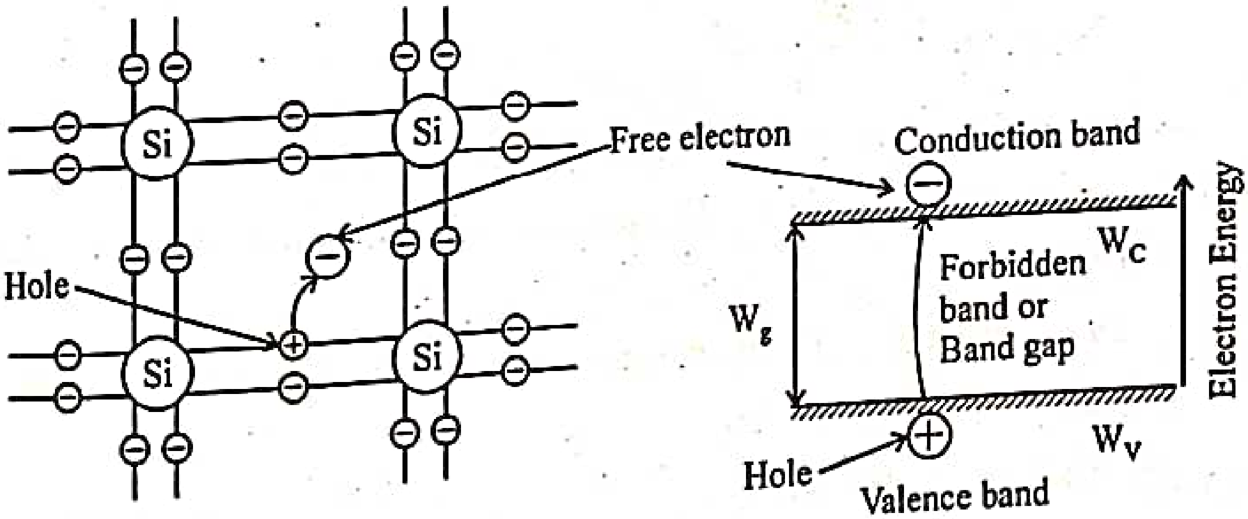
5 RF SYSTEM DESIGN CONCEPTS

5.1 ACTIVE RF COMPONENTS: SEMICONDUCTOR BASICS IN RF

5.1.1 Physical Properties of Semiconductors

The operation of semiconductor devices is dependent on physical behaviour of semiconductor themselves. Three most commonly used semiconductors are silicon (si), germanium (Ge) and gallium arsenide (GaAs). Figure 5.1(a) shows the bonding structure of pure silicon. Each silicon atom shares its four valance electrons with four neighboring atoms, forming four covalent bonds.



(a) Planar representation of covalent bonds

(b) Energy band levels

Figure 5.1 Lattice structure and energy levels of silicon

In the absence of thermal energy i.e., when the temperature is equal to zero degree Kelvin ($T = 0K = -273.15^{\circ}C$) all electrons are bonded to the corresponding atoms and semiconductor is not conductive. When the temperature increases, some of electrons obtain sufficient energy to break up the covalent bond and cross the energy gap $W_g = W_C - W_V$ as shown in figure 5.1(b). At room temperature $T = 300$ K, the band gap energy is equal to 1.12 eV for Si, 0.62 eV for Ge, 1.42 eV for GaAs. These free electrons form negative charge carriers that allow electric current conduction. The concentration of conduction electrons semiconductor is denoted as n. when electron breaks the covalent bond, it leaves behind a positively charged vacancy, which can be occupied by another free electron. These type of vacancies are called holes and their concentration is denoted by p. In thermal equilibrium, we have equal number of recombination and generation of holes and electrons.

The concentration obey the Fermi statistics according to

$$n = N_C e^{-\left(\frac{W_C - W_F}{KT}\right)} \quad \text{... (1)}$$

$$p = N_V e^{-\left(\frac{W_F - W_V}{KT}\right)} \quad \text{... (2)}$$

where

$$N_{C,V} = 2\left[\frac{2 m^*_{n,p} \pi KT}{h^2}\right]^{3/2} \quad \text{... (3)}$$

are the effective carrier concentration in conduction band (N_C) and valance band (N_V). W_C and W_V is energy levels associated with conduction and valance band W_F is Fermi energy level.

m^*_n and m^*_p refer to the effective mass of electrons and holes in the semiconductor.

K is Boltzmann's constant

h is Planck's constant

T is absolute temperature measured in Kevin

The electron and hole concentrations are described by concentration law

$$np = n_i^2 \quad \text{... (4)}$$

where n_i is the intrinsic concentration

Substitution of equation (1) and (2) in (4) results in the expression for intrinsic carrier concentration

$$n_i = \sqrt{N_C N_V}\, e^{-\left(\frac{W_C - W_V}{2KT}\right)} = \sqrt{N_C N_V}\, e^{-\left(\frac{W_g}{2KT}\right)} \quad \text{... (5)}$$

Classical electromagnetic theory specifies electrical conductivity in a material to be $\sigma = \frac{J}{E}$ where J is current density & E is applied electric field

$$\sigma = \frac{qNV_d}{E} \qquad \text{... (6)}$$

where N is carrier concentration

q is elementary charge

V_d is drift velocity

E is applied electric field

We can rewrite equation (6) as

$$\sigma = qn\mu_n + qp\mu_p \qquad \text{... (7)}$$

where μ_n, μ_p are mobilities of electrons & holes

simplify the equation (7) using $n = p = n_i$

$$\sigma = qn_i(\mu_n + \mu_p)$$

$$= q\sqrt{N_C N_V}\, e^{-\left(\frac{W_g}{2KT}\right)}(\mu_n + \mu_p) \qquad \text{... (8)}$$

Consider n-type semiconductor in which the electron concentration is related to the hole concentration as

$$n_n = N_D + p_n \qquad \text{... (9)}$$

where N_D is the donor concentration

p_n is minority hole concentration

To find n_n and p_n solve equation (9) in conjunction with (4). The result is

$$n_n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \qquad \text{... (10)}$$

$$p_n = \frac{-N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \qquad \text{... (11)}$$

If the donor concentration N_D is much greater than the intrinsic electron concentration n_i then

$$n_n = N_D \qquad \text{... (12)}$$

$$p_n = \frac{-N_D + N_D\left(1 + \frac{2n_i^2}{N_D^2}\right)}{2} = \frac{n_i^2}{N_D} \quad \text{... (13)}$$

Consider p type semiconductor with

$$p_p = N_A + n_p \quad \text{... (14)}$$

where N_A, n_p are acceptor and minority electron concentrations. Solving equation (14) together with (4)

$$p_p = \frac{N_A + \sqrt{N_A^2 + 4n_i^2}}{2} \quad \text{... (15)}$$

$$n_p = \frac{-N_A + \sqrt{N_A^2 + 4n_i^2}}{2} \quad \text{... (16)}$$

5.1.2 The pn-Junction

The physical contact of a p-type with n-type semiconductor leads to one of the most important concepts when dealing with active semiconductor devices pn junction. Because of difference in the carrier concentration between two types of semiconductor, a current flow will be initiated across the interface. This current is known as diffusion current and is composed of electrons and holes. Consider one dimensional model of pn Junction as shown in Figure 5.2.

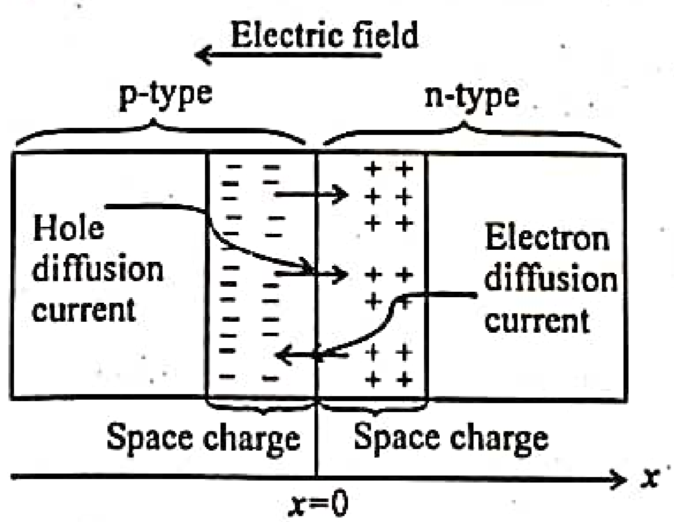


Figure 5.2 Current flows in the pn-Junction

The diffusion current is composed of $I_{n\,diff}$ and $I_{p\,diff}$ components

$$I_{diff} = I_{n\,diff} + I_{p\,diff} = qA\left(D_n \frac{dn}{dx} + D_p \frac{dp}{dx}\right) \qquad \text{... (1)}$$

where A is the semiconductor cross-sectional area orthogonal to the x-axis and D_n, D_p are diffusion constants for electrons and holes in the form

$$D_{n,p} = \mu_{n,p} \frac{KT}{q} = \mu_{n,p} V_T \qquad \text{... (2)}$$

The thermal potential $V_T = \frac{KT}{q}$ is approximately 26 mV at room temperature of 300K.

Since p type semiconductor was initially neutral, the diffusion current of holes is going to leave behind a negative space charge. Similarly electron current flow from n semiconductor will leave behind positive space charges. As the diffusion current flow takes place an electric field E is crated between net positive charge in the n semiconductor and net negative charge in the p semiconductor. This field in turn induces a current $I_F = \sigma AE$ which opposes the diffusion current such that $I_F + I_{diff} = 0$.

Substituting equation (7) for the conductivity

$$I_F = qA(n\mu_n + p\mu_p)E = I_{nF} + I_{pF} \qquad \text{... (3)}$$

since total current is equal to zero, the electron portion of the current is also equal to zero; that is

$$I_{ndiff} + I_{nF} = q\,D_n A\frac{dn}{dx} + q_n \mu_n AE = q\mu_n A\left(V_T \frac{dn}{dx} - \frac{ndV}{dx}\right) = 0 \qquad \text{... (4)}$$

where electric field E has been replaced by the derivative of the potential $E = \frac{-dV}{dx}$.

Integrating equation (4) we obtain the diffusion barrier voltage or built in potential

$$\int_0^{V_{diff}} dV = V_{diff} = V_T \int_{n_p}^{n_n} n^{-1}\,dn = V_T \ln\left[\frac{n_n}{n_p}\right] \qquad \text{... (5)}$$

where n_n is electron concentration in n-type

n_p is electron concentration in p-type

$$V_{diff} = V_T \ln\left[\frac{p_p}{p_n}\right] \quad \ldots (6)$$

If the concentration of acceptor in the p-semiconductor is $N_A >> n_i$, and the concentration of donors in the n semiconductor is $N_D >> n_i$, then $n_n \simeq N_D$, $n_p = \frac{n_i^2}{N_A}$.

By using $n_p = \frac{-N_A + N_A\left(1+\frac{2n_i^2}{N_A^2}\right)}{2} \simeq \frac{n_i^2}{N_A}$ and equation (5) we obtain

$$V_{diff} \simeq V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) \quad \ldots (7)$$

If we desire to determine the potential distribution along the x-axis, we can employ Poisson's equation which for one-dimensional analysis is written as

$$\frac{d^2 V(x)}{dx^2} = \frac{-\rho(x)}{\varepsilon_r \varepsilon_o} = -\frac{dE}{dx} \quad \ldots (8)$$

where $\rho(x)$ is the charge density and ε_r is the relative dielectric constant of the semiconductor

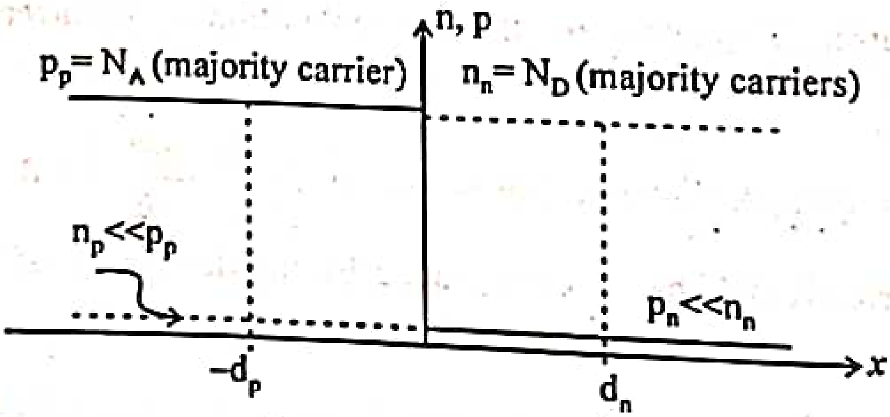


Figure 5.3 Acceptor and donor concentration

Assuming uniform doping and abrupt junction approximation as shown in figure 5.3 the charge density in each material is

$$\rho(x) = -qN_A, \text{ for } -d_p \leq x \leq 0 \quad \ldots (9)$$

$$\rho(x) = q N_D, \text{ for } 0 \le x \le d_n \qquad \text{... (10)}$$

where d_p and d_n are extents of space charges in p and n type semiconductor

The electric field in the semiconductor is found by integrating equation (8) with spatial limits $-d_p \le x \le d_n$ such that

$$E(x) = \int_{-d_p}^{x} \frac{\rho(x)dx}{\varepsilon_r \varepsilon_o} = \begin{cases} \dfrac{-q N_A}{\varepsilon_r \varepsilon_o}(x+d_p), & \text{for } -d_p \le x \le 0 \\ \dfrac{-q N_D}{\varepsilon_r \varepsilon_o}(d_n - x), & \text{for } 0 \le x \le d_n \end{cases} \qquad \text{... (11)}$$

To obtain voltage distribution profile, integrating equation (11) as follows

$$V(x) = \int_{-d_p}^{x} E(x)\,dx = \begin{cases} \dfrac{q N_A}{2\varepsilon_r \varepsilon_o}(x+d_p)^2, \text{ for} -d_p \le x \le 0 \\ \dfrac{q}{2\varepsilon_r \varepsilon_o}(N_A d_p^2 + N_D d_n^2) \dfrac{-q N_D}{2\varepsilon_r \varepsilon_o}(d_n - x)^2, \text{for } 0 \le x \le d_n \end{cases} \qquad \text{... (12)}$$

Since total voltage drop must be equal to diffusion voltage V_{diff} it is found that

$$V(d_n) = V_{diff} = \frac{q N_A d_p^2}{2\varepsilon_r \varepsilon_o} + \frac{q N_D d_n^2}{2\varepsilon_r \varepsilon_o} \qquad \text{... (13)}$$

substituting $d_p = \dfrac{d_n N_D}{N_A}$ and solving (13) for d_n

$$d_n = \left[\frac{2\varepsilon V_{diff}}{q} \frac{N_A}{N_D}\left(\frac{1}{N_A + N_D}\right)\right]^{\frac{1}{2}} \qquad \text{... (14)}$$

where $\varepsilon = \varepsilon_o \varepsilon_r$. An identical derivation involving $d_n = \dfrac{d_p N_A}{N_D}$ gives us the space charge extent into p-semiconductor

$$d_p = \left[\frac{2\varepsilon V_{diff}}{q} \frac{N_D}{N_A}\left(\frac{1}{N_A + N_D}\right)\right]^{1/2} \qquad \text{... (15)}$$

Entire length is addition of (14) & (15)

$$d_s = d_n + d_p = \left[\frac{2\varepsilon\, V_{diff}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\right]^{1/2} \quad \dots (16)$$

Junction capacitance $C = \dfrac{\varepsilon A}{d_s}$ ... (17)

substituting equation (16) in (17)

$$C = A\left[\frac{q\varepsilon}{2V_{diff}} \frac{N_A N_D}{N_A + N_D}\right]^{1/2} \quad \dots (18)$$

5.2 BIPOLAR JUNCTION TRANSISTORS (BJT)

Transistor Definition: A non linear three terminal active semiconductor device where the flow of electrical current between two of the terminals is controlled by the third terminal. The name is an acronym for transfer resistor.

Transistors may be used in circuits as amplifiers oscillators, detector, switches and so on.

Bipolar Junction transistor consists of three layers of semiconductors and two junctions. The semiconductor layer may be alternate N-type and P-type or vice versa. Thus BJTs are either emitter, base and collector and two Junctions are emitter base junction (EBJ) and collector base Junction (CBJ) as shown in figure 5.4 and 5.5

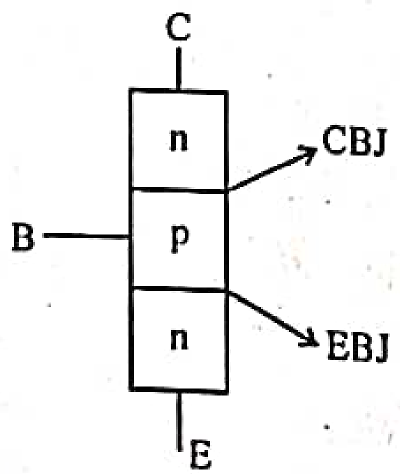
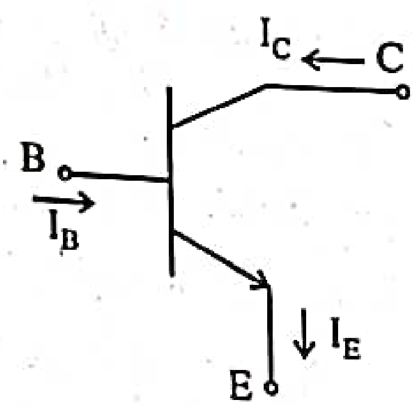


Figure 5.4 (a) NPN Transistor



(b) An NPN circuit symbol

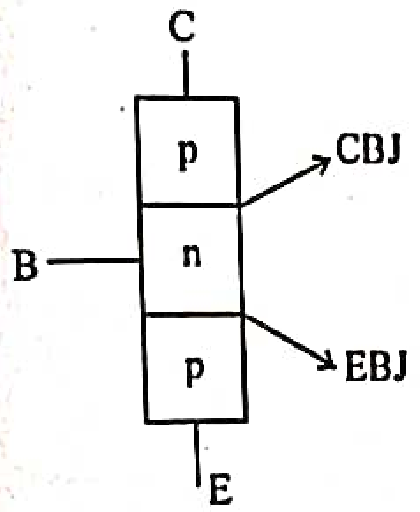
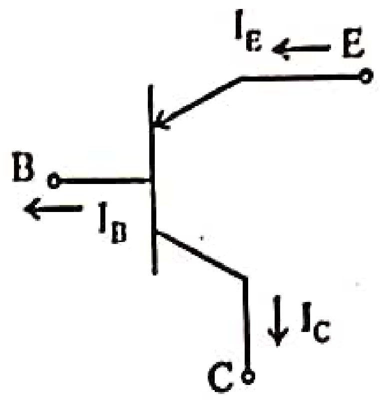


Figure 5.5 (a) PNP Transistor



(b) PNP circuit symbol

5.2.1 Graphical Representation of BJT Characteristics

It is often useful to describe a BJT graphically in terms of its I-V characteristics. Consider the following conceptual circuit as shown in figure 5.6 where the transistor is connected in common emitter configuration. In this setup the DC voltage sources are independent variables and create three dependent variables (I_C, I_B, V_{CE}).

To obtain a single transistor characteristic curve that is plotted in the $I_C - V_{CE}$ plane with I_B as a parameter, DC voltage source is set to a voltage and then V_{CE} voltage source setting is theoretically varied from 0 to infinity while measuring corresponding collector current (I_C). This is shown in figure 5.7.

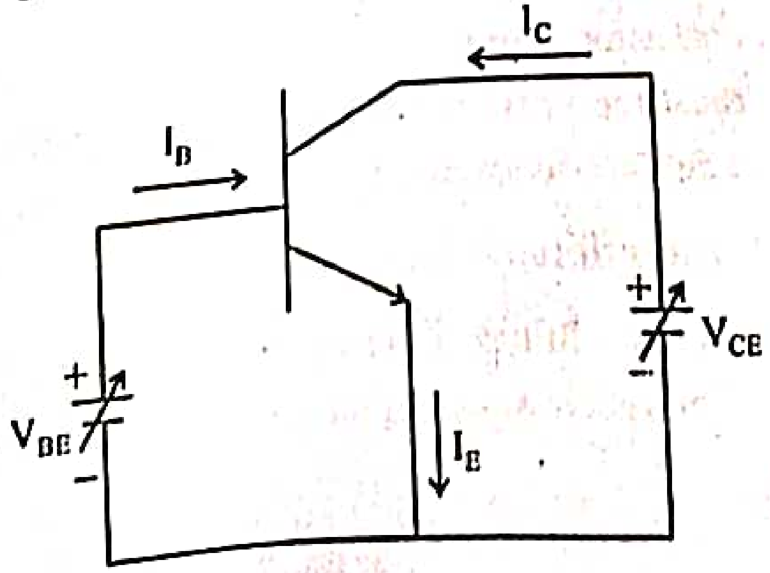


Figure 5.6 A conceptual circuit

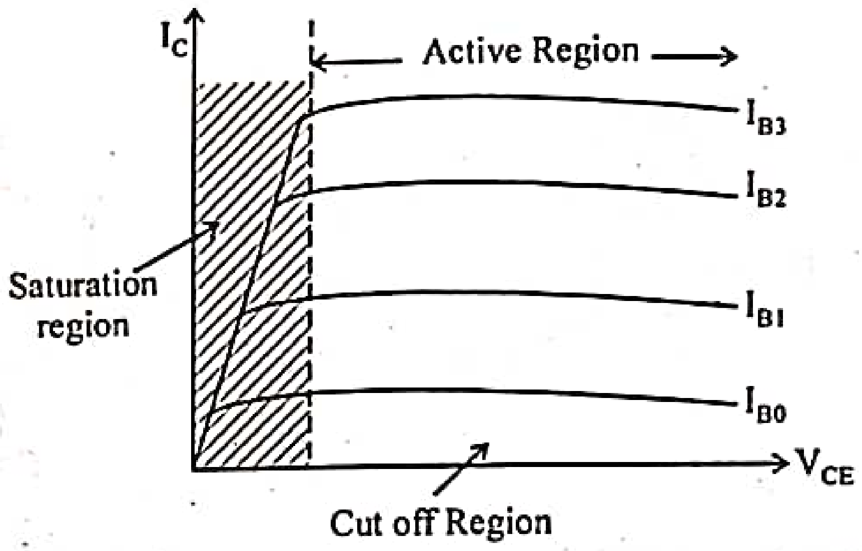


Figure 5.7 $I_C - V_{CE}$ characteristics of a BJT

$I_C - V_{CE}$ family of characteristic curves can be subdivided into several regions active, saturation and cut off.

5.2.2 BJT DC Biasing

The DC biasing of a transistor plays a major role in the operation and proper function of an active circuit.

DC Biasing Definition: It is the setting of the DC voltages at each of the two transistor junctions (EBJ or CBJ) such that the transistor will perform in a stable fashion in the intended mode (e.g., active mode for amplifiers, etc). Setting the proper values for each of the two transistor junction voltages can be translated equivalently into terminal current values such as emitter or collector current, which can alternatively be used to specify the DC bias values of the transistor. These currents in conjunction with the bias voltage values of junctions make the DC bias specification of a transistor complete.

5.2.3 BJT Modes of Operation

Depending on the bias conditions on each of the two Junctions (EBJ or CBJ), there can be four modes of operation. Assuming the following notations,

FWD = forward bias

REV = reverse bias

these four modes of operation are shown in figure 5.8.

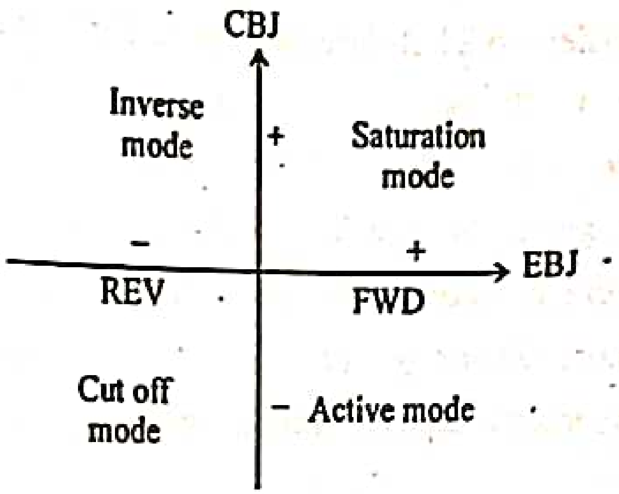


Figure 5.8 Four modes of operation of a BJT

Each mode can be defined as follows.

Saturation mode is the mode in which both EBJ and CBJ are forward biased. In this mode an increase in Base current (I_B) produces no further increase in collector current (I_C).

Cut off mode is the mode in which both EBJ and CBJ are reverse biased. Thus there is no current of any kind through the circuit. $I_E = 0$, $I_C = 0$ and $I_B = 0$. Active mode is the mode where EBJ is forward biased and CBJ is reverse biased. In this mode the collector current (I_C) is proportional to base current.

$$I_C = \beta I_B$$

where β is common emitter current gain

KCL gives $I_E = I_B + I_C = \left(1+\frac{1}{\beta}\right) I_C$

$$I_C = \frac{\beta}{1+\beta} I_E = \alpha I_E$$

where $\alpha = \frac{\beta}{1+\beta}$ is called common base current gain

A first order model for the operation of transistor in the active mode can be represented by hybrid–π equivalent circuit as shown in figure 5.9

$$I_C = I_S \, e^{\left(\frac{V_{BE}}{V_T}\right)}$$

where I_S is the reverse saturation current, V_T is thermal voltage defined to be $V_T = \frac{KT}{q}$, where K is Boltzmann's constant (1.38×10^{-23} J/K), T is absolute temperature in Kelvin and q is magnitude of electronic charge (1.602×10^{-19} C).

In the first order model of the active mode the forward voltage (V_{BE}) causes an exponentially related collector current (I_C) to flow, and as long as the CBJ remains reverse biased ($V_{CB} > 0$), the collector terminal behave as a non linear voltage controlled current source depending exponentially on V_{BE}. β is common emitter current gain and is much larger than unity for a good transistor, $\beta >> 1$. As a result I_B is seen to be much smaller than I_C. Furthermore because α is very close to unity for a good transistor, the collector current I_C is approximately equal to emitter current (I_E): $I_C = I_E$.

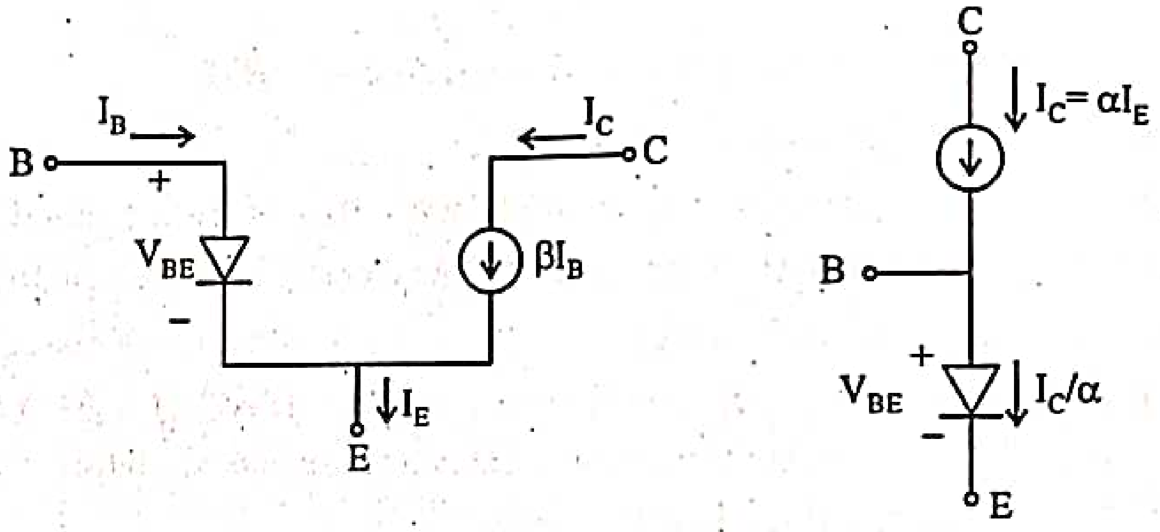


Figure 5.9 Two possible large-signal equivalent circuit models of NPN BJT in active mode.

Inverse mode is a mode in which the EBJ is reverse biased and CBJ is forward biased; that is emitter's and collector's roles are reversed. This mode may theoretically be used in the same manner as active mode.

5.3 RF FIELD EFFECT TRANSISTORS

Field effect transistors are monopolar devices meaning that only one carrier type, either holes or electrons contributes to the current flow through the channel. If hole contributions are involved it is of p-channel otherwise of n-channel FETs. FET is voltage controlled device. A variable electric field controls the current flow from source to drain by changing the applied voltage on the gate electrode.

5.3.1 Construction

FETs are classified according to how the gate is connected to the conducting channel. Specifically the following four types are used.

1. **Metal Insulator semiconductor FET (MISFET):** Here gate is separated from the channel through an insulation layer one of the most widely used types, the metal oxide semiconductor FET (MOSFET) belongs to this class.

2. **Junction FET(JFET):** This type relies on a reverse biased pn-junction that isolates gate from the channel.

3. **Metal semiconductor FET(MESFET):** If the reverse biased pn-junction is replaced by a schottky contact, the channel can be controlled just as in the JFET case.

4. **Hetero FET:** As the name implies heterostructure utilize abrupt transition between layers of different semiconductor materials. Examples are GaAlAs to GaAs. The High electron mobility Transistor (HEMT) belongs to this class.

Due to presence of a large capacitance formed by the gate electrode and the insulator or reverse biased pn junction, MISFET and JFET have a relatively low cut off frequency and are usually operated in low and medium frequency ranges of typically up to 1GHz. GaAs MESFET find application up to 60 – 70 GHz, and HEMT can operate beyond 100 GHz.

5.3.2 Functionality

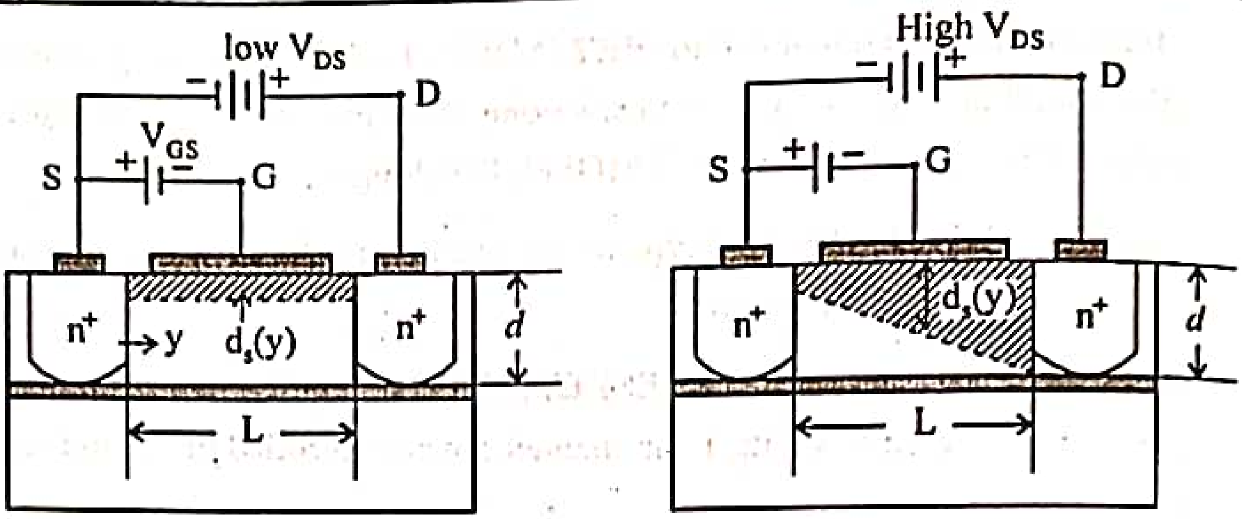
Because of its importance in RF and microwave amplifier, mixer and oscillator circuits, we focus our analysis on the MESFET, whose physical behaviour is in many ways similar to the JFET. The analysis is based on the geometry shown in figure 5.10 where the transistor is operated in depletion mode.

The schottky contact builds up a channel space charge domain that affect the current flow from source to drain. The space charge extent d_s can be controlled via the gate voltage.

$$d_s = \left(\frac{2\varepsilon}{q}\,\frac{V_d - V_{GS}}{N_D}\right)^{1/2} \quad \text{... (1)}$$

For instance the barrier voltage V_d is 0.9 V for GaAs–Au interface. The resistance R between source and drain is predicted by

$$R = \frac{L}{\sigma(d - d_s)W} \quad \text{... (2)}$$



(a) Operation in linear region (b) Operation in saturation region

Figure 5.10 Functionality of MESFET for different drain-source voltages

$\sigma = q\mu_n N_D$, W being gate width.

substituting (1) in (2) yields the drain current equation

$$I_D = \frac{V_{DS}}{R} = G_0\left[1-\left(\frac{2\varepsilon}{qd^2}\frac{V_d - V_{GS}}{N_D}\right)^{1/2}\right]V_{DS} \quad \text{... (3)}$$

where $G_0 = \dfrac{\sigma Wd}{L}$. This equation shows that the drain current depends linearly on drain source voltage, a fact that is only true for small V_{DS}.

As the drain source voltage increases, the space change domain near the drain contact increases as well, resulting in a non uniform distribution of the deflection region along the channel.

If we assume that the voltage along the channel changes from 0 at the source location to V_{DS} at the drain end then we can compute the drain current for non uniform space charge region. This approach is also known as gradual-channel approximation. The approximation rests primarily on the assumption that the cross sectional area at a particular location y along the channel is given by $A(y) = (d - d_s(y))$ W and electric field E is only y-directed.

The channel current is thus

$$I_D = -\sigma E A(y) = \sigma \frac{dV(y)}{dy}(d - d_s(y))W \quad \text{... (4)}$$

where the difference between V_d and V_{GS} in the expression for $d_s(y)$ has to be augmented by the additional drop in voltage V(y) along the channel. So equation (1) becomes

$$d_s(y) = \left[\frac{2\varepsilon}{q N_D}(V_d - V_{GS} + V(y)\right]^{1/2} \quad \text{... (5)}$$

substituting (5) in (4) and carrying out the integration on both sides of equation yields

$$\int_0^L I_D\, d_y = I_D L = \sigma W \int_0^{V_{DS}} \left[d - \left[\frac{2\varepsilon}{q N_D}(V + V_d - V_{GS})\right]^{1/2}\right] dV \quad \text{... (6)}$$

The result is the output characteristic of the MESFET in terms of drain current as a function of V_{DS} and V_{GS} or

$$I_D = G_0\left[V_{DS} - \frac{2}{3}\sqrt{\frac{2\varepsilon}{q N_D d^2}}\left[V_{DS} + V_d - V_{GS}\right]^{3/2} - (V_d - V_{GS})^{3/2}\right] \text{... (7)}$$

We note that this equation reduces to (3) for small V_{DS}. When the space charge extends over the entire channel depth d, the drain source voltage for this situation is called drain saturation voltage $V_{DS\,sat}$ and is given by

$$d_s(L) = d = \sqrt{\frac{2\varepsilon}{q N_D}(V_d - V_{GS} + V_{DSsat})} \quad \text{... (8)}$$

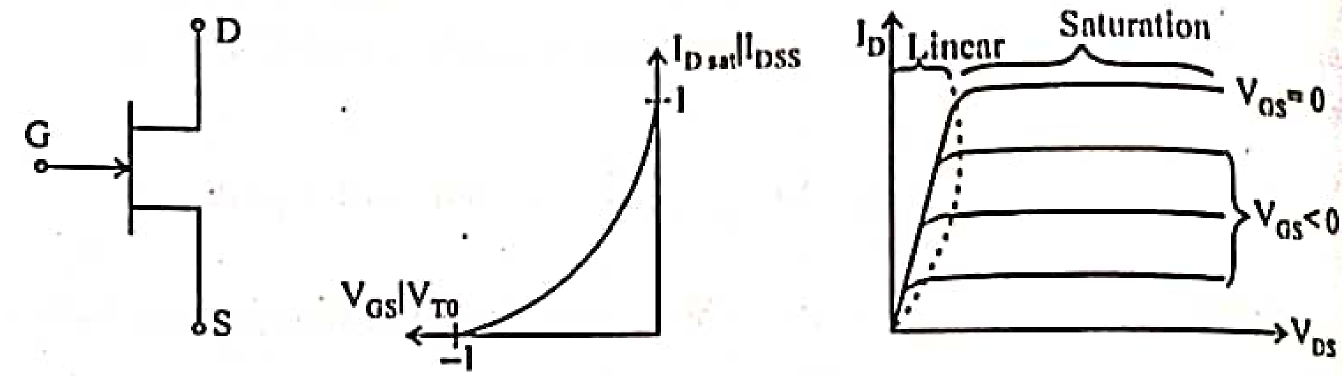
or explicitly

$$V_{DS\,sat} = \frac{q N_D d^2}{2\varepsilon} - (V_d - V_{GS}) = V_p - V_d + V_{GS} = V_{GS} - V_{T0} \quad \text{... (9)}$$

where V_p pinch off voltage $= \dfrac{q N_D d^2}{2\varepsilon}$ and threshold voltage $V_{T0} = V_d - V_p$. The associated drain saturation current is found by inserting (9) into (7) with result

$$I_{D\,sat} = G_0\left[\frac{V_p}{3} - (V_d - V_{GS}) + \frac{2}{3\sqrt{V_p}}(V_d - V_{GS})^{3/2}\right] \quad \text{... (10)}$$

The maximum saturation current in equation (10) is obtained when $V_{GS} = 0$, which we define as $I_{Dsat}(V_{GS} = 0) = I_{DSS}$. Input and output transfer as well as output characteristic behaviour is shown in figure 5.11.



(a) Circuit symbol (b) Transfer characteristics (c) Output characteristics

Figure 5.11 Transfer and output characteristics of an n-channel MESFET

The saturation drain current is often approximated by simple relation.

$$I_{Dsat} = I_{DSS}\left(1-\frac{V_{GS}}{V_{T0}}\right)^2 \qquad \text{...(11)}$$

Example 1

For a particular Si pn-junction, the doping concentrations are given as $N_A = 10^{18}$ cm^{-3} and $N_D = 5 \times 10^{15}$cm^{-3}, with an intrinsic concentration of $n_i = 1.5 \times 10^{10}$ cm^{-3}. Find the barrier voltage for T = 300 K

Solution:

Given data,

$N_A = 10^{18}$ cm^{-3}, $N_D = 5 \times 10^{15}$cm^{-3}

$n_i = 1.5 \times 10^{10}$ cm^{-3}, T = 300 K

$$V_{diff} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_{diff} = 0.796 \text{ V.}$$

Example 2

A GaAs MESFET has following parameters $N_D = 10^{16}\ cm^{-3}$, $d = 0.75\ \mu m$, $W = 10\ \mu m$, $L = 2\ \mu m$, $\varepsilon_r = 12.0$, $V_d = 0.8\ V$ and $\mu_n = 8500\ cm^2/VS$. Determine

(i) The pinch off voltage

(ii) Threshold voltage

(iii) Maximum saturation current.

Solution:

Given data:

$N_D = 10^{16}\ cm^{-3}$, $d = 0.75\ \mu m$, $W = 10\ \mu m$, $L = 2\ \mu m$, $\varepsilon_r = 12.0$ $V_d = 0.8\ V$ & $\mu_n = 8500\ cm^2/VS$

(i) Pinch off voltage

$$V_p = \frac{q N_D d^2}{2\varepsilon}$$

$$V_p = 4.24\ V$$

(ii) Threshold voltage

$$V_{T0} = V_d - V_p$$

$$= 0.8 - 4.24$$

$$V_{T0} = -3.44\ V$$

(iii) Maximum saturation current

$$I_{DSS} = G_0\left[\frac{V_p}{3} - V_d + \frac{2}{3\sqrt{V_p}} V_d^{3/2}\right]$$

$$G_0 = \frac{\sigma q N_D W d}{L} = \frac{q^2 \mu_n N_D^2 W d}{L} = 8.16$$

$$I_{DSS} = 6.89\ A$$

5.4 HIGH ELECTRON MOBILITY TRANSISTORS

It is also known as modulation doped field effect transistor exploits the differences in bandgap energy between dissimilar semiconductor materials such as GaAlAs and GaAs in an effort to substantially surpass the upper frequency limit of MESFET while maintaining low noise performance and high power rating. Transition frequencies of 100 GHz and above have been achieved. The high frequency behaviour is due to a separation of carrier electrons from their donor sites at the interface between doped GaAlAs and undoped GaAs layer, where they are confined to a very narrow layer in which motion is possible only parallel to the interface. Here a two dimensional electron gas (2DEG) or plasma of very high mobility upto 9000 $cm^2/V\text{-}S$.

5.4.1 Construction

The basic heterostructure is shown in figure 5.12 where a GaAlAs n-doped semiconductor is followed by an undoped GaAlAs spacer layer, an undoped GaAs layer and a highly resistive semi-insulating GaAs substrate.

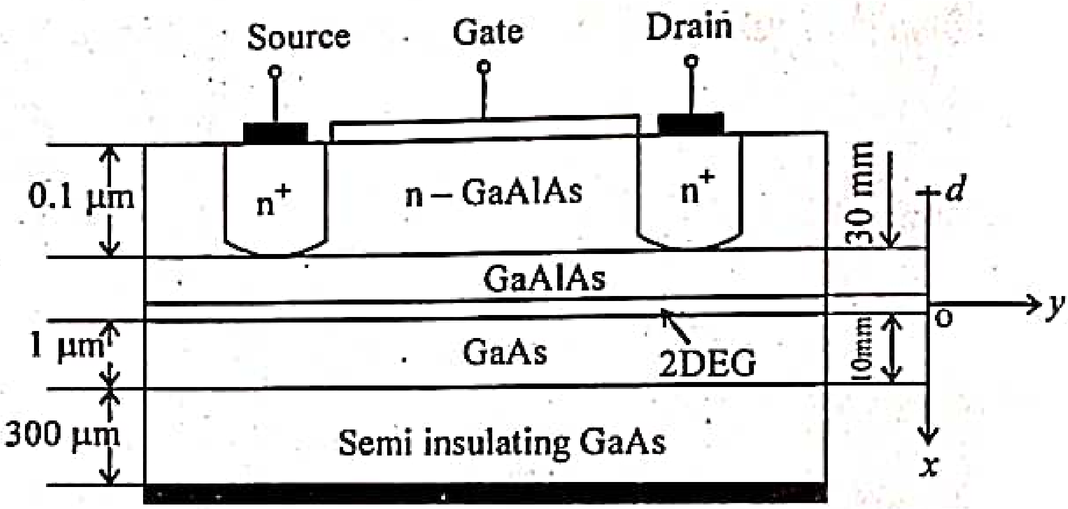


Figure 5.12 Generic heterostructure of a depletion-mode HEMT

The 2DEG forms in the undoped GaAs layer at zero gate bias condition because the fermi level is above the conduction band, so that electrons accumulate in this narrow potential well. The electron concentration can be depleted by applying an increasingly negative gate voltage.

HEMTS are primarily constructed of heterostructures with matching lattice constants to avoid mechanical tensions between layers. Examples are GaAlAs-GaAs and InGaAs-Inp interfaces. A larger In GaAs lattice is compressed onto a smaller GaAs lattice. Such a device configuration are known as pseudomorphic HEMTs or PHEMTs.

5.4.2 Functionality

The key issue that determines the drain current flow in a HEMT is the narroe interface between GaAlAs & GaAs layers.

A mathematical model can be developed by writing the one dimensional poisson equation in the form

$$\frac{\partial^2 V}{\partial x^2} = \frac{-q N_D}{\varepsilon_H} \quad \text{... (1)}$$

where N_D & ε_H are donor concentration and dielectric constant in the GaAlAs heterostructure. The boundary condition for the potential are imposed such that $V(x=0)=0$ and at the metal semiconductor side $V(x=-d) = -V_b + V_G + \Delta W_C / q$. Here V_b is barrier voltage, ΔW_C is energy difference in the conduction levels between the n-doped GaAlAs and GaAs and V_G is composed of the gate source voltage as well as the channel voltage drop $V_G = -V_{GS} + V(y)$. To find potential equation (1) is integrated twice. At the metal-semiconductor interface, we set

$$V(-d) = \frac{q N_D}{2\varepsilon_H} x^2 - E_y(0) d \quad \text{... (2)}$$

which yields

$$E(0) = \frac{1}{d}\left(V_{GS} - V(y) - V_{T0}\right) \quad \text{... (3)}$$

where defined the HEMT threshold voltage V_{T0} as $V_{T0} = \frac{V_b - \Delta W_C}{q - V_p}$

$$\text{where } V_p = \frac{q N_D d^2}{2\varepsilon_H}$$

$$I_D = \sigma E_y A = -q\mu_n N_D E W d = q\mu_n N_D \left(\frac{dV}{dy}\right) W d \quad \text{... (4)}$$

the current flow is restricted to a very thin layer so that it is appropriate to carry out the integration over a surface charge density Q_s at $x=0$.

The result is $\sigma = \frac{-\mu_n Q}{WL d} = \frac{-\mu_n Q_s}{d}$. For the surface charge density, find with Gauss's law $Q_s = \varepsilon_H E(0)$

substitute in (4)

$$\int_0^L I_D \, dy = \mu_n W \int_0^{V_{DS}} Q_s \, dV \quad \text{... (5)}$$

Using (3) It is seen that drain current can be found

$$I_D L = \mu_n W \int_0^{V_{DS}} \frac{\varepsilon_H}{d}(V_{GS} - V - V_{T0})\, dV \quad \dots (6)$$

or

$$I_D = \mu_n \frac{W\varepsilon_H}{Ld}\left[V_{DS}(V_{GS} - V_{T0}) - \frac{V_{DS}^2}{2}\right] \quad \dots (7)$$

pinch off occur when drain-source voltage is equal to or greater than difference of gate–source and threshold voltages $(V_{DS} \geq V_{GS} - V_{T0})$. If the equality of this condition is substituted in (7) it is seen that

$$I_D = \mu_n \frac{W\varepsilon_H}{2Ld}(V_{GS} - V_{T0})^2 \quad \dots (8)$$

The threshold voltage allows us to determine if the HEMT is operated as an enhancement or depletion type. For the depletion type, we require $V_{T0} < 0$ or $V_b - \left(\frac{\Delta W_C}{a}\right) - V_p < 0$.

substituting the pinch off voltage $V_p = \frac{qN_D d}{2\varepsilon_H}$ and solving for d, this implies

$$d > \left[\frac{2\varepsilon_H}{qN_D}\left(V_b - \frac{\Delta W_C}{q}\right)\right]^{1/2} \quad \dots (19)$$

and if d is less than the $V_{T0} > 0$, we deal with an enhancement HEMT.

5.5 BASIC CONCEPTS OF RF DESIGN

5.5.1 RF Circuit Design Consideration

Low RF circuits have to go through a three – step design process. In this design process, the effect of wave propagation on the circuit operation is negligible and the following facts in connection with design process can be stated:

1. The length of the circuit (l) is generally much smaller than wavelength ($l \ll \lambda$)
2. Propagation delay time (t_d) is approximately zero ($t_d = 0$)
3. Maxwell's equations simplify into all of the low frequency laws such as KVL, KCL. Therefore at RF frequencies, the delay time of propagation is approximately zero when $l \ll \lambda$ and all elements in the circuit can be considered to be lumped.

The design process has following three steps

Step 1: The design process starts with selecting a suitable device and performing a DC design to obtain a proper Q-point.

Step 2: The device will be characterized to obtain its AC small signal parameters based on the specific DC operating point selected earlier.

Step 3: It consist of designing two matching circuits that transition this device to the outside world, the signal source at one end and load at the other.

The design process for RF circuit is summarized and shown in figure 5.13.

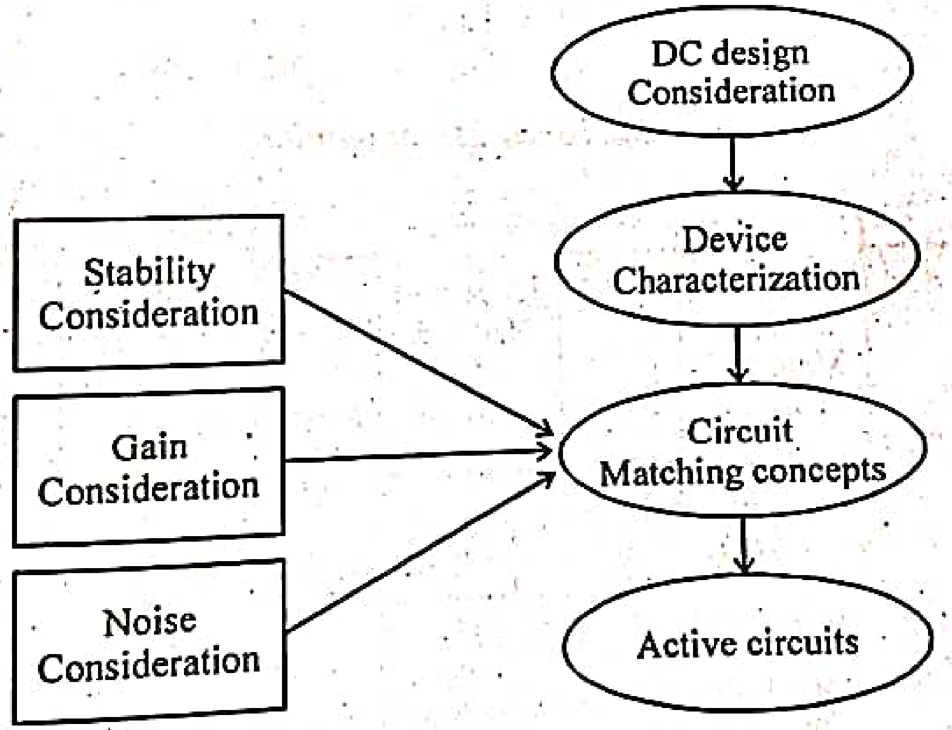


Figure 5.13 RF circuit design steps

5.5.2 High RF and Microwave Circuit Design Consideration

Step 1: The design process starts with the design of DC circuit to establish a stable operating point.

Step 2: Characterize the device at the operating point using electrical waves to measure the percentage of reflection and transmission that device presents at each port.

Step 3: Designing the matching network that transition the device to the outside world such that the required specification such as stability, overall gain, etc., are satisfied.

5.6 MIXER

Definition: A non linear 3 port circuit (Two in, uts and one output) that generates a spectrum of output frequencies equal to sum or difference of two input frequencies and their harmonics. The two input ports are referred as RF and LO where as the output is called IF port.

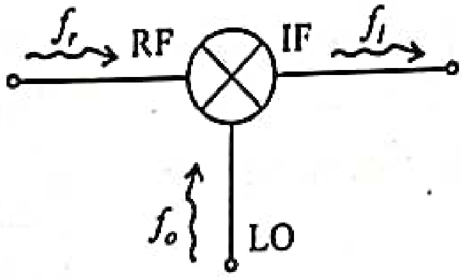


Figure 5.14 (a) Mixer symbol

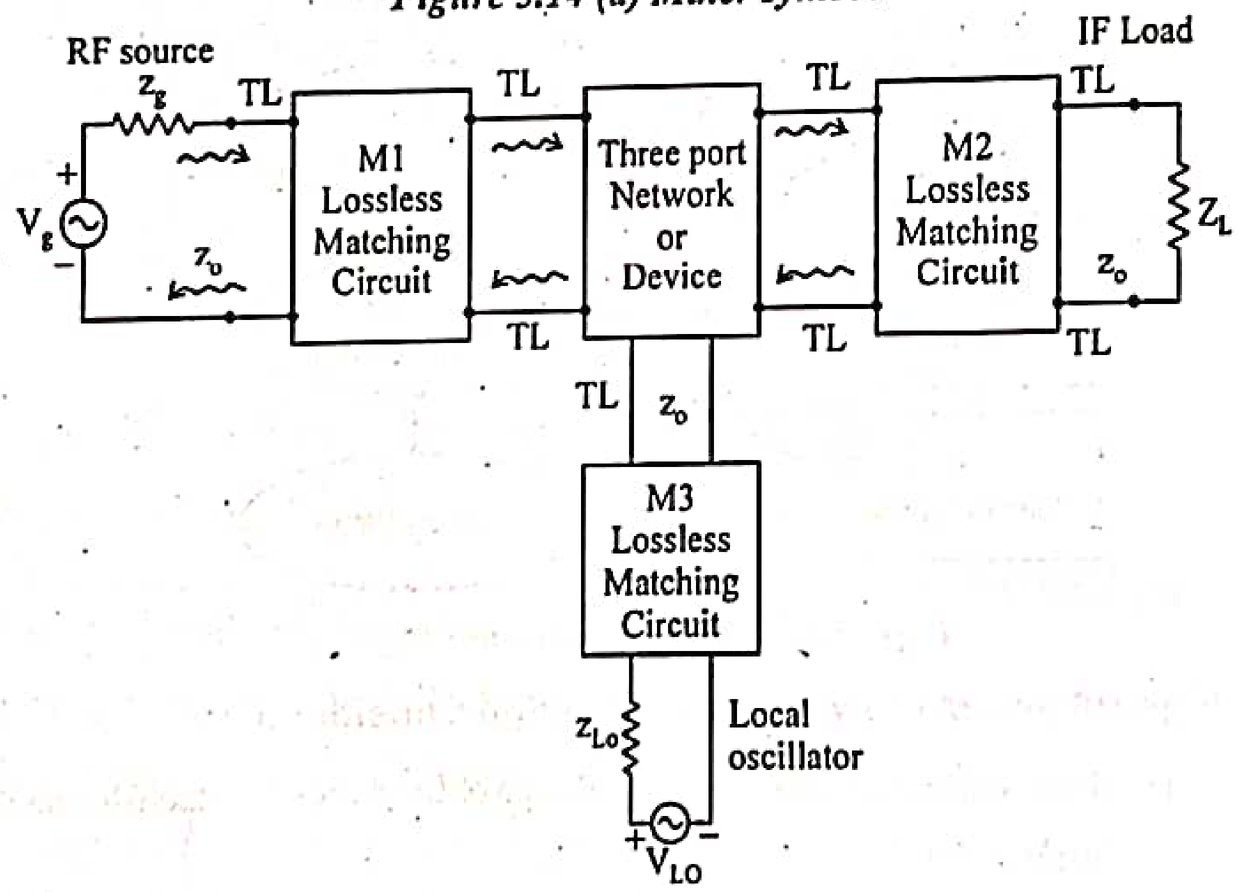


Figure 5.14 (b) Mixer block diagram

Mixer uses the non linearity of a device to generate a spectrum of frequencies (at the IF port) based on sum and difference of harmonics of the RF signal and local oscillator signal frequencies as shown in figure 5.14. The non linear device is flanked on three sides by three matching circuits,, which need to be designed properly for maximum conversion efficiency.

In general, mixers utilize one or more non linear device, properly pumped with a relatively large signal (called LO) to mix with an RF signal in order to generate a spectrum of frequencies based on the sum and difference of harmonics of the RF & LO frequencies.

$$\omega_i = m\omega_r \pm n\omega_o$$

Where m & n are positive integers.

The most important terms for mixer operation are those with frequencies at $\omega_r + \omega_o$ and at $\omega_r - \omega_o$..

5.6.1 Types of Mixers

5.6.1.1 Up - Converters

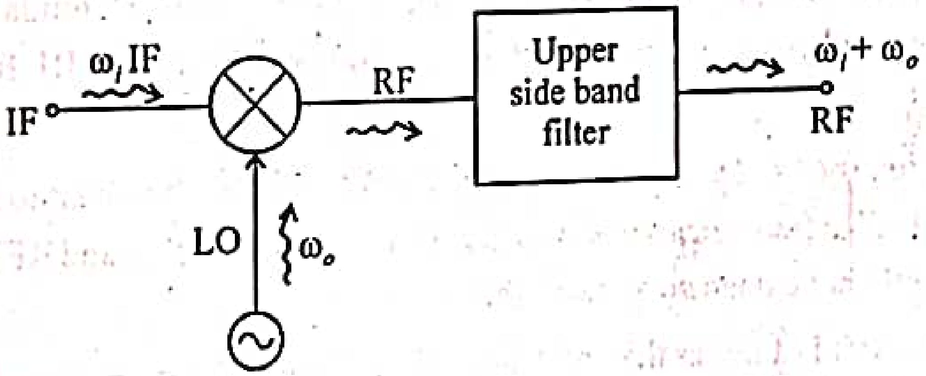


Figure 5.15 (a) Up conversion in a transmitter

When a mixer is used as an up converter the sum frequency $f_i = f_o + f_r$ is utilized and the difference frequency is rejected. In an up converter shown in figure 5.15 (a) the IF oscillator is modulated with desired information signal which when mixed with LO signal will generate the desired frequency conversion. Use of mixer as an up converter particularly in case of a radar or a transceiver is advantageous because it allows the use of a single local oscillator for both receiver and transmitter.

$$\omega_r = \omega_o + \omega_i$$

Use of proper filtering or an image rejection mixer is needed to generate the sum frequency ($\omega_r = \omega_o + \omega_i$). An up converters is used in a transmitter to modulate a carrier wave (LO signal) with an information bearing signal (IF signal) in order to generate an RF signal for transmission.

5.6.1.2 Down Converter

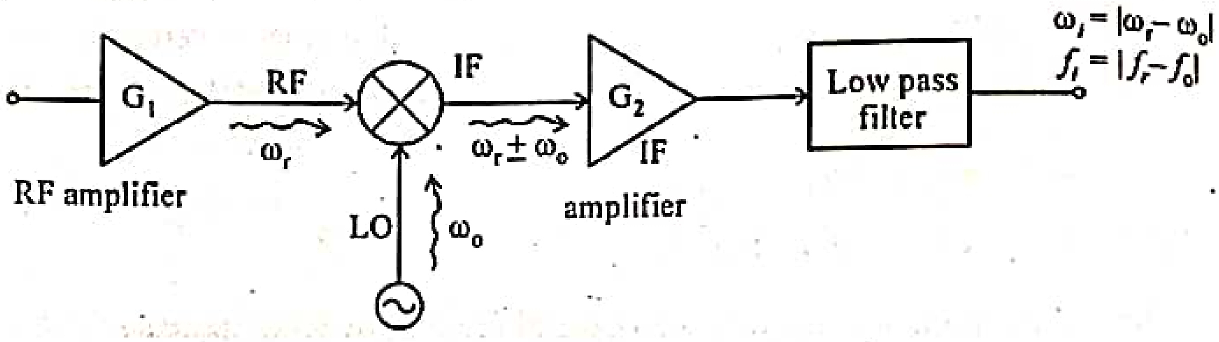


Figure 5.15 (b) Down conversion in a heterodyne receiver

When application requires a down – conversion the difference frequency $f_i = f_o - f_r$ is used and sum frequency is filtered out as shown in figure 5.15 (b). Use of mixer as a down converter in heterodyne receiver has several advantages.

✓ The IF signal being in the range of $10\text{ MHz} \le f_{IF} \le 100\text{MHz}$ lends itself for low noise amplification because 1/f noise is lower in IF than in RF frequency range.

✓ By changing the LO frequency, a heterodyne receiver can be tuned to receive a wide band of RF frequencies without a need for a high gain wideband RF amplifier which would have been necessary otherwise.

A down converter is a mixer that, with help of an LO shift the frequency of the RF signal substantially down to an IF signal ready for further signal processing. The IF signal frequency is given by

$$\omega_i = |\omega_r - \omega_o|$$

Such a mixer is used in receiver as a demodulator to remove the carrier wave from the transmitted signal in order to obtain the information carrying signal.

5.6.1.3 Harmonic Mixers

A simple method of down converting a high frequency RF signal when only a low frequency LO exist is the use of harmonic mixers. Frequency down conversion is achieved by mixing the high frequency signal with an appropriate harmonic of LO frequency.

$$\omega_i = n\omega_o - \omega_r$$

Where $n = 1, \ldots N$ is an integer, when $n = 1$ a fundamental down converter is obtained.

Applications of harmonic mixers are in millimeter wave instrumentation where the use of high frequency LO source, which can generate substantial power to satisfy LO power needs is impractical or very expensive.

5.7 LOW NOISE AMPLIFIERS

A Low noise amplifier is an electronic amplifier that amplifies very low power signal without significantly degrading its signal to noise ratio. LNAs are designed to amplify a signal while minimizing additional noise.

LNA are found in radio communication system, medical instruments and electronic test equipment.

A typical LNA may supply a power gain of 100 while decreasing the signal to noise ratio by less than factor of two. LNA as shown in figure 5.16.

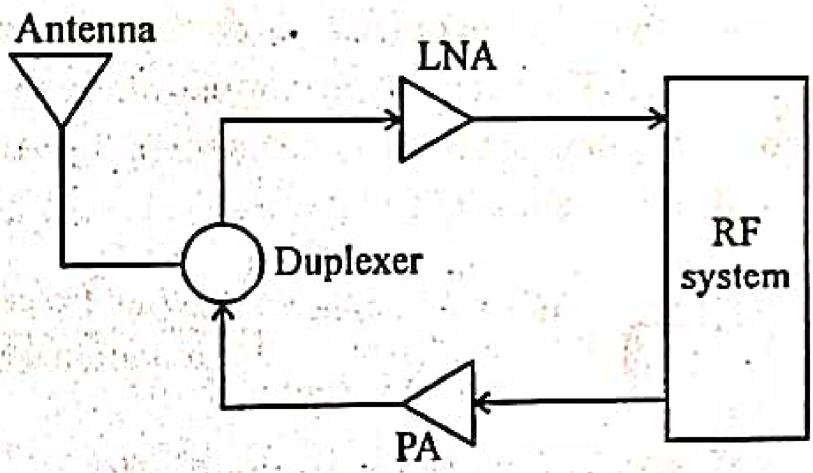


Figure 5.16 Block diagram of LNA

The low noise amplifier of receiver path and power amplifier of transmit path connected to the antenna via duplexer, which separate the two signals and prevents the relatively powerful power amplifier output from overloading the sensitive LNA input.

For LNA, the primary parameters are noise figure (NF), gain, and non linearity. Noise is due to thermal and other sources with typical noise figure 0.5 to 1.5 dB range. The noise figure helps determine the efficiency of particular LNA. Low noise figure results in better signal reception. With low noise figure, an LNA must have high gain. Typical gain is between 10 and 20 dB for a single stage.

LNAs are used in applications such as industrial scientific and medical band (ISM) radios, cellular telephones, GPS receivers, wireless LANs, satellite communication.

5.8 VOLTAGE CONTROL OSCILLATORS

A voltage controlled oscillator is an oscillator with an output signal whose output can be varied over a range, which is controlled by the input DC voltage. It is an oscillator whose output frequency is directly related to the voltage at its input. The oscillation frequency varies from few hert Z to hundred of GHz.

Types of voltage controlled oscillators.

Harmonic oscillators. The output is a signal with sinusoidal waveform. Examples are crystal oscillators and tank oscillators.

Relaxation oscillators:

The output is a signal with saw tooth or triangular waveform and provides a wide range of operational frequencies. The output frequency depends on time of charging and discharging of capacitor. Block diagram of VCO is shown in figure 5.17 and saw tooth wave generator VCO is shown in figure 5.18.

Figure 5.17 Block diagram of VCO

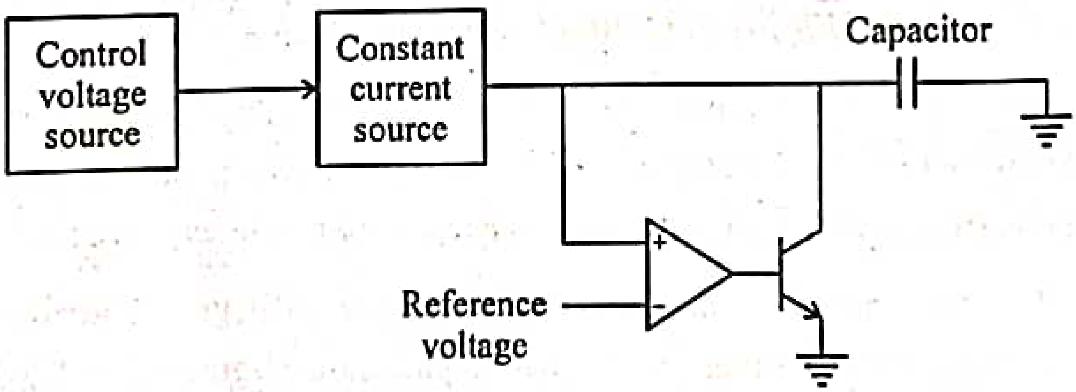


Figure 5.18 Basic working principle of saw tooth wave generator VCO

For a voltage controlled oscillator generating a saw tooth waveform, main component is capacitor who's charging & discharging actually decides formation of output waveform. The input is given in the form of voltage which can be controlled. This voltage is converted into a current signal and is applied to capacitor. As current passes through capacitor it starts charging and voltage starts building across it. As the capacitor charges and voltage across it increases gradually, the voltage is compared with a reference voltage using a comparator.

When capacitor voltage exceeds reference voltage, the comparator generate high logic output which trigger the transistor and capacitor is connected to ground and starts discharging. Thus the output waveform generated is the representation of charging and discharging of capacitor and frequency is controlled by input dc voltage.

Applications of VCO

- Electronic Jamming equipment
- Function generator
- Phase locked loop
- Frequency synthesizer, used in communication circuits.

5.9 POWER AMPLIFIER

The RF power amplifier is the last component of transmitter chain. The purpose of transmitter is to deliver an RF signal with required properties and specified power level to the antenna and need for the PA (Power Amplifier) is in amplification of that signal to level expected at antenna port.

Requirements of power amplifier

✓ It has to have sufficient gain

✓ It has to have sufficient power handling capability

✓ It has to be stable.

5.10 AMPLIFIER POWER RELATIONS

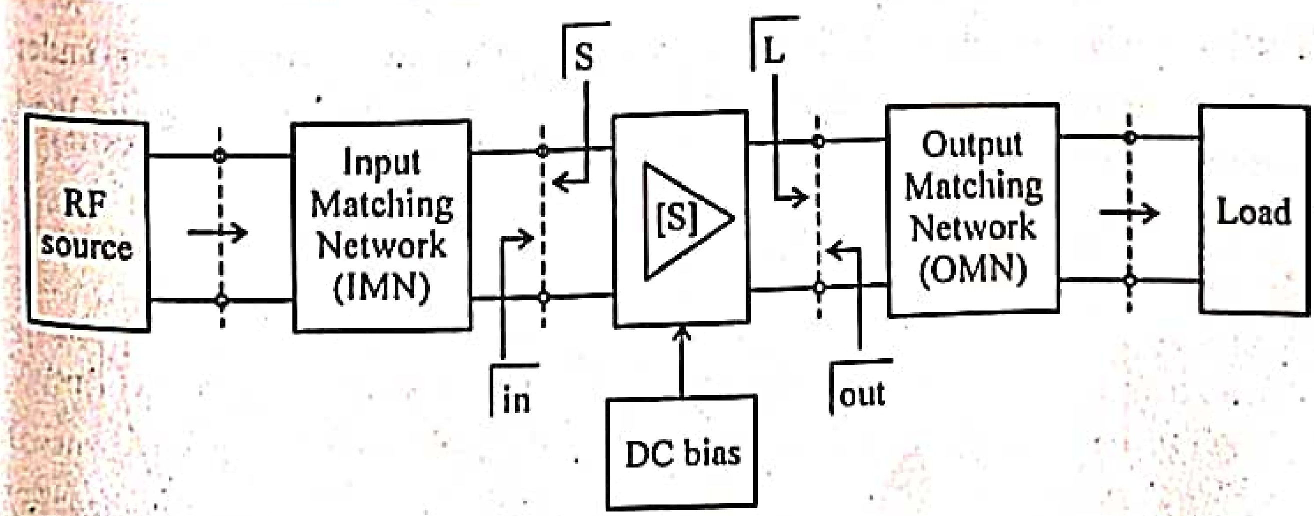


Figure 5.19 Generic amplifier system

A generic single stage amplifier configuration embedded between input and output matching networks is shown in figure 5.19. Input and output matching networks are needed to reduce undesired reflections and thus improve the power flow capabilities. The amplifier is characterized through its S parameter matrix at a particular DC bias point. The following list constitute a set of key amplifier parameters:

- ✓ Gain and gain flatness (in dB)
- ✓ Operating frequency and bandwidth (in Hz)
- ✓ Output power (in dBm)
- ✓ Power supply requirements (in V and A)
- ✓ Noise figure (in dB)

5.10.1 RF Source

There are various power gain definitions that are critical to the understanding of how an RF amplifier functions.

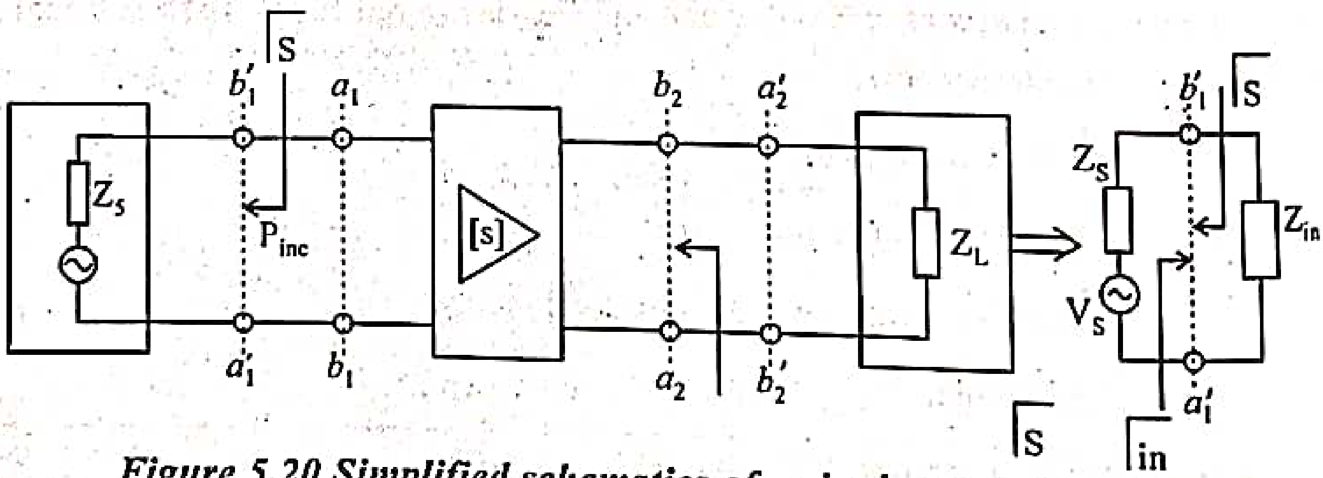


Figure 5.20 Simplified schematics of a single stage amplifier

For this reason let us examine figure 5.19 in terms of its power flow relations under the assumption that the two matching networks are included in the source and load impedances.

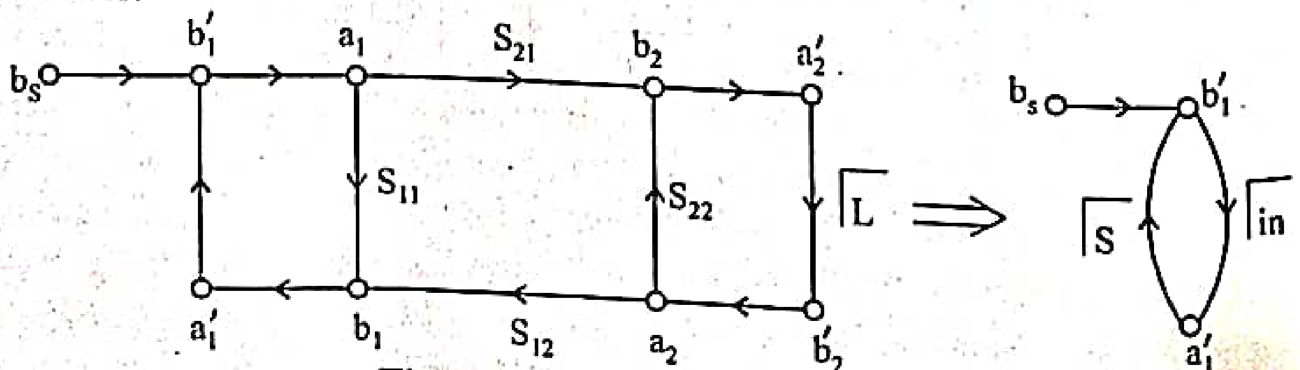


Figure 5.21 Signal flow graph

$$b_S = \frac{\sqrt{Z_o}}{Z_S + Z_o} V_S = b_1' - a_1' \Gamma_S = b_1' \left(1 - \Gamma_{in} \Gamma_S\right) \quad \ldots(1)$$

The incident power wave associated with b_1' is given as

$$P_{inc} = \frac{|b_1'|^2}{2} = \frac{1}{2} \frac{|b_S'|^2}{|1 - \Gamma_{in}\Gamma_S|^2} \quad \ldots(2)$$

Which is the power launched towards the amplifier. The actual input power P_{in} observed at the input terminal of the amplifier is composed of the incident and reflected power waves with the aid of the input reflection coefficient Γ_{in}.

$$P_{in} = P_{inc}\left(1 - |\Gamma_{in}|^2\right) = \frac{1}{2} \frac{|b_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} \left(1 - |\Gamma_{in}|^2\right) \quad \ldots(3)$$

The maximum power transfer from the source to the amplifier is achieved if the input impedance is complex conjugate matched $(Z_{in} = Z_s')$ or in terms of the reflection coefficients, if $\Gamma_{in} = \Gamma_S^*$. Under maximum power transfer condition, the available power P_A as

$$P_A = P_{in}\Big|_{\Gamma_{in} = \Gamma_S^*} = \frac{1}{2} \frac{|b_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2}\Bigg|_{\Gamma_{in} = \Gamma_S^*} \left(1 - |\Gamma_{in}|^2\right)$$

$$= \frac{1}{2} \frac{|b_S|^2}{1 - |\Gamma_S|^2} \quad \ldots(4)$$

This expression makes clear the dependence on Γ_S. If $\Gamma_{in} = 0$ and $\Gamma_S \neq 0$, it is seen from (2) that $P_{inc} = \frac{|b_S|^2}{2}$.

5.10.2 Transducer Power Gain

It quantifies the gain of the amplifier placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}} = \frac{P_L}{P_A}$$

or with $P_L = \frac{1}{2}|b_2|^2 \cdot \left(1 - |\Gamma_L|^2\right)$ we obtain

$$G_T = \frac{P_L}{P_A} = \frac{|b_2|^2}{|b_S|^2}(1-|\Gamma_L|^2)\left(1-|\Gamma_S|^2\right) \quad ...(5)$$

In this expression, the ratio b_2/b_S has to be determined, with help of signal flow graph and based on figure we establish

$$b_2 = \frac{S_{21}\,a_1}{1-S_{22}\Gamma_L} \quad ...(6)$$

$$b_S = \left[1-\left(S_{11}+\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right)\Gamma_S\right]a_1 \quad ...(7)$$

The required ratio is given by

$$\frac{b_2}{b_S} = \frac{S_{21}}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_L\Gamma_S} \quad ...(8)$$

Inserting (8) into (5) results in

$$G_T = \frac{(1-|\Gamma_L|^2)\,|S_{21}|^2\,(1-|\Gamma_S|^2)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_L\Gamma_S|^2} \quad ...(9)$$

Which can be rearranged by defining the input and output reflection coefficients

$$\Gamma_{in} = S_{11}+\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L} \quad ...(10)$$

$$\Gamma_{out} = S_{22}+\frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} \quad ...(11)$$

With these two definitions two more transducer power gain expressions can be derived. First by incorporating (10) into (9) it is seen that

$$G_T = \frac{(1-|\Gamma_L|^2)\,|S_{21}|^2\,(1-|\Gamma_S|^2)}{|1-\Gamma_S\Gamma_{in}|^2\,|1-S_{22}\Gamma_L|^2} \quad ...(12)$$

Second using (11) into (9) results in

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L\Gamma_{out}|^2\,|1-S_{11}\Gamma_S|^2} \quad ...(13)$$

An often employed approximation for the transducer power gain is so called unilateral power gain G_{TU}, which neglect the feedback effect of the amplifier ($S_{12} = 0$). This simplifies (13) into

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L S_{22}|^2\,|1-S_{11}\Gamma_S|^2} \qquad ...(14)$$

5.10.3 Additional Power Relations

The transducer power gain is a fundamental expression from which additional important power relations can be derived. For instance, the available power gain for load side matching $\left(\Gamma_L = \Gamma_{out}^*\right)$ is defined as

$$G_A = G_T\big|_{\Gamma_L = \Gamma_{out}^*}$$

$$= \frac{\text{Power available from the network}}{\text{Power available from the source}} = \frac{P_N}{P_A}$$

or

$$G_A = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left(1-|\Gamma_{out}|^2\right)|1-S_{11}\Gamma_S|^2} \qquad ...(15)$$

Further power gain (operating power gain) is defined as the ratio of the power delivered to the load to the power supplied to the amplifier.

$$G = \frac{\text{Power deliverd to load}}{\text{Power supplied to the amplifier}}$$

$$= \frac{P_L}{P_{in}} = \frac{P_L}{P_A}\cdot\frac{P_A}{P_{in}} = G_T\frac{P_A}{P_{in}}$$

Combining (3), (4) and (12)

$$G = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{\left(1-|\Gamma_{in}|^2\right)|1-S_{22}\Gamma_L|^2} \qquad ...(16)$$

Example: 3

An RF amplifier has the following S-parameters:

$S_{11} = 0.3\angle{-70^o}$, $S_{21} = 3.5\angle{85^o}$, $S_{12} = 0.2\angle{-10^o}$, $S_{22} = 0.4\angle{-45^o}$. Furthermore, the amplifier is connected to a voltage source with $V_s = 5V\angle{0^o}$ and source impedance $Z_S = 40\ \Omega$. The output is utilized to drive an antenna, which has an impedance of $Z_L = 73\ \Omega$. Assuming that the S parameters of the amplifier are measured with reference to a $Z_0 = 50\Omega$

Find the following quantities.

i) ***Transducer gain G_T***
ii) ***Unilateral transducer gain G_{TU}***
iii) ***Available gain G_A***
iv) ***Operating power gain G***
v) ***Incident power to the amplifier P_{inc}***
vi) ***Available power from the source P_A***
vii) ***Power delivered to the load P_L***

Solution:

Source reflection coefficient $\Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0}$

$$= \frac{40 - 50}{40 + 50}$$

$$\Gamma_S = -0.111$$

Load reflection coefficient $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{73 - 50}{73 + 50}$$

$$\Gamma_L = 0.187$$

$$\Gamma_{in} = S_{11} + \frac{S_{21} S_{12} \cdot \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$= 0.146 - j\,0.151$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}\, S_{21}\, \Gamma_S}{1 - S_{11}\, \Gamma_S}$$

$$= 0.265 - j\,0.358$$

i) $$G_T = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{\left|1 - \Gamma_L\, \Gamma_{out}\right|^2 \left|1 - S_{11}\Gamma_S\right|^2} = 12.56 \text{ or } 10.99 \text{ dB}$$

ii) $$G_{TU} = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{\left|1 - \Gamma_L\, S_{22}\right|^2 \left|1 - S_{11}\Gamma_S\right|^2} = 12.67 \text{ or } 11.03 \text{ dB}$$

iii) $$G_A = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{\left|1 - |\Gamma_{out}|^2\right| \left|1 - S_{11}\Gamma_S\right|^2} = 14.74 \text{ or } 11.68 \text{ dB}$$

iv) $$G = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2}{\left|1 - |\Gamma_{in}|^2\right| \left|1 - S_{22}\Gamma_L\right|^2} = 13.74 \text{ or } 11.38 \text{ dB}$$

v) $$P_{inc} = \frac{1}{2} \frac{|b_S|^2}{\left|1 - \Gamma_{in}\Gamma_S\right|^2} = \frac{1}{2} \frac{Z_0}{(Z_S + Z_0)} \frac{|V_S|^2}{\left|1 - \Gamma_{in}\Gamma_S\right|^2} = 74.7 \text{ mw}$$

$$P_{inc}(dBm) = 10 \log (P_{inc}/(1 \text{ mw})) = 18.73 \text{ dBm}$$

vi) $$P_A = \frac{1}{2} \frac{|b_S|^2}{1 - |\Gamma_S|^2} = 78.1 \text{mw or } 18.93 \text{ dBm}$$

vi) $$P_L = P_A\, G_T = 981.4 \text{ mw or } 29.92 \text{ dBm}$$

5.11 STABILITY CONSIDERATIONS

5.11.1 Stability Circles

An amplifier circuit must be stable over the entire frequency range. The RF circuit tend to oscillate depending on operating frequency and termination. The phenomenon of oscillation can be understood in the context of voltage wave along a transmission line.

If $|\Gamma| > 1$ then return voltage increases in magnitude possibly causing instability.

If $|\Gamma| < 1$ causes a diminished return voltage wave (negative feedback).

Amplifier as a two port network characterized through its S - parameters. Amplifier is stable, when magnitude of reflection coefficient are less than unity.

$$|\Gamma_L| < 1, \ |\Gamma_S| < 1 \qquad ...(1)$$

$$|\Gamma_{in}| = \left|\frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_S}\right| < 1 \qquad ...(2)$$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1 \qquad ...(3)$$

where $\quad \Delta = S_{11} S_{22} - S_{12} S_{21} \qquad ...(4)$

Since S parameters are fixed for a particular frequency, the only factor that have a parametric effect on stability are Γ_L and Γ_S

In terms of amplifier output port, the condition is established for which (2) is satisfied.

To this end, the complex quantities

$$\left.\begin{aligned} S_{22} &= S_{11}^R + j S_{11}^I, \qquad & S_{22} &= S_{22}^R + j S_{22}^I \\ \Delta &= \Delta^R + j\Delta^I, \qquad & \Gamma_L &= \Gamma_L^R + j\Gamma_L^I \end{aligned}\right\} \qquad ...(5)$$

substituting equation (5) into (2) output stability circle equation is

$$\left(\Gamma_L^R - C_{out}^R\right)^2 + \left(\Gamma_L^I - C_{out}^I\right)^2 = r_{out}^2 \qquad ...(6)$$

where circle radius is given by

$$r_{out} = \frac{|S_{12} S_{21}|}{||S_{22}|^2 - |\Delta|^2|} \qquad ...(7)$$

and center of circle is located at

$$C_{out} = C_{out}^R + jC_{out}^I = \frac{(S_{22} - S_{11}^{*} \Delta)^{*}}{|S_{22}|^2 - |\Delta|^2} \qquad ...(8)$$

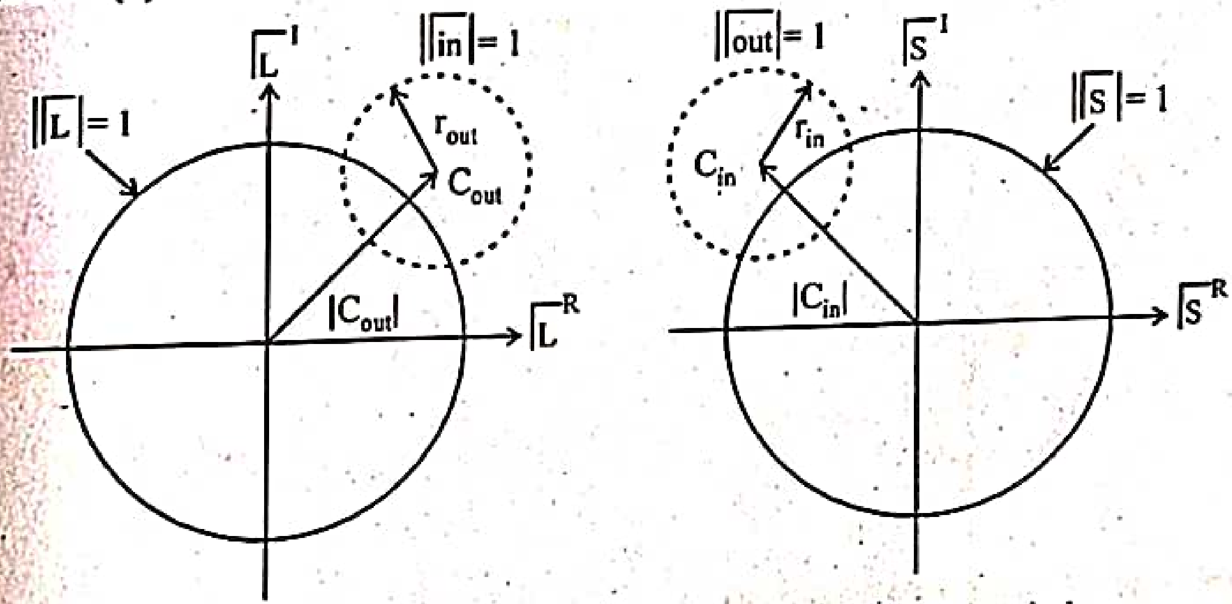
as depicted in figure 5.22(a) Interms of input port, substituting (5) into (3) yields the input stability circle equation

$$\left(\Gamma_S^R - C_{in}^R\right)^2 + \left(\Gamma_S^I - C_{in}^I\right)^2 = r_{in}^2 \qquad ...(9)$$

where $$r_{in} = \frac{S_{12}\,S_{21}}{\left|\,|S_{11}|^2 - |\Delta|^2\right|} \quad ...(10)$$

and $$C_{in} = \frac{(S_{11} - S_{22}^{*}\,\Delta)^{*}}{|S_{11}|^2 - |\Delta|^2} = C_{in}^{R} + jC_{in}^{I} \quad ...(11)$$

When plotted in Γ_S plane we obtain response as schematically shown in figure 5.22 (b).



(a) output stability circle (b) Input stability circle

Figure 5.22 Stability circle $|\Gamma_{in}| = 1$ in the complex Γ_L-plane and stability circle $|\Gamma_{out}| = 1$ in the complex Γ_S-plane

5.11.2 Unconditional Stability

Unconditional stability refers to the situation where the amplifier remains stable for any passive source and load at the selected frequency and bias conditions.

For $|S_{11}| < 1$ and $|S_{22}| < 1$, it is stated as

$$\left|\,|C_{in}| - r_{in}\right| > 1 \quad ...(12)$$

$$\left|\,|C_{out}| - r_{out}\right| > 1 \quad ...(13)$$

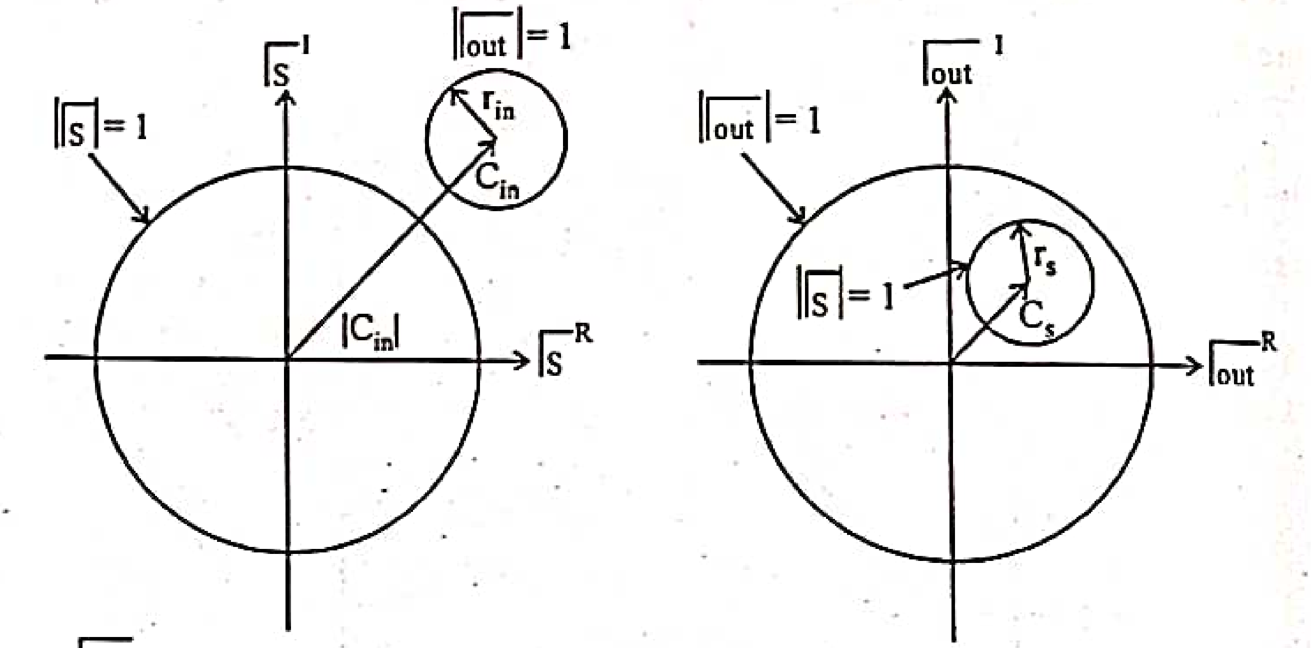
In other words, the stability circles have to reside completely outside the $|\Gamma_S| = 1$ and $|\Gamma_L| = 1$ circles.

The condition for stability is expressed in terms of stability factor K as

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} > 1 \qquad ...(14)$$

The stability factor K is also referred as Rollet factor.

It applies for both input and output ports.



(a) $|\Gamma_{out}| = 1$ circle must reside outside (b) $|\Gamma_S| = 1$ circle must reside inside

Figure 5.23 Unconditional stability in the Γ_S and Γ_{out} planes for $|S_{11}| < 1$

5.11.3 Stabilization Methods

If the operation of a FET or BJT is found to be unstable, an attempt can be made to stabilize the transistor. The conditions $|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$ can be written in terms of input and output impedances.

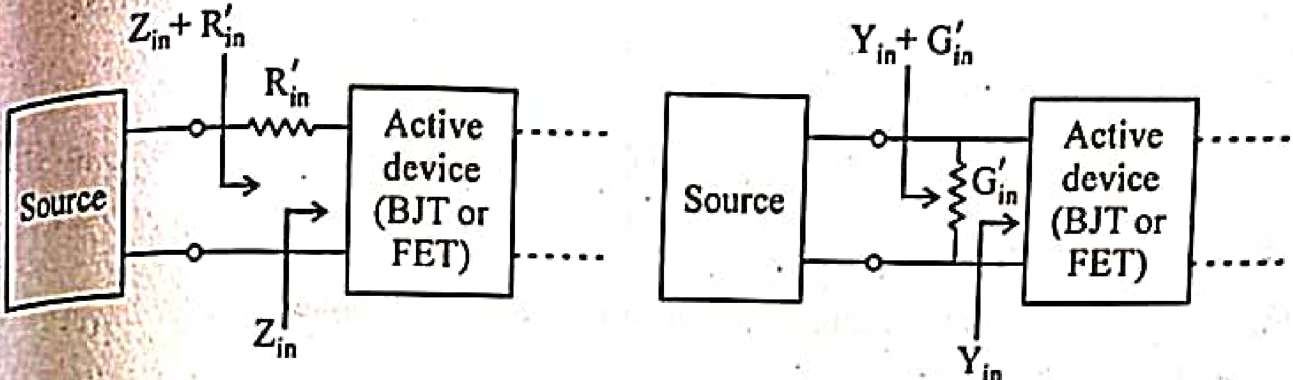
$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_o}{Z_{in} + Z_o}\right| > 1 \quad |\Gamma_{out}| = \left|\frac{Z_{out} - Z_o}{Z_{out} + Z_o}\right| > 1$$

Which imply $Re(Z_{in}) < 0$ and $Re(Z_{out}) < 0$. One way to stabilize the active device is to add a series resistance or a shunt conductance to port.

Figure 5.24 shows the configuration for the input port. This loading in conjunction with $Re(Z_S)$ must compensate the negative contribution of $Re(Z_{in})$. Thus we require

$$\text{Re}\,(Z_{in} + R'_{in} + Z_S) > 0 \text{ (or)}$$

$$\text{Re}\,(Y_{in} + G'_{in} + Y_S) > 0$$



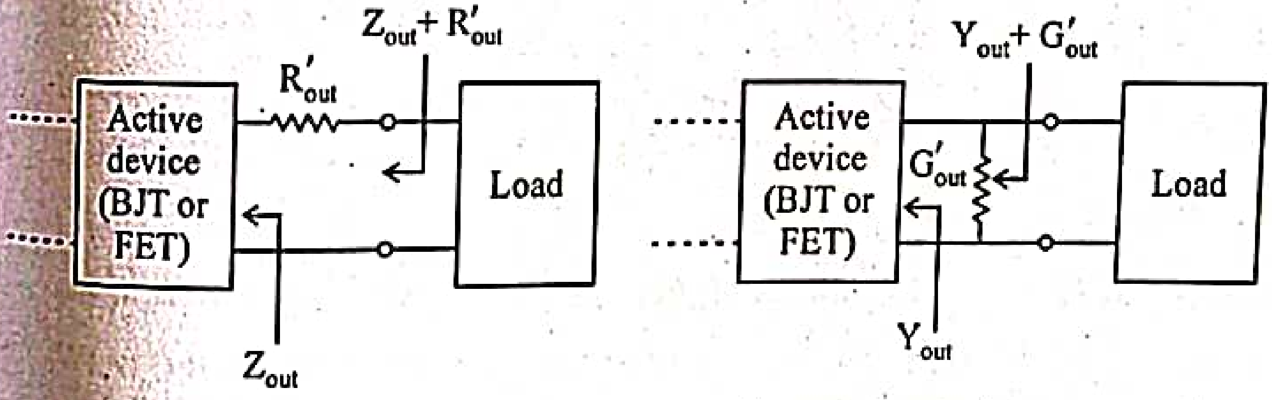
(a) Series Resistance (b) Shunt conductance

Figure 5.24 Stabilization of input port through series resistance or shunt conductance

Figure 5.25 shows the stabilization of the output port. The corresponding condition is

$$\text{Re}\,(Z_{out} + R'_{out} + Z_L) > 0 \text{ (or)}$$

$$\text{Re}\,(Y_{out} + G'_{out} + Y_L) > 0$$



(a) Series resistance (b) Shunt conductance

Figure 5.25 Stabilization of output port through series resistance or shunt conductance

Example: 4

A MESFET operated at 5.7 GHz has following S parameters. $S_{11} = 0.5\angle{-60^\circ}$, $S_{12} = 0.02\angle{0^\circ}$, $S_{21} = 6.5\angle{115^\circ}$, $S_{22} = 0.6\angle{-35^\circ}$. Verify the circuit whether it is unconditionally stable or not?

Solution:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}|\,|S_{21}|}$$

$$K = 2.17$$

$$|\Delta| = |S_{11}\,S_{22} - S_{12}\,S_{21}|$$

$$|\Delta| = 0.42$$

$K > 1$ and $|\Delta| < 1$ so the transistor is unconditionally stable.

TWO MARK QUESTIONS AND ANSWERS

1. **What are the key parameters used to evaluate the performance of an amplifier?**

   i) Gain and gain flatness

   ii) Operating frequency and bandwidth

   iii) Output power

   iv) Power supply requirements

   v) Input and output reflection coefficient

   vi) Noise figure.

2. **Define transducer power gain.**

   It is gain of the amplifier when placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}} = \frac{P_L}{P_A}$$

3. **Define unilateral power gain.**

   It is amplifier power gain when feedback effect of amplifier in neglected $S_{12} = 0$.

4. **Define unconditional stability.**

   Unconditional stability refers to the situation where the amplifier remains stable for Unconditional stability refers to the situation where the amplifier remains stable for any passive source and load at the selected frequency and bias conditions.

5. **What is the need of matching network?**

   It can help stabilize the amplifier by keeping the source and load impedances in the appropriate range.

6. **What are factors used for selecting a matching network?**

   ✓ Complexity

   ✓ Bandwidth requirement

   ✓ Adjustability

   ✓ Implementation

7. Define operating power gain.

It is defined as ratio of power delivered to the load to the power supplied to the amplifier

$$G = \frac{\text{Power deliverd to load}}{\text{Power supplied to the amplifier}} = \frac{P_L}{P_{in}}$$

8. What are the advantages of microwave transistors?

Microwave transistors are miniaturized designs to reduce device and package parasitic capacitances and inductances and to overcome the finite transit time of the charge carriers in the semiconductor materials.

9. What is bipolar transistor?

Bipolar is a three semiconductor (pnp or npn) region structure where charge carriers of both negative (electrons) and positive (holes) polarities are involved in transistor operation.

10. Write the applications of bipolar transistors.

Bipolar transistors are suitable for oscillator and power amplifier applications in addition to small signal amplifiers.

11. What are the different modes of bipolar transistor?

✓ Normal (active) mode

✓ Saturation mode

✓ Cut off mode

✓ Inverse (Inverted mode)

12. What is referred as unipolar transistor?

In field effect transistors, the current flow is carried by majority carriers either electrons or holes, this type is referred to as unipolar transistor.

13. Write the advantages of unipolar transistor?

✓ Efficiency is higher

✓ Noise figure is low

✓ It may have voltage gain in addition to current gain.

✓ Its operating frequency is upto X band.

✓ Its input resistance is very high upto several mega ohms.

14. What is MESFET?

Field effect transistors at microwave frequencies are mostly fabricated in GaAs and use a metal semiconductor schottky junction for gate contact. This device is referred as MESFET (Metal Semiconductor FET)

15. Define pinch off voltage.

It is the gate reverse voltage that removes all the free charge from the channel.

16. What is called high electron mobility transistor?

The field effect transistor made using hetero junction is called high electron mobility transistor.

17. Define threshold voltage.

A minimum gate voltage is required to induce the channel and it is called threshold voltage.