Chapter - 2 :- Discrete-Time Signals and Systems

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Problems :-

1, A discrete time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3} & , -3 \le n \le -1 \\ 1 & , 0 \le n \le 3 \\ 0 & , \text{elsewhere} \end{cases}$$

a) Determine the values and sketch the signal $x[n]$.

Sol:- for $n=-3 \Rightarrow x[n] = 1+\frac{(-3)}{3} = 1-1 = 0$

$n=-2 \Rightarrow x[n] = 1+\frac{(-2)}{3} = 1-\frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$

$n=-1 \Rightarrow x[n] = 1+\frac{(-1)}{3} = 1-\frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

$n=0 \Rightarrow x[n]=1$

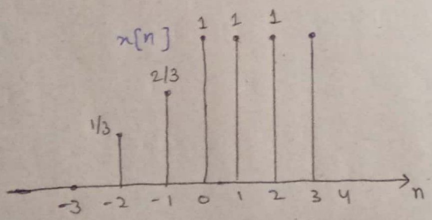
$n=1 \Rightarrow x[n]=1$

$n=2 \Rightarrow x[n]=1$

$n=3 \Rightarrow x[n]=1$

$x[n]=0$ for elsewhere

$$x[n] = \{\ldots 0, \tfrac{1}{3}, \tfrac{2}{3}, 1, 1, 1, 1, 0 \ldots\}$$
$\uparrow$ $n=0$



b) Sketch the signals that result if we:

1. First fold $x[n]$ and then delay the resulting signal by four samples.

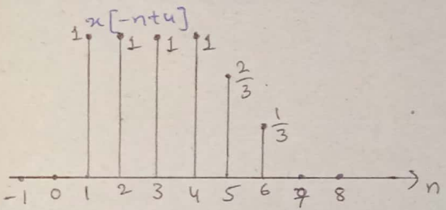
2. First delay $x[n]$ by four samples and then fold the resulting signal.

Sol:-

1. $x[-n] = \{\ldots 0, 1, 1, 1, 1, \tfrac{2}{3}, \tfrac{1}{3}, 0, \ldots\}$ ($\uparrow$)

delaying the folded signal by four samples

$x[-(n-4)] = x[-n+4] = \{\ldots 0, 0, 1, 1, 1, 1, \tfrac{2}{3}, \tfrac{1}{3}, 0, \ldots\}$ ($\uparrow$)



2. delaying $x[n]$ by four samples

$x[n-4] = \{\ldots 0, 0, \tfrac{1}{3}, \tfrac{2}{3}, 1, 1, 1, 1, 0, \ldots\}$ ($\uparrow$)

folding $x[n-4] \Rightarrow x[-n-4]$

$x[-n-4] = \{\ldots 0, 1, 1, 1, 1, \tfrac{2}{3}, \tfrac{1}{3}, 0, 0, \ldots\}$ ($\uparrow$)

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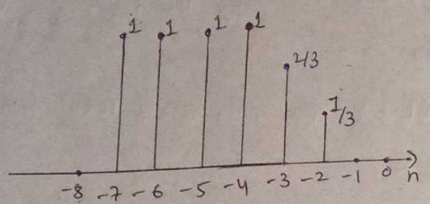
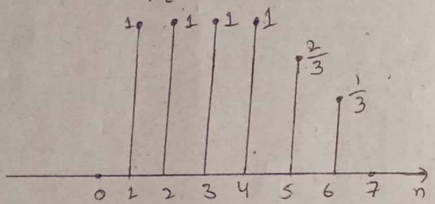
$x[-n-4]$



c) sketch the signal $x[-n+4]$.

Sol: $x[-n+4] = \{\ldots, 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots\}$ ($\uparrow$ under the first 1)

$x[-n+4]$



d) compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x[-n+k]$ from $x[n]$.

Sol:- By comparing the results in parts (b) and (c) we can get $x[-n+k]$ by first folding $x[n] \to x[-n]$ and delaying it with k samples if $k>0$, or if $k<0$, advancing it with k samples.

e) can you express the signal $x[n]$ in terms of signals $\delta[n]$ and $u[n]$?

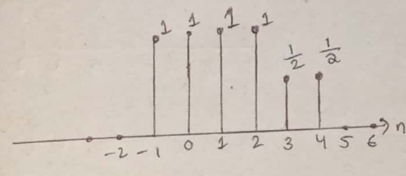
Sol:- yes, we can express the signal $x[n]$ in terms of signals $\delta[n]$ and $u[n]$

$$x[n] = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u[n] - u[n-4]$$

2.2 A discrete-time signal $x[n]$ is shown in Fig. P2.2. Sketch and label carefully each of the following signals.

Sol:-

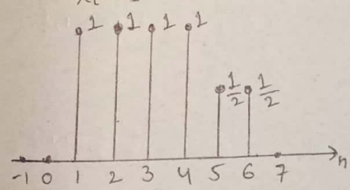
$x[n]$



a) $x[n-2]$

$x[n-2] = \{\ldots, 0, 0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\}$ ($\uparrow$)

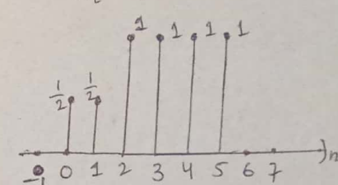
$x[n-2]$



(b) $x[4-n]$

$x[4-n] = \{\ldots, 0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0, \ldots\}$ ($\uparrow$)

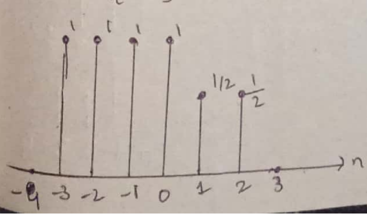
$x[4-n]$



c) $x(n+2]$

$x(n+2) = \{\ldots 0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\}$ ($\uparrow$)

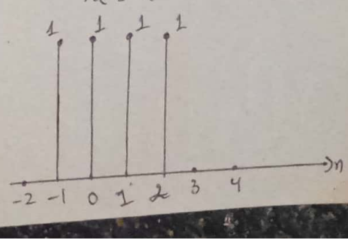
$x[n+2]$



d) $x[n]u[2-n]$

$x[n]u[2-n] = \{\ldots 0, 1, 1, 1, 1, 0, 0, \ldots\}$ ($\uparrow$)

$x[n]u[2-n]$



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e) $x[n-1]\delta[n-3]$

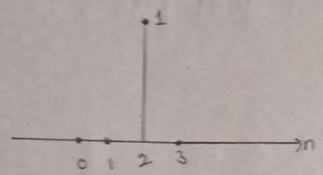
$x[n-1]\delta[n-3]$

$= x[3-1]\delta[n-3]$

$= x[2]\delta[n-3]$

$= \{\ldots 0, 0, 1, 0, \ldots\}$

$x[n-1]\delta[n-3]$

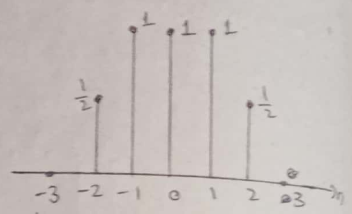


f) $x[n^2]$

$= \{\ldots 0, x(4), x(1), x(0), x(1) \ldots\}$

$= \{\ldots 0, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, \ldots\}$

$x[n^2]$



g) even part of $x[n]$

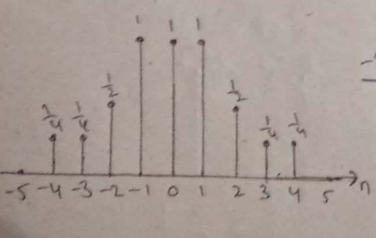
$x_e[n] = \frac{x[n] + x[-n]}{2}$

$x[n] + x[-n] =$

$\{\ldots 0, \frac{1}{2}, \frac{1}{2}, 1, 2, 2, 2, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\}$

$x_e[n] = \{\ldots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0 \ldots\}$

$x_e[n]$



h) odd part of $x[n]$

$x_o[n] = \frac{x[n] - x[-n]}{2}$

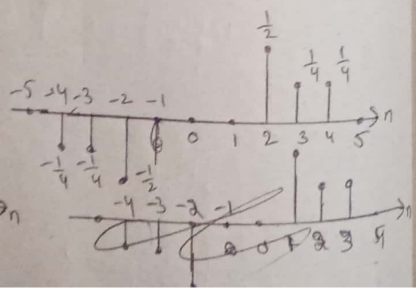
$x[n] - x[-n]$

$= \{\ldots 0, -\frac{1}{2}, -\frac{1}{2}, -1, 0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}, 0 \ldots\}$

$x_o[n] =$

$\{\ldots 0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\}$

$x_o[n]$



2.3 Show that

a) $\delta[n] = u[n] - u[n-1]$

Sol: we know that $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$ $=$ L.H.S. $= \{\ldots 0, 1, 0, \ldots\}$

$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} = \{\ldots 0, 1, 1, 1, 1, \ldots\}$

$u[n-1] = \begin{cases} 1, & n \geq 1 \\ 0, & n < 1 \end{cases} = \{0, 0, 1, 1, 1, \ldots\}$

R.H.S $= u[n] - u[n-1] = \{\ldots 0, 1, 1, 1, 1 \ldots\} - \{\ldots 0, 0, 1, 1, 1, \ldots\}$

R.H.S. $= \{\ldots 0, 1, 0, \ldots\}$

R.H.S $= \delta[n]$

L.H.S = R.H.S

Hence proved $\delta[n] = u[n] - u[n-1]$

b) $u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$

Sol: $u[n] = \{\ldots 0, 1, 1, 1, 1, \ldots\} = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=-\infty}^{n} \delta[k] = \{\ldots 0, 1, 1, 1, 1, \ldots\} = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta[n-k] = \{\ldots 0, 1, 1, 1, 1, \ldots\} = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$\therefore$ Hence proved that $u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$

2.4. Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal

$$x[n] = [2, 3, 4, 5, 6]$$
$$\uparrow$$

Sol:- By the properties of even and odd signals

we know that $x_e[-n] = x_e[n]$

&

$x_o[-n] = -x_o[n]$

And also let we

$$x_e[n] = \frac{1}{2}\left[x[n] + x[-n]\right] \rightarrow (1)$$

$$x_o[n] = \frac{1}{2}\left[x[n] - x[-n]\right]. \rightarrow (2)$$

Add eq (1) & eq (2)

$$x_e[n] + x_o[n] = \frac{1}{2}\left[x[n] + x[-n]\right] + \frac{1}{2}\left(x[n] - x[-n]\right)$$

$$= \frac{x[n] + \cancel{x[-n]} + x[n] - \cancel{x[-n]}}{2}$$

$$= \frac{2x[n]}{2}$$

$$\boxed{\Rightarrow x[n] = x_e[n] + x_o[n]}$$

The decomposition is unique.

Given $x[n] = \{2, 3, 4, 5, 6\}$
$\uparrow$

$$x_e[n] = \frac{1}{2}\left[x[n] + x[-n]\right]$$

$$= \frac{1}{2}\{8, 8, 8, 8, 8\}$$
$\uparrow$

$$x_e[n] = \{4, 4, 4, 4, 4\}$$
$\uparrow$

$$x_o[n] = \frac{1}{2}\left[x[n] - x[-n]\right]$$

$$= \frac{1}{2}\{-4, -2, 0, 2, 4\}$$
$\uparrow$

$$x_o[n] = \{-2, -1, 0, 1, 2\}$$
$\uparrow$

2.5. Show that the energy (power) of a real-valued signal is equal to the sum of the energies (powers) of its even and odd components.

Sol:- We have to prove that

(Energy) power of real valued signal = Sum of powers (energies) of its even & odd components

L.H.S = ~~Pow~~ Energy of real valued signal $= \sum_{n=-\infty}^{\infty} x^2[n]$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} [x_e[n]+x_o[n]]^2$$ $\left\{ \because \text{we know} \atop x[n] = x_e[n]+x_o[n] \right\}$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} [x_e[n]]^2 + \sum_{n=-\infty}^{\infty} [x_o[n]]^2 + \sum_{n=-\infty}^{\infty} 2x_e[n]\, x_o[n] \rightarrow (1)$$

Let us simply $2\sum_{n=-\infty}^{\infty} x_e[n]\, x_o[n] = 2\sum_{n=-\infty}^{\infty} x_e[-n]x_o[-n]$

$\{\because$ Replacing n by $-n\}$

$$= -2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n]$$

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0 \rightarrow (2)$$

Substuting eq(2) in eq(1)

$$\Rightarrow \underbrace{\sum_{n=-\infty}^{\infty} x^2[n]}_{E_{total}} = \underbrace{\sum_{n=-\infty}^{\infty} x_e^2[n]}_{E_{even}} + \underbrace{\sum_{n=-\infty}^{\infty} x_o^2[n]}_{E_{odd}}$$

Hence proved that,

$$\Rightarrow \boxed{E_{total} = E_{even} + E_{odd}}$$

2.6 consider the system

$$y[n] = T\{x[n]\} = x[n^2]$$

a) Determine if the system is time invariant

Sol:-

i/p : $\rightarrow x[n]$

o/p : $\rightarrow y[n]$

$y[n] = x[n^2]$.

giving delay k samples $\Rightarrow x[n-k]$ to s/m produce $y_1[n]$

$$\Rightarrow y_1[n] = x[(n-k)^2]$$

$$y_1[n] = x[n^2+k^2-2nk] \rightarrow eq(1)$$

giving delay k samples to o/p $\Rightarrow$ produce $y[n-k]$

as $y[n-k] \neq y_1[n]$

The system is not time invariant.

$\therefore$ Hence, the system is time variant.

b) To clarify the result in part (a) assume that the signal

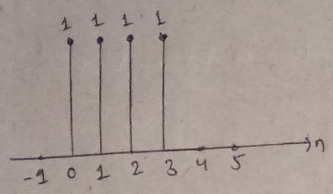
$$x[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$ is applied into the s/m

(i) sketch the signal $x[n]$

$$x[n] = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$$

$x[n]$

Sol:-

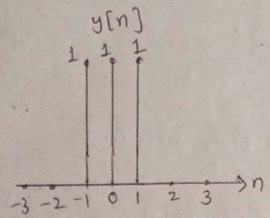


2) Determine and sketch the signal $y[n] = T[x[n]]$

Sol:- $y[n] = T[x[n]] = x[n^2] = \{ \ldots 0, x[(-3)^2], x[(-2)^2], x[(-1)^2], \underset{\uparrow}{x[0^2]}, x[1^2], x[2^2], x[3^2], 0 \ldots \}$

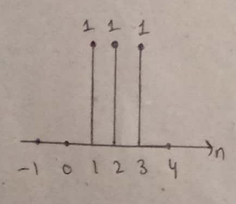
$= \{ 0, x[9], x[4], x[1], \underset{\uparrow}{x[0]}, x[1], x[4], x[9], 0, \ldots \}$

$y[n] = \{ \ldots 0, 0, 0, 1, \underset{\uparrow}{1}, 1, 0, 0, 0, \ldots \}$



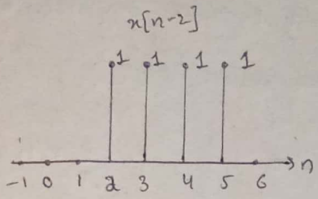
3) sketch the signal $y_2'[n] = y[n-2]$

Sol:- $y_2'[n] = y[n-2] = \{ \ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 0, \ldots \}$



4) Determine and sketch the signal $x_2[n] = x[n-2]$

Sol:- $x_2[n] = x[n-2] = \{ \ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots \}$

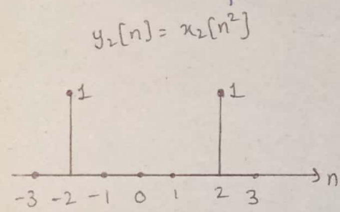


5) Determine and sketch the signal $y_2[n] = T[x_2[n]]$

Sol:- $y_2[n] = T[x_2[n]] = x_2[n^2]$

$= \{ \ldots 0, x_2[4], x_2[1], \underset{\uparrow}{x_2[0]}, x_2[1], x_2[4], 0 \ldots \}$

$y_2[n] = \{ \ldots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \ldots \}$



6) compare the signals $y_2[n]$ and $y[n-2]$. what is your conclusion?

Sol:- $y_2[n] \neq y[n-2]$

Hence, we can say that the system is time variant and not time invariant.

c) Repeat part (b) for the system

$y[n] = x[n] - x[n-1]$

Can you see this result to make any statement about the time invariance of this system? Why?

Sol:- $y[n] = \{\cdots 0, 1, 1, 1, 1, 0, \cdots\} - \{\cdots 0, 1, 1, 1, 1, 0, \cdots\}$

$y[n] = \{\cdots 0, 1, 0, 0, 0, 0, -1, 0, \cdots\}$

$y_2 = y[n-2] = \{0, 0, 1, 0, 0, 0, 0, -1\}$

$x_2 = x[n-2] = \{0, 0, 1, 1, 1, 1, 0\}$

$y_2[n] = \{0, 0, 1, 0, 0, 0, 0, 0, -1\}$

Here $y_2[n] = y[n-2]$. So, the system is time invariant

d) Repeat parts (b) and (c) for the system

$y[n] = T[x[n]] = nx[n]$

Sol:- Given s/m $y[n] = nx[n]$

$x[n] = \{\cdots 0, 1, 1, 1, 1, 0, \cdots\}$

$y[n] = nx[n] = \{\cdots 0, 1, 2, 3, 0, \cdots\}$

$y_2[n] = y[n-2] = \{\cdots 0, 0, 0, 1, 2, 3, 0, \cdots\}$

$x[n-2] = \{\cdots 0, 0, 0, 1, 1, 1, 1, 0, \cdots\}$

$y_2[n] = T[x[n-2]] = \{\cdots 0, 0, 0, 2, 3, 4, 5, 0, \cdots\}$

$y_2[n] \neq y[n-2]$

Hence, the s/m is time variant.

2.7 A discrete-time system can be

(1) static or dynamic

(2) Linear or nonlinear

(3) Time invariant or time varying

(4) casual or non casual

(5) stable or unstable

Examine the following systems with respect to the properties above.

a) $y[n] = \cos[x[n]]$

Sol:- → $y[n]$ is static, as it is memoryless.

→ $\cos[x_1[n] + x_2[n]] \neq \cos[x_1[n]] + \cos[x_2[n]]$

⇒ It is non-linear

→ $y[n-k] = \cos[x[n-k]]$

→ It is time invariant

→ It input only depends on present i/ps, it is causal As s/m

→ It is stable.

b) $y[n] = \sum_{k=-\infty}^{n+1} x[k]$

Sol:- → It is Dynamic, coz it is depending on future i/ps, which needs memory

→ $\sum_{k=-\infty}^{n+1} [x_1[k] + x_2[k]] = \sum_{k=-\infty}^{n+1} x_1[k] + \sum_{k=-\infty}^{n+1} x_2[k]$

so, it is linear

→ It is time invariant

→ It is noncausal as it needs future i/ps

→ S/m is unstable

Bcoz for bounded i/p $x[k] = u[k]$

$$y[n] = \sum_{k=-\infty}^{n+1} u[k] = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

Since $y[n] \to \infty$ as $n \to \infty$, the s/m is unstable.

c) $y[n] = x[n] \cos[w_0 n]$

Sol:- → It is static, as n → i/p values doesn't need memory

→ $\{x_1[n] + x_2[n]\} \cos[w_0 n] = x_1[n] \cos[w_0 n] + x_2[n] \cos w_0 n$

so, s/m is linear

→ $x_1[n-k] \cos[w_0 n] = y[n-k] \neq x_1[n-k]$

So, s/m is time variant

→ It is causal, no need of future i/ps

→ It satisfies BIBO stability

So, it is stable.

d) $y[n] = x[-n+2]$

Sol:- → It is Dynamic, bcoz it needs past values which requires memory.

→ $y_1[n] + y_2[n] = x_1[-n+2] + x_2[-n+2]$

So, s/m is linear.

→ $y[n-k] = x[-n+2-k]$

So, s/m is time invariant

→ It ~~doesn't~~ needs future values bcoz $y[-n] = x[n+2]$

So, s/m is ~~cau~~ non-causal

→ It is stable

e) $y[n] = Trun[x[n]]$, where $Trun[x[n]]$ denotes the integer part of $x[n]$, obtained by truncation.

Sol:- → It is static, as no need of memory

→ $Trun[x_1[n] + x_2[n]] \neq Trun[x_1[n]] + Trun[x_2[n]]$

So, s/m is non-linear

→ $Trun[x[n-k]] = y[n-k]$

So, s/m is time invariant

→ as s/m doesn't needs any future values,

s/m is causal.

→ for bounded i/p, we will get bounded o/p

So, s/m is stable.

f) $y[n] = Round[x[n]]$, where $Round[x[n]]$ denotes the integer part of $x[n]$ obtained by rounding.

Sol:- → It is static

→ It is non-linear

→ It is time invariant

→ It is causal

→ It is stable.

g) $y[n] = |x[n]|$

Sol:- → The s/m is static, no need of memory

→ $\cancel{y_1[n] + y_2[n] = |x_1[n]|}$

⇒ $|x_1[n] + x_2[n]| \neq |x_1[n]| + |x_2[n]|$

So, s/m is non-linear

⇒ $y[n-k] = |x[n-k]|$

So, s/m is time invariant

→ as it not needs future i/ps, s/m is causal.

→ s/m is stable.

h) $y[n] = x[n]u[n]$

Sol:- → The s/m is static, bcoz of no need of memory

→ $[x_1[n] + x_2[n]]u[n] = x_1[n]u[n] + x_2[n]u[n]$

So, the s/m is linear

→ $y[n-k] = x[n-k]u[n]$ ⇒ So, s/m is time invariant

→ as it no needs future i/ps, s/m is causal

→ s/m is stable.

i) $y[n] = x[n] + n\,x[n+1]$

Sol:- → The s/m is dynamic, as it needs future i/ps which also needs memory & it is non-causal

→ $x_1[n] + n x_2[n+1] + x_2[n] + n x_2[n+1] = y_1[n] + y_2[n]$

So, s/m is linear

→ $y[n-k] = x[n-k] + (n-k)\,x[n+1-k] \neq x[n-k] + n\,x[n+1-k]$

So, s/m is time variant

→ for bounded i/p $x[n] = u[n]$

$y(n) = u[n] + n\,u[n+1]$ produces an unbounded o/p.

So, the s/m is unstable.

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j) $y[n] = x[2n]$

Sol:- → s/m is non-causal & also dynamic as it needs future i/ps

→ $x_1[2n] + x_2[2n] = y_1[n] + y_2[n]$

so, s/m is linear.

→ $y[n-k] = x[2n-k] \neq x[2(n-k)]$

so, s/m is time variant

→ s/m is stable.

k) $y[n] = \begin{cases} x[n] & , \text{if } x[n] \geq 0 \\ 0 & , \text{if } x[n] < 0 \end{cases}$

Sol:- → s/m is static & also causal as no memory & future i/ps are needed

→ s/m is non-linear

→ s/m is time invariant

→ s/m is stable

l) $y[n] = x[-n]$

Sol:- → s/m is dynamic & non-causal as it needs past, future values and also memory.

→ s/m is linear & time invariant & stable.

m) $y[n] = \text{sign}[x[n]]$

Sol:- → s/m is causal and static

→ s/m is non-linear as

$\text{sign}[x_1[n] + x_2[n]] \neq \text{sign}[x_1[n]] + \text{sign}[x_2[n]]$

→ $y[n-k] = \text{sign}[x[n-k]]$

so, s/m is time invariant

→ s/m is stable

n) The ideal sampling system with input $x_a(t)$ and o/p

$x[n] = x_a[nT]$, $-\infty < n < \infty$

Sol:- → s/m is static & causal as no future, memory needed

→ s/m is linear as

$x_a[nT] + x_b[nT] = x_a[nT] + x_b[nT]$

→ s/m is time invariant & stable.

Q9 Let $\mathcal{T}$ be an LTI related, and BIBO stable system with input $x[n]$ and output $y[n]$. Show that,

a) If $x[n]$ is periodic with period N [i.e., $x[n] = x[n+N]\ \forall n \geq 0$], the output $y[n]$ leads to a periodic signal with the same period

Sol:- i/p : $x[n]$

Impulse response of LTI s/m : $h[n]$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{n} h[k]\, x[n-k]$$

$\Rightarrow$ advancing o/p through period N

$$\Rightarrow y[n+N] = \sum_{k=-\infty}^{n+N} h[k]\, x[n+N-k] = \sum_{k=-\infty}^{n+N} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{n} h[k]\, x[n-k] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k]$$

$$= y[n] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k]$$

Given, is BIBO stable s/m $\Rightarrow \lim_{n\to\infty} |h(n)| = 0$.

$$\therefore \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h[k]\, x[n-k] = 0$$

$$\Rightarrow \lim_{n\to\infty} y(n+N) = y[N].$$

$$\therefore y[n+N] = y[n]$$

$\therefore$ o/p is also periodic

b) If $x[n]$ is bounded and tends to a constant, the output will also tend to a constant.

Sol:- Let $x[n] = x_0[n] + a u[n]$, $a \to$ constant

$x_0[n]$ is bounded sig with $\lim_{n\to\infty} x_0[n] = 0$

$$\therefore y[n] = a\sum_{k=0}^{\infty} h[k]u[n-k] + \sum_{k=0}^{\infty} h[k]\, x_0[n-k]$$

$$= a\sum_{k=0}^{\infty} h[k] + y_0[n]$$

$$\Rightarrow \sum_n x_0^2[n] < \infty \Rightarrow \sum_n y_0^2[n] < \infty$$

$$\Rightarrow \lim_{n\to\infty} |y_0(n)| = 0$$

$$\Rightarrow y[n] = y_0[n] + \underbrace{a\sum_{k=0}^{\infty} h[k]}_{\text{constant} \to R}$$

$$y[n] = y_0[n] + R$$

$$\lim_{n\to\infty} y[n] = R$$

c) If $x(n)$ is an energy signal, the output $y[n]$ will also be an energy signal.

Sol:-
$$y[n] = \sum_{k=0}^{\infty} h[k]\, x[n-k]$$

$$\Rightarrow \text{Energy} \Rightarrow \sum_{n=-\infty}^{\infty} y^2(n) = \sum_{n=-\infty}^{\infty}\left[\sum_k h[k]\, x[n-k]\right]^2$$

$$= \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} h[k]\,h[l] \sum_{n=-\infty}^{\infty} x[n-k]x[n-l]$$

But
$$\sum_n x[n-k]x[n-l] \le \sum_n x^2[n] = E_x$$

$$\therefore \sum_n y^2[n] \le E_x \sum_k |h[k]| \sum_l |h[l]|$$

For a BIBO stable system
$$\sum_{k=-\infty}^{\infty} |h[k]| \le M$$

$\therefore$ $E_y \le M^2 E_x$, so that

$E_y < \infty$ if $E_x < \infty$

2.10 The following input-output pairs have been observed during the operation of a time-invariant system.

$$x_1[n] = [\underset{\uparrow}{1},0,2] \xleftrightarrow{T} y_1[n] = [0,\underset{\uparrow}{1},2]$$

$$x_2[n] = [\underset{\uparrow}{0},0,3] \xleftrightarrow{T} y_2[n] = [\underset{\uparrow}{0},1,0,2]$$

$$x_3[n] = [\underset{\uparrow}{0},0,0,1] \xleftrightarrow{T} y_3[n] = [1,\underset{\uparrow}{2},1]$$

Can you draw any conclusion regarding the linearity of the system. What is the impulse response of the system?

Sol:-

→ If you assume system is time invariant,

then a shift of x_2, say

$$x_2^{s} = \{0, 0, 0, 3\}$$

should give the shifted version of $y_2[n]$,

$$y_2^{s}[n] = \{0, 0, 1, 0, 2\}$$

But $x_2^{s} = 3x_3$ and y_2^{s} is not a multiple of y_3

Linearity is not preserved.

→ If the system were linear, then, since

$$x_2[n] = 3x_3[n+1], \text{ then } y_2[n] = 3y_3[n+1]$$

Impulse response of s/m is

$$x_3[n+3] = \{0, 0, 0, 1, 0\} \rightarrow y_3[n] = \{1, 2, 1, 0, 0, 0\}$$

2.11. The following input-output pairs have been observed during the operation of a linear system:-

$$x_1[n] = \{-1, 2, 1\} \xleftrightarrow{T} y_1[n] = \{1, 2, -1, 0, 1\}$$

$$x_2[n] = \{1, -1, 1\} \xleftrightarrow{T} y_2[n] = \{-1, 1, 0, 2\}$$

$$x_3[n] = \{0, 1, 1\} \xleftrightarrow{T} y_3[n] = \{1, 2, 1\}$$

Can you draw any conclusions about the time invariance of the system?

Sol:-. By observing that

$$x_1[n] + x_2[n] = \delta[n]$$

and the system is linear,

Impulse response of s/m is

$$y_1[n] + y_2[n] = \{0, 3, -1, 2, 1\}$$

If system were time invariant, response to $x_3[n]$ would be $\{3, 2, 1, 3, 1\}$

2.12 ~~The only available information about a system consists of N input-output pairs of signals $y_i[n] = T[x_i[n]]$, $i = 1, 2, \ldots N$.~~

~~(a) what is the class of input signals for which we can determine the output, using the information above, if the system is known to be linear?~~

Sol:-

2.13. Show that the necessary condition for a relaxed LTI system to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \le M_h < \infty$$

for some constant M_h.

Sol:- An arbitary relaxed s/m is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded of output.

Mathematicaly, $|x(n)| \le M_x < \infty$, $|y(n)| \le M_y < \infty$

we know, $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

$$|y[n]| \le \sum_{k=-\infty}^{\infty} |h[k]|\,|x[n-k]|$$

$$\le M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

where $|x[n-k]| \le M_x$. Therefore, $|y[n]| < \infty$ $\forall n$, if and only if

$$\boxed{\sum_{k=-\infty}^{\infty} |h[k]| < \infty.}$$

2.14. Show that

(a) A relaxed linear system is causal if and only if for any input $x[n]$ such that

$x[n] = 0$ for $n < n_0 \Rightarrow y[n] = 0$ for $n < n_0$

Sol:- $x[n] = 0$ for $n < n_0$

$y[n] = 0$ for $n < n_0$.

If the system is causal if the output is non zero of the system at any time 'n' depends only on present ~~value~~ and past i/p's of the non zero.

$\therefore x[n] = 0$ for $n < n_0$ is equal to

$y[n] = 0$ for $n < n_0$

b) A relaxed LTI system is causal if and only if

$h[n] = 0$ for $n < 0$

Sol:- $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$, where $x[n] = 0$ for $n < 0$.

If $h[k] = 0$ for $k < 0$ then,

$y[n] = \sum_{k=0}^{\infty} h[k]\,x[n-k]$, and hence $y[n] = 0$ for $n < 0$.

on the other hand, if $y[n] = 0$ for $n < 0$, then

$$\sum_{k=-\infty}^{n} h[k]\,x[n-k] \Rightarrow h[k] = 0,\ k < 0$$

2.15 a) show that for any real or complex constant a, and any finite integer numbers M and N, we have

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{(1-a)} & ; \text{if } a \neq 1 \\ N-M+1 & ; \text{if } a = 1 \end{cases}$$

Sol:-

For $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$, $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \dots + a^N$

$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} - a^{M+1} + \dots + a^N - a^N - a^{N+1}$

$= a^M - a^{N+1}$

b) Show that if $|a| < 1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Sol: For $M=0$, $|a|<1$, and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

2.16 a) If $y[n] = x[n] * h[n]$, show that $\sum_y = \sum_x \sum_h$, where $\sum_x = \sum_{n=-\infty}^{\infty} x[n]$.

Sol:- Given $y[n] = x[n] * h[n]$

means

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} h[k]\,x[n-k] = \sum_{k=-\infty}^{\infty} h[k] \sum_{n=-\infty}^{\infty} x[n-k]$$

$$= \sum_k h[k] \sum_n x[n]$$

$$\Rightarrow \boxed{\sum_y = \sum_x \sum_h}$$

b) compute the convolution $y[n] = x[n] * h[n]$ of the following signals and check the correctness of the results by using the test in (a).

i) $x[n] = \{1, 2, 4\}$, $h[n] = \{1, 1, 1, 1, 1\}$

Sol: $y[n] = h[n] * x[n]$

$= \{1, 3, 7, 7, 7, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$\sum_n y[n] = 1+3+7+7+7+6+4$

$= 35$

$\sum_k h[k] = 5 \qquad \sum_n x[n] = 7$

$\Rightarrow \sum_y = \sum_x \sum_h$

2) $x[n] = \{1, 2, -1\}$, $h[n] = x[n]$

Sol:- $y[n] = h[n] * x[n]$

$= x[n] * x[n]$

	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y[n] = \{1, 4, 2, -4, 1\}$

$\sum_n y[n] = 4 \quad \sum_k h[k] = 2 \quad \sum_n x[n] = 2$

$\sum_y = \sum_h \sum_x$

3) $x[n] = \{0, 1, -2, +3, -4\}$, $h[n] = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

Sol:- $y[n] = h[n] * x[n]$

	0	1	-2	+3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$+\frac{3}{2}$	-2
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$+\frac{3}{2}$	-2
1	0	1	-2	+3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$+\frac{3}{2}$	-2

$y[n] = \{0, \frac{1}{2}, -\frac{1}{2}, +\frac{3}{2}, -2, 0, -\frac{5}{2}, -2\}$

$\sum_n y[n] = -5, \quad \sum_k h[k] = 2.5$

$\sum_n x[n] = -2$

$\sum_y = \sum_h \sum_x$

4) $x[n] = \{1, 2, 3, 4, 5\}$, $h[n] = \{1\}$

Sol:- $y[n] = h[n] * x[n]$ $\quad \sum_n y[n] = 15 \quad \sum_k h[k] = 1$

$y[n] = \{1, 2, 3, 4, 5\}$ $\quad \sum_n x[n] = 15$

$\sum_y = \sum_x \sum_h$

5) $x[n] = \{1, -2, 3\}$, $h[n] = \{0, 0, 1, 1, 1, 1\}$

Sol:- $y[n] = h[n] * x(n)$

	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$y[n] = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y[n] = 8 \quad \sum_k h[k] = 4$

$\sum_n x[n] = 2$

$\sum_y = \sum_x \sum_h$

6) $x[n] = \{0, 0, 1, 1, 1, 1\}$, $h[n] = \{1, -2, 3\}$

Sol:- $y[n] = h[n] * x[n]$

	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y[n] = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y[n] = 8, \quad \sum_k h[k] = 4 \quad \sum_n x[n] = 2 \quad \sum_y = \sum_x \sum_h$

7) $x[n] = \{0, 1, 4, -3\}$, $h[n] = \{1, 0, -1, -1\}$

Sol:- $y[n] = h[n] * x[n]$

	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$y[n] = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y[n] = -2 \quad \sum_k h[k] = -1 \quad \sum_n x[n] = 2$

$\sum_y = \sum_x \sum_h$

8) $x[n] = \{1, 1, 2\}$, $h[n] = u[n]$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = u[n] + u[n-1] + 2u[n-2]$

$\sum_n y[n] = \infty$, $\sum_n h[n] = \infty$, $\sum_n x[n] = 4$

9) $x[n] = \{1, 1, 0, 1, 1\}$, $h[n] = \{1, -2, -3, 4\}$

Sol:- $y[n] = h[n] * x[n]$

	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
3	-3	-3	0	-3	-3
4	4	4	0	4	4

$y[n] = \{1, -1, -5, \cancel{-6}\ 2, \cancel{-8}\ 3, -5, 1, 4\}$

$\sum_n y[n] = 0$; $\sum_n x[n] = 4$; $\sum_k h[k] = 0$

10) $x[n] = \{1, 2, 0, 2, 1\}$, $h[n] = x[n]$

Sol:- $x[n] = \{1, 2, 0, 2, 1\}$, $h[n] = \{1, 2, 0, 2, 1\}$

$y[n] = h[n] * x[n]$

	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$\Rightarrow y[n] = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y[n] = 36$, $\sum_k h[k] = 6$, $\sum_n x[n] = 6$

Sol:- $\sum x[k] h[n-k]$

11) $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $h[n] = \left(\frac{1}{4}\right)^n u[n]$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = \left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n]$

$x[n] = \{0.5, 0.25, 0.125, 0.0625, 0.03125\}$

$h[n] = \{0.25, \cancel{0.125}, 0.0625, \cancel{0.03125}\ 0.015625\}$

$h[n] * x[n]$

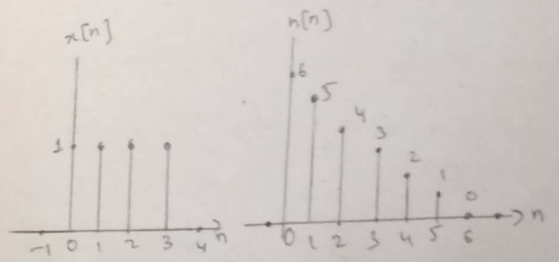
	0.5	0.25	0.125	0.0625	0.03125
0.25	0.25	0.25	0.0625		
0.0625					
0.015625					

$y[n] = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u[n]$

$\sum_n y[n] = \frac{8}{3}$, $\sum_n h[n] = \frac{4}{3}$, $\sum_n x[n] = 2$

2.17 Compute and plot the convolutions $x[n] * h[n]$ and $h[n] * x[n]$ for the pairs of signals shown in Fig. P2.17

a)



Sol:-

$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$

$h[n] = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1\}$

$$y[n] = \sum_{k=0}^{n} x[k]\, h[n-k]$$

$y[0] = x[0]h[0] = 6$

$y[1] = x[0]h[1] + x[1]h[0] = 11$

$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] = 15$

$y[3] = x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0] = 18$

$y[4] = x[0]h[4] + x[1]h[3] + x[2]h[2] + x[3]h[1] + x[4]h[0] = 14$

$y[5] = x[0]h[5] + x[1]h[4] + x[2]h[3] + x[3]h[2] + x[4]h[1]$

$+ x[5]h[0] = 10$

$y[6] = x[1]h[5] + x[2]h[4] + x[3]h[2] = 6$

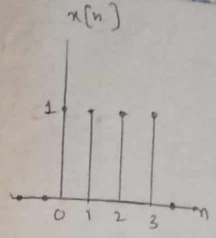
$y[7] = x[2]h[5] + x[3]h[4] = 3$

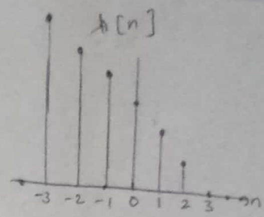
$y[8] = x[3]h[5] = 1$

$y[n] = 0, \; n \geq 9$

$y[n] = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

b)



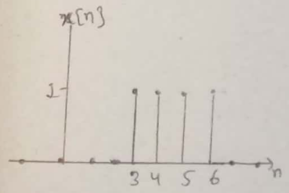


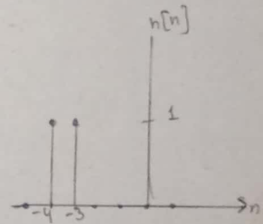
Sol: $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $\quad h[n] = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1\}$

	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
→ 3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

$y[n] = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

c)





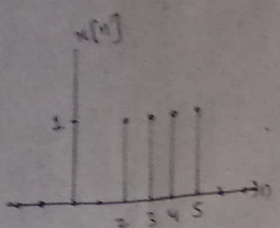
Sol:-

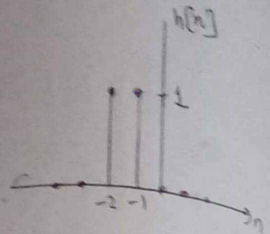
$x[n] = \{1, 1, 1, 1\}$ $\quad y[n] = \{1, 1\}$

	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y[n] = \{1, \underset{\uparrow}{2}, 2, 2, 1\}$

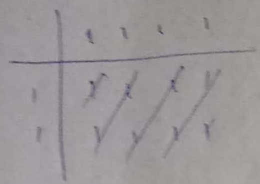
5 d)





Sol^n $x[n] = \cancel{\{2,3,4,5\}}$ $h[n] = \{1,1\}$

$\{1,1,1,1\}$



$y[n] = \{\underset{\uparrow}{1}, 2, 2, 2, 1\}$

2.18 Determine and sketch the convolutions y[n] of the signals

$$x[n] = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Graphically

(b) Analytically

a) $x[n] = \{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$y[n] = x[n] * h[n]$

$= \{\frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

b) $x[n] = \frac{1}{3} n[u[n] - u[n-7]]$

$h[n] = u[n+2] - u[n-3]$

$y(n) = x[n] * h[n]$

$= \frac{1}{3} n[u[n] - u[n-7]] * [u[n+2] - u[n-3]]$

$= \frac{1}{3} n\,[u[n] * u[n+2] - u[n] * u[n-3] - u[n-7] * u[n+2]$
$+ u[n-7] * u[n-3]]$

$y[n] = \frac{1}{3}\delta[n+1] + \delta[n] + 2\delta[n-1] + \frac{10}{3}\delta[n-2] + 5\delta[n-3] + \frac{20}{3}\delta[n-4]$
$+ 6\delta[n-5] + 5\delta[n-6] + 5\delta[n-6] + \frac{11}{3}\delta[n-7] + \delta[n-8]$

2.19. compute the convolution $y[n]$ of the signals

$$x[n] = \begin{cases} \alpha^n , & -3 \le n \le 5 \\ 0 , & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 1 , & 0 \le n \le 4 \\ 0 , & \text{elsewhere} \end{cases}$$

Sol:- $y[n] = \sum_{k=0}^{4} h[k]x[n-k]$

$x[n] = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \dots, \alpha^5\}$

$h[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$y[n] = \sum_{k=0}^{4} x[n-k] , \quad -3 \le n \le 9$

$= 0$, otherwise

Therefore $y[-3] = \alpha^{-3}$

$y[-2] = x[-3] + x[-2] = \alpha^{-3} + \alpha^{-2}$

$y[-1] = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y[0] = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y[1] = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$

$y[2] = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2$

$y[3] = \alpha^{-1} + 1 + \alpha + \alpha^2 + \alpha^3$

$y[4] = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$

$y[5] = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y[6] = \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y[7] = \alpha^3 + \alpha^4 + \alpha^5$

$y[8] = \alpha^4 + \alpha^5$

$y[9] = \alpha^5$

2.20. consider the following three operations

a) Multiply the integer numbers 131 and 122.

sol:- $y[n] = \sum_{k=0}^{n} a^k u[k] b^{n-k} u[n-k] = b^n \sum_{k=0}^{n} (ab^{-1})^k$

$$y[n] = \begin{cases} \frac{b^{n+1} - a^{n+1}}{b-a} u[n], & a \ne b \\ b^n [n+1] u[n], & a = b \end{cases}$$

$131 \times 122 = 15982$

b) compute the convolution of signals: $\{\underset{\uparrow}{1}, 3, 1\} * \{\underset{\uparrow}{1}, 2, 2\}$

Sol: $x[n] = \{1, 2, 1, 1\}$

$h[n] = \{1, -1, 0, 0,$

$\{\underset{\uparrow}{1}, 3, 1\} * \{\underset{\uparrow}{1}, 2, 2\}$

$\Rightarrow \{\underset{\uparrow}{1}, 5, 9, 8, 2\}$

	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the polynomials: $1+3z+z^2$ and $1+2z+2z^2$

Sol:- $(1+3z+z^2)(1+2z+2z^2) = 1+5z+9z^2+8z^3+2z^4$

d) Repeat part (a) for the numbers 1.31 and 12.2

Sol:- $1.31 \times 12.2 = 15.982$

e) comment on your results.

Sol:- These are different ways to perform convolution.

2.21. compute the convolution $y[n] = x[n] * h[n]$ of the following pair of signals.

a) $x[n] = a^n u[n]$, $h[n] = b^n u[n]$ when $a \neq b$ and when $a=b$

Sol:- $y[n] = \sum_{k=0}^{n} a^k u[k] b^{n-k} u[n-k]$

$= b^n \sum_{k=0}^{n} (ab^{-1})^k$

$$y[n] = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a} u[n], & a \neq b \\ b^n u[n+1]\, u[n], & a=b \end{cases}$$

b) $x[n] = \begin{cases} 1, & n=-2,0,1 \\ 2, & n=-1 \\ 0, & \text{elsewhere} \end{cases}$ $h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$

Sol:- $x[n] = \{1, 2, 1, 1\}$ (with $\uparrow$ under the third entry)

$h[n] = \{1, -1, 0, 0, 1, 1\}$ (with $\uparrow$ under the first entry)

$y[n] = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$ (with $\uparrow$ under the third entry)

(c) $x[n] = u[n+1] - u[n-4] - \delta[n-5]$

$h[n] = [u[n+2] - u[n-3]] \cdot (3 - |n|)$

Sol:- $x[n] = \{1, 1, 1, 1, 1, 0, -1\}$

$h[n] = \{1, 2, 3, 2, 1\}$

$y[n] = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

d) $x[n] = u[n] - u[n-5]$; $h[n] = u[n-2] - u[n-8] + u[n-11] - u[n-17]$

Let $x[n]$ be the i/p signal to a discrete-time filter with impulse response $h_i[n]$ and let $y_i[n]$ be the corresponding signal.

Sol:- $x[n] = \{1, 1, 1, 1, 1\}$

$h'[n] = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$h[n] = h'[n] + h'[n-9]$,

$y[n] = y'[n] + y'[n-2]$, where

$y'[n] = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2.22. (a) compute and sketch $x[n]$ and $y_1[n]$ in the following cases, using the same scale in all figures.

$x[n] = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1[n] = \{1, 1\}$

$h_2[n] = \{1, 2, 1\}$

$h_3[n] = \{\frac{1}{2}, \frac{1}{2}\}$

$h_4[n] = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5[n] = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

Sketch $x[n]$, $y_1[n]$, $y_2[n]$ on one graph and $x[n]$, $y_3[n]$, $y_4[n]$, $y_5[n]$ on another graph.

Sol:- $y[n] = h[n] * x[n]$

$y_1[n] = x[n] + x[n-1]$

$= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

$y_2[n] = \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\}$

$y_3[n] = \{0, 5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

$y_4[n] = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\}$

$y_5[n] = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$

(b) what is the difference between $y_1[n]$ and $y_2[n]$, and between $y_3[n]$ and $y_4[n]$?

Sol:- $y_3[n] = \frac{1}{2} y_1[n]$, because

$h_3[n] = \frac{1}{2} h_1[n]$

$y_4[n] = \frac{1}{4} y_2[n]$, because

$h_4[n] = \frac{1}{4} h_2[n]$

c) comment on the smoothness of $y_2[n]$ and $y_4[n]$. which factors affect the smoothness.

Sol:- $y_2[n]$ and $y_4[n]$ are smoother than $y_1[n]$, but $y_4[n]$ will appear even smoother because of the smaller scale factor.

d) compare $y_4[n]$ with $y_5[n]$. what is the difference? can you explain it?

Sol:- System 4 results in a smoother output. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5[n]$

c) Let $h_6[n] = \{ \frac{1}{2}, -\frac{1}{2} \}$. compute $y_6[n]$. sketch $x[n]$, $y_2[n]$, and $y_6[n]$ on the same figure and comment on the results.

Sol:- $$y_6[n] = \left\{ \frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2} \right\}$$

$y_2[n]$ is smoother than $y_6[n]$.

2.23. Express the output $y[n]$ of a LTI s/m with impulse response $h[n]$ in terms of its step response $s[n] = h[n] * u[n]$ and the input $x[n]$.

~~The discrete time s/m~~

$$\cancel{y[n] = ny[n-1] + x[n], \quad n \geq 0}$$

~~is at rest [i.e., $y(-1) = 0$]. check if the s/m is LTI and BIBO stable.~~

Sol:- we can express the unit sample in terms of the unit step function as $\delta[n] = u[n] - u[n-1]$.

then,
$$h[n] = h[n] * \delta[n]$$
$$= h[n] * [u[n] - u[n-1]]$$
$$= h[n] * u[n] - h[n] * u[n-1]$$
$$= s[n] - s[n-1]$$

using this definition of $h[n]$

$$y[n] = h[n] * x[n]$$
$$= [s[n] - s[n-1]] * x[n]$$
$$= \delta[n] * x[n] - s[n-1] * x[n]$$

2.24. The discrete time system

$$y[n] = ny[n-1] + x[n], \quad n \geq 0$$

is at rest [i.e., $y(-1) = 0$]. check if the system is linear time invariant and BIBO stable.

Sol:- $y_1[n] = ny_1[n-1] + x_1[n]$ and

$y_2[n] = ny_2[n-1] + x_2[n]$ then

$x[n] = ax_1[n] + bx_2[n]$

produces the output

$y[n] = ny[n-1] + x[n]$, where

$y[n] = ay_1[n] + by_2[n]$

Hence, the s/m is linear. If the input is $x[n-1]$, we have

$y[n-1] = [n-1]\, y[n-2] + x[n-1]$. But

$y[n-1] = ny[n-2] + x[n-1]$.

Hence, the s/m is time variant. If $x[n] = u[n]$, then $|x[n]| \leq 1$. But for this bounded input, the o/p is

$y[0]=1, \quad y[1]=1+1=2, \quad y[2]=2\times2+1=5, \ldots,$

which is unbounded. Hence, the s/m is unstable.

2.25 Consider the signal $\gamma[n]=a^n u[n], \; 0<a<1$.

a) show that any sequence $x[n]$ can be decomposed as

$$x[n]=\sum_{n=-\infty}^{\infty} c_k \gamma[n-k]$$

and express c_k in terms of $x[n]$.

Sol:-

$\delta[n]=\gamma[n]-a\gamma[n-1]$ and,

$\delta[n-k]=\gamma[n-k]-a\gamma[n-k-1]$. Then,

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$=\sum_{k=-\infty}^{\infty} x[k]\left[\gamma[n-k]-a\gamma[n-k-1]\right]$$

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\,\gamma[n-k]-a\sum_{k=-\infty}^{\infty} x[k]\,\gamma[n-k-1]$$

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\,\gamma[n-k]-a\sum_{k=-\infty}^{\infty} x[k-1]\,\gamma[n-k]$$

$$=\sum_{k=-\infty}^{\infty}\left[x[k]-ax[k-1]\right]\gamma[n-k]$$

Thus, $c_k=x[k]-ax[k-1]$

b) Use the properties of linearity and time invariance to express the o/p $y[n]=T[x[n]]$ in terms of the i/p $x[n]$ and the signal $g[n]=T[\gamma[n]]$, where $T[\cdot]$ is an LTI system.

Sol:-

$$y[n]=T[x[n]]$$

$$=T\left[\sum_{k=-\infty}^{\infty} c_k\gamma[n-k]\right]$$

$$=\sum_{k=-\infty}^{\infty} T[\gamma[n-k]]$$

$$=\sum_{k=-\infty}^{\infty} c_k\, g[n-k]$$

c) Express the impulse response $h[n]=T[\delta[n]]$ in terms of $g[n]$.

Sol:-

$$h[n]=T[\delta[n]]$$

$$=T\left[\gamma[n]-a\gamma[n-1]\right]$$

$$=g[n]-ag[n-1]$$

2.26. Determine the zero-input response of the s/m described by the second-order difference equation

$$x[n]-3y[n-1]-4y[n-2]=0$$

Sol:- with $x[n]=0$, we have

$$y[n-1]+\frac{4}{3}y[n-2]=0$$

$$y[-1]=-\frac{4}{3}y[-2]$$

$$y[0] = \left(-\frac{4}{5}\right)^2 y[-2]$$

$$y[1] = \left[-\frac{4}{3}\right]^3 y[-2]$$

$$\vdots$$

$$y[n] = \left[\frac{-4}{3}\right]^{n+2} y[-2] \leftarrow \text{zero-input response.}$$

2.37. Determine the particular solution of the difference equation

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n]$$

when the forcing ~~equat~~ function is $x[n] = 2^n u[n]$.

Sol:- Consider the homogeneous equation:

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 0.$$

The characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0. \lambda = \frac{1}{2}, \frac{1}{3}$$

Hence,

$$y_h[n] = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

The particular solution is

$$x[n] = 2^n u[n] \text{ is}$$

$$y_p[n] = k[2^n]u[n].$$

Substitution this solution into the difference equation. Then, we obtain

$$k(2^n)u[n] - k\left(\frac{5}{6}\right)(2^{n-1})u[n-1] + k\left(\frac{1}{6}\right)(2^{n-2})u[n-2] = 2^n u[n]$$

For $n = 2$,

$$4k - \frac{5k}{3} + \frac{k}{6} = 4 \Rightarrow k = \frac{8}{5}.$$

Therefore, the total solution is

$$y[n] = y_p[n] + y_h[n] = \frac{8}{5}(2^n)u[n] + c_1\left(\frac{1}{2}\right)^n u[n] + c_2\left(\frac{1}{3}\right)^n u[n].$$

To determine c_1 and c_2, assume that $y[-2] = y[-1] = 0$.

Then, $y[0] = 1$ and

$$y[1] = \frac{5}{6}y[0] + 2 = \frac{17}{6}$$

Thus,

$$\frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = \frac{-3}{5}$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = -\frac{11}{5}$$

and therefore,

$$c_1 = -1, c_2 = \frac{2}{5}$$

The total solution is

$$y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u[n].$$

2.28. ~~In~~

2.29. Determine the impulse response ~~$y[n], n\geq 0$, of~~ for the cascade of two linear time-invariant systems having impulse responses.

$$h_1[n] = a^n\,[u[n]-u[n-N]] \text{ and } h_2[n] = [u[n]-u[n-M]]$$

Sol:-

$$h[n] = h_1[n] * h_2[n]$$

$$= \sum_{k=-\infty}^{\infty} a^k [u[k]-u[k-N]]\,[u[n-k]-u[n-k-M]]$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k]\,u[n-k] - \sum_{k=-\infty}^{\infty} a^k u[k]\,u[n-k-M]$$

$$- \sum_{k=-\infty}^{\infty} a^k u[k-N]\,u[n-k] + \sum_{k=-\infty}^{\infty} a^k u[k-N]\,u[n-k-M]$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-M} a^k\right)$$

$$= 0$$

2.30. Determine the response $y[n]$, $n\geq 0$, of the system described by the second-order difference equation

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$$

to the input $x[n] = 4^n u[n]$.

Sol:-

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$$

The characteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0$$

Hence, $\lambda = 4, -1$ and

$$y_h[n] = c_1[n]4^n + c_2(-1)^n$$

Since 4 is a characteristic root and the excitation is

$$x[n] = 4^n u[n]$$

we assume a particular solution of the form

$$y_p[n] = kn4^n u[n].$$

Then

$$kn4^n u[n] - 3k[n-1]\,4^{n-1} u[n-1] - 4k[n-2]\,4^{n-2} u[n-2]$$

$$= 4^n u[n] + 2(4)^{n-1} u[n-1]$$

For $n = 2$,

$$k(32-12) = 4^2 + 8 = 24 \rightarrow k = \frac{6}{5}$$

The total solution is

$$y[n] = y_p[n] + y_h[n]$$

$$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2(-1)^n\right] u[n]$$

To solve for c_1 and c_2, we assume $y[-1] = y[-2] = 0$

then, $y[0] = 1$ and

$$y[1] = 3y[0] + 4 + 2 = 9$$

Hence,

$$c_1 + c_2 = 1 \text{ and}$$

$$\frac{24}{5} + 4c_1 - c_2 = 9$$

$$4c_1 - c_2 = \frac{21}{5}$$

Therefore,

$$c_1 = \frac{21}{25} \text{ and } c_2 = \frac{-1}{25}$$

the total solution is

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u[n]$$

~~27~~. 2.31) Determine the impulse response of the following causal system:

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$$

Sol:- the characteristic values from previous problem are $\lambda = 4, -1$. Hence

$$y_h[n] = c_1 4^n + c_2 [-1]^n$$

when $x[n] = \delta[n]$, we find that

$$y[0] = 1 \text{ and}$$

$$y[1] - 3y[0] = 2 \text{ or}$$

$$y[1] = 5.$$

Hence,

$$c_1 + c_2 = 1 \text{ and } 4c_1 - c_2 = 5$$

This yields, $c_1 = \frac{6}{5}$ and $c_2 = -\frac{1}{5}$. Therefore,

$$h[n] = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u[n]$$

2.32 Let $x[n]$, $N_1 \le n \le N_2$ and $h[n]$, $M_1 \le n \le M_2$ be two finite-duration signals.

a) Determine the range $L_1 \le n \le L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2.

Sol:- $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$

b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from the right. For convenience, assume that $h[n]$ has shorter duration than $x[n]$.

Sol:- Partial overlap from left:

low $N_1 + M_1$ high $N_1 + M_2 - 1$

Full overlap: low $N_1 + M_2$ high $N_2 + M_1$

Partial overlap from right:

low $N_2 + M_1 + 1$ high $N_2 + M_2$

c) Illustrate the validity of your results by computing the convolution of the signals

$$x[n] = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases} \qquad h[n] = \begin{cases} 2, & -1 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Sol:- $x[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h[n] = \{2, \underset{\uparrow}{2}, 2, 2\}$

$N_1 = -2$ $\qquad M_1 = -1$

$N_2 = 4$ $\qquad M_2 = 2$

Partial overlap from left: $n = -3$ $\quad n = -1$ $\quad L_1 = -3$

Full overlap: $n = 0$ $\quad n = 3$

Partial overlap from right: $n = 4$ $\quad n = 6$ $\quad L_2 = 6$

2.33 Determine the impulse response and the unit step response of the system described by the difference equation

a) $y[n] = 0.6y[n-1] - 0.08y[n-2] + x[n]$

Sol:- $y[n] - 0.6y[n-1] + 0.08y[n-2] = x[n]$

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2, 0.4$ Hence,

$$y_h[n] = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n$$

with $x[n] = \delta[n]$, the initial conditions are

$y[0] = 1$,

$y[1] - 0.6y[0] = 0 \Rightarrow y[1] = 0.6$.

Hence, $c_1 + c_2 = 1$ and

$$\frac{1}{5}c_1 + \frac{2}{5} = 0.6 \Rightarrow c_1 = -1, \; c_2 = 3.$$

Therefore. $h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n \right] u[n]$

the step response is

$$s[n] = \sum_{k=0}^{n} h[n-k], \; n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k} \right]$$

$$= \left\{ \frac{1}{0.12}\left[\left(\frac{2}{5}\right)^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1} - 1\right] \right\} u[n]$$

b) $y[n] = 0.7y[n-1] - 0.1y[n-2] + 2x[n] - x[n-2]$

Sol:- the characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = \frac{1}{2}, \frac{1}{5}$. Hence,

$$y_h[n] = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x[n] = \delta[n]$, we have

$$y[0] = 2,$$

$$y[1] - 0.7y[0] = 0 \Rightarrow y[1] = 1.4.$$

Hence, $c_1 + c_2 = 2$ and

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

these equations yield

$$c_1 = \frac{10}{3}, \; c_2 = -\frac{4}{3}.$$

$$h[n] = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n \right] u[n]$$

The step response is

$$s[n] = \sum_{k=0}^{n} h[n-k],$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\left(\frac{1}{2}\right)^n (2^{n+1} - 1)\right)u[n] - \frac{4}{3}\left(\frac{1}{5}\right)^n \left[5^{n+1} - 1\right]u[n]$$

2.34. consider a system with impulse response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the input $x[n]$ for $0 \leq n \leq 8$ that will generate the output sequence

$$y[n] = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0, \dots\}$$

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Sol:

$$h[n] = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right\}$$
$\uparrow$

$$y[n] = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$$

$$x[0]h[0] = y[0] \Rightarrow x[0] = 1$$

$$\frac{1}{2}x[0] + x[1] = y[1] \Rightarrow x[1] = \frac{3}{2},$$

By continuing this process, we obtain

$$x[n] = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \dots\right\}$$

2.35. Consider the interconnection of LTI systems as shown in Fig. P2.35

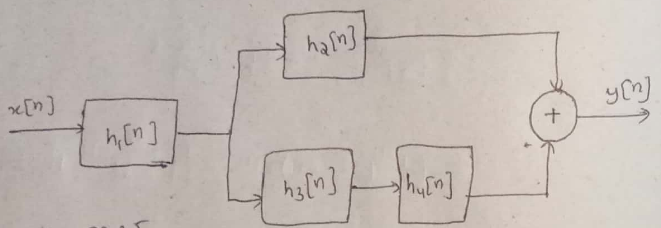


Fig P2.35

a) Express the overall impulse response in terms of $h_1[n]$, $h_2[n]$, $h_3[n]$, and $h_4[n]$.

$$h_o[n] = h_1[n] * [h_2[n] - h_3[n] * h_4[n]]$$

b) Determine $h[n]$ when

$$h_1[n] = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\right\}$$

$$h_2[n] = h_3[n] = [n+1]u[n]$$

$$h_4[n] = \delta[n-2]$$

Sol:

$$h_3[n] * h_4[n] = [n-1]u[n-2]$$

$$h_2[n] - h_3[n] * h_4[n] = 2u[n] - \delta[n]$$

$$h_1[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n-2]$$

Hence
$$h[n] = \left[\frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n-2]\right] * [2u[n] - \delta[n]]$$

$$= \frac{1}{2}\delta[n] + \frac{5}{4}\delta[n-1] + 2\delta[n-2] + \frac{5}{2}u[n-3]$$

c) Determine the response of the system in part (b) if

$$x[n] = \delta[n+2] + 3\delta[n-1] - 4\delta[n-3]$$

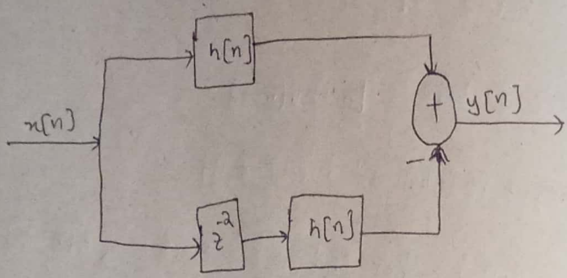
Sol:

$$x[n] = \{1, 0, 0, 3, 0, -4\}$$
$\uparrow$

$$y[n] = \left\{\frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots\right\}$$

2.36. Consider the system in given Fig. with $h[n] = a^n u[n]$, $-1 < a < 1$. Determine the response $y[n]$ of the system to the excitation

$$x[n] = u[n+5] - u[n-10]$$



Soln:- First, we determine

$$s[n] = u[n] * h[n]$$

$$s[n] = \sum_{k=0}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=0}^{n} h[n-k]$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1} - 1}{a-1}, \quad n \geq 0$$

For $x[n] = u[n+5] - u[n-10]$, we have the response

$$s[n+5] - s[n-10] = \frac{a^{n+6} - 1}{a-1} u[n+5] - \frac{a^{n-9} - 1}{a-1} u[n-10]$$

From figure,

$$y[n] = x[n] * h[n] - x[n] * h[n-2]$$

Hence, $y[n] = \dfrac{a^{n+6} - 1}{a-1} u[n+5] - \dfrac{a^{n-9} - 1}{a-1} u[n-10]$

$\sum^{\infty} x[n]\, x[n-l]$

$$= \frac{a^{n+4} - 1}{a-1} u[n+3] + \frac{a^{n-11} - 1}{a-1} u[n-12]$$

2.37. Compute and sketch the step response of the system.

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$$

Soln:- $h[n] = [u[n] - u[n-M]] / M$

$$s[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=0}^{\infty} h[n-k] = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

2.38. Determine the range of values of the parameter a for which the linear time-invariant system with impulse response

$$h[n] = \begin{cases} a^n, & n \geq 0,\ n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$ is stable.

Soln:- $$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0,\, n \text{ is even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

stable if $|a| < 1$.

2.39. Determine the response of the system with impulse response

$h[n] = a^n u[n]$

to the i/p signal $x[n] = u[n] - u[n-10]$

[Hint: The solution can be ~~determined~~ obtained easily and quickly by applying the linearity and time-invariance properties to the result in Example 2.3.5]

Sol:- $h[n] = a^n u[n]$. The response to $u[n]$ is

$$y_1[n] = \sum_{k=0}^{\infty} u[k] h[n-k]$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u[n]$$

Then, $y[n] = y_1[n] - y_1[n-10]$

$$= \frac{1}{1-a} \left[(1-a^{n+1}) u[n] - (1-a^{n-9}) u[n-10]\right]$$

2.40. Determine the response of the (relaxed) s/m characterized by the impulse response

$h[n] = \left(\frac{1}{2}\right)^n u[n]$

to the i/p signal

$$x[n] = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$$

Sol:- ~~we may use the rep~~

result from previous $a = \frac{1}{2}$.

So, $y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u[n] - 2\left[1 - \left(\frac{1}{2}\right)^{n-9}\right] u[n-10]$

2.41. Determine the response of the (relaxed) system characterized by the impulse response

$h[n] = \left(\frac{1}{2}\right)^n u[n]$ to the i/p signals

(a) $x[n] = 2^n u[n]$

(b) $x[n] = u[-n]$

Sol:- a) $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] \left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$$

b) $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

$$= \sum_{k=0}^{\infty} h[k] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \; n < 0$$

$$y[n] = \sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)^k$$

$$= 2 - \frac{\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(\frac{1}{2}\right)}$$

$$= 2\left(\frac{1}{2}\right)^n , n \geq 0$$

2.42. Three s/ms with impulse responses $h_1[n] = \delta[n] - \delta[n-1]$, $h_2[n] = h[n]$, and $h_3[n] = u[n]$, are connected in cascade.

a) what is the impulse response, $h_c[n]$, of the overall s/m?

Sol:-

$$h_c[n] = h_1[n] * h_2[n] * h_3[n]$$

$$= [\delta[n] - \delta[n-1]] * u[n] * h[n]$$

$$= [u[n] - u[n-1]] * h[n]$$

$$= \delta[n] * h[n]$$

$$h_c[n] = h[n]$$

2.43. a) Prove and explain graphically the difference b/w the relations $x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$ and $x[n] * \delta[n-n_0] = x[n-n_0]$

Sol:- $x[n]\,\delta[n-n_0] = x[n_0]$. Thus only the value of $x[n]$ at $n = n_0$ is of interest.

$x[n] * \delta[n-n_0] = x[n-n_0]$. Thus, we obtain the shifted version of the sequence $x[n]$.

b) show that a discrete-time s/m, which is described by a convolution summation, is LTI and relaxed.

Sol:-

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$= h[n] * x[n]$$

Linearity ; $x_1[n] \rightarrow y_1[n] = h[n] * x_1[n]$

$$x_2[n] \rightarrow y_2[n] = h[n] * x_2[n]$$

then $x[n] = \alpha x_1[n] + \beta x_2[n] \rightarrow y[n] = h[n] * x[n]$

$$y[n] = h[n] * [\alpha x_1[n] + \beta x_2[n]]$$

$$= \alpha h[n] * x_1[n] + \beta h[n] * x_2[n]$$

$$= \alpha y_1[n] + \beta y_2[n]$$

Time Invariance:

$$x[n] \rightarrow y[n] = h[n] * x[n]$$

$$x[n-n_0] \rightarrow y_1[n] = h[n] * x[n-n_0]$$

$$= \sum_k h[k]\,x[n-n_0-k]$$

$$= y[n-n_0]$$

c) what is the impulse response of the s/m described by $y[n] = x[n-n_0]$?

Sol:- $h[n] = \delta[n-n_0]$

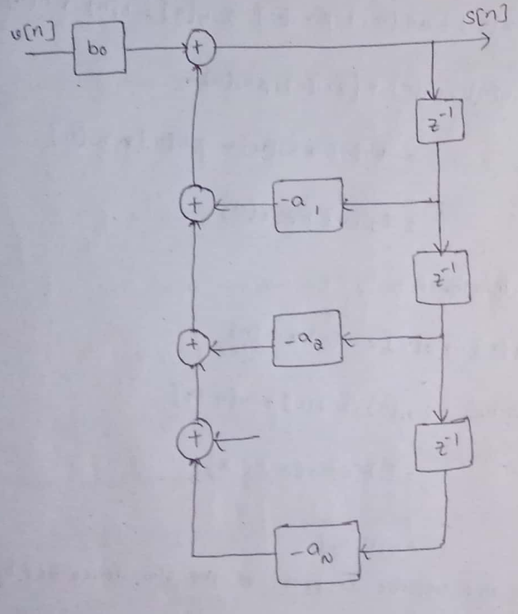
2.44. Two signals s[n] and v[n] are related through the following difference equations.

$$s[n] + a_1 s[n-1] + \cdots + a_N s(n-N) = b_0 v[n]$$

Design the block diagram realization of

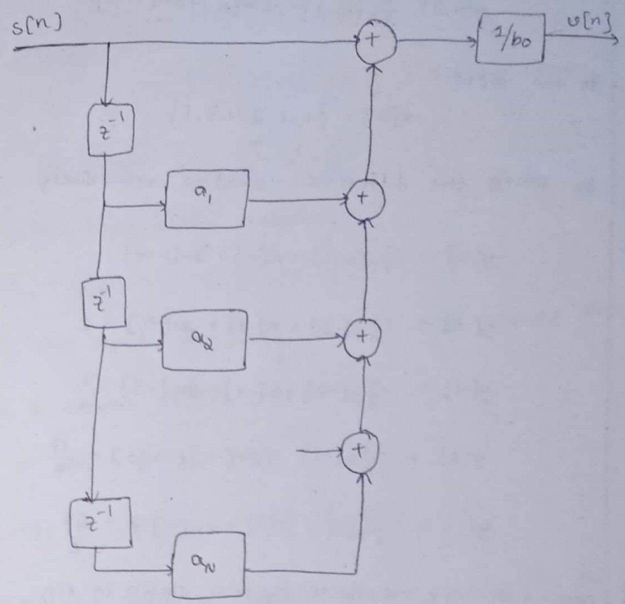
a) the s/m that generates s[n] when excited by v[n].

Sol:- $s[n] = -a_1 s[n-1] - a_2 s[n-2] - \ldots - a_N s(n-N) + b_0 v[n]$



b) The s/m that generates v[n] when excited by s(n).

Sol: $v[n] = \frac{1}{b_0}\left[s[n] + a_1 s[n-1] + a_2 s[n-2] + \ldots + a_N s[N-N]\right]$.



2.45. Compute the zero state response of the system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n] + 2x[n-2]$$

to the input

$$x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$$

by solving the difference equation recursively.

Sol:-

$$y[n] = -\frac{1}{2}y[n-1] + x[n] + 2x[n-2]$$

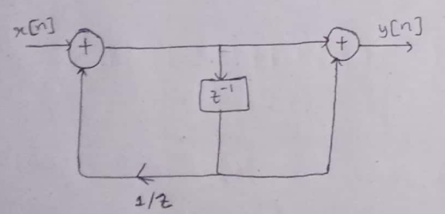
$$y[-2] = -\frac{1}{2}y[-3] + x[-2] + 2x[-4] = 1$$

$$y[-1] = -\frac{1}{2}y[-2] + x[-1] + 2x[-3] = \frac{3}{2}$$

$$y[0] = -\frac{1}{2}y[-1] + 2x[-2] + x[0] = \frac{17}{4}$$

$$y[1] = -\frac{1}{2}y[0] + x[1] + 2x[-1] = \frac{47}{8}, \text{ etc.}$$

2.47. Consider the discrete-system shown in fig



a) Compute the 10 first samples of its impulse response

Sol:-

$$x[n] = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$$

$$y[n] = \frac{1}{2}y[n-1] + x[n] + x[n-1]$$

$$y[0] = x[0] = 1$$

$$y[1] = \frac{1}{2}y[0] + x[1] + x[0] = \frac{3}{2}$$

$$y[2] = \frac{1}{2}y[1] + x[2] + x[1] = \frac{3}{4}.$$ Thus, we obtain

$$y[n] = \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \ldots\right\}$$

b) Find the input-output relation.

Sol:- $y[n] = \frac{1}{2}y[n-1] + x[n] + x[n-1]$

c) Apply the input $x[n] = \{\underset{\uparrow}{1}, 1, 1, \ldots\}$ and compute the first 10 samples of the output.

Sol:- As in part (a), we obtain

$$y[n] = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \ldots\right\}$$

d) Compute the first 10 samples of the output for the i/p given in part (c) by using convolution.

Sol:-

$$y[n] = u[n] * h[n]$$

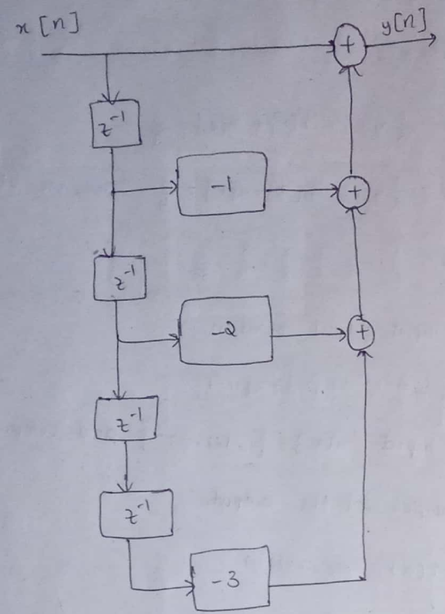
$$= \sum_k u[k]\, h[n-k]$$

$$= \sum_{k=0}^{n} h[n-k]$$

$$y[0] = h[0] = 1$$

$$y[1] = h[0] + h[1] = \frac{5}{2}$$

$$y[2] = h[0] + h[1] + h[2] = \frac{13}{4}. \text{ etc.}$$



e) Is the system causal ? Is it stable?

Sol:- from part (a), $h[n] = 0$ for $n < 0$ $\Rightarrow$ the s/m is causal

$$\sum_{n=0}^{\infty} |h[n]| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) = 4 \Rightarrow \text{s/m is stable.}$$

2.48. Consider the s/m described by the difference equation

$$y[n] = ay[n-1] + bx[n]$$

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h[n] = 1$$

Sol:- $y[n] = ay[n-1] + bu[n]$

$\Rightarrow h[n] = ba^n u[n]$

$$\sum_{n=0}^{\infty} h[n] = \frac{b}{1-a} = 1$$

$\Rightarrow b = 1-a$

b) Compute the zero-state step response $s[n]$ of the s/m and choose b so that $s[\infty] = 1$.

Sol:- $$s[n] = \sum_{k=0}^{n} h[n-k]$$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u[n]$$

$$s[\infty] = \frac{b}{1-a} = 1$$

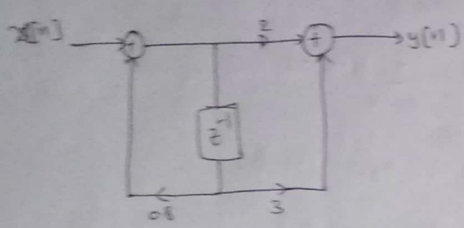
$\Rightarrow b = 1-a$

c) Compare the values of b obtained in parts (a) and (b). What did you notice?

Sol:- $b = 1-a$ in both cases.

2-49 A discrete-time s/m is realized by the structure shown in Fig.

a) Determine the impulse response



Sol:-

$$y[n] = 0.8y[n-1] + 2x[n] + 3x[n-1]$$

$$y[n] - 0.8y[n-1] = 2x[n] + 3x[n-1]$$

the characteristic equation is

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h[n] = c(0.8)^n$$

Let us first consider the response of the s/m.

$$y[n] - 0.8y[n-1] = x[n]$$

to $x[n] = \delta[n]$. Since $y[0] = 1$. It follows that $c = 1$.

then, the impulse response of the original s/m is

$$h[n] = 2(0.8)^n u[n] + 3(0.8)^{n-1} u[n-1]$$

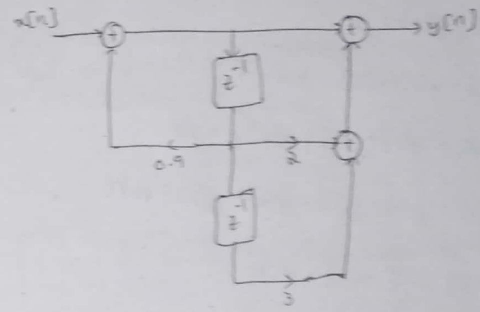
$$= 2\delta[n] + 4.6(0.8)^{n-1} u[n-1]$$

b) Determine a realization for its inverse system, that is, the system which produces $x[n]$ as an output when $y[n]$ is used as an input.

Sol:- the inverse s/m is characterized by the difference equation

$$x[n] = -1.5x[n-1] + \frac{1}{2}y[n] - 0.4y[n-1]$$

2-50. consider the discrete-time system shown in Fig



a) compute the first 6 values of the impulse response of the s/m.

Sol:- For $x[n] = \delta[n]$, we have

$$y[0] = 1.$$

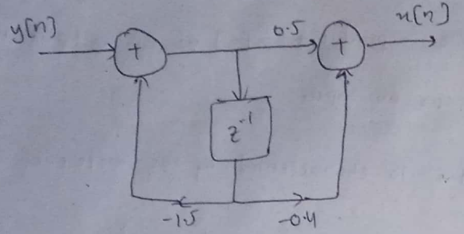
$$y[1] = 2.9$$

$$y[2] = 5.61$$

$$y[3] = 5.049,$$

$$y[4] = 4.544,$$

$$y[5] = 4.090 \ldots$$



b) compute the first 6 values of the zero-state step response of the slm.

Sol:- $s[0] = y[0] = 1$

$s[1] = y[0]+y[1] = 3.91$

$s[2] = y[0]+y[1]+y[2] = 9.51$

$s[3] = y[0]+y[1]+y[2]+y[3] = 14.56$

$s[4] = \sum_{0}^{4} y[n] = 19.10$

$s[5] = \sum_{0}^{5} y[n] = 23.19$

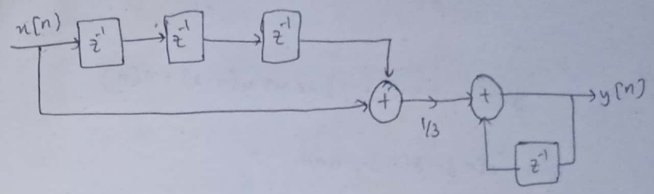
c) Determine an analytical expression for the impulse response of the slm.

Sol:- $h[n] = (0.9)^n u[n] + 2(0.9)^{n-1} u[n-1] + 3(0.9)^{n-2} u[n-2]$

$= \delta[n] + 2.9\delta[n-1] + 5.61\,(0.9)^{n-2} u[n-2]$

2.51. Determine and sketch the impulse response of the following slms for $n = 0, 1, \ldots, 9$.

a)

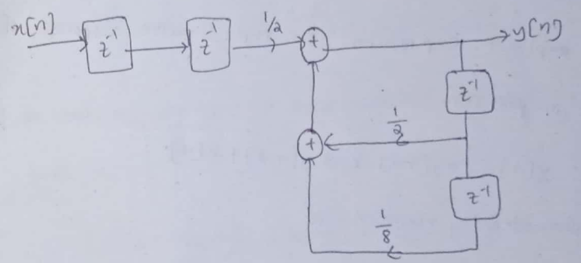


Sol:- $y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-3] + y[n-1]$

for $x[n] = \delta[n]$, we have

$h[n] = \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \ldots\right\}$

b)



Sol:- $y[n] = \frac{1}{2}y[n-1] + \frac{1}{8}y[n-2] + \frac{1}{2}x[n-2]$

with $x[n] = \delta[n]$, and

$y[-1] = y[-2] = 0$, we obtain

$h[n] = \left\{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024}, \ldots\right\}$

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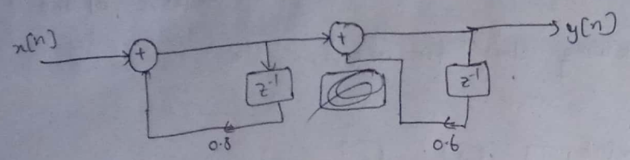
c)



Sol:-

$$y[n] = 1.4y[n-1] - 0.48\,y(n-2) + x[n]$$

with $x[n] = \delta[n]$, and

$y[-1] = y[-2] = 0$, we obtain

$$h[n] = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086, \dots\}$$

d) classify the systems above as FIR or IIR.

Sol:- All 3 s/ms are IIR.

e) Find an explicit expression for the impulse response of the s/m in part (c).

Sol:-

$$y[n] = 1.4y[n-1] - 0.48\,y[n-2] + x[n]$$

the characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ Hence

$\lambda = 0.8, 0.6$. and

$$y_h[n] = c_1[0.8]^n + c_2[0.6]^n$$

For $x[n] = \delta[n]$. we have

$c_1 + c_2 = 1$ and

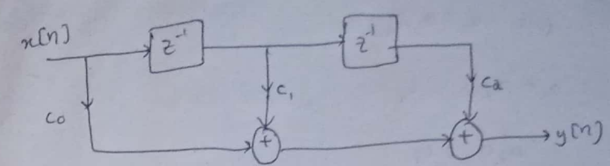
$0.8c_1 + 0.6c_2 = 1.4$

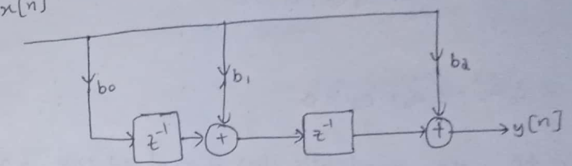
$\Rightarrow c_1 = 4$

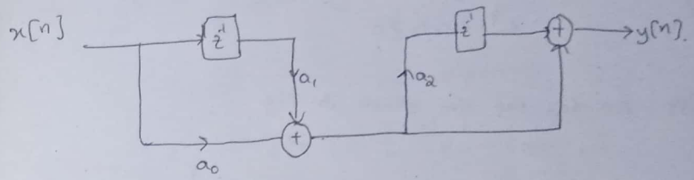
$c_2 = -3$. Therefore

$$h[n] = [4(0.8)^n - 3(0.6)^n]u[n]$$

2.52 Consider the s/ms shown in fig







a) Determine and sketch their impulse response $h_1[n], h_2[n]$ and $h_3[n]$.

Sol:-

$$h_1[n] = c_0\delta[n] + c_1\delta[n-1] + c_2\delta[n-2]$$

$$h_2[n] = b_2\delta[n] + b_1\delta[n-1] + b_0\delta[n-2]$$

$$h_3[n] = a_0\delta[n] + (a_1 + a_0a_2)\delta[n-1] + a_1a_2\delta[n-2]$$

b) Is it possible to choose the coefficients of these s/ms in such a way that

$$h_1[n] = h_2[n] = h_3[n]$$

Sol:- The only question is whether

$h_3[n] = h_2[n] = h_1[n]$

Let $a_0 = c_0$,

$a_1 + a_2 c_0 = c_1$

$a_2 a_1 = c_2$

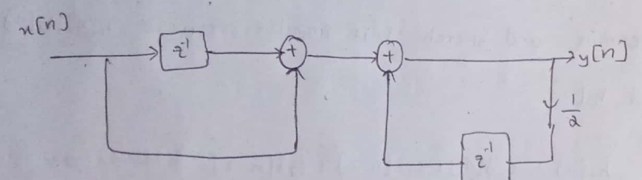
Hence $\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

For $c_0 \neq 0$, the quadratic has a real solution if and only if

$c_1^2 - 4c_0 c_2 \geq 0$

2.53 Consider the s/m shown in Fig



a) Determine the impulse response h[n].

Sol:- $y[n] = \frac{1}{2} y[n-1] + x[n] + x[n-1]$

For $y[n] - \frac{1}{2} y[n-1] = \delta[n]$, the solution is

$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]$

b) Show that h[n] is equal to the convolution of the following signals:

$h_1[n] = \delta[n] + \delta[n-1]$

$h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

Sol:- $h_1[n] * [\delta[n] + \delta[n-1]] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]$

2.54. Compare and sketch the ~~signal~~ convolution $y_i[n]$ and correlation $r_i[n]$ sequences for the following pair of signals and comment on the results obtained.

a) $x_1[n] = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1[n] = \{1, \underset{\uparrow}{1}, 1, 1, 1\}$

Sol:-

	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution:-

$\Rightarrow y[n] = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

Correlation:-

$r_1[n] = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) $x_2[n] = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $h_2[n] = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

Sol:- convolution $y_1[n] = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

correlation : $r_1[n] = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, 2\}$

Note that $y_2[n] = r_2[n]$, because $h_2[-n] = h_2[n]$

c) $x_3[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_3[n] = \{4, 3, 2, 1\}$

Sol:- Convolution, $y_3[n] = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation, $r_3[n] = \{1, 4, 10, 20, 25, 24, \underset{\uparrow}{16}\}$

d) $x_4[n] = \{1, 2, 3, 4\}$ $\quad h_4[n] = \{1, 2, 3, 4\}$

Sol:- Convolution $y_4[n] = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

correlation $r_4[n] = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

Note that $h_3[-n] = h_4[n+3]$.

hence $r_3[n] = y_4[n+3]$.

and $h_4[-n] = h_3[n+3]$

$\Rightarrow r_4[n] = y_3[n+3]$.

2.55 The zero-state response of a causal LTI s/m to the i/p $x[n] = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y[n] = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$. Determine its impulse response.

Sol:- obviously, the length of $h[n]$ is 2,

i.e.,

$$h[n] = \{h_0, h_1\}$$

$$h_0 = 1$$

$$3h_0 + h_1 = 4$$

$$\Rightarrow h_0 = 1,\ h_1 = 1$$

2.56 Prove by direct substitution the equivalence of equations. $w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n]$ and $y[n] = \sum_{k=0}^{M} b_k w[n-k]$, which describe the direct form II structure, to the relation.

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$, which describes the direct form I structure.

Sol:-

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \rightarrow (1)$$

$$w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n] \rightarrow (2)$$

$$y[n] = \sum_{k=0}^{M} b_k w[n-k] \rightarrow (3)$$

from eq (2) we obtain

$$x[n] = w[n] + \sum_{k=1}^{N} a_k w[n-k] \rightarrow (4)$$

By substituting eq (3) for $y[n]$ and eq (4) into (1)

we obtain L.H.S = R.H.S

2.57 Determine the response $y[n]$, $n \geq 0$ of the s/m described by the second-order difference equation

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$$

where the i/p is

$$x[n] = (-1)^n u[n]$$

and the initial conditions are $y[-1] = y[-2] = 0$

Sol:-

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$$

the characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence,

$$y_h[n] = c_1 2^n + c_2 n 2^n$$

the particular solution is

$$y_p[n] = k(-1)^n u[n].$$

Substituting this solution into the difference equation we obtain

$$k(-1)^n u[n] - 4k(-1)^{n-1} u[n-1] + 4k(-1)^{n-2} u[n-2]$$

$$= (-1)^n u[n] - (-1)^{n-1} u[n-1]$$

For $n = 2$, $k[1+4+4] = 2 \Rightarrow k = \frac{2}{9}$. The total solution is

$$y[n] = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u[n]$$

From the initial conditions, we obtain $y[0] = 1, y[1] = 2$.

Then, $c_1 + \frac{2}{9} = 1$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow \frac{14}{9} - \frac{2}{9} + 2c_2 = 2 \Rightarrow 2c_2 = 2 - \frac{12}{9}$$

$$2c_2 = \frac{6}{9}$$

$$\Rightarrow c_2 = \frac{3}{9} = \frac{1}{3}$$

2.58 Determine the impulse response $h[n]$ for the s/m described by the second-order differential equation

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$$

Sol:- from previous problem

$$h[n] = [c_1 2^n + c_2 n 2^n] u[n]$$

with $y[0] = 1$, $y[1] = 3$, we've

$$c_1 = 1$$

$$2c_1 + 2c_2 = 3$$

$$\Rightarrow c_2 = \frac{1}{2}$$

Thus, $h[n] = \left[2^n + \frac{1}{2} n 2^n\right] u[n]$.

2.59. Show that any discrete-time signal $x[n]$ can be expressed as

$$x[n] = \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]]\, u[n-k]$$

where $u[n-k]$ is a unit step delayed by k units in time, that is

$$u[n-k] = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

Sol:-

$$x[n] = x[n] * \delta[n]$$

$$= x[n] * [u[n] - u[n-1]]$$

$$= [x[n] - x[n-1]] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]]\, u[n-k]$$

2.60 Show that the o/p of an LTI s/m can be expressed in terms of its unit step response $s[n]$ as follows

$$y[n] = \sum_{k=-\infty}^{\infty} [s[k]-s[k-1]]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} [x[k]-x[k-1]]\, s[n-k]$$

Sol:- Let $h[n]$ be the impulse response of the s/m.

$$s[k] = \sum_{m=-\infty}^{k} h[m]$$

$$\Rightarrow h[k] = s[k]-s[k-1]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} [s[k]-s[k-1]]\, x[n-k]$$

2.61. Compute the correlation sequences $r_{xx}[l]$ and $r_{xy}[l]$ for the following signal sequences.

$$x[n] = \begin{cases} 1, & n_0-N \le n \le n_0+N \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

Sol:-

$$x[n] = \begin{cases} 1, & n_0-N \le n \le n_0+N \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$$

the range of non-zero values of $r_{xx}[l]$ is determined by

$$n_0-N \le n \le n_0+N$$

$$n_0-N \le n-l \le n_0+N$$

which implies

$$-2N \le l \le 2N$$

For a given shift l, the number of terms in the summation for which both $x[n]$ and $x[n-l]$ are non-zero is $2N+1-|l|$, and the value of each term is 1.

Hence,

$$r_{xx}[l] = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

For $r_{xy}[l]$ we've

$$r_{xy}[l] = \begin{cases} 2N+1-|l-n_0|, & n_0-2N \le l \le n_0+2N \\ 0, & \text{otherwise} \end{cases}$$

2.62. Determine the autocorrelation sequences of the following signals

a) $x[n] = \{1, 2, 1, 1\}$ (with ↑ under the first element)

Sol:-

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$$

$$r_{xx}[-3] = x[0]\, x[3] = 1$$

$r_{xx}[-2] = x[0]x[2] + x[1]x[3] = 3$

$r_{xx}[-1] = x[0]x[1] + x[1]x[2] + x[2]x[3] = 5$

$r_{xx}[0] = \sum_{n=0}^{3} x^2[n] = 7$

Also $r_{xx}[-l] = r_{xx}[l]$

Therefore $r_{xx}[l] = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

b) $y[n] = \{1, 1, 2, 1\}$

Sol:- $r_{yy}[l] = \sum_{n=-\infty}^{\infty} y[n]y[n-l]$

we obtain

$r_{yy}[l] = \{1, 3, 5, 7, 5, 3, 1\}$

we observe that $y[n] = x[-n+3]$, which is equivalent to reversing the sequence $x[n]$. This has not changed the autocorrelation sequence.

2.63. what is the normalized autocorrelation sequence of the signal $x[n]$ given by

$$x[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

Sol:- $r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l]$

$$= \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

$r_{xx}[0] = 2N+1$

therefore, the normalized autocorrelation is

$$\rho_{xx}[l] = \frac{1}{2N+1}\{2N+1-|l|\}, \quad -2N \le l \le 2N$$

$$= 0 \quad , \text{otherwise}$$

2.64. An audio signal $s[t]$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$$x[n] = s[n] + r_1 s[n-k_1] + r_2 s[n-k_2]$$

where k_1 and k_2 are the delays of the two echoes.

a) Determine the autocorrelation $r_{xx}[l]$ of the signal $x[n]$.

Sol:- $r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l]$

$$= \sum_{n=-\infty}^{\infty} [s[n] + r_1 s[n-k_1] + r_2 s[n-k_2]]$$

$$* [s[n-l] + r_1 s[n-l-k_1] + r_2 s[n-l-k_2]]$$

$$= [1 + r_1^2 + r_2^2] r_{ss}[l] + r_1 [r_{ss}[l+k_1] + r_{ss}[l-k_1]]$$

$$+ r_2 [r_{ss}[l+k_2] + r_{ss}[l-k_2]]$$

$+ \gamma_1 \gamma_2 \left[\gamma_{nn}[l+k_1-k_2] + \gamma_{nn}[l+k_2-k_1] \right]$

b) can we obtain γ_1, γ_2, k_1, and k_2 by observing $\gamma_{nn}[l]$?

Sol: $\gamma_{nn}[l]$ has peaks at $l=0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$.

Suppose that $k_1 < k_2$. then, we've determine γ_1 and k_1.

The problem is to determine γ_2 and k_2 from the other peak

c) what happens if $\gamma_2 = 0$?

Sol: If $\gamma_2 = 0$, the peaks occur at $l=0$ and $l = \pm k_1$.

then, it is easy to obtain γ_1 and k_1.