CP468 AI – TERM PROJECT

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How to Run
To run this program, you must have python 3.7 or higher installed. https://www.python.org/downloads/

Next, install matplotlib using pip. https://pip.pypa.io/en/stable/quickstart/

Then, you can run either dejong_test.py, himmelblau_test.py, or rosenbrock_test.py to see the test's reults display.
This calls the SGA funtion in simple_genetic_algorithm.py and performs mating, reproduction and mutation operations
on as many generations of a population of specified sizes. The program will plot each generation's fittest member's fitness measure.
This graph can be saved as a .png file.

#Note
It is possible to compare 2 different bit mutation methods by alternating the commented out sections in the attempt_mutation() function.

Design Decisions
The only data structure which we used were Python Lists. Lists are dynamic arrays that take O(n) time to initialize, O(1) time to update values and
O(1) amortized time to append to the end of the list (only used for the plot).

As all lists are of fixed size, we could have used Numpy Arrays but we wanted to use vanilla Python as much as possible. The entire script is
written in vanilla Python except for plotting.

We used Python modules to make the code easier to follow. Each objective function is in its own module, along with constants that the SGA function
uses. Functions.py contains several helper functions that are not directly part of the SGA. The SGA function itself calls a different function for
each main step: Reproduction, crossover and mutation.

```
#simple_genetic_algorithm.py

from typing import List
import random
from functions import general_decoder
import matplotlib.pyplot as plt


LENGTH_OF_DECIMAL=4

def initialize(pop_size: int, alnum_set: List[str], var_string_length) -> List[str]:
    """
    Generates pop_size of random strings with characters of their alphanumeric character set.

    Args:
        pop_size (int): Number of strings to be generated
        alnum_set (List[str]): Valid characters to genrate random string from
        var_string_length (int): Number of characters expected to be in each string
    Returns:
        (List[str]): List of pop_size random strings made with characters of their alphanumeric
                     character set
    """

    return ["".join(random.choices(alnum_set, k=var_string_length)) for _ in range(pop_size)]


def perform_reproduction(population, inverse_fitness_func) -> List:
    """
    Takes in a population of strings and probabalistically candidates for mating based on the
    relative fitness of each string (i.e. the more fit members of the population will be picked more often
    and will therefore make up a bigger portion of the mating pool).

    Args:
        population (List[str]): List of strings that make up the mating pool
        inverse_fitness_func (<function>): Benchmark function with known global minima; returned values
                                           closer to zero mean the given arguments are
```

```
                                        closer to optimum
                                    Args:
                                        (str): Alphanumeric string
                                        representing number system
                                        value(s)
                                    Returns:
                                        (float): floating point
                                        value between 0 to 1;
                                        results closer to zero
                                                    are closer to
                                                    optimization
        Returns:
        (List[str]): List of strings that make up the new mating
pool generation
    """

        #compile a list of floating point values, numbers closer to 0 have
a higher fitness
        inverse_fitnesses = [inverse_fitness_func(s) for s in population]
        min_inv_fit = min(inverse_fitnesses)
        max_inv_fit =
max(inverse_fitnesses)

        #compile a list of inverse fitness values that are normalized
        to [0,1] with regard to
        #the range of the population's fitness measures, numbers closer
        to 0 have a higher fitness
        inverse_fitnesses = [(X-min_inv_fit)/(max_inv_fit-min_inv_fit+1)
        for X in
inverse_fitnesses]              #adding 1 to avoid division by zero

        #invert list of normalized inverse fitness values so that
        numbers closer to 1 have a higher fitness
        fitnesses = [1 / ((s)+1) for s in
inverse_fitnesses]

        #sum up all found fitness measures
        total_fitne
ss = sum(fitnesses)

        #compile a list of each fitness measure's proportion to
        the sum of all found fitness measures
        #of the mating pool; to be used as a probability that the condidate
        will mate
        probabilities_of_reproduction = [fit / total_fitness
for fit in fitnesses]

        #create list of randomly chosen strings that are weight biased
        return random.choices(population=population,
        weights=probabilities_of_reproduction, k=len(population))


    def
perform_mating(po
pulation):
    """
        Takes a list of strings, returns a list of strings after potential
mating operations.

        Args:
            population (List[str]): List of strings that make up the mating
            pool
```

```
        Returns:
            (List[str]): List of strings that make up the now
            potentially modified mating pool
        """

        new_pop = []

        while (len(population) > 0):
            s1 = population.pop(random.randint(0, len(population) - 1))
            s2 = population.pop(random.randint(0,
len(population) - 1))
            s1p, s2p = crossover_pair(s1, s2)
            new_pop.append(s1p)
            n
ew_pop.append
(s2p)
        return new_pop

    def
crossover_pa
ir(s1, s2):
        """
        Takes two strings for crossover, randomly determines a crossover
point
        between 1 and len(s1)-1 then performs crossover of character data
at crossover point.
        Assu
mes len(s1) =
len(s2)
        Args:
            s1 (str): string for mating crossover
            s2 (str): string for mating crossover
        Returns:
            s1p (str): string from mating crossover
        s2p (str): string from
mating crossover
        """
        crossover_point = random.randint(1, len(s1) - 2)
        s1p = s1[:crossover_point] + s2[crossover_point:]
        s2p = s2[:crossover_point] +
s1[crossover_point:]
        return (s1p, s2p)

    def perform_mutations(population,
probability_of_mutation, alnum_set):
        """
Takes a list of strings, returns a list of strings after potential
mutation operation.

        Args:
            population (List[str]): List of strings that make up the mating
            pool
            probability_of_mutation (float): decimal number between 0 and 1
            that represents
                                                the liklihood
                                                that a character will
                                                mutate into another
                                                character from
                                                its alphanumeric character
```

```
                                                  set
            alnum_set (List[str]): Valid alphanumeric characters to
            genrate number-system value-strings from
        Returns:
        (List[str]): List of strings that make up a potentially
mutated mating pool
        """

        return [attempt_mutation(s, probability_of_mutation, alnum_set)
for s in population]

    def attempt_mutation(s,
probability_of_mutation, alnum_set):
        """
        Takes a string and probabilisticly performs
character mutation
        Args:
            s (str): string to mutate
            probability_of_mutation (float): decimal number between 0
            and 1 that represents the liklihood
                                                  that a character will
                                                  mutate into another
                                                  character from
                                                  its alphanumeric character
                                                  set
            alnum_set (List[str]): Valid alphanumeric characters to
            genrate number-system value-strings from
        Returns:
            None
        """

        #bit flipping
        #on average, method 2 seems to
outperform method 1
        # #method 1
        # #choose one bit randomly if random chance falls into the
        probability

        that the string should mutate
        # if random.random() < probability_of_mutation:
        #     bit_to_flip = random.randint(0, len(s) - 1)
        #     s = list(s)
        #     s[bit_to_flip] = random.choice(alnum_set)
        #     return "".join(s)
        # else:
        #
        return s
        #method 2
        #bit by bit, perform mutation if random chance falls into the
        probability

        that the bit should mutate
        for i in range(len(s)):
            if random.random() < probability_of_mutation:
                s = s[:i] + str(alnum_set[random.randint(0, len(alnum_set)-
                1)]) + s[i +
                1:]
        r
```

```
eturns

# program starts here:
#--------------------------------------------------------------------------------------------------------------------------------------------------

def SGA(test_function, pop_size, alnum_set, var_string_length, variable_length, domain_min, domain_max,
    number_of_generations, probability_of_mutation):
    """
    Simple Genetic Algorithm that finds global minima of test functions through generational mating,
    reproduction and bit mutations while printing out each generation's performance results.

    Args:
        test_function (<function>): Benchmark function that has known optimum values (global min)
        pop_size (int): the number of strings to create for population
        alnum_set (List[str]): Valid alphanumeric characters to genrate number system value-strings from
        var_string_length (int): character length of string that contains one or more number system value-string string variables
        variable_length (int): character length of one number system value-string variable
        domain_min (Union[float,int]): min value of operational domain
        domain_max (Union[float,int]): max value of operational domain
        number_of_generations (int): number of times to mate / mutate the poulation in the attempt to hone in
                                     on the optimal input values for the given test_function
        probability_of_mutation (float): decimal number between 0 and 1 that represents the liklihood
                                         that a character will mutate into another character from
                                         its alphanumeric character set
    Prints:
        table: generational performances, avoiding repeat max performance levels between contiguous generations
    """

    #initialize a random population of pop_size values to be the starting point for optimization attempt
    #returns string with (var_string_length * pop_size) number of
```

```
    characters
    population = initialize(pop_size, alnum_set, var_string_length)

    #small anonymous function used to find fitness measures of a member of a mating pool population,
    #determined by the given benchmark test function
    inverse_fitness = lambda string: test_function(*general_decoder(string, variable_length, domain_min, domain_max, len(alnum_set)))


    #print perfromance of poulation
    #header
    print("\nTested population size: ", pop_size, " Number of generations: ", number_of_generations)
    pad = str((4 + LENGTH_OF_DECIMAL) * int(var_string_length / variable_length))
    print(("\n{:<16s}{:<" + pad + "s}\t{:}").format("Generation", "Strongest Candidate", "Fitness"))
    print("="*80)

    #print off generational performances, avoiding repeat performance levels between contiguous generations
    last_fit_individual = fittest_individual = [] #coordinate values
    last_max_fit = 0 #fitness value
    new_fit = True #is the found fitness value different than the last
    max_fitness_list = [] #list of all found fitness values to be used for a graph
    global_max_found = (0, [], 0) #to record the overall best found fitness measure form all generations
    first_gen_repeat = last_gen_repeat = 0 #to keep track of how many generations have had repeated max_fitness measures

    for i in range(number_of_generations):

        #create new generation of population
        population = perform_reproduction(population, inverse_fitness)
        population = perform_mating(population)
        population = perform_mutations(population, probability_of_mutation, alnum_set)

        #determine the max fitness measure of this generation
        max_fitness = max(1/(inverse_fitness(m)+1) for m in population)
        max_fitness_list.append(max_fitness)

        #determine the string variable values that have max_fitness of
            this generation
        for m in population:
            if 1/(inverse_fitness(m)+1) == max_fitness:
                fittest_individual = general_decoder(m, variable_length, domain_min, domain_max, len(alnum_set))

    #case: if this generation is the last, or it is more fit than its predecessor
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        if (i == number_of_generations - 1) or not (fittest_individual == last_fit_individual):
            #case: if is the new fittest member after repeated max peformance
            if (first_gen_repeat != last_gen_repeat) and (last_gen_repeat - first_gen_repeat > 1) :
                print(("\n\tFor generation {} to {}, the max fitness level was {:."+str(LENGTH_OF_DECIMAL)+"f}.\n").format(first_gen_repeat, last_gen_repeat, last_max_fit))

            #print generation's performance results
            print(("{:<16d}[{:<" + pad + "s}]\t {:>}").format(i, " ".join([("{:." + str(LENGTH_OF_DECIMAL) + "f},").format(x) for x in fittest_individual]), max_fitness))

            last_fit_individual = fittest_individual
            last_max_fit = max_fitness

            #record the overall best found fitness measure form all generations
            if max_fitness > global_max_found[2]:
                global_max_found = ("Gen: " + str(i), fittest_individual, max_fitness)

            first_gen_repeat = last_gen_repeat = i
            new_fit = True

        #case: first repeat of same fittest member between generations
        elif last_fit_individual == fittest_individual:
            new_fit = False
            last_gen_repeat = i

    print("="*80)
    print("Highest fitness acheived by:\n", global_max_found)
    print("="*80)
    print("")
    plt.plot(max_fitness_list)
    plt.show()
```

```
#functions.py
def num_in_interval(lo, hi, value, needed_increments):
    """
    Maps (normalizes) the given value from the domain of (0, (number_system_base ^ number_of_digits_of_value))
    to a value in the operational domain that coincides to the incremental position that the given
    value had in its original domain.

    Args:
        lo (Union[float,int]): min value of operational domain
        hi (Union[float,int]): max value of operational domain
        value (int): the value that is to be mapped to the operational domain
        needed_increments (int): how many possible values can be represented given the same
                                 number system, and number of digits, as the given value
    Returns:
        (float): value within the operational domain that coincides to the incremental position that
    the given value had in its original domain
    """

    #determine increment size that splits the operational domain into the number needed
    #increments of equal portion
    increment_size = (hi - lo) / needed_increments

    #return value within the operational domain that coincides to the incremental position that
    #the given value had in its original domain
    return lo + value * increment_size


def general_decoder(string, var_length, domain_min, domain_max, number_system_base):
    """
    Takes in binary string and splits it into several string variables of length var_length
    and returns a list of floating point decimal number values representing each within their
    operational domain.

    Args:
        string (str): alphanumeric string representing a number system value
        var_length (int): length of each string variable
        domain_min (Union[float,int]): min value of operational domain
        domain_max (Union[float,int]): max value of operational domain
        number_system_base (int): base of the string variable's utilized numbering system alphanumeric
                                  character set
    Returns:
        (List[float]): list of floating point decimal number values representing each of the string variables
```

```
    within their operational domain
(domain_min, domain_max)
    """

    #splits string into separate variables of var_length from given
string
    str_var_list = [string[i:i + var_length] for i in range(0,
len(string), var_length)]

    #convert each variable from original alphanumeric character set to
decimal
    dec_list = [(int(num, number_system_base)) for num
in str_var_list]

    #map each decimal to a floating point number in their
    operational domain (domain_min, domain_max)
    max_var_val = number_system_base ** var_length
    xs = [(num_in_interval(domain_min, domain_max, dec_list[i],
    max_var_val)) for i in
    range(len(dec_list))]
  return xs
```

```python
#dejong_test.py

from simple_genetic_algorithm import SGA

ALNUM = ["0", "1"]
VAR_STRING_LEN = 20
NUMBER_OF_VARIABLES = 4
VARIABLE_LEN = int(VAR_STRING_LEN / NUMBER_OF_VARIABLES)
PROBABILITY_OF_MUTATION = 0.05
POP_SIZE = 40
NUM_GENERATIONS = 1000
DOMAIN_MIN = -5.12
DOMAIN_MAX = 5.12

def dejong(*xs):
    """
    The function is defined on n-dimensional space.
    The function can be defined on any input domain but it is usually evaluated on x_i element of [-5.12, 5.12] for i = 1, ..., n.

    Takes in n dimensional coordinates and returns output of:
        f(x,y) = sum[b(x_i + 1 - (x_i)^2)^2 + (a - x_i)^2]]
    for i = 1, ..., n; and the parameters a and b are constants set to a = 1 and b = 100..

    The function has one global minimum f(x^*) = 0 at x^* = (0, ..., 0).

    Args:
        xs (List[num]): n dimensional coordinates for Euclidean (n + 1)-space
    Returns:
        (float): f(x,y) = value closer to 0 indicates a coordinate closer to know global minimum
    """

    return sum(xi ** 2 for xi in xs)

print("Running Simple Genetic Algorithm on De Jong Sphere benchmark function")
SGA(dejong, POP_SIZE, ALNUM, VAR_STRING_LEN, VARIABLE_LEN, DOMAIN_MIN, DOMAIN_MAX, NUM_GENERATIONS,
    PROBABILITY_OF_MUTATION)

print("\nDe Jong benchmark test complete\n")
```

```
#himmelblau_test.py

from simple genetic algorithm import SGA

ALNUM = ["0", "1"]
VAR_STRING_LEN = 16
NUMBER OF VARIABLES = 2
VARIABLE_LEN = int(VAR_STRING_LEN / NUMBER_OF_VARIABLES)
PROBABILITY_OF_MUTATION = 0.005
POP SIZE = 100
NUM GENERATIONS = 1000
DOMAIN_MIN = -6
DOMAIN_MAX = 6

def himmelblau(*xs):
    """
    The function is defined on the 2-dimensional space.
    The function can be defined on any input domain but it is usually evaluated on
    x_i element of [-6, 6] for i = 1, 2.

    Takes in x, y cartesian values and returns function output of:
        f(x,y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2

    The function has four local minima at:
        f(x^*) = 0 at x^* = (3, 2)
        f(x^*) = 0 at x^* = (-2.805118, 3.283186)
        f(x^*) = 0 at x^* = (-3.779310, -3.283186)
        f(x^*) = 0 at x^* = (3.584458, -1.848126)

    Args:
        xs (List[num]): [x,y] cartesian coordinates
    Returns:
        (float): f(x,y) = value closer to 0 indicates a coordinate closer to know
        global minimum
    """

    x = xs[0]
    y = xs[1]
    return (x ** 2 + y - 11) ** 2 + (x + y ** 2 - 7) ** 2

print("Running Simple Genetic Algorithm on Himmelblau benchmark function")
SGA(himmelblau, POP SIZE, ALNUM, VAR STRING LEN, VARIABLE LEN, DOMAIN MIN, DOMAIN MAX,
NUM GENERATIONS,
    PROBABILITY_OF_MUTATION)

print("\nHimmelblau benchmark test complete\n")
```

```python
#rosenbrock_test.py
from simple_genetic_algorithm import SGA

ALNUM = ["0", "1"]
VAR_STRING_LEN = 16
NUMBER_OF_VARIABLES = 2
VARIABLE_LEN = int(VAR_STRING_LEN / NUMBER_OF_VARIABLES)
PROBABILITY_OF_MUTATION = 0.01
POP_SIZE = 100
NUM_GENERATIONS = 1000
DOMAIN_MIN = -2
DOMAIN_MAX = 2


def rosenbrock(*xs):
    """
    The function is defined on n-dimensional space.
    The function can be defined on any input domain but it is usually evaluated on
    x_i element of [-5, 10] for i = 1, ..., n.

    Takes in n dimensional coordinates and returns output of:
        f(x,y) = sum[b(x_i + 1 - (x_i)^2)^2 + (a - x_i)^2]]
    for i = 1, ..., n; and the parameters a and b are constants set to a = 1 and b =
    100..

    The function has one global minimum f(x^*) = 0 at x^* = (1, ..., 1).

    Args:
        xs (List[num]): n dimensional coordinates for Euclidean (n + 1)-space
    Returns:
        (float): f(x,y) = value closer to 0 indicates a coordinate closer to know
        global minimum
    """

    x = xs[0]
    y = xs[1]

    return (1-x)**2+100*(y-x**2)**2


print("Running Simple Genetic Algorithm on Rosenbrock benchmark function")

SGA(rosenbrock, POP_SIZE, ALNUM, VAR_STRING_LEN, VARIABLE_LEN, DOMAIN_MIN, DOMAIN_MAX,
NUM_GENERATIONS,
    PROBABILITY_OF_MUTATION)

print("\nRosenbrock benchmark test complete\n")
```

Benchmark Test Results

Comparing 2 methods of character mutation

Method 1
Chooses one bit randomly if random chance falls into the probability that the string should mutate,
if random.random() < probability_of_mutation.

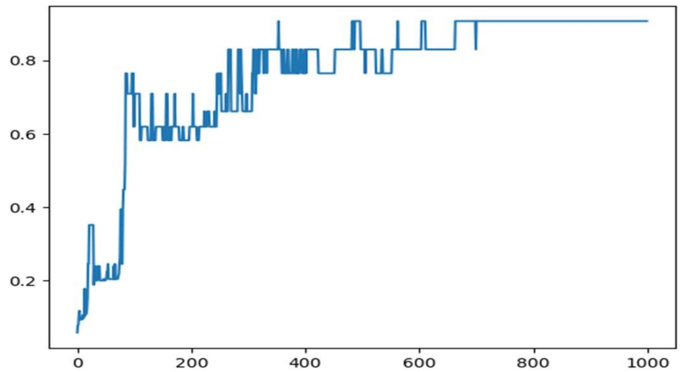
Method 2
Bit by bit, perform mutation if random chance falls into the probability that the bit should mutate,
if random.random() < probability_of_mutation.

All graphs are plotted as Fitness Measures (Y-axis [0, 1]) against Generations (X-axis [0, 1000]).

Several tests have shown that method 1 is capable of occasionally outperforming method 2, method 2 is far more consistent and is therefore more reliable.

<u>De Jong</u>

<u>Method 1</u>

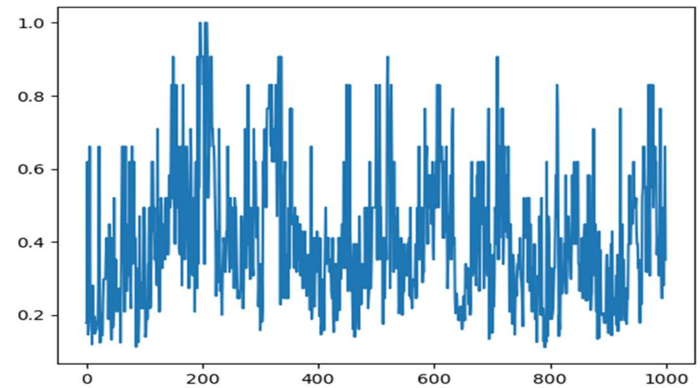


```
Running Simple Genetic Algorithm on De Jong Sphere benchmark function

Tested population size:  40  Number of generations:  1000

Generation      Strongest Candidate                         Fitness
=================================================================================
0               [-1.9200, 1.9200, -1.9200, 2.2400,]         0.05855898060526562
1               [-1.6000, 1.6000, 2.5600, -0.3200,]         0.07827175954915469
2               [-1.9200, 1.9200, -1.9200, 0.6400,]         0.08020017964840241
3               [-0.3200, -1.9200, -1.9200, 0.6400,]        0.11255177381595535
4               [-0.3200, -1.9200, -1.9200, 0.0000,]        0.11799131583915423
5               [-1.6000, -1.6000, -1.9200, 0.9600,]        0.09321401938851603
6               [-1.2800, 1.9200, -1.9200, 0.0000,]         0.09988812529966437
7               [-1.2800, 2.2400, -1.6000, -0.3200,]        0.0969142502713599
9               [-1.2800, 0.6400, 2.2400, -1.6000,]         0.09411233248004816
10              [-0.6400, 0.6400, 2.2400, -1.6000,]         0.10641920653839604
12              [-2.5600, 0.6400, 0.6400, 1.2800,]          0.09988812529966438
13              [-1.2800, 0.9600, -0.6400, 1.2800,]         0.17831669044222537
14              [-1.2800, 0.9600, 0.9600, -2.2400,]         0.10527202290719216
15              [-0.6400, 1.6000, -1.6000, 1.6000,]         0.11001584228128854
17              [-0.6400, 0.9600, 0.9600, -2.2400,]         0.12091313600309538
18              [-1.2800, 0.9600, 0.6400, 1.6000,]          0.15314873805439846
19              [-1.2800, 0.9600, 0.6400, 0.3200,]          0.24557956777996065
21              [-0.6400, 0.9600, 0.6400, 0.3200,]          0.35171637591446264

        For generation 21 to 28, the max fitness level was 0.3517.

29              [-0.6400, 0.9600, 0.6400, 1.6000,]          0.18865076969514044
```

…

```
552             [-0.3200, 0.0000, 0.3200, 0.0000,]          0.8300132802124832

        For generation 552 to 561, the max fitness level was 0.8300.

562             [-0.3200, 0.0000, 0.0000, 0.0000,]          0.9071117561683597
563             [-0.3200, 0.0000, 0.3200, 0.0000,]          0.8300132802124832

        For generation 563 to 603, the max fitness level was 0.8300.

604             [-0.3200, 0.0000, 0.0000, 0.0000,]          0.9071117561683597

        For generation 604 to 610, the max fitness level was 0.9071.

611             [-0.3200, 0.0000, 0.3200, 0.0000,]          0.8300132802124832

        For generation 611 to 662, the max fitness level was 0.8300.

663             [-0.3200, 0.0000, 0.0000, 0.0000,]          0.9071117561683597

        For generation 663 to 698, the max fitness level was 0.9071.

699             [-0.3200, 0.0000, 0.3200, 0.0000,]          0.8300132802124832
700             [-0.3200, 0.0000, 0.0000, 0.0000,]          0.9071117561683597

        For generation 700 to 998, the max fitness level was 0.9071.

999             [-0.3200, 0.0000, 0.0000, 0.0000,]          0.9071117561683597
=================================================================================
Highest fitness acheived by:
 ('Gen: 353', [-0.3200000000000003, 0.0, 0.0, 0.0], 0.9071117561683597)
=================================================================================
```

<u>Method 2</u>



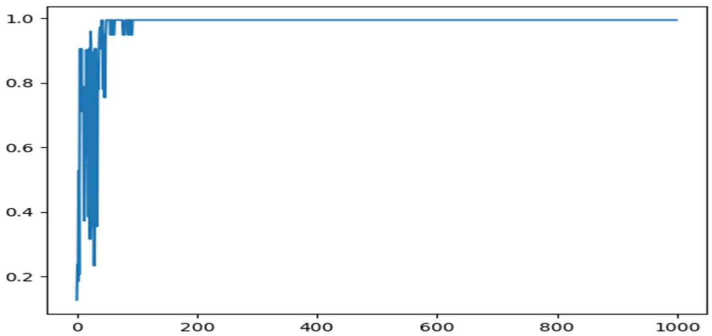
```
Running Simple Genetic Algorithm on De Jong Sphere benchmark function

Tested population size:  40  Number of generations:  1000

Generation      Strongest Candidate                         Fitness
=================================================================================
0               [0.6400, -1.2800, 0.0000, 1.6000,]          0.17831669044222542
1               [1.2800, -0.6400, 0.0000, 1.6000,]          0.17831669044222542
2               [0.6400, -0.3200, 0.0000, 0.3200,]          0.6194251734390486
3               [0.6400, -1.2800, 1.9200, 0.3200,]          0.146267259536253
4               [-0.6400, -0.3200, -0.9600, 1.9200,]        0.16339869281045755
5               [1.2800, 0.6400, 0.0000, 0.9600, ]          0.25191455058444173
6               [0.0000, -0.6400, 0.0000, 0.3200,]          0.6613756613756615
7               [0.0000, -0.6400, 1.9200, 0.3200,]          0.19236688211757466
8               [0.0000, -1.9200, -0.6400, -0.6400,]        0.18163324614937523
9               [0.0000, -1.9200, -0.6400, -0.3200,]        0.19236688211757466
10              [0.0000, -2.5600, 0.6400, -0.6400,]         0.11943435887636158
11              [0.6400, 0.6400, -1.2800, 0.3200,]          0.2808988764044944
12              [0.0000, -1.9200, 0.6400, -0.9600,]         0.1661792076575379
13              [0.9600, -1.9200, 0.0000, -0.9600,]         0.15314873805439844
14              [0.0000, -1.9200, 0.0000, -0.6400,]         0.19623233908948198
15              [1.2800, -1.9200, 0.0000, -0.6400,]         0.14849132810643859
16              [0.6400, -1.9200, 0.0000, -0.6400,]         0.18163324614937523
17              [1.2800, -1.2800, 1.2800, -0.6400,]         0.15810776625347833
18              [1.2800, -1.2800, 1.2800, -0.3200,]         0.16617920765753783
19              [0.6400, -1.9200, -0.6400, -0.3200,]        0.17831669044222542
20              [-0.6400, 0.6400, -1.2800, 1.2800,]         0.19623233908948196
21              [-0.6400, -1.2800, 0.0000, -0.6400,]        0.28921795465062466
```

…

```
966             [0.0000, -0.6400, 0.3200, -1.2800,]         0.3174200101574403
967             [0.0000, -0.3200, 0.6400, -0.6400,]         0.5203996669442134
968             [1.2800, -0.6400, 0.0000, -0.3200,]         0.31742001015744026
969             [0.0000, -0.3200, 0.0000, -0.6400,]         0.6613756613756615
970             [0.0000, -0.3200, 0.0000, -0.3200,]         0.8300132802124832
971             [0.6400, -0.3200, 0.3200, -0.3200,]         0.5824790307548928
972             [1.2800, -0.6400, 0.3200, -0.3200,]         0.307427447122479
973             [0.0000, 0.0000, 0.3200, -0.3200,]          0.8300132802124832
974             [0.0000, -0.3200, 0.0000, -0.3200,]         0.8300132802124832
975             [0.0000, -0.6400, 0.0000, -0.6400,]         0.5496921723834656
976             [0.0000, -0.3200, 0.6400, -0.3200,]         0.6194251734390486
977             [0.0000, -0.3200, 0.0000, -0.3200,]         0.8300132802124832
978             [0.6400, -0.3200, 0.6400, -0.3200,]         0.4940711462450594
979             [0.0000, -0.3200, 0.3200, -0.6400,]         0.6194251734390486
981             [0.0000, -0.3200, 0.0000, -0.6400,]         0.6613756613756615
982             [0.0000, -0.3200, 0.3200, -0.6400,]         0.6194251734390486
984             [0.0000, -0.9600, 0.6400, -0.6400,]         0.3648569760653825
985             [0.0000, -0.3200, 0.9600, -0.6400,]         0.41091387245233 4
986             [0.0000, -1.2800, 0.0000, -0.3200,]         0.36485697606538225
988             [0.0000, -0.9600, 0.9600, -0.6400,]         0.3074274471224791
989             [0.6400, 0.6400, 0.0000, -0.6400,]          0.4486719310839916
990             [0.0000, -0.3200, 0.3200, -0.3200,]         0.7649938800489593
991             [1.2800, -0.3200, 0.0000, -0.6400,]         0.31742001015744026
993             [1.2800, -0.3200, 0.6400, -0.9600,]         0.24557956777996065
994             [0.3200, -0.3200, 0.6400, -0.6400,]         0.4940711462450594
995             [0.0000, -0.3200, 0.6400, -0.9600,]         0.410913872452334
996             [1.2800, -0.6400, 0.3200, -0.6400,]         0.28089887640449435
997             [0.0000, -0.6400, 0.0000, -0.9600,]         0.42896362388469467
998             [0.0000, -0.6400, 0.0000, -0.3200,]         0.6613756613756615
999             [0.6400, -0.3200, 0.6400, -0.9600,]         0.35171637591446264
=================================================================================
Highest fitness acheived by:
 ('Gen: 196', [0.0, 0.0, 0.0, 0.0], 1.0)
=================================================================================


De Jong benchmark test complete
```

Himmelblau

Method 1

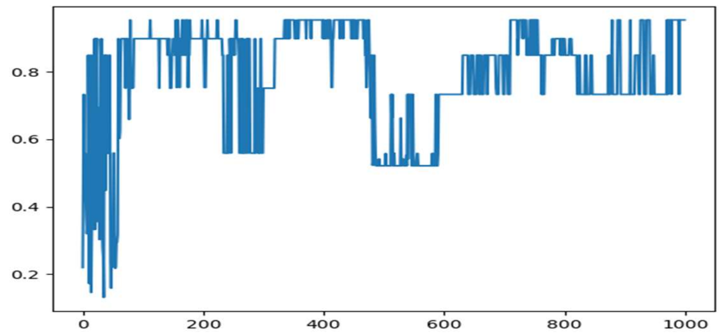



```
Running Simple Genetic Algorithm on Himmelblau benchmark function

Tested population size:  100  Number of generations:  1000

Generation      Strongest Candidate      Fitness
==================================================================
0               [3.4219, -0.9844,]       0.12685092257472316
1               [3.4219, -2.1562,]       0.23576390181437498
2               [-2.4375, 3.0000,]       0.18418870798658832
3               [3.0469, 1.7344, ]       0.5281303795684011
4               [-2.4375, 3.0938,]       0.2049805991176992
5               [-2.7656, 3.0938,]       0.9057085681453921
7               [-2.7656, 3.0469,]       0.7546138259377453
8               [-2.7656, 3.0938,]       0.9057085681453921
9               [-2.9062, 3.0938,]       0.7123313727807917
10              [3.5625, -1.9688,]       0.7879095273411465
12              [3.5625, -2.1562,]       0.3724650854831554
14              [3.5625, -2.0625,]       0.5542531418616059
15              [3.5625, -2.0156,]       0.6684277361309007
16              [3.6094, -1.7812,]       0.9024394166606019
17              [3.5625, -2.0156,]       0.6684277361309007
19              [3.6094, -1.7812,]       0.9024394166606019
20              [3.6094, -2.1562,]       0.38445216363670154
21              [3.6094, -1.9219,]       0.9066419898073936
22              [3.6094, -2.2031,]       0.31530059654322834
23              [3.6094, -1.8281,]       0.9595585892857286
24              [3.6094, -1.7812,]       0.9024394166606019
25              [3.6094, -2.0156,]       0.6887587173430882
26              [3.5625, -1.4531,]       0.35525716342591057
```

…

```
57              [-2.8125, 3.1406,]       0.9948443921995289

        For generation 57 to 59, the max fitness level was 0.9948.

60              [-2.7656, 3.1406,]       0.9487880473133589

        For generation 60 to 62, the max fitness level was 0.9488.

63              [-2.8125, 3.1406,]       0.9948443921995289

        For generation 63 to 76, the max fitness level was 0.9948.

77              [-2.7656, 3.1406,]       0.9487880473133589

        For generation 77 to 80, the max fitness level was 0.9488.

81              [-2.8125, 3.1406,]       0.9948443921995289

        For generation 81 to 85, the max fitness level was 0.9948.

86              [-2.7656, 3.1406,]       0.9487880473133589
87              [-2.8125, 3.1406,]       0.9948443921995289
89              [-2.7656, 3.1406,]       0.9487880473133589
90              [-2.8125, 3.1406,]       0.9948443921995289
91              [-2.7656, 3.1406,]       0.9487880473133589
93              [-2.8125, 3.1406,]       0.9948443921995289

        For generation 93 to 998, the max fitness level was 0.9948.

999             [-2.8125, 3.1406,]       0.9948443921995289
==================================================================
Highest fitness acheived by:
 ('Gen: 42', [-2.8125, 3.140625], 0.9948443921995289)
==================================================================


Himmelblau benchmark test complete
```

Method 2



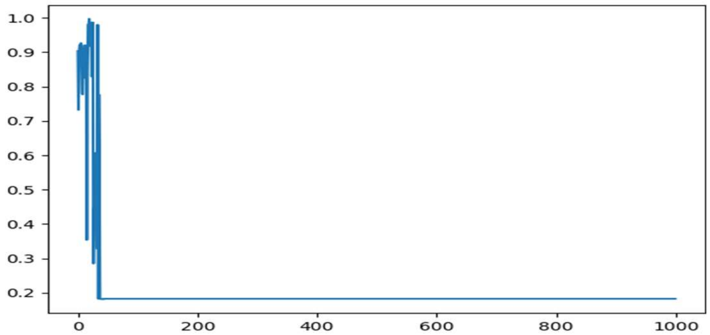

```
Running Simple Genetic Algorithm on Himmelblau benchmark function

Tested population size:  100  Number of generations:  1000

Generation      Strongest Candidate      Fitness
==================================================================
0               [3.1875, 2.2500, ]       0.21972996442665219
1               [-3.6562, -3.1875,]      0.521534039504397
2               [-3.7500, -3.1875,]      0.7333848099282685
3               [-3.7500, -3.0938,]      0.41835122118981877
4               [-3.7500, -3.1406,]      0.5580541659818725
5               [-2.7656, 2.9531,]       0.44446231772630235
6               [3.0000, 2.2969, ]       0.3682727405489496
7               [3.1406, 2.2031, ]       0.3198419604143811
8               [3.2344, 1.9219, ]       0.3426966317380834
9               [-3.7500, -3.3281,]      0.8495425659298472
10              [3.0000, 2.2031, ]       0.5649189289312718
11              [-3.0469, 2.8594,]       0.17223202916858715
12              [-2.8125, 2.9531,]       0.452451961679104
13              [-3.7500, -3.3281,]      0.8495425659298472
14              [2.5312, 2.2031, ]       0.1458012412004613
15              [3.7031, -1.5469,]       0.3147154801968908
16              [3.6562, -2.1094,]       0.43676270558425573
17              [-3.7500, -3.3281,]      0.8495425659298472
18              [3.4688, -2.1094,]       0.3329600109200428
19              [-3.7500, -3.3281,]      0.8495425659298472
20              [-3.7500, -3.2344,]      0.898507261095179
21              [3.4688, -2.1094,]       0.3329600109200428
```

…

```
        For generation 936 to 941, the max fitness level was 0.9541.

942             [-3.7500, -3.1875,]      0.7333848099282685

        For generation 942 to 946, the max fitness level was 0.7334.

947             [-3.7500, -3.3281,]      0.8495425659298472
949             [-3.7500, -3.1875,]      0.7333848099282685

        For generation 949 to 969, the max fitness level was 0.7334.

970             [-3.7500, -3.2812,]      0.9540826957320063
971             [-3.7500, -3.1875,]      0.7333848099282685
972             [-3.7500, -3.2812,]      0.9540826957320063

        For generation 972 to 976, the max fitness level was 0.9541.

977             [-3.7500, -3.1875,]      0.7333848099282685
978             [-3.7500, -3.2812,]      0.9540826957320063

        For generation 978 to 989, the max fitness level was 0.9541.

990             [-3.7500, -3.1875,]      0.7333848099282685
991             [-3.7500, -3.2812,]      0.9540826957320063

        For generation 991 to 998, the max fitness level was 0.9541.

999             [-3.7500, -3.2812,]      0.9540826957320063
==================================================================
Highest fitness acheived by:
 ('Gen: 79', [-3.75, -3.28125], 0.9540826957320063)
==================================================================


Himmelblau benchmark test complete
```

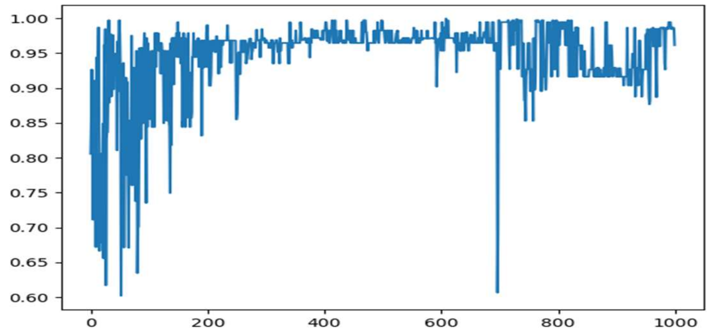
Rosenbrock

Method 1




```
Running Simple Genetic Algorithm on Rosenbrock benchmark function

Tested population size:  100  Number of generations:  1000

Generation      Strongest Candidate       Fitness
==========================================================================
0               [1.2031, 1.4219, ]        0.9033636766002906
1               [0.9531, 0.9688, ]        0.7321505844648348
2               [0.6719, 0.4531, ]        0.9025612245710402
3               [1.2500, 1.5781, ]        0.9200359389038635
4               [1.2500, 1.5469, ]        0.9200359389038635
5               [0.7188, 0.5156, ]        0.9266149414114279
6               [0.7500, 0.5781, ]        0.9200359389038635
7               [0.7500, 0.5156, ]        0.7798933739527799
8               [0.9219, 0.7969, ]        0.7771361403442459
9               [0.9844, 0.9375, ]        0.9095603828204
10              [0.7969, 0.5938, ]        0.8254253515527755
11              [0.7500, 0.5781, ]        0.9200359389038635
12              [0.7500, 0.5469, ]        0.9200359389038635
14              [0.9375, 0.9219, ]        0.8413701021927797
15              [0.7344, 0.6719, ]        0.35360758557227123
16              [0.9375, 0.9219, ]        0.8413701021927797
17              [0.9531, 0.9219, ]        0.9801733757561382
19              [0.9531, 0.9062, ]        0.9973271002925899
20              [0.8906, 0.7656, ]        0.9190567751421268
21              [0.9531, 0.9219, ]        0.9801733757561382
22              [0.8906, 0.7969, ]        0.9868706773126782
23              [0.5469, 0.2969, ]        0.8293214471223307
24              [0.8750, 0.7969, ]        0.8982456140350877
25              [0.8906, 0.7969, ]        0.9868706773126782
26              [0.9531, 0.7500, ]        0.2846771771491518
27              [0.9531, 0.7969, ]        0.44503103266984945
29              [0.9531, 0.7812, ]        0.38166311708082545
30              [0.9531, 0.8281, ]        0.6070304195138754
32              [0.9531, 0.7656, ]        0.328729236951222
33              [0.9531, 0.9219, ]        0.9801733757561382
34              [-1.1094, 1.2500,]        0.18226017435828668
35              [0.9219, 0.7969, ]        0.7771361403442459
36              [0.9219, 0.7656, ]        0.5829041895321386
37              [-1.1094, 1.2500,]        0.18226017435828668
38              [-1.1250, 1.2656,]        0.1813031161473088

        For generation 38 to 42, the max fitness level was 0.1813.

43              [-1.1094, 1.2500,]        0.18226017435828668
44              [-1.1250, 1.2656,]        0.1813031161473088
45              [-1.1094, 1.2500,]        0.18226017435828668

        For generation 45 to 998, the max fitness level was 0.1823.

999             [-1.1094, 1.2500,]        0.18226017435828668
==========================================================================
Highest fitness acheived by:
 ('Gen: 19', [0.953125, 0.90625], 0.9973271002925899)
==========================================================================


Rosenbrock benchmark test complete
```

Method 2




```
Running Simple Genetic Algorithm on Rosenbrock benchmark function

Tested population size:  100  Number of generations:  1000

Generation      Strongest Candidate       Fitness
==========================================================================
0               [0.5781, 0.3594, ]        0.8056634380357375
1               [0.7188, 0.5469, ]        0.8541535654355582
2               [0.7188, 0.5156, ]        0.9266149414114279
3               [0.6094, 0.3906, ]        0.8404864689282009
4               [1.1250, 1.2031, ]        0.7111111111111111
5               [0.6250, 0.3438, ]        0.7351040918880115
6               [1.2969, 1.6719, ]        0.9106188394120442

        For generation 6 to 8, the max fitness level was 0.9106.

9               [0.3750, 0.1719, ]        0.6719160104986877
10              [1.0938, 1.2188, ]        0.9440745049032495
11              [0.8750, 0.7969, ]        0.8982456140350877
12              [0.8906, 0.7656, ]        0.9190567751421268
13              [0.8906, 0.7969, ]        0.9868706773126782
15              [0.4219, 0.2188, ]        0.6664623251239198
16              [0.5312, 0.2969, ]        0.8056821639430923

        For generation 16 to 18, the max fitness level was 0.8057.

19              [0.3125, 0.0938, ]        0.6783422349190577
20              [0.5000, 0.2344, ]        0.7846743295019157
```

...

```
969             [0.8750, 0.7656, ]        0.9846153846153847
971             [0.9375, 0.8594, ]        0.9596438821531073
972             [0.8750, 0.7656, ]        0.9846153846153847
974             [0.9375, 0.8594, ]        0.9596438821531073
975             [0.9062, 0.8281, ]        0.9867167533019916

        For generation 975 to 979, the max fitness level was 0.9867.

980             [0.8750, 0.7656, ]        0.9846153846153847
981             [0.9062, 0.8281, ]        0.9867167533019916
982             [0.9062, 0.7969, ]        0.9359845468859904
983             [0.7188, 0.5156, ]        0.9266149414114279
984             [0.8750, 0.7656, ]        0.9846153846153847
986             [0.8906, 0.7969, ]        0.9868706773126782
987             [0.8750, 0.7656, ]        0.9846153846153847
989             [0.9375, 0.8750, ]        0.9945972196928307

        For generation 989 to 992, the max fitness level was 0.9946.

993             [0.8750, 0.7656, ]        0.9846153846153847

        For generation 993 to 995, the max fitness level was 0.9846.

996             [0.8906, 0.7969, ]        0.9868706773126782
998             [0.8750, 0.7656, ]        0.9846153846153847
999             [0.8750, 0.7500, ]        0.9615023474178404
==========================================================================
Highest fitness acheived by:
 ('Gen: 608', [0.984375, 0.96875], 0.9997499614454374)
==========================================================================


Rosenbrock benchmark test complete
```