

Mata Kuliah Dasar Teknik Digital

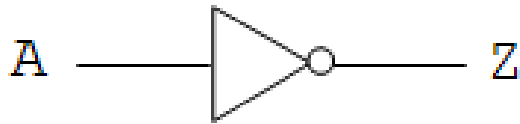
GERBANG LOGIKA dan ALJABAR BOOLEAN





Gerbang Dasar - NOT

- penyangkalan dengan kata-kata "***tidak***" (NOT)
 $1' = 0$ dan $0' = 1$



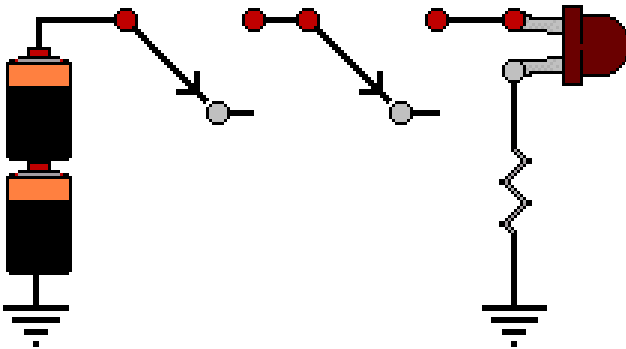
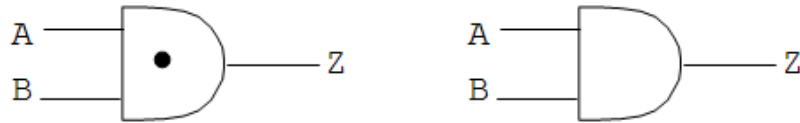
Tabel Kebenaran **NOT**

A	$Z = \bar{A}$
0	1
1	0



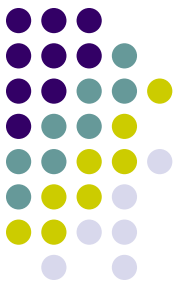
Gerbang Dasar - AND

- AND : $Z = A.B = AB$



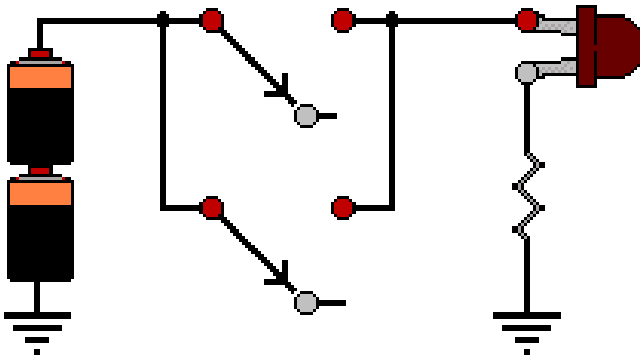
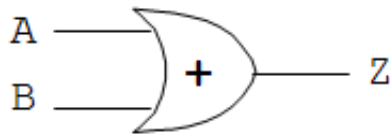
animasi

masukan		keluaran
A	B	$Z = A.B$
0	0	0
0	1	0
1	0	0
1	1	1



Gerbang Dasar - OR

- OR : $Z = A+B$



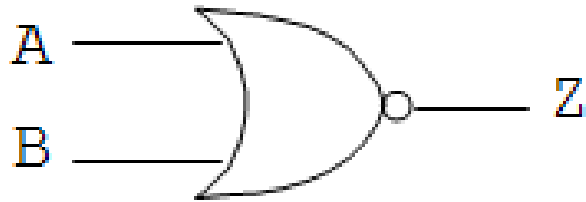
animasi

masukan		keluaran
A	B	$Z = A+B$
0	0	0
0	1	1
1	0	1
1	1	1



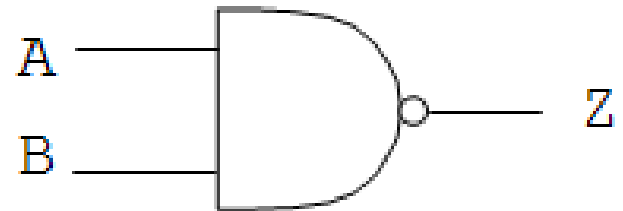
Gerbang Tambahan

- NOR : $Z = (A + B)'$



A	B	$Z = \overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

- NAND : $Z = (A B)'$

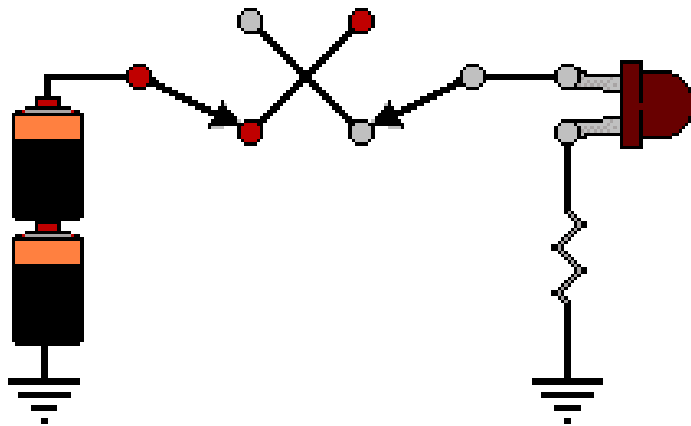
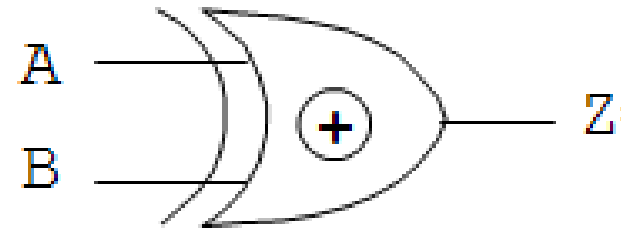


A	B	$Z = \overline{A B}$
0	0	1
0	1	1
1	0	1
1	1	0



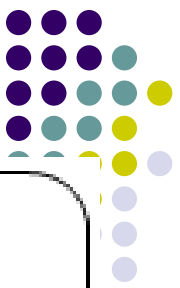
Gerbang Tambahan

- EXOR : $Z = A \oplus B$



animasi

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0



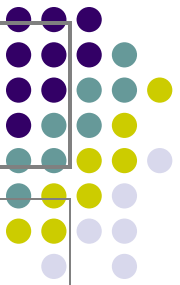
Logika Kombinasi

- + Rangkaian kombinasi adalah rangkaian aplikasi yang terbentuk dari berbagai macam gerbang logika dan dapat merupakan kombinasi dari satu jenis gerbang logika atau lebih.
- + Penyederhanaan rangkaian terintegrasi dapat menggunakan aljabar boole atau peta karnaugh

CONTOH.

Buatlah rangkaian dengan Gerbang Logika untuk kombinasi sbb.

$$Q = X \cdot (X' + Y)$$

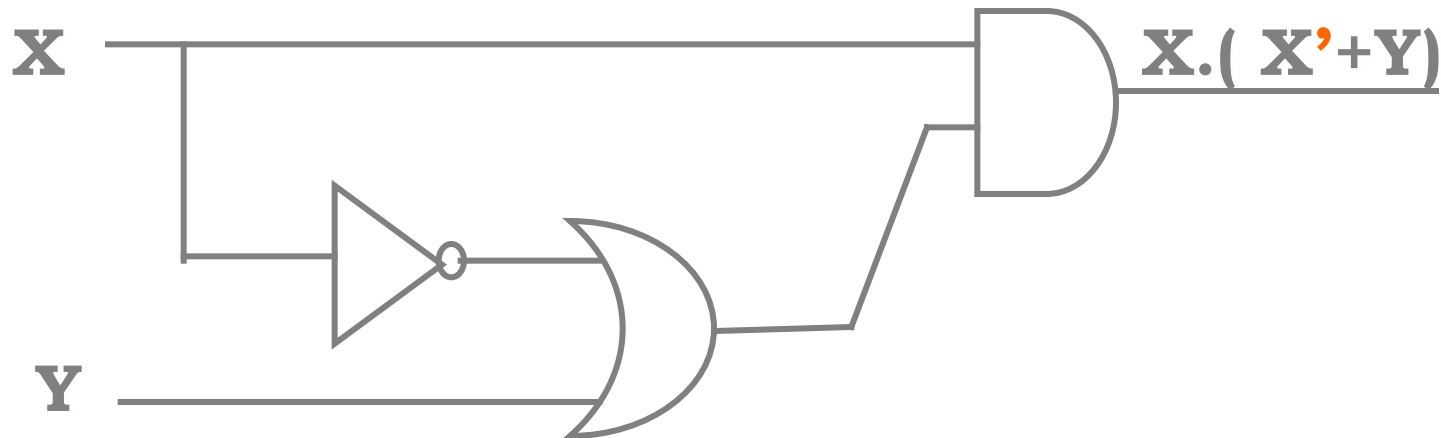


CONTOH.

Buatlah rangkaian dengan Gerbang Logika untuk kombinasi sbb.

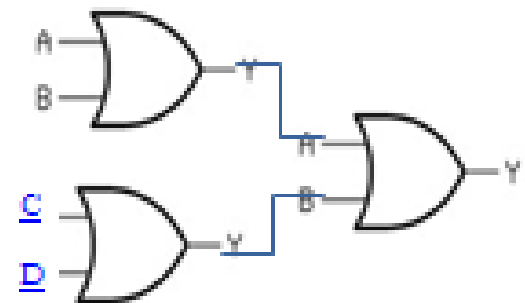
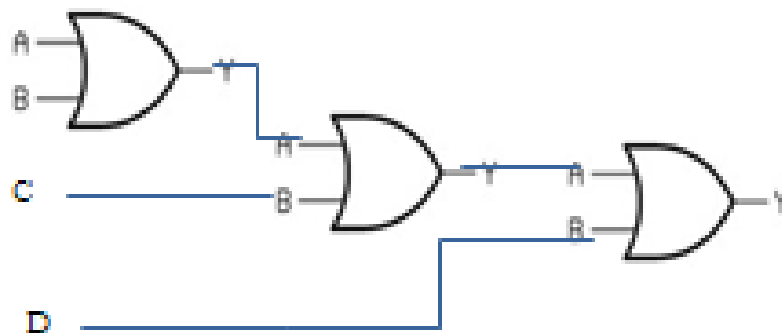
$$Q = X \cdot (X' + Y)$$

Jawab.



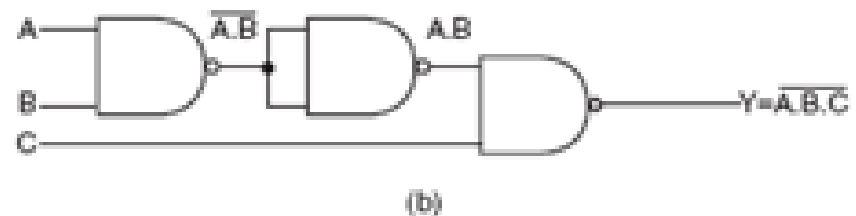
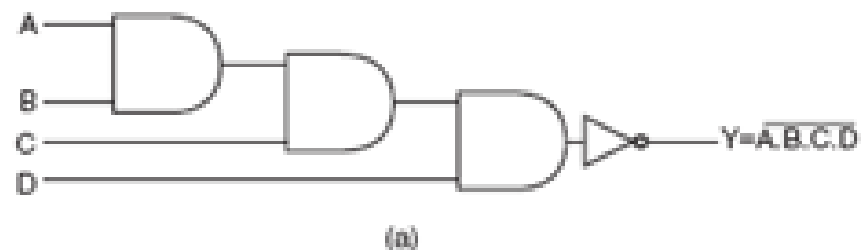
Contoh 2.

- ✚ Bagaimana cara mengaplikasikan gerbang OR 4 masukan dengan menggunakan gerbang OR 2 masukan?



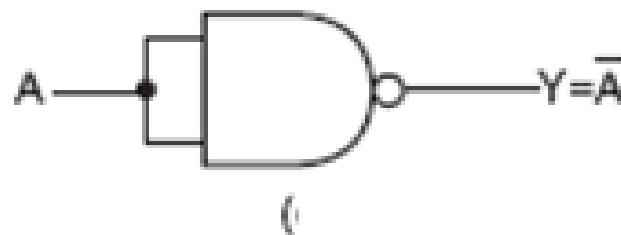
Contoh 3 .

- ✚ Gerbang NAND 4 input menggunakan gerbang AND 2 input dan 1 inverter
- ✚ Gerbang NAND 3 input menggunakan gerbang NAND 2 input



Contoh 4

- + Rangkaian NOT menggunakan 2 input gerbang NAND
- + Rangkaian NOT menggunakan 2 input gerbang NOR
- + Rangkaian NOT menggunakan 2 input gerbang XOR

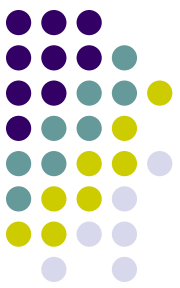




Teorema Dasar Boole

1. Operasi dengan 0 dan 1	$x + 0 = x$ $x + 1 = 1$ $x \cdot 0 = 0$ $x \cdot 1 = x$
2. Hukum Idempoten	$x + x = x$ $x \cdot x = x$
3. Hukum Involusi	$\overline{\overline{x}} = x$
4. Hukum Komplement	$x + \overline{x} = 1$ $x \cdot \overline{x} = 0$
5. Hukum Komutatif	$x + y = y + x$ $x \cdot y = y \cdot x$
6. Hukum Asosiatif	$(x+y)+z = x+(y+z) = x+y+z$ $(xy)z = x(yz) = xyz$
7. Hukum Distributif	$x(y+z) = xy+xz$ $x+yz = (x+y)(x+z)$

Teorema Tambahan Boole



1. Teorema penyederhanaan:

$$x y + x \bar{y} = x$$

$$x + x y = x$$

$$(x + \bar{y}) y = x y$$

$$(x+y)(x+\bar{y}) = x$$

$$x (x+\bar{y}) = x$$

$$x\bar{y} + y = x + y$$

2. Hukum de Morgan:

$$\overline{x + y + z + \dots} = \bar{x} \bar{y} \bar{z} \dots$$

$$\overline{x \cdot y \cdot z \cdot \dots} = \bar{x} + \bar{y} + \bar{z} + \dots$$

3. Teorema Konsensus:

$$xy + yz + \bar{x}z = xy + \bar{x}z$$

$$(x+y)(y+z)(\bar{x}+z) = (x+y)(\bar{x}+z)$$

$$(x+y)(\bar{x}+z) = xz + \bar{x}y$$

4. Dualitas

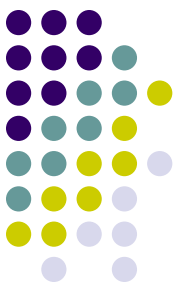
$$(x + y + z + \dots)^D = x y z$$

$$(x y z \dots)^D = x + y + z + \dots$$

$$[f(x_1, x_2, x_3, \dots, x_n, 0, 1, +, \cdot)]^D = f(x_1, x_2, x_3, \dots, x_n, 0, 1, +, \cdot)$$

Hukum de Morgan

$$\overline{A + B} = \bar{A} . \bar{B}$$



(i) When $A = 0, \quad B = 0,$

$$\overline{A + B} = \overline{0 + 0} = \bar{0} = 1$$

and $\bar{A} . \bar{B} = \bar{0} . \bar{0} = 1 . 1 = 1$

Hence $\overline{A + B} = \bar{A} . \bar{B}$

(ii) When $A = 0, \quad B = 1,$

$$\overline{A + B} = \overline{0 + 1} = \bar{1} = 0$$

and $\bar{A} . \bar{B} = \bar{0} . \bar{1} = 1 . 0 = 0$

Hence $\overline{A + B} = \bar{A} . \bar{B}$

(iii) When $A = 1, \quad B = 0,$

$$\overline{A + B} = \overline{1 + 0} = \bar{1} = 0$$

and $\bar{A} . \bar{B} = \bar{1} . \bar{0} = 0 . 1 = 0$

Hence $\overline{A + B} = \bar{A} . \bar{B}$

(iv) When $A = 1, \quad B = 1,$

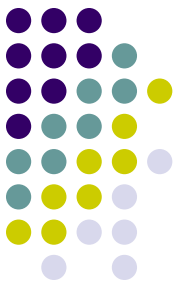
$$\overline{A + B} = \overline{1 + 1} = \bar{1} = 0$$

and $\bar{A} . \bar{B} = \bar{1} . \bar{1} = 0 . 0 = 0$

Hence $\overline{A + B} = \bar{A} . \bar{B}$

Hukum de Morgan

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$



(i) When $A = 0, \quad B = 0,$

$$\overline{A \cdot B} = \overline{0 \cdot 0} = \bar{0} = 1$$

$$\text{and} \quad \bar{A} + \bar{B} = \bar{0} + \bar{0} = 1 + 1 = 1$$

$$\text{Hence} \quad \overline{A \cdot B} = \bar{A} + \bar{B}$$

(iii) When $A = 1, \quad B = 0,$

$$\overline{A \cdot B} = \overline{1 \cdot 0} = \bar{0} = 1$$

$$\text{and} \quad \bar{A} + \bar{B} = \bar{1} + \bar{0} = 0 + 1 = 1$$

$$\text{Hence} \quad \overline{A \cdot B} = \bar{A} + \bar{B}$$

(ii) When $A = 0, \quad B = 1,$

$$\overline{A \cdot B} = \overline{0 \cdot 1} = \bar{0} = 1$$

$$\text{and} \quad \bar{A} + \bar{B} = \bar{0} + \bar{1} = 1 + 0 = 1$$

$$\text{Hence} \quad \overline{A \cdot B} = \bar{A} + \bar{B}$$

(iv) When $A = 1, \quad B = 1,$

$$\overline{A \cdot B} = \overline{1 \cdot 1} = \bar{1} = 0$$

$$\text{and} \quad \bar{A} + \bar{B} = \bar{1} + \bar{1} = 0 + 0 = 0$$

$$\text{Hence} \quad \overline{A \cdot B} = \bar{A} + \bar{B}$$



Penyajian Fungsi Boole

- **sukumin** (singkatan dari "suku minimum"
minterm, minimum term)

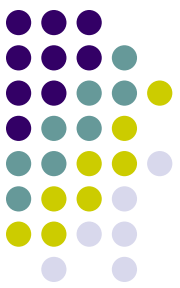
→ Sum Of Product, nilai 1

$$m_0 + m_1 + m_2 + \dots + m_{n-1} = \sum_{i=0}^{n-1} m_i = \sum m(0,1,2,\dots,n-1)$$

- **sukumax** (singkatan dari "suku maksimum"
maxterm, maximum term)

→ Product of sum, nilai 0

$$M_0 M_1 M_2 \dots M_{n-1} = \prod_{i=0}^{n-1} M_i = \prod M(0,1,2,\dots,n-1)$$



contoh

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$f = \sum m(1, 3, 4, 6)$$

$$= \prod M(0, 2, 5, 7)$$

$$f = \bar{A} \bar{B} C + \bar{A} B C + A \bar{B} \bar{C} + A B \bar{C}$$

$$f = (A + B + C)(A + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})$$



Fungsi Tidak Lengkap

- 'd' (dont care), dapat dianggap 1 ataupun 0 tergantung pertimbangan desain
- Contoh : $y = \sum m(0,3,7) + \sum d(1,6)$,

A	B	C	y
0	0	0	1
0	0	1	x
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	x
1	1	1	1

Latihan



1. Gambarkan Gerbang Logika AND, OR, NOT, EXOR dan EXNOR [simbol dan table kebenaran]
2. Gambarkan Gerbang Logika NAND, NOR, [simbol dan table kebenaran]
3. Buatlah rangkaian dan tabel kebenaran dari soal latihan berikut ini: $Z = A + (B.C)$
4. Buatlah rangkaian dan tabel kebenaran dari soal latihan berikut ini: $Z = (A+B). A'$

NB:

symbol matematis dari gerbang logika **AND** diwakili dengan simbol (.)

OR diwakili dengan simbol (+) **NOT** diwakili dengan simbol (')