

Today's content

- (i) Unique element
- (ii) Sqrt of a number
- (iii) Search in sorted and rotated array.

10. Every element occurs twice except for one element, find the unique element.

Note: Duplicates are adjacent to each other.

ex: ar:

0	1	2	3	4	5	6	7	8
3	3	1	1	8	8	10	10	19

9	10	11	12	13	14
6	6	2	2	4	4

← Before unique → ← After unique ele →

[1st occurrence lies on even index] [1st occurrence lies on odd index].

idea:

using xor.	}	TC: $O(N)$.
linear search		SC: $O(1)$.

Idea 2:

Can we apply BS?

Case 1:



If $ar[mid]$ is unique.

if $(ar[mid-1] \neq ar[mid] \text{ \& } ar[mid+1] \neq ar[mid])$,

Make mid land on 1st occurrence.

(i) if $(ar[mid-1] == ar[mid])$

{
mid = mid-1

}

(ii) if $(mid \% 2 == 0)$

{
// we are left side of unique ele
goto right

}

else

{
// we are on right side of unique ele
goto left

}

(8 8).



arr. \rightarrow [0 1 2 3 4 5 6 7 8 9 10 11 12 13 14]
 [3 3 1 1 8 8 10 10 19 6 6 2 2 4 4]
 L
 h.

l	h	m	isUnique	1 st occurrence (arr[m] == arr[m-1]).
0	14	7	x	m = m-1 m = 6, m%2 = 0, goto right, l = m+2.
8	14	11	x	m = 11, m%2 = 1, goto left, h = m-1.
8	10	9	x	m = 9, m%2 = 1, goto left, h = m-1.
8	8	8	✓	return arr[m].

Code:

```
def uniqueElement(arr, n)
{
```

```
    if (n == 1)
        return arr[0]; } There's only one element.
```

```
    if (arr[0] != arr[1])
        return arr[0]
```

```
    if (arr[n-1] != arr[n-2])
        return arr[n-1].
```

```
[0          n-1]
l           h.
↑           ↑
m = 0,      m = n-1.
```

```
    l = 1, h = n-2.
```

```
    while (l <= h).
```

```
        m = (l+h)/2;
```

```
        if (arr[m] != arr[m-1] && arr[m] != arr[m+1]).
```

```
            return arr[m] // unique element.
```

```
        if (arr[m] == arr[m-1])
```

```

        m = m-1 // since arr[m] is second occurrence.
    if (m%2 == 0)
        // unique element lies on right
        l = m+2
    else
        h = m-1
}
}

```

Doubts.

To avoid overflow.

$$m = \frac{l+h}{2}, \quad m = l + \frac{(h-l)}{2}, \quad l=126, \quad h=127$$

inds. \longrightarrow 128.

$l = \text{integer}$
 $h = \text{integer}$

$$m = 128 + \frac{(128-128)}{2} = \underline{\underline{128}}$$

$$\begin{aligned}
 m &= \frac{l+h}{2} \\
 &= \frac{128+128}{2} = \frac{256}{2} \\
 &= \underline{\underline{\quad}}
 \end{aligned}$$

$$\begin{aligned}
 m &= 126 + \frac{(127-126)}{2} = 126 + 0.5 \\
 &\equiv \underline{\underline{126}}
 \end{aligned}$$

28. Given a +ve number N , find $\text{sqrt}(N)$.

↳ Greatest i such that $i * i \leq N$.
 ↓
 integer.

Ex: $\text{sqrt}(25) = 5$
 $\text{sqrt}(30) = 5$

Idea:

$N = 30$.

i	$i * i \leq N$	ans.
1	✓	1
2	✓	2
3	✓	3
4	✓	4
5	✓	5
6	✗	return 5.

Idea 2:

Binary search

- Target : $\text{sqrt}(N)$.
- Search space : $1 \dots N$.

[$\text{sqrt}(N)$ will definitely lie in $[1 \dots N]$]

Can I apply BS?

Case 1:

$\text{mid} * \text{mid} = N$,
 return mid.

$\text{mid} * \text{mid} = N$.



Case 2:

$\text{mid} * \text{mid} < N$
 $\text{ans} = \text{mid}$
 goto right.

$\text{mid} * \text{mid} < N$



Case 3:

$\text{mid} * \text{mid} > N$
 goto left.

$\text{mid} * \text{mid} > N$.



Tracing.

[1 _ _ _ 6 7 9 _ _ _ 11 12 _ 24 25 _ 49 50 _ 100]

$N=100$, $ans=1$.

l	h	m	update answer.
1	100	50	$50 * 50 > 100$, goto left, $h = m - 1$
1	49	25	$25 * 25 > 100$, goto left, $h = m - 1$.
1	24	12	$12 * 12 > 100$, goto left, $h = m - 1$.
1	11	6	$6 * 6 < 100$, $ans = 6$, goto right, $l = m + 1$.
7	11	9	$9 * 9 < 100$, $ans = 9$, goto right, $l = m + 1$.
10	11	10	$10 * 10 = 100$, return ans.

Hlw, do a dry for $N=99$, $ans=9$.

code:

Hlw.

Break.

10:13:00

10:19:00

10:20:00

38. Given a sorted and rotated array, search for an element k .

ex: ① ar : [10 28 30 80 90 8 9], $k = 30$.

[8 9 10 28 30 80 90] 8 9, O/P = 2.

② ar : [200 300 400 10 20], $k = 40$.

[10 20 200 300 400] 10 20, O/P = -1.

Idea: linear search for the element k .

TC: $O(N)$, SC: $O(1)$.

Idea 2:

[50 100 150 200 500 5 10].

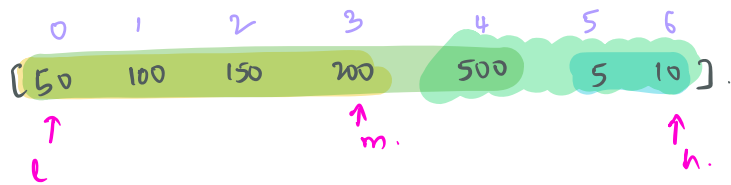
$x = 50$..

Claim: Put the middle at any position, one half of the array is sorted.

Question: 1) Which half is sorted?

compare mid element with $ar[0]$ ($ar[l]$).

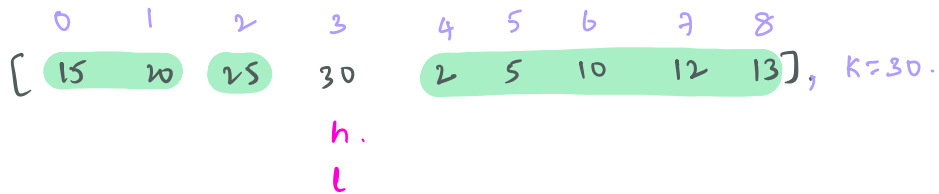
2) Can you figure out in which half the element ' k ' lies in?



$k = 50 \rightarrow [200 > 50, 1^{st} \text{ half is sorted, } 50 \text{ lies in } [50--200], \text{ Search left}]$

$k = 5 \rightarrow [200 > 5, 1^{st} \text{ half is sorted, } 5 \text{ does lie in } [50--200], \text{ Search right}]$

Tracing



l	h	$m = \left(\frac{l+h}{2}\right)$	Sorted Part	k lies in which part	goto	update
0	8	4	R	L	left	$h = m-1$
0	3	1	L	R	right	$l = m+1$
2	3	2	L	R	right	$l = m+1$
3	3	3	$(k = ar(mid)) \Rightarrow \text{return mid.}$			

def SortedAndRotatedSearch (ar, n, k)

{

l = 0, h = n-1

while (l ≤ h)

{

m = (l+h)/2

if (ar[m] == k)

return m

// left half is sorted.

if (ar[m] ≥ ar[l])

// check if k lies in left half.

if ($k \geq ar[l] \ \&\& \ k \leq ar[m]$) // lies in first half.

h = m-1

else

l = m+1

else // right half is sorted.

// check if k lies in right half.

if ($k \geq ar[m] \ \&\& \ k \leq ar[h]$) // lies in right half.

l = m+1

else

h = m-1

}

return -1

}

