**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with ** = 45 minutes and ** = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Mean (μ) = 45 minutes

Standard Deviation (σ) = 8 minutes

Time allowed = 60 minutes (1 hour)

Using the standard normal distribution, we need to calculate the z-score for the value 60 and then find the probability corresponding to that z-score.

The formula for the z-score is:

**z = (X - μ) / σ**

Calculate the z-score:

**z = (60 - 45) / 8 = 1.875**

**Then the probability of z value is 0.0306**

This probability corresponds to the service manager not meeting his commitment. However, the question asks for the probability that he cannot meet his commitment, which means we need to find the complement of this probability.

**So, the probability that the service manager cannot meet his commitment is approximately:**

**1 - 0.0306 = 0.9694**

Among the given options, none of them exactly matches this probability. However, the closest option is:

**D. 0.6987**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean ** = 38 and Standard deviation ** =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Given:

Mean (μ) = 38

Standard Deviation (σ) = 6

We can use the standard normal distribution to calculate the z-scores for these age values and then find the corresponding probabilities.

For age > 44:

z = (44 - 38) / 6 = 1

Using a standard normal distribution table or calculator, the probability for z = 1 is approximately 0.8413.

For 38 < age < 44:

z1 = (38 - 38) / 6 = 0

z2 = (44 - 38) / 6 = 1

**Using a standard normal distribution table or calculator, the probability for z between 0 and 1 is approximately 0.3413.**

Comparing these probabilities:

**Probability for age > 44: 0.8413**

**Probability for 38 < age < 44: 0.3413**

**Clearly, the statement is false**. More employees fall between the ages of 38 and 44 than are older than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Given:

Mean (μ) = 38

Standard Deviation (σ) = 6

We want to find the probability that an employee's age is under 30.

Calculate the z-score for age 30:

z = (30 - 38) / 6 = -1.33

Probability of z is 0.0912

No of employees under 30 = 0.0912 \* 400 = 36.48 # multiplying probability with total no of employees

So, according to the calculation, the expected number of employees under the age of 30 is around 36.48, which is approximately 36. Therefore, **the statement is true.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Mean (μ) = 100

Variance (σ^2) = 20^2 = 400

First, we need to find the standard deviation (σ) because 99% of the data lies within approximately 2.576 standard deviations from the mean in a normal distribution (using the empirical rule).

σ = √(σ^2) = √400 = 20

Now, we can find a and b symmetrically around the mean (μ) so that the range [a, b] captures 99% of the data:

a = μ - 2.576 \* σ

b = μ + 2.576 \* σ

Calculate a and b:

a = 100 - 2.576 \* 20 ≈ 48.5

b = 100 + 2.576 \* 20 ≈ 151.5

Among the given options, the closest to the calculated values are:

D. 48.5, 151.5

**So, the answer is option D: 48.5 and 151.5.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

For Division 1, the annual profit range centered on the mean of Rs. 225 million is approximately Rs. 218.12 million to Rs. 231.88 million, with a 95% probability.

For Division 2, the range centered on the mean of Rs. 315 million is roughly Rs. 306.16 million to Rs. 323.84 million, with a 95% probability.

1. Specify the 5th percentile of profit (in Rupees) for the company

The 5th percentile of profit for Division 1 is estimated to be around Rs. 219.07 million, and for Division 2, it's about Rs. 308.62 million.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Division 1, with a mean profit of Rs. 225 million, has a larger likelihood of making a loss in a given year compared to Division 2, which has a higher mean profit of Rs. 315 million.