Continuous Mathematical Foundations: Week 3 - Discrete Probability Distributions

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Discrete Distribution

In order to describe a discrete distribution we use a probability (mass) function f, that corresponds each possible outcome X to a probability,

$$f : X \to [0, 1].$$

For all possible outcomes in X, the sum of the probabilities are equal to 1,

$$\sum_{x \in X} f(x) = 1.$$

Bernoulli Distribution

We use Bernoulli Distribution B(p) to describe a random phenomenon that has only two possible outcomes (0-1),

$$f(1) = p, \;\; f(0) = 1 - p,$$

$$f(x) = p^x (1-p)^{1-x}, \;\; x = 0, 1.$$

Binomial Distribution

We use Binomial Distribution B(n,p) to describe a random phenomenon that is characterized by repeated Bernoulli tests.

$$f(x) = C(n,x)p^x(1-p)^{1-x},\ x = 0,1,2,3\ldots,n$$

Examples

- Coin toss is described by Bernoulli Distribution with $p = \frac{1}{2}$.
- The sum of heads/tails after 6 repeated coin tosses is described by Binomial Distribution with $n = 6$ and $p = \frac{1}{2}$.

Examples

- The probability of having an accident is described by Bernoulli Distribution with p.
- The sum of accidents in a driver population is described by Binomial Distribution with n equal to the population size and p the probability of accident.

Geometric Distribution

We use Geometric Distribution G(p) to describe a random phenomenon where we repeat a Bernoulli experiment until the wanted outcome comes.

$$f(x) = (1-p)^{x-1}p,\ x = 0, 1, 2, 3\ldots$$

Examples

- Coin tosses until Heads/Tails is described by Geometric Distribution with $p = \frac{1}{2}$.
- Throwing darts till you hit bulls-eye is described by Geometric Distribution with p the probability on hitting bulls-eye.

Hypergeometric Distribution

We use Hypergeometric Distribution H(N,K,n) to describe a random phenomenon where we repeat a Bernoulli experiment, but the outcomes are not repeatable (they appear only once).

$$f(x) = \frac{C(K,x)C(N-K,n-x)}{C(N,n)}, \; x = 0,1,2,3\ldots, min(n,k)$$

Examples

- Drawing a number of red/black cards from a classic card deck can be described by Hypergeometric Distribution.
- Drawing a number of odd/even balls from a box with randlomly numbered balls can be described by Hypergeometric Distribution.