Penyederhanaan Gerbang Logika

Latihan Soal

Sederhanakan fungsi logika berikut lalu buatlah tabel kebenaran serta lengkapi dengan gambar gerbang logika sebelum dan sesudah penyederhanaan

$$z = ABC + A\overline{B} \cdot (\overline{\overline{A}\,\overline{C}})$$

Teorema Dasar Boole

1. Operasi dengan 0 dan 1	$x + 0 = x$ $\quad x.0 = 0$ $x + 1 = 1$ $\quad x.1 = x$
2. Hukum Idempoten	$x + x = x$ $x.x = x$
3. Hukum Involusi	$\bar{\bar{x}} = x$
4. Hukum Komplement	$x + \bar{x} = 1$ $\quad x.\bar{x} = 0$
5. Hukum Kumutatif	$x + y = y + x$ $x.y = y.x$
6. Hukum Assosiatif	$(x+y)+z = x+(y+z) = x+y+z$ $(xy)z = x(yz) = xyz$
7. Hukum Distributif	$x(y+z) = xy+xz$ $x+yz = (x+y)(x+z)$

Teorema Tambahan Boole

1. Teorema penyederhanaan:	$x\,y + x\,\bar{y} = x$ $x + x\,y = x$ $(x + \bar{y})\,y = x\,y$ $(x+y)(x+\bar{y}) = x$ $x\,(x+\bar{y}) = x$ $x\bar{y} + y = x + y$
2. Hukum de Morgan:	$\overline{x + y + z + \ldots} = \bar{x}\,\bar{y}\,\bar{z}\ldots$ $\overline{x \,.\, y \,.\, z \,.\, \ldots} = \bar{x} + \bar{y} + \bar{z} + \ldots$
3. Teorema Konsensus:	$xy + yz + \bar{x}z = xy + \bar{x}z$ $(x+y)(y+z)(\bar{x}+z) = (x+y)(\bar{x}+z)$ $(x+y)(\bar{x}+z) = xz + \bar{x}y$
4. Dualitas	$(x + y + z + \ldots)^D = x\,y\,z$ $(x\,y\,z\ldots)^D = x + y + z + \ldots.$ $[f(x_1, x_2, x_3, \ldots, x_n, 0, 1, +, .)]^D = f(x_1, x_2, x_3, \ldots, x_n, 0, 1, +, .)$

Hukum de Morgan

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

(i) *When* $A = 0, \quad B = 0,$

$$\overline{A+B} = \overline{0+0} = \overline{0} = 1$$

and $\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{0} = 1 \cdot 1 = 1$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

(ii) *When* $A = 0, \quad B = 1,$

$$\overline{A+B} = \overline{0+1} = \overline{1} = 0$$

and $\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{1} = 1 \cdot 0 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

(iii) *When* $A = 1, \quad B = 0,$

$$\overline{A+B} = \overline{1+0} = \overline{1} = 0$$

and $\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{0} = 0 \cdot 1 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

(iv) *When* $A = 1, \quad B = 1,$

$$\overline{A+B} = \overline{1+1} = \overline{1} = 0$$

and $\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{1} = 0 \cdot 0 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

Hukum de Morgan

$$\overline{A \,.\, B} = \bar{A} + \bar{B}$$

(i) *When* $A = 0, \quad B = 0,$

$$\overline{A \,.\, B} = \overline{0 \,.\, 0} = \bar{0} = 1$$

and $\bar{A} + \bar{B} = \bar{0} + \bar{0} = 1 + 1 = 1$

Hence $\overline{A \,.\, B} = \bar{A} + \bar{B}$

(ii) *When* $A = 0, \quad B = 1,$

$$\overline{A \,.\, B} = \overline{0 \,.\, 1} = \bar{0} = 1$$

and $\bar{A} + \bar{B} = \bar{0} + \bar{1} = 1 + 0 = 1$

Hence $\overline{A \,.\, B} = \bar{A} + \bar{B}$

(iii) *When* $A = 1, \quad B = 0,$

$$\overline{A \,.\, B} = \overline{1 \,.\, 0} = \bar{0} = 1$$

and $\bar{A} + \bar{B} = \bar{1} + \bar{0} = 0 + 1 = 1$

Hence $\overline{A \,.\, B} = \bar{A} + \bar{B}$

(iv) *When* $A = 1, \quad B = 1,$

$$\overline{A \,.\, B} = \overline{1 \,.\, 1} = \bar{1} = 0$$

and $\bar{A} + \bar{B} = \bar{1} + \bar{1} = 0 + 0 = 0$

Hence $\overline{A \,.\, B} = \bar{A} + \bar{B}$

Sum of Product Form

Metode penyederhanaan rangkaian logika salah satunya adalah SOP (Sum of Product)

Setiap pernyataan dalam bentuk sum of products terdiri dari dua atau lebih operasi AND yang semuanya di OR kan. Contoh berikut ini:

1. $ABC + \overline{A}B\overline{C}$
2. $AB + \overline{A}B\overline{C} + \overline{C}\,\overline{D} + D$
3. $\overline{A}B + C\overline{D} + EF + GK + H\overline{L}$

Product of Sum

Bentuk persamaan logika juga menggunakan POS (Products of Sum). Terdiri dari 2 atau lebih operasi OR yang kemudian di AND kan.
Contoh:

1. $(A + \overline{B} + C)(A + C)$
2. $(A + \overline{B})(\overline{C} + D)F$
3. $(A + C)(B + \overline{D})(\overline{B} + C)(A + \overline{D} + \overline{E})$

Latihan Soal

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$$z = ABC + A\overline{B} \cdot (\overline{\overline{A}\,\overline{C}})$$

Contoh Penyederhanaan dengan aljabar boole

$$z = ABC + A\overline{B} \cdot (\overline{\overline{A}\,\overline{C}})$$

Langkah-langkah:
Menggunakan teori de Morgan – > Menghasilkan SOP

$$z = ABC + A\overline{B}(\overline{\overline{A}} + \overline{\overline{C}}) \quad \text{(Hukum de Morgan)}$$
$$= ABC + A\overline{B}(A + C) \quad \text{(teorema Involusi)}$$
$$= ABC + A\overline{B}A + A\overline{B}C \quad \text{(Hukum distributif)}$$
$$= ABC + A\overline{B} + A\overline{B}C \quad \text{(Hukum Idempoten)}$$

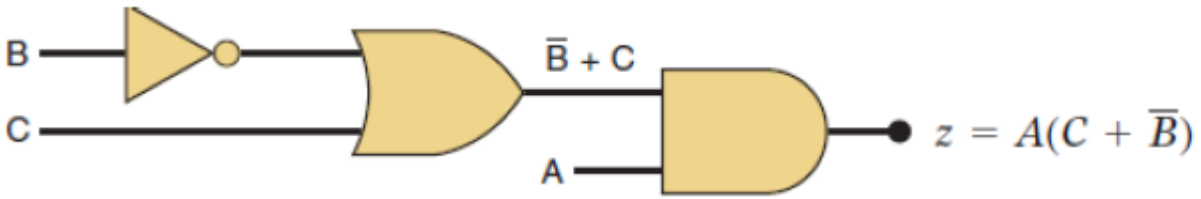
$$z = AC(B + \overline{B}) + A\overline{B} \quad \text{(Hukum distributif)}$$

Karena $B + \overline{B} = 1$ **maka** (Hukum komplemen)

BUKTIKAN dengan Tabel Kebenaran

$$z = AC(1) + A\overline{B} \quad \text{(operasi dengan 0 dan 1)}$$
$$= AC + A\overline{B}$$

$$z = A(C + \overline{B}) \quad \text{(Hukum distributif)}$$



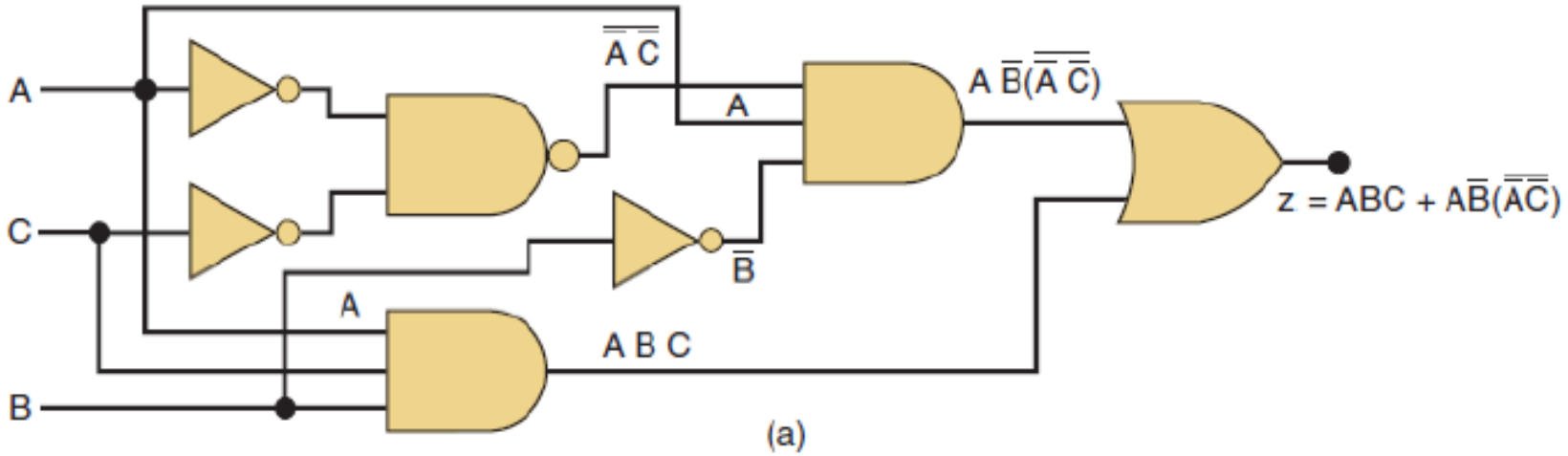
PEMBUKTIKAN dengan Tabel Kebenaran

$$z = ABC + A\overline{B} \cdot (\overline{\overline{A}\,\overline{C}})$$

$$z = A(C + \overline{B})$$

A	B	C	A.B.C	A. B'	(A'.C')'	z = ABC + AB'.(A'C')'	z = A(C+B')
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	0
0	1	1	0	0	1	0	0
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	0	1	0	0
1	1	1	1	0	1	1	1

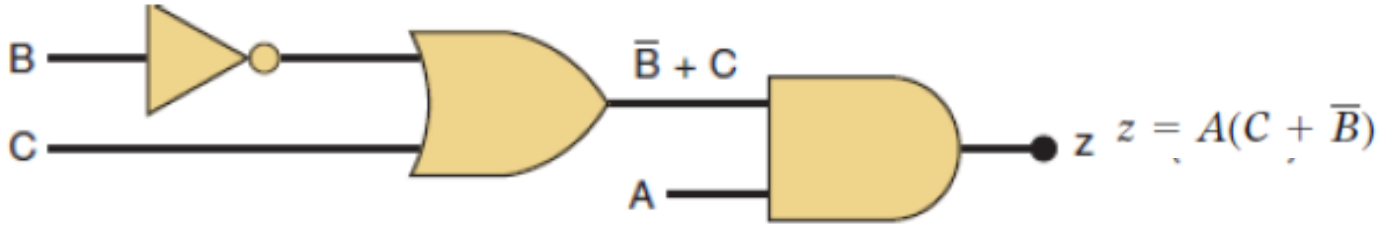
Sebelum Penyederhanaan



(a)

Persamaan : $z = ABC + A\overline{B} \cdot (\overline{\overline{A}\,\overline{C}})$

Sesudah Penyederhanaan



Latihan Soal

Sederhanakan fungsi logika berikut lalu buatlah tabel kebenaran serta lengkapi dengan gambar gerbang logika sebelum dan sesudah penyederhanaan

1. $F(A,B,C) = (A \cdot B) + (\overline{A} \cdot C) + (B \cdot \overline{C})$
2. $F(A,B) = \overline{(A \cdot B)} + (A \cdot \overline{B})$
3. $F(A,B,C) = (A \cdot B) + (\overline{A} \cdot C) + (\overline{B} \cdot \overline{C})$
4. $F(A,B,C) = \overline{(A \cdot B)} \cdot C + A \cdot \overline{B}$
5. $F(A,B,C) = A \cdot (B + C) + \overline{A} \cdot (B + \overline{C})$