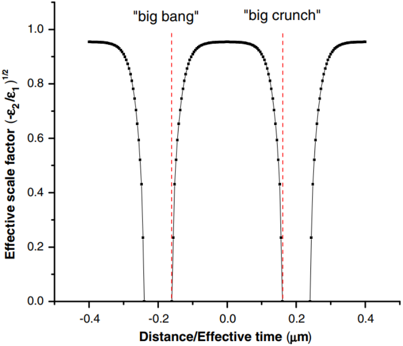
- In $\Gamma \ll 1$ limit, when A = 30, B = 40




- The divergence of the electric field: Lighting rod effect at tips of silver nanowires

# Optical Space-time

- Space-time interval is given as
  $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
- Effective 2+1 - dimensional "optical space-time" interval
  $ds^2 = -dz^2 + \left(-\frac{\varepsilon_2}{\varepsilon_1}\right)(dx^2 + dy^2)$
  $\left(-\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2}$ plays the role of a scale factor



# Activation of Grating

- At interfaces we observe big bang or big crunch
- The inflation-like behavior of the effective scale near interface $\Longrightarrow$ Extremely large field $\Longrightarrow$ Strong non-linearities at interface
- Strong non-linearities $\Longrightarrow$ The grating gets turned ON $\Longrightarrow$Can alter the state of signal photon

# Analog Gravity Study

- Hyperbolic metamaterials exhibit nonlinear optics analogous to gravity in 2+1-dimensional 'optical space-time.'
- Interface of hyperbolic metamaterials with conventional dielectric media resembles horizons in Minkowski space-time.
- Strong localized optical field divergences near these horizons promote robust nonlinear photon-photon interactions.
- At the interfaces let us take the translational symmetry in the x and y direction and the $k_x$ and $k_x$ can be a good set of quantum numbers.
- $\varepsilon$ has only z dependence, so we will consider the $\psi$ as the z component of the displacement vector.
- As you can see, the equation resembles the Schrodinger therefore the coefficient of the derivative acts as the analog of potential.

# Bloch Waves

- The solution of the equation, when assumed to be Bloch waves can satisfy the equation.
- kz in this case is defined between $-\pi/L$ and $\pi/L$ .
- Strong Bragg is scattering is observed at these $k_z$ approximately $\pi/L$ which is much greater than $\pi/\lambda$ leading to bandgaps in the wavenumber and frequency domains.

# Effective potential

- By using the resemblance of the equation with the 2+1D Minkowski space we can reach out to the final expression of the $\varepsilon_z$.
- Following the above expression we can reach out to the final expression of the effective potential where the $\varepsilon_z$ follow the above z dependence while $\varepsilon_1$ is constant given to us.
- Following the above expression we can have the following graphs:

# Final Expression

- If we substitute the expression of $\varepsilon_z$ into the Schrodinger equation and the limit we are talking of is the perfect loss compensation limit $\Gamma << 1$ we will obtain the following expression:
  For one case if we put the value of A and B along with limit of gamma we get the following graph for electric field :
- As apparent from the above graph we can see that for the extraordinary wave's electric field diverges at the interface.

# Effect of photon introduction on Effective potential

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# NONLINEAR OPTICAL INTERACTION

- Let us now consider how the introduction of photons into the individual hyperbolic resonators alters the effective potential $V = \frac{\varepsilon_1}{\varepsilon_2}$
- The diagonal components of the permittivity tensor of the wire-array metamaterial may be obtained as

$$\varepsilon_1 = \varepsilon_{x,y} = \frac{2\alpha\varepsilon_m\varepsilon_d + (1-\alpha)\varepsilon_d(\varepsilon_d + \varepsilon_m)}{(1-\alpha)(\varepsilon_d + \varepsilon_m) + 2\alpha\varepsilon_d} \approx \frac{1+\alpha}{1-\alpha}\varepsilon_d$$

$$\varepsilon_2 = \varepsilon_z = \alpha\varepsilon_m + (1-\alpha)\varepsilon_d,$$

  $\alpha$ is the volume fraction of the metallic phase, and $\varepsilon_m < 0$ and $\varepsilon_d > 0$ are the dielectric permittivities of the metal and dielectric component of the metamaterial, respectively
- We will assume that the nonlinear optical effects do not affect $\varepsilon_m$

# Main effect of nonlinearities

- Dielectric permittivities of Al2O3 and LiNbO3 will be influenced by the interfacial electric field divergence via the nonlinear Kerr effect as follows

$$n = n_0 + n_2 I.$$

- We may assume that near the interface located at $z = 0$

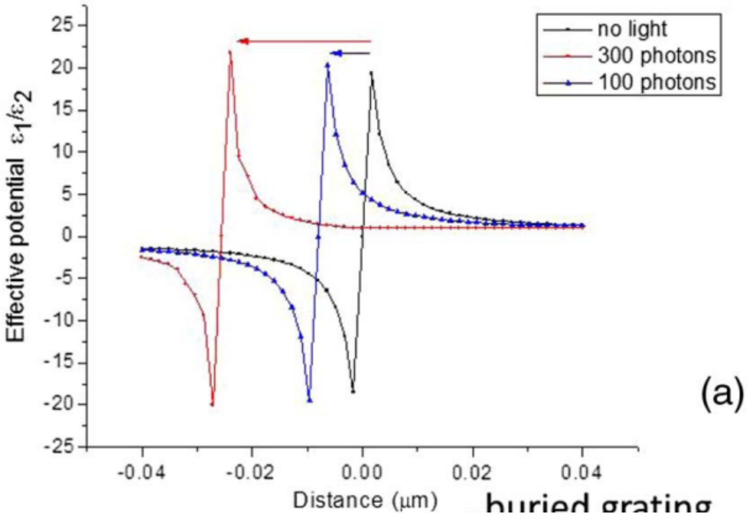
$$n_2(z) = \frac{(n_{2+} + n_{2-})}{2} - \frac{(n_{2+} - n_{2-})}{2} \frac{(1 - \exp(z/\delta))}{(1 + \exp(z/\delta))}$$
$$\approx \frac{n_{2+} \exp(z/\delta)}{(1 + \exp(z/\delta))}$$

# Shifting the position of the potential wall

- The effective potential in the presence of a photon field may be estimated as follows:

$$V(z) = \frac{\varepsilon_1}{\varepsilon_2} \approx \frac{\varepsilon_1^0 + 2\sqrt{\varepsilon_1^0} n_2(z) I_z}{\varepsilon_2^0 + 2\sqrt{\varepsilon_2^0} n_2(z) I_z} \approx \frac{\varepsilon_1^0}{\varepsilon_2^0 + 2\sqrt{\varepsilon_2^0} n_2(z) I_z}$$

- Equation makes clear that the main effect of nonlinearities consists of shifting the position of the potential wall with respect to $z = 0$


no light
300 photons
100 photons
Effective potential $\varepsilon_1/\varepsilon_2$
Distance (μm)
25
20
15
10
5
0
-5
-10
-15
-20
-25
-0.04
-0.02
0.00
0.02
0.04
(a)
buried grating

# Effect of positioning the potential wall

- In the bit encoding scheme based on the $k_{xy}$ direction, mode coupling properties of a buried grating may be altered between the "on" and "off " states via positioning the potential wall of the $V = \frac{\varepsilon_1}{\varepsilon_2}$ potential barrier either in front or behind the grating
- In the absence of control light, the 45° grating embedded into the nonlinear layer is exposed, and therefore it mixes the "0" and "1" states in equal proportions. Under the illumination with control light, the effective "potential walls" move beyond the grating so that the optical bit mixing is switched off.

# Effect of broadening of tunneling gaps

In a classical optical computing scheme, control light may be used to drastically change transmission of the photonic hyper-crystal via broadening of the tunneling gaps around the nonlinear dielectric layers

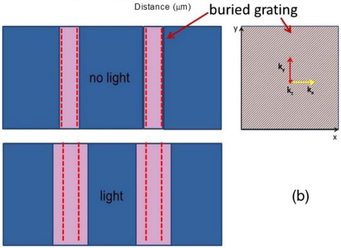



Figure: Caption for Image 1

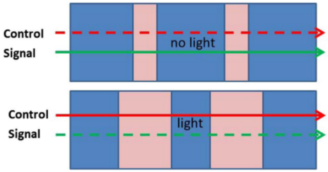



Figure: Caption for Image 2

In the optical limiting scheme, intense light drastically changes transmission of the photonic hyper-crystal via broadening of the effective tunnelling barriers around the nonlinear dielectric layers

# Single-photon level

An obvious way to increase $n_2$ by 2 to 3 orders of magnitude compared to LiNbO3 and to reach the single-photon level is to implement a photonic hyper-crystal cavity design and make use of the EIT effects in either atomic impurities or quantum dots as is typically done in conventional photonic crystal cavity geometries

Much higher levels of $n_2$ nonlinearity are achieved due to considerably smaller mode volumes in such nanocavities

Considerable photon–photon interaction may be observed at a single-photon level

# Use in the Optical Computing

- As discussed above that the kx and ky are encoded for the optical bits representation.
- The buried grating can switch off and on via the control light.For example: the grating when switch off is at 45 degree will mix half of both the state 0 and 1.
- With the control light the grating is on thus switching off the mixing.

# Uses in the Optical Computing

- Using the correspondence principle we can see the wave function of the extraordinary photon follow:
- In this $\psi_w$ represent the wavefunction of the photon and the effective mass being equal to $m^*$.

$$\left(-\frac{1}{\varepsilon_1}\frac{\partial^2}{\partial z^2}+\frac{1}{|\varepsilon_2|}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-\frac{m*^2c^2}{\hbar^2}\right)\varphi_\omega=0,$$

$$m* = \hbar\omega/c^2.$$

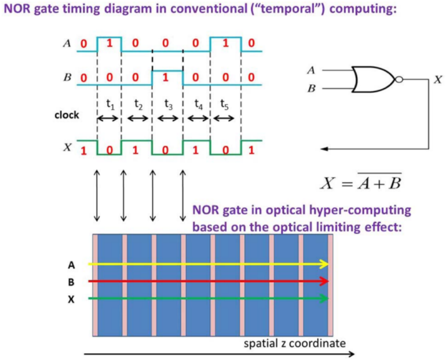
$$i\frac{\hbar c}{\varepsilon_1}\frac{\partial}{\partial z}\varphi_\omega=\hat{H}\varphi_\omega=\pm\left(m*c^2-\frac{\hbar^2}{|\varepsilon_2|2m*}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right)\varphi_\omega$$
$$=\pm\left(m*c^2+\frac{(\hat{p}_x^2+\hat{p}_y^2)}{|\varepsilon_2|2m*}\right)\varphi_\omega. \qquad \textbf{(22)}$$

- In the non relativistic limit the Kinetic Energy is much smaller than the effective rest energy $mc^2$, hence the equation reduces to standard schrodinger equation:
- However in this case the Hamiltonian Operator is given:

$$\hat{H} = i\frac{\hbar c}{\varepsilon_1}\frac{\partial}{\partial z}$$

- Now using the same analogy of the 2+1D Minkowski Space we can see the z coordinate to resemble with the time hence one spatial term can be dealt as temporal term.
- Using the above fact, we can say that since the time can be represented as a space therefore a certain length will show the time interval of a certain length.

- In Conventional computing when there is a time signal of length N we will wait for that much time that is there will be N steps however in the crystal of length N steps it will simultaneously show the clock signal of N steps simultaneously.
- Now the optical bits operation are done using the optical limiting effect where A and the B input are the control light and the X signal showing the output.



# Potential Limitations

- Large density of optical states in hyperbolic metamaterials implies reduced group velocity compared to the velocity of light in a vacuum, may impede the optical hypercomputing computation scheme.
- However $v_g$ approx. $0.1c_0$ $0.3c_0$, where $c_0$ is the velocity of light in a vacuum.
- Information propagation time between the resonators $L/v_g$ in the femtosecond range, which far exceeds the gigahertz (GHz) clock speed of both classical and quantum state-of-the-art computers.
- Balancing losses: Too much propagation loss will make the individual optical bits non-interacting, while too little loss may lead to undesirable back-scattering effects.
- The proper balance between these opposite effects may be found by properly choosing the gain and the length of the individual hyperbolic resonators in the photonic hyper-crystal structure.

# Conclusion

- Photonic hypercrystals exhibit light localization on deep subwavelength scales, leading to considerable enhancement of nonlinear photon–photon interaction.
- Nonlinear photonic hyper-crystals appear to be very promising in optical limiting and optical computing applications.
- Since the spatial coordinate plays the role of time in photonic hypercrystal, an optical hyper-computing scheme can accelerate computation time greatly.