Continuous Mathematical Foundations: Continuous Probability Distributions

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Random Variable

In the previous class we discussed about the notation of distributions such as $B(p)$, $B(n,p)$, $G(p)$ and $H(N,K,n)$.

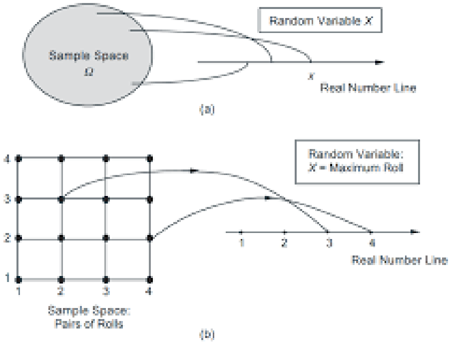
We have also declared random phenomena as coins with the notation $Coin_1 = 0, 1$ for heads or tails encoded.

Random Variable

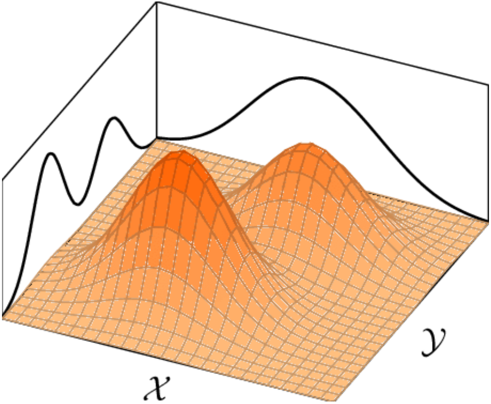
In general when we want to describe a random variable for a coin, dice, etc, we will use capital letters as notation, X, Y,. . . ,
i.e. $C_1 = 1$ for coin being equal to heads, $D_1 = 3$ for dice equal to 3,. . .

If a Random Variable X follows a known distribution like Bernoulli, we will write $X \sim B(p)$.

Random Variable in comparison with sample space



Random Variable in comparison with sample space



From Bernoulli to Binomial Distribution

From what we discussed in the previous class,

$$X_1, X_2, \ldots, X_n \sim B(p) \Rightarrow \sum_{i=1}^{n} X_i \sim B(p, n).$$

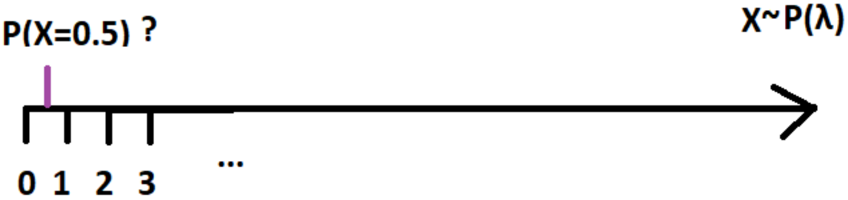
From Binomial to Poisson Distribution

Later mathematicians wanted to see what happens when $n \to +\infty$ for the previous sum and they noticed,

$$X = \lim_{n \to +\infty} \sum_{i=1}^{n} X_i \sim P(\lambda).$$

From Poisson to Exponential Distribution

So then people thought, ok, why not to search what happens in between the observations?



Disccrete vs Continuous Probability Distributions

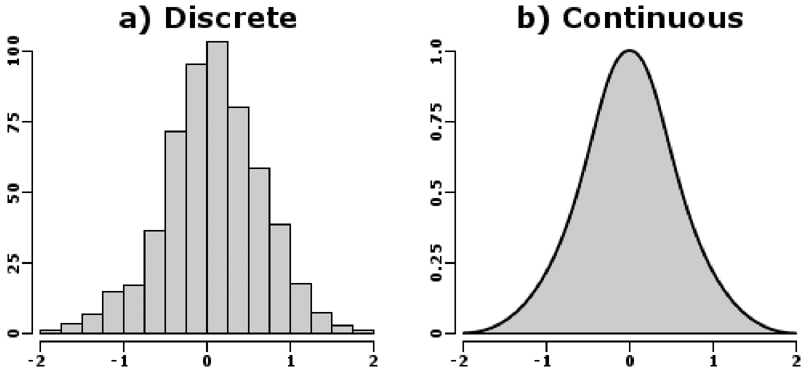
	Discrete	Continuous
Probability Distribution	x_1 p_1 x_2 p_2 $\cdots$ $\cdots$ x_n p_n	pdf: f(x)
F(x)	$\sum_{i=1}^{n} p_i = 1$	$\int_{-\infty}^{\infty} f(x)dx = 1$
Mean μ	$\sum_{i=1}^{n} x_i p_i$	$\int_{-\infty}^{\infty} xf(x)dx$
Variance σ^2	$\sum_{i=1}^{n} (x_i - \mu)^2 p_i$	$\int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Continuous Probability Distributions

In continuous probability distributions we can't calculate probabilities directly through functions like in the discrete case.

We have the probability density function f and in order to calculate probability we integrate it in respective intervals.

Geometric Comparison



Uniform Distribution U(a,b)

$$f(x) = \frac{1}{b-a}, x \in [a, b]$$

Exponential Distribution Exp(λ)

$$f(x) = \lambda e^{-\lambda x}, x \in [0, \infty)$$

Gaussian/Normal Distribution

https://georgstag.shinyapps.io/gaussiand/?_ga=2.195706476.1111206572.1636210550-382330877.1634652153

Goals of the Week

Read for the coming Quiz.