Continuous Mathematical Foundations: Week 2 - Set Theory

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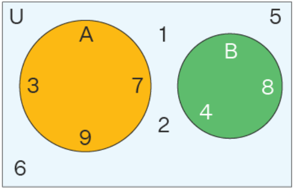
What is a Set?

Set Definition

In mathematics, a set is a collection of elements.

Simply, it is something that is consisted from other components.



Set Notation

We use capital letters to notate sets, such as A,B,C,Γ,Ω.

We use the following notation in order to describe what are the components $a_1, a_2, ...$ of a set A.

$$A = \{a_1, a_2, ...\}$$

Set Components

What can a set contain?

Anything, i.e. numbers, words and Anything else you might imagine (abstract).

Examples,

- $A = \{1, 2, 3\}$,
- $B = \{4, \sqrt{12}\}$,
- $\Gamma = \{\frac{17}{7}, \text{``}\textit{apple}\text{''}, \text{``}\textit{disco}\text{''}\}$.

Abstract Set example

If the set Ω contains all the sets that don't contain their subsets, is Ω inside Ω?

Why we need Sets?

Set Theory is highly used both in Computer Science and in Probability Theory as we will see next week.

In general, the probability of a set A in comparison with the all the possible outcomes Ω, is defined as

$$p = \frac{|A|}{|\Omega|},$$

where $|\cdot|$ returns the number of elements in a set.

Empty Set

By empty set $\emptyset$ we mean a set that contains nothing.

Complementary Set

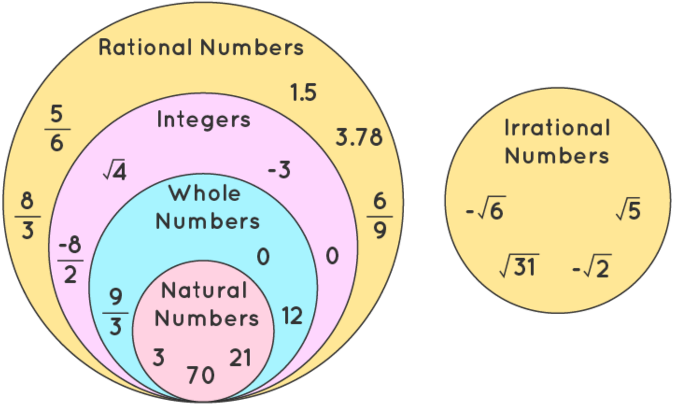
The complementary set A^c or A' of a set A in a space Ω is equal to the elements of Ω without the elements of A,

$$A^c = \Omega \setminus A.$$

For complementary sets, Ω must contain A, or else A being a subset of Ω.

Number sets

Number sets are sets that contain only numbers.



Set equality

Two sets A, B are equal if and only if they have the same components,

$$A \equiv B.$$

Set Comparison

A is a subset of B, if and only if B has all the components of A,

$$A \subseteq B,$$

or

$$B \supseteq A.$$

Set Comparison Properties

- If $A \equiv B$, then A is subset of B and B is subset of A.
- If A is subset of B and B is subset of A, then $A \equiv B$.
- If A has k components, then the number of subsets of A is 2^k.
- If A is not subset of B and B is not subset of A, then you can't compare them.

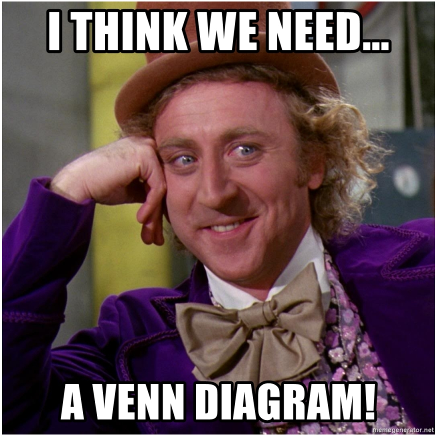
Cardinality

Cardinality of a set A, $card(A)$, is the number of components of A.

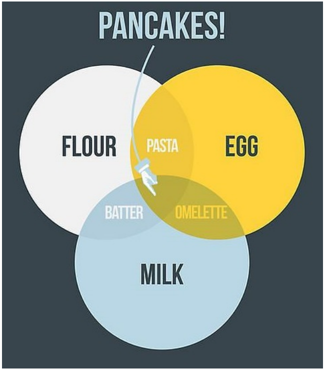
Cardinality of infinite sets

- If $A \equiv B$, then A is subset of B and B is subset of A.
- If A is subset of B and B is subset of A, then $A \equiv B$.
- If A has k components, then the number of subsets of A is 2^k.
- If A is not subset of B and B is not subset of A, then you can't compare them.

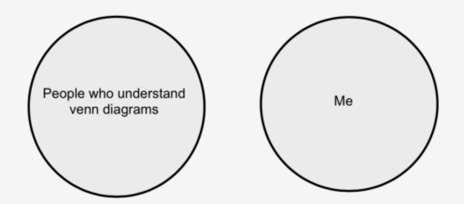
Set Representation

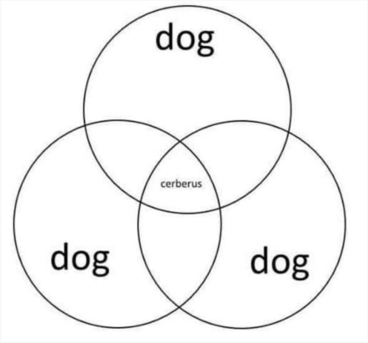


Venn Diagrams

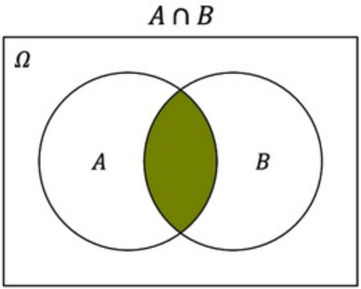


Funny Venn Diagrams

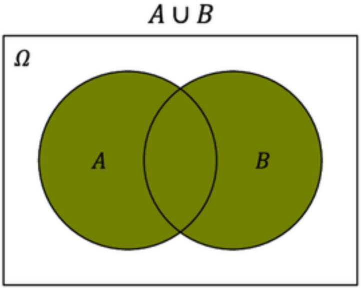




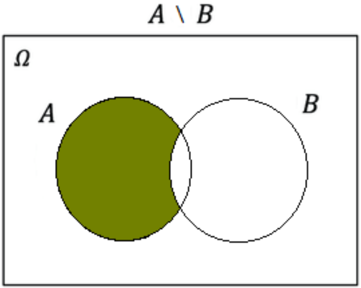
Intersection $A \cap B$



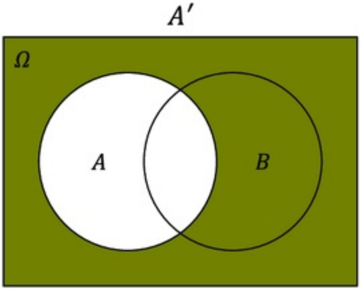
Union $A \cup B$



Complement $A \setminus B$



Complementary $A' = \Omega \setminus A$



Goals of the Week t-shirt

Goals of the Week

- Understand what sets are.
- Interpret Venn Diagrams.
- Being able to perform set operations.