



Inspire...Educate...Transform.

Clustering and IBL

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DISTANCE METRICS

Desiderata for proximity

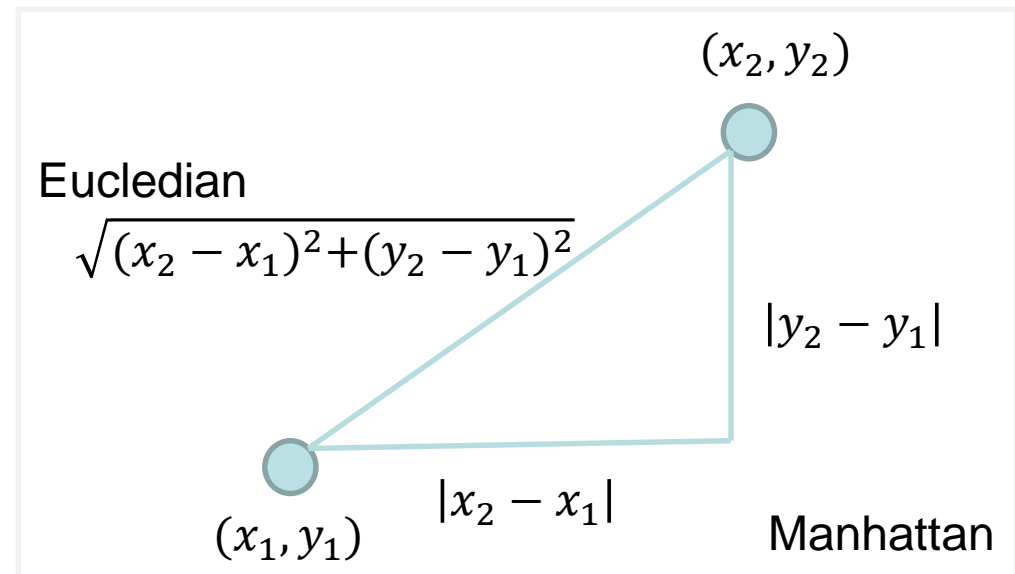
- If d_1 is near d_2 , then d_2 is near d_1 .
- If d_1 near d_2 , and d_2 near d_3 , then d_1 is not far from d_3 .
- No document is closer to d than d itself.

Distance for numeric attributes

- We denote distance with: $dist(\mathbf{x}_i, \mathbf{x}_j)$
 - Where \mathbf{x}_i and \mathbf{x}_j are data points (vectors)
- Minkowski distance

$$dist(\mathbf{x}_i, \mathbf{x}_j) = ((x_{i1} - x_{j1})^h + (x_{i2} - x_{j2})^h + \dots + (x_{ir} - x_{jr})^h)^{\frac{1}{h}}$$

- Where h is positive integer.
- $h = 2$ is **Euclidean** distance
- $h = 1$ is **Manhattan** distance





When to choose what?

- When all attributes have similar scale: (1,2), (2,1)
 - Manhattan = $\text{Abs}(1-2) + \text{Abs}(2-1) = 2$
 - Euclidean = $\sqrt{2}$

Choosing the distance metric



- When attributes have different ranges (10, 100), (50, 500)
 - Manhattan = 440
 - Euclidean = 401.99
- Manhattan is more stable than Euclidean
 - Scaling is better

Squared Euclidean and Chebyshev distance



- **Squared Euclidean distance:** Place greater weight on data points that are further apart

$$\text{dist}(\mathbf{x}_i, \mathbf{x}_j) = (x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \dots + (x_{ir} - x_{jr})^2$$

- **Chebyshev distance:** Two data points are "different" if they are different on any one of the attributes.

$$\text{dist}(\mathbf{x}_i, \mathbf{x}_j) = \max(|x_{i1} - x_{j1}|, |x_{i2} - x_{j2}|, \dots, |x_{ir} - x_{jr}|)$$

– **Security alerts**

More metrics

- Weighted squares
 - A particular attribute may be a lot more important than other attributes
- Text: Cosine similarity

$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| |\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \cdot \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

Diagram illustrating the components of the cosine similarity formula:

- Dot product**: $\vec{q} \cdot \vec{d}$
- Unit vectors**: $\frac{\vec{q}}{|\vec{q}|}$ and $\frac{\vec{d}}{|\vec{d}|}$

Doc	Team	Coach	Hockey	Baseball	Soccer	Penalty	Score	Win	loss
Doc1	5	0	3	0	2	0	0	2	0
Doc2	3	0	2	0	1	1	0	1	0
Doc3	0	7	0	2	1	0	0	3	0
Doc4	0	1	0	0	1	2	2	0	3



Distance functions for Binary & Nominal attributes

- We use a confusion matrix to introduce the distance functions/measures.

Confusion matrix



		Data point j		
		1	0	
Data point i	1	a	b	$a+b$
	0	c	d	$c+d$
		$a+c$	$b+d$	$a+b+c+d$

$$\text{Hamming distance} = \frac{\text{\#of dissimilar attributes}}{\text{\#of dissimilar} + \text{\#of similar}} = \frac{b + c}{b + c + a + d}$$

Confusion matrix



	x_1	x_2	x_3	x_4	x_5
R_1	1	0	0	1	1
R_2	0	0	0	1	0

What is the Manhattan Distance for R_1 - R_2 ?

2

What is the distance normalized for # of attributes?

2/5

		R_2	
		1	0
R_1	1	1 (a)	2 (b)
	0	0 (c)	2 (d)

$$Distance = \frac{b + c}{a + b + c + d} = \frac{2}{5}$$

Symmetric binary attributes



- A binary attribute is **symmetric** if both of its states (0 and 1) have equal importance, and carry the same weights, e.g., male and female of the attribute Gender

Asymmetric binary attributes

- **Asymmetric**: if one of the states is more important or more valuable than the other.
 - By convention, state 1 represents the more important state, which is typically the rare or infrequent state.
 - **Jaccard coefficient** is a popular measure

$$dist(\mathbf{x}_i, \mathbf{x}_j) = \frac{b + c}{a + b + c}$$

Data point i

	Data point j		
	1	0	
1	a	b	$a+b$
0	c	d	$c+d$
	$a+c$	$b+d$	$a+b+c+d$

- We can have some variations, adding weights

Dissimilarity between Binary Variables

- Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y (Yes) and P (Positive) be set to 1, and the value N (Negative) be set to 0

$$d(jack, mary) = \frac{0 + 1}{2 + 0 + 1} = 0.33$$

$$d(jack, jim) = \frac{1 + 1}{1 + 1 + 1} = 0.67$$

$$d(jim, mary) = \frac{1 + 2}{1 + 1 + 2} = 0.75$$

Categorical variables with multiple levels

- A generalization of the binary variable in that it can take more than 2 states, e.g., red, yellow, blue, green
- Method 1: Simple matching
 - m : # of matches, p : total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2: use a large number of binary variables
 - creating a new binary variable for each of the M nominal states

Another distance metric used in supervised learning



Value difference measure (VDM): d_{ij}

All classes

$$\sum_{h=1} |P(h|val_i) - P(h|val_j)|$$

ID	Age	Income	Family	CCAvg	Personal Loan
1	Young	Low	4	Low	0
2	Old	Low	3	Low	0
3	Middle	Low	1	Low	0
4	Middle	Medium	1	Low	0
5	Middle	Low	4	Low	0
6	Middle	Low	4	Low	0
10	Middle	High	1	High	1
17	Middle	Medium	4	Medium	1
19	Old	High	2	High	1
30	Middle	Medium	1	Medium	1
39	Old	Medium	3	Medium	1
43	Young	Medium	4	Low	1
48	Middle	High	4	Low	1

Distance between F1 and F2

$$\begin{aligned}
 &= |P(0|F1) - P(0|F2)| + |P(1|F1) - P(1|F2)| \\
 &= |0.5 - 0| + |0.5 - 1| \\
 &= 1
 \end{aligned}$$



Ordinal variables

- Same as numeric
- Look up is better than computation

Look up matrix for ordinal with 3 states


$$\begin{bmatrix} & 1 & 2 & 3 \\ 1 & 0 & 1 & 4 \\ 2 & 1 & 0 & 1 \\ 3 & 4 & 1 & 0 \end{bmatrix}$$



Clustering

Unsupervised learning

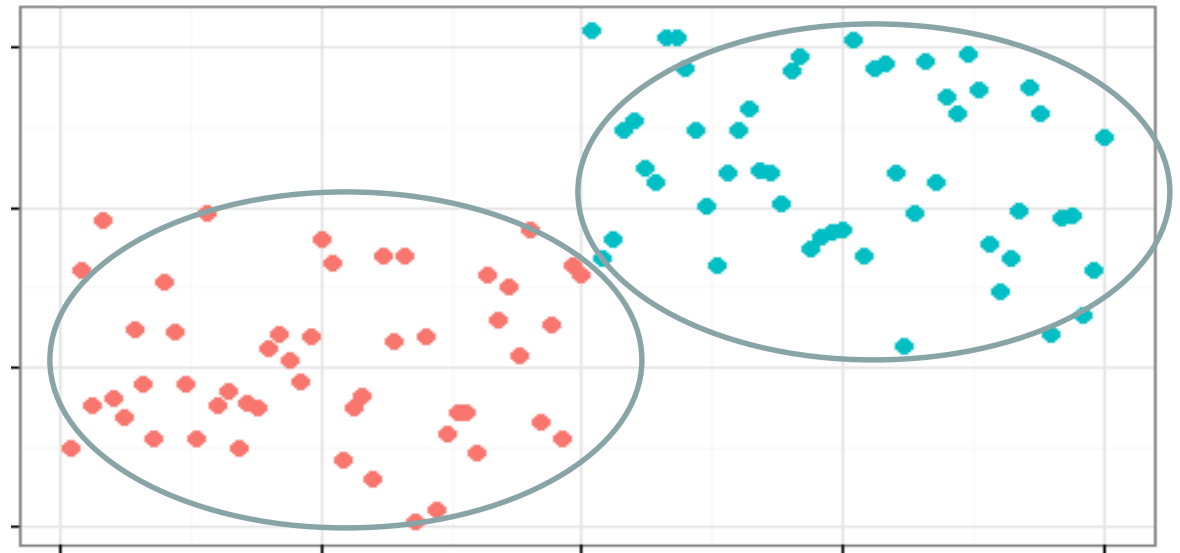
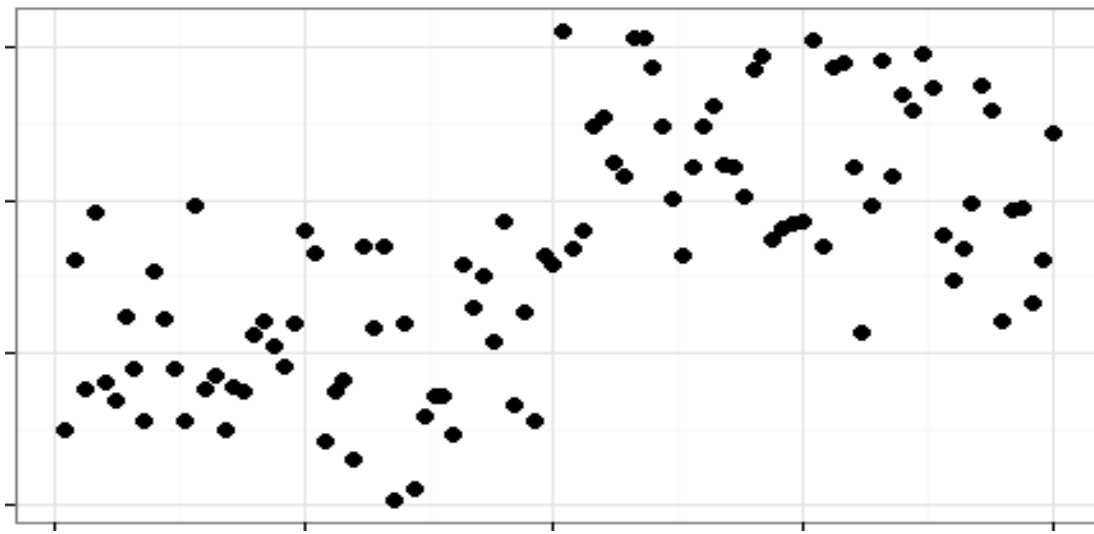


- Supervised: Data and target
- Unsupervised: Just data

Clustering



- One of the unsupervised learning techniques
- Finding **similarity groups** in data, called **clusters**, i.e.,
 - Data instances that are **similar** to (near) each other are in the **same cluster**
 - Data instances that are very **different** (far away) from each other fall in **different clusters**.



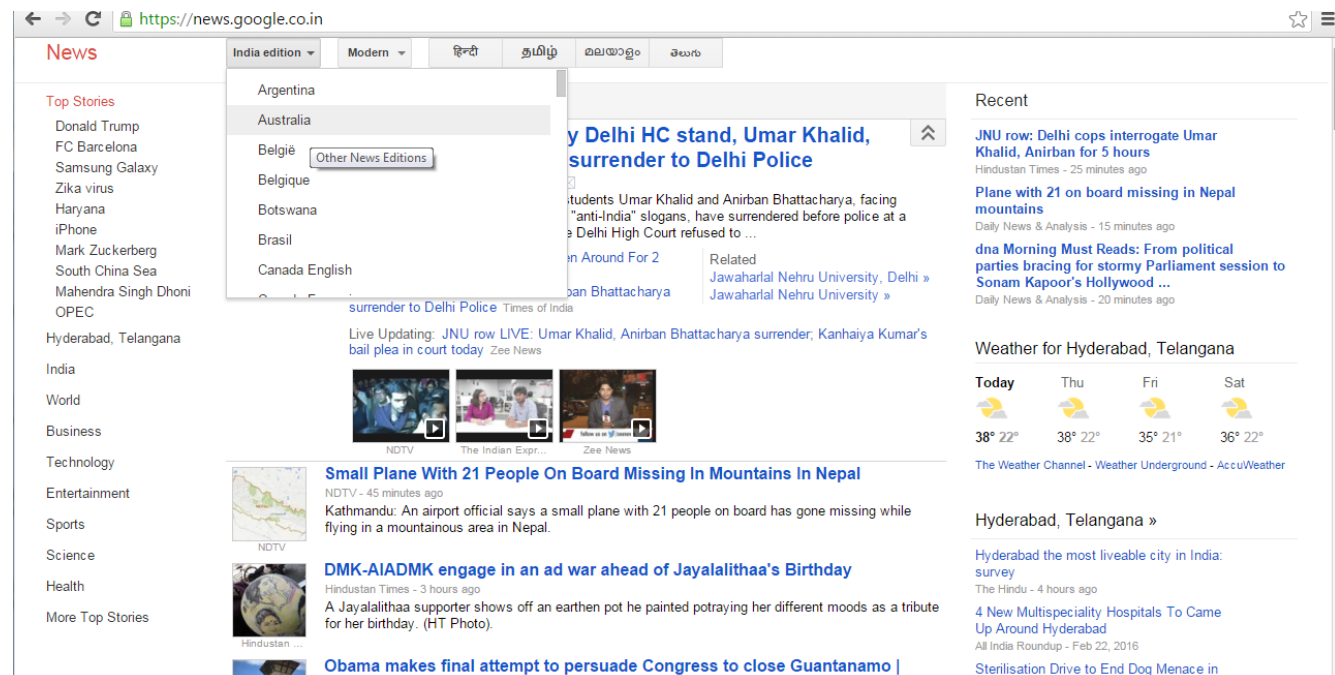
A few clustering applications

- In marketing, segment customers according to their similarities
 - To do targeted marketing
 - It is not uncommon to have over 100,000 segments in insurance clustering

Google search



- Given a collection of text documents, organize them according to their content similarities
 - e.g., Google news



Algorithms



- **Hierarchical approach**: Create a hierarchical decomposition of the set of data (or objects) using some criterion (Wald)
- **Partitioning approach**: Construct various partitions and then evaluate them by some criterion, e.g., minimizing the sum of square errors (K-means, Spectral clustering)
- **Model-based methods**: A model is hypothesized for each of the clusters and tries to find the best fit of that model to each other (EM)



HIERARCHICAL (AGGLOMERATIVE) CLUSTERING

Agglomerative clustering (Hierarchical)

- Assign each item to its own cluster, so that if you have N items, you now have N clusters, each containing just one item.
- Merge most similar clusters into a single cluster, so that now you have one less cluster.
- Compute distances (similarities) between the new cluster and each of the old clusters.
- Repeat steps 2 and 3 until all items are clustered into a single cluster of size N .

Example of agglomerative clustering



	BOS	NY	DC	MIA	CHI	SEA	SF	LA	DEN
BOS	0	206	429	1504	963	2976	3095	2979	1949
NY	206	0	233	1308	802	2815	2934	2786	1771
DC	429	233	0	1075	671	2684	2799	2631	1616
MIA	1504	1308	1075	0	1329	3273	3053	2687	2037
CHI	963	802	671	1329	0	2013	2142	2054	996
SEA	2976	2815	2684	3273	2013	0	808	1131	1307
SF	3095	2934	2799	3053	2142	808	0	379	1235
LA	2979	2786	2631	2687	2054	1131	379	0	1059
DEN	1949	1771	1616	2037	996	1307	1235	1059	0

- No assignment of centroid upfront.
- Each point is considered a cluster.
- Find the closest clusters and merge them.



	BOS/NY	DC	MIA	CHI	SEA	SF	LA	DEN
BOS/NY	0	223	1308	802	2815	2934	2786	1771
DC	223	0	1075	671	2684	2799	2631	1616
MIA	1308	1075	0	1329	3273	3053	2687	2037
CHI	802	671	1329	0	2013	2142	2054	996
SEA	2815	2684	3273	2013	0	808	1131	1307
SF	2934	2799	3053	2142	808	0	379	1235
LA	2786	2631	2687	2054	1131	379	0	1059
DEN	1771	1616	2037	996	1307	1235	1059	0



	BOS/NY/DC	MIA	CHI	SEA	SF	LA	DEN
BOS/NY/DC	0	1075	671	2684	2799	2631	1616
MIA	1075	0	1329	3273	3053	2687	2037
CHI	671	1329	0	2013	2142	2054	996
SEA	2684	3273	2013	0	808	1131	1307
SF	2799	3053	2142	808	0	379	1235
LA	2631	2687	2054	1131	379	0	1059
DEN	1616	2037	996	1307	1235	1059	0



	BOS/ NY/DC	MIA	CHI	SEA	SF/LA	DEN
BOS/NY/DC	0	1075	671	2684	2631	1616
MIA	1075	0	1329	3273	2687	2037
CHI	671	1329	0	2013	2054	996
SEA	2684	3273	2013	0	808	1307
SF/LA	2631	2687	2054	808	0	1059
DEN	1616	2037	996	1307	1059	0



	BOS/NY/DC/ CHI	MIA	SEA	SF/LA	DEN
BOS/NY/DC/CHI	0	1075	2013	2054	996
MIA	1075	0	3273	2687	2037
SEA	2013	3273	0	808	1307
SF/LA	2054	2687	808	0	1059
DEN	996	2037	1307	1059	0

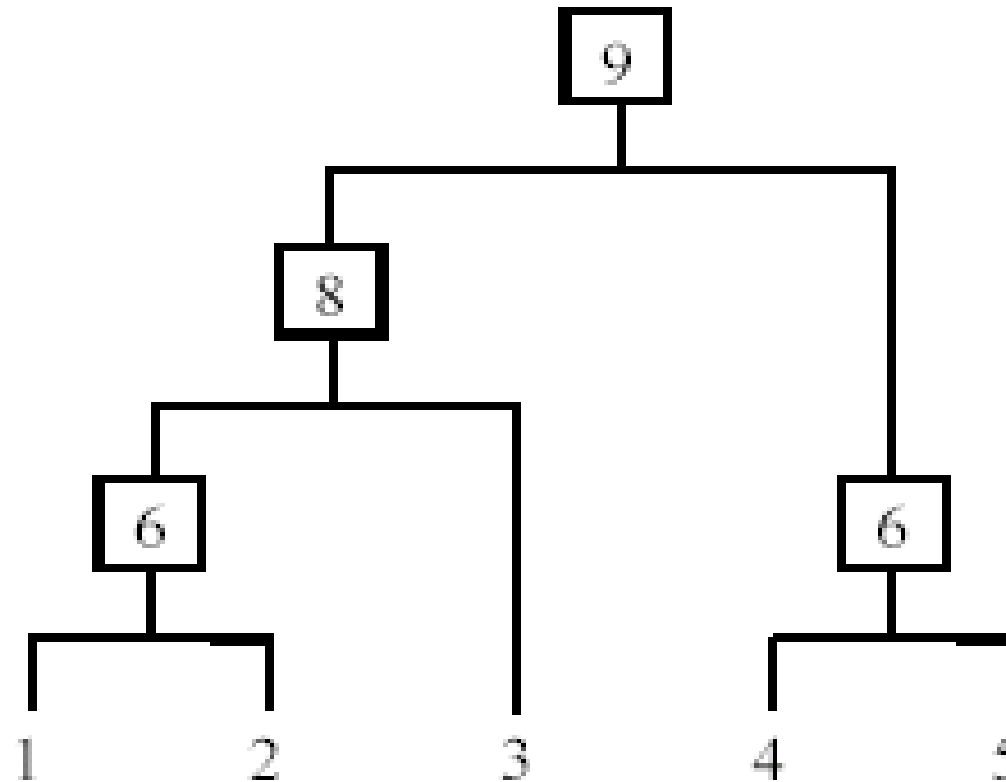


	BOS/NY/DC/CHI	MIA	SF/LA/SEA	DEN
BOS/NY/DC/CHI	0	1075	2013	996
MIA	1075	0	2687	2037
SF/LA/SEA	2054	2687	0	1059
DEN	996	2037	1059	0

	BOS/NY /DC/CHI/DEN	MIA	SF/LA/SEA
BOS/NY/DC/CHI/DEN	0	1075	1059
MIA	1075	0	2687
SF/LA/SEA	1059	2687	0

	BOS/NY /DC/CHI /DEN/SF /LA/SEA	MIA
BOS/NY/DC/CHI/DEN/SF/LA/SEA	0	1075
MIA	1075	0

Hierarchical Clustering



Decomposes data objects into a several levels of nested partitioning (tree of clusters).

A clustering of the data objects is obtained by cutting the dendrogram at the desired level, then each connected component forms a cluster.

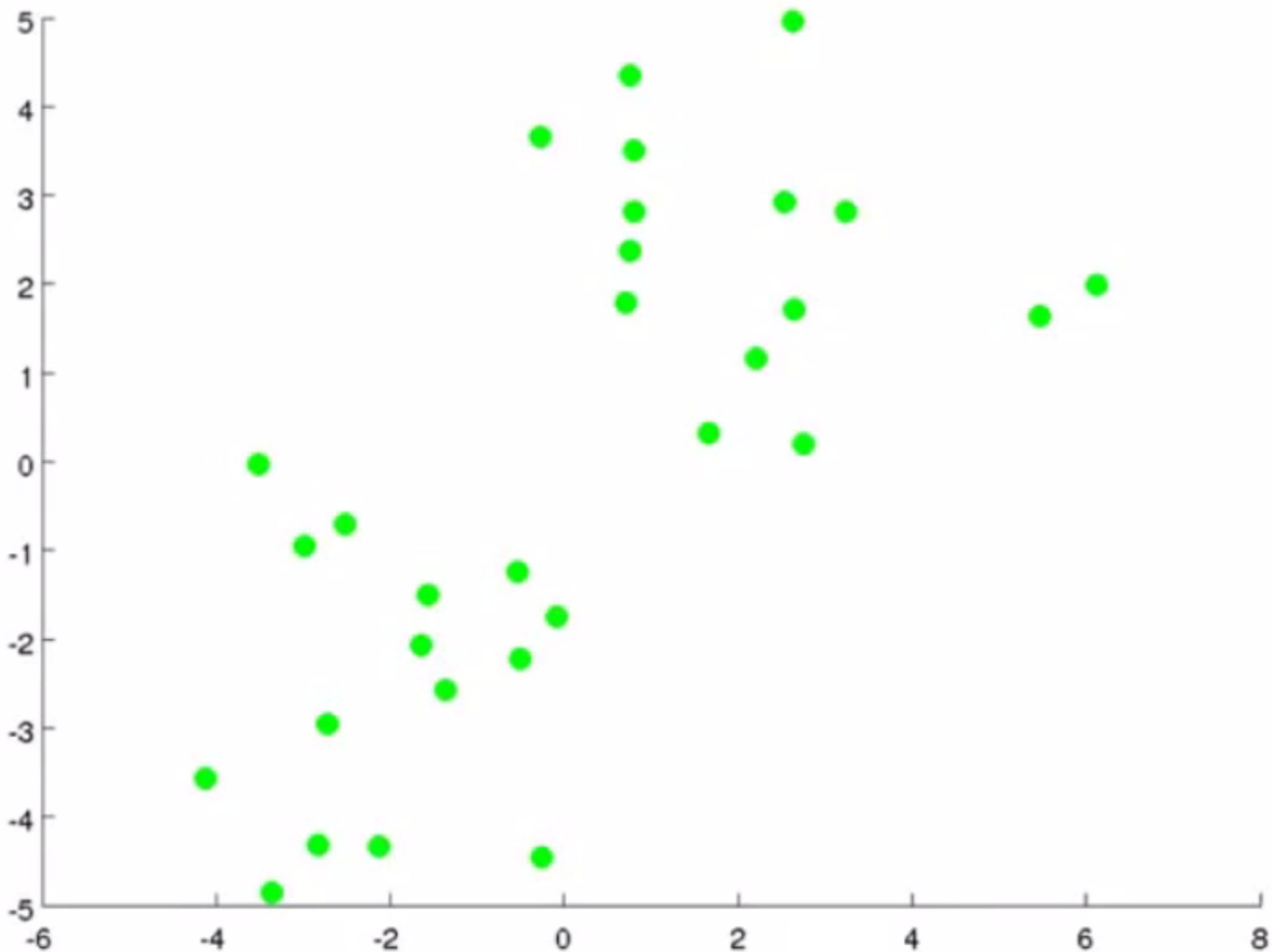


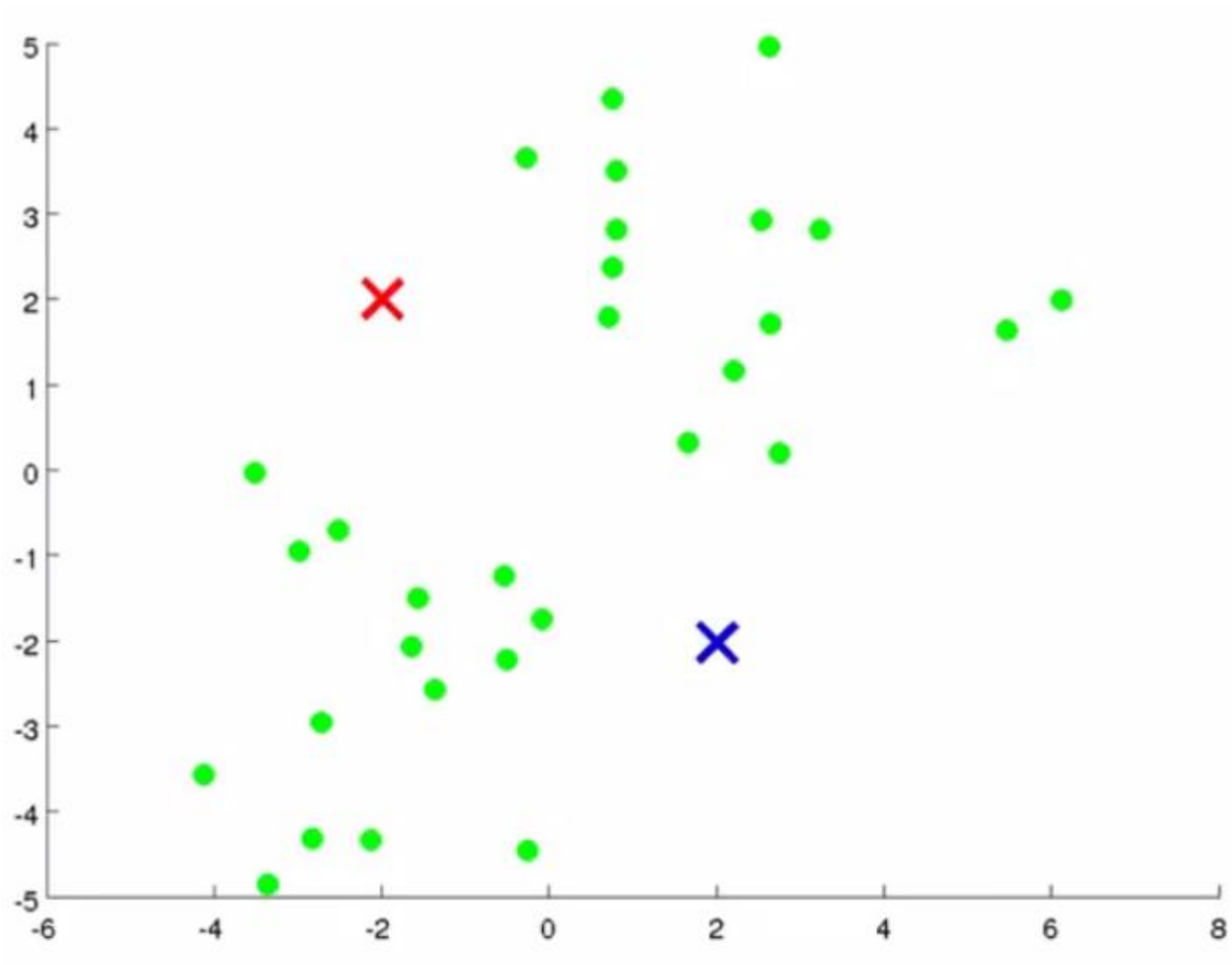
Partitioning algorithms

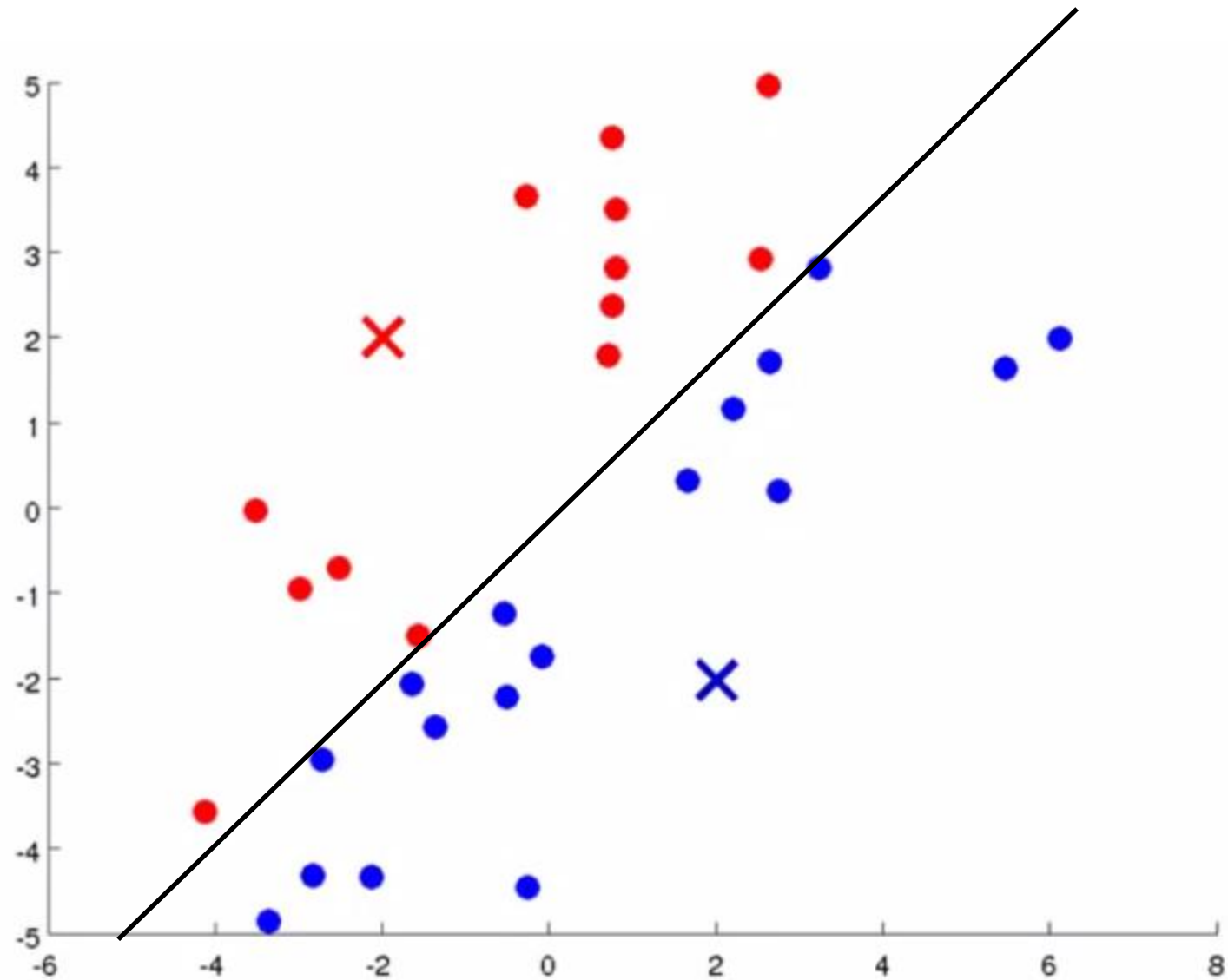
K-MEANS AND K-MEDOIDS

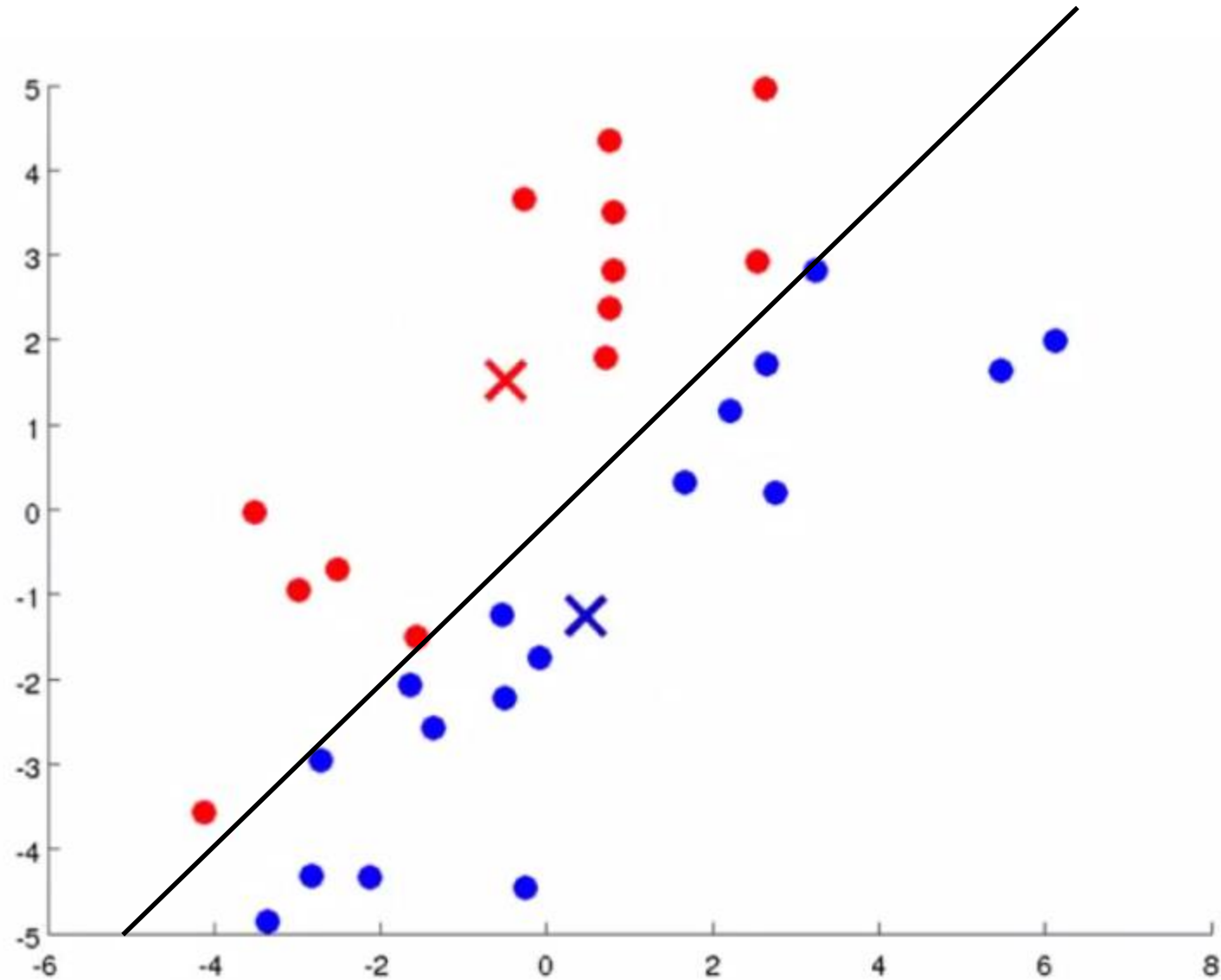
K-means clustering

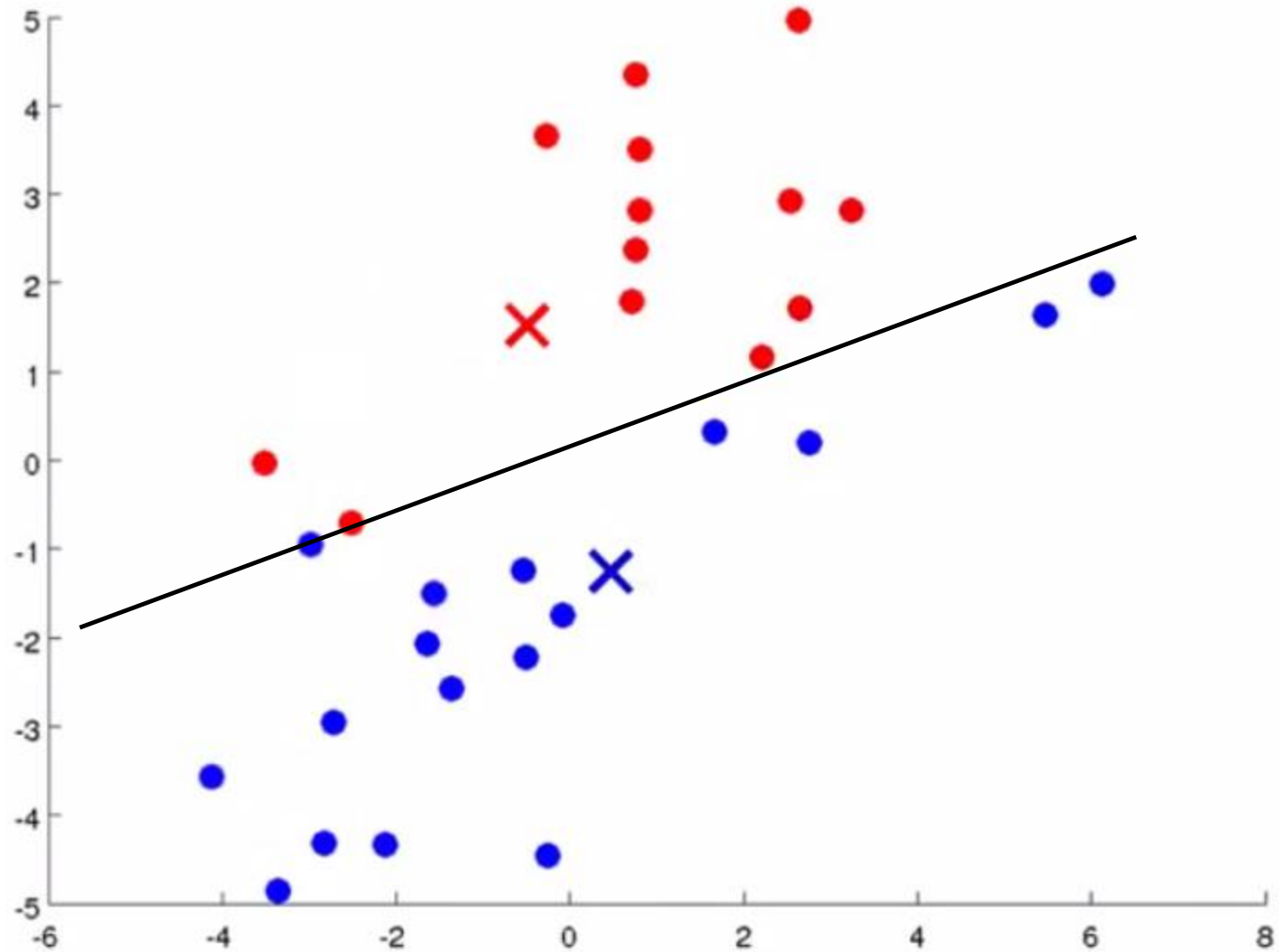
- K-means is a **partitional clustering** algorithm as it partitions the given data into k clusters.
 - Each cluster has a cluster **center**, called **centroid**.
 - k is specified by the user

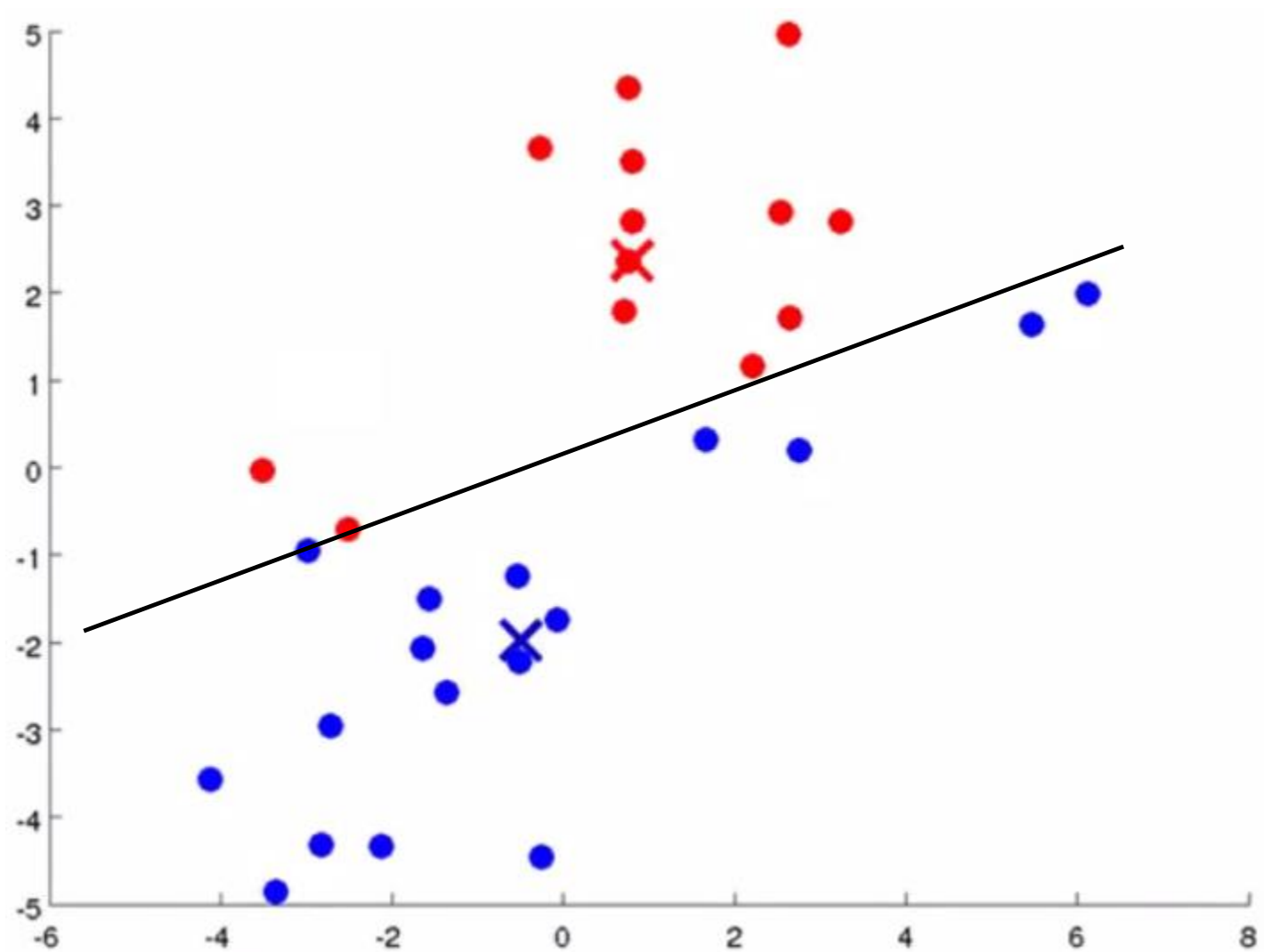


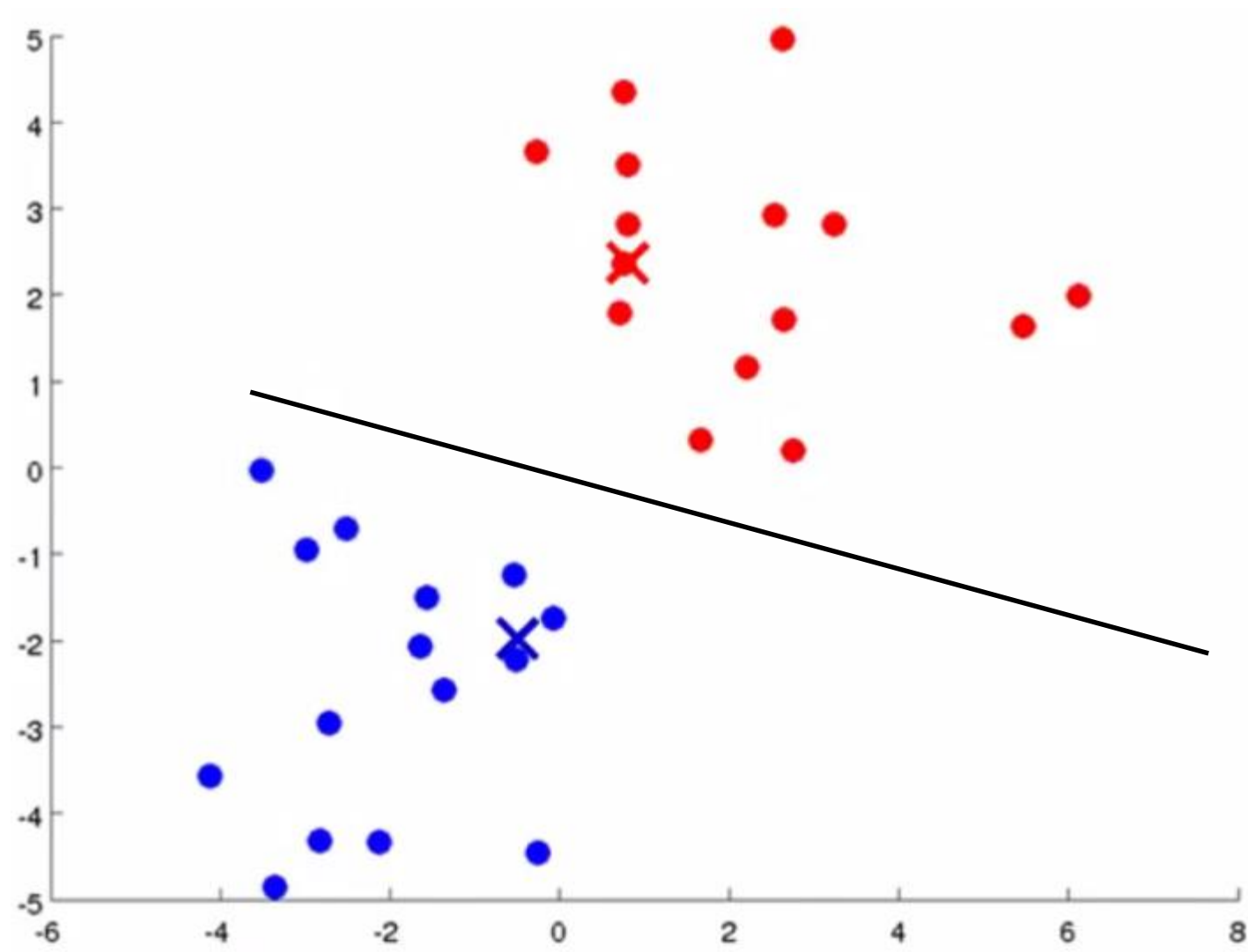


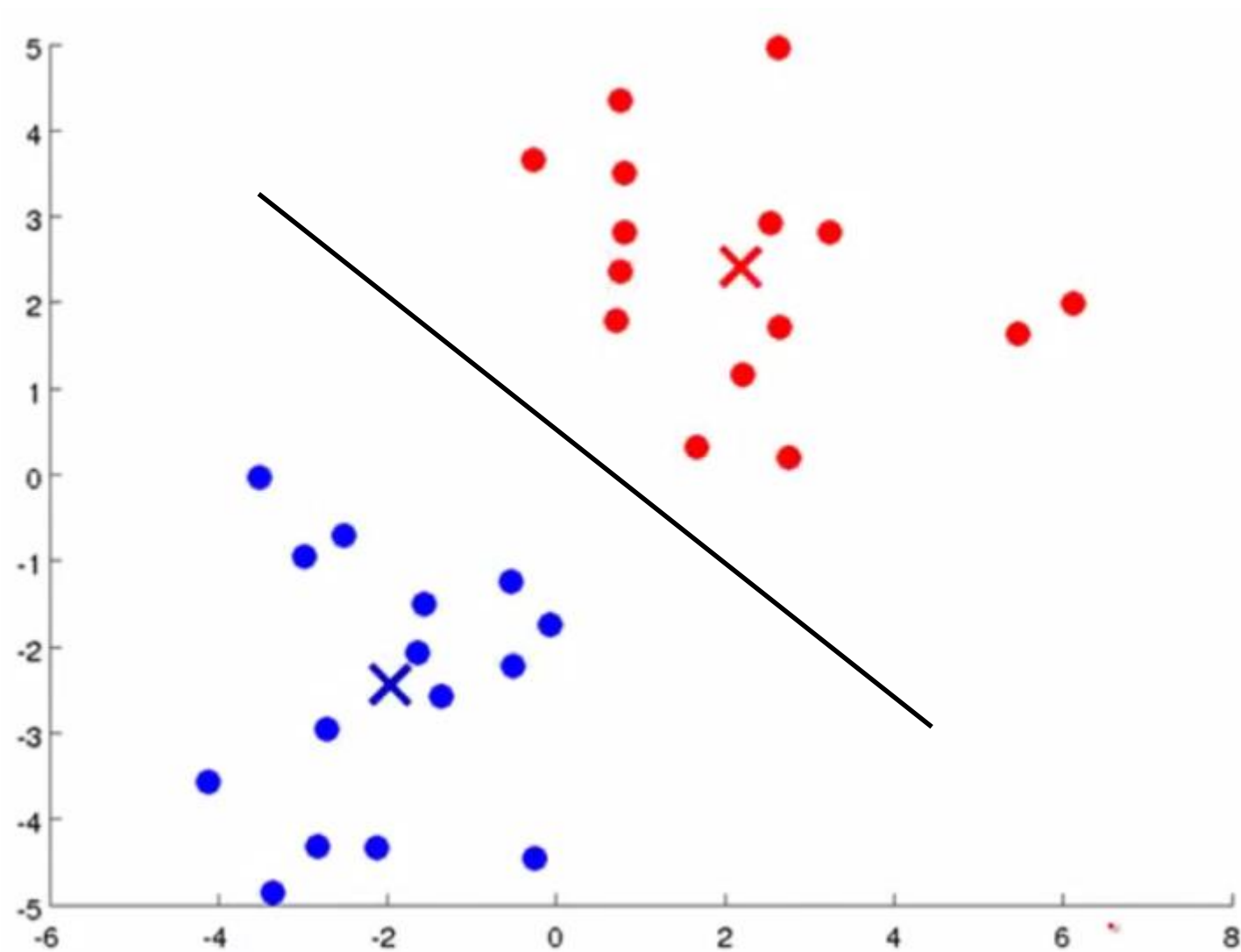












K-means algorithm



- Given k , the *k-means* algorithm works as follows:
 1. Randomly choose k data points (**seeds**) to be the initial **centroids**, cluster centers
 2. Assign each data point to the closest **centroid**
 3. Re-compute the **centroids** using the current cluster memberships.
 4. If a convergence criterion is not met, or **if some clusters don't get any points** go to **2**.

Optimizing



$$\frac{1}{m} \sum_{i=1}^m ||x^{(i)} - \mu_{c(i)}||^2$$

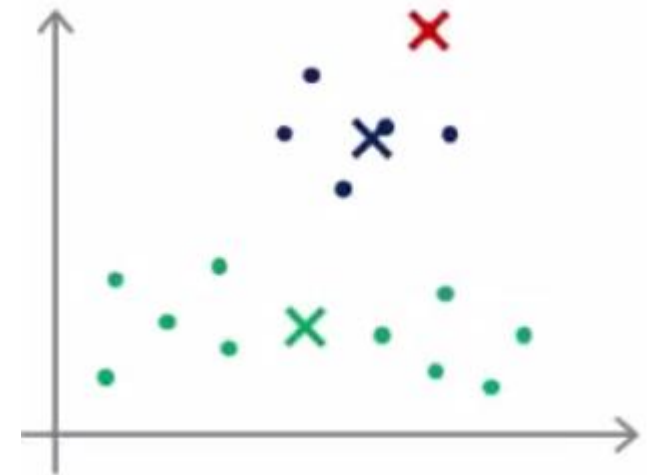
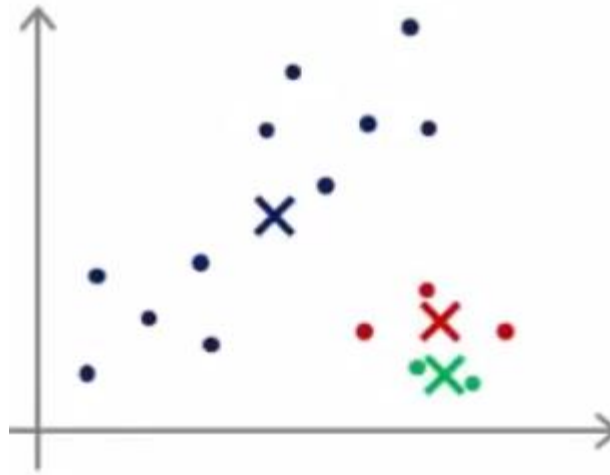
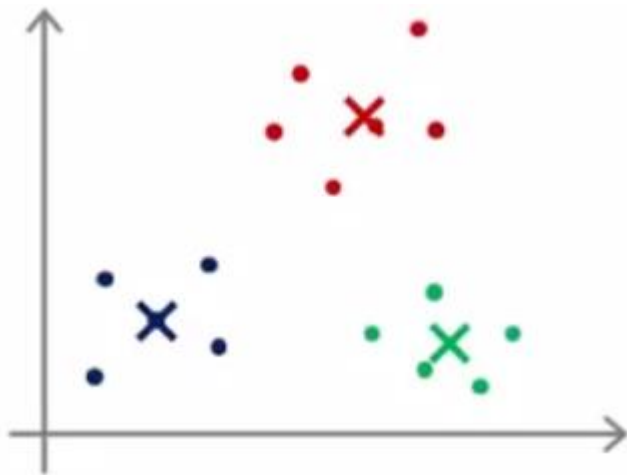
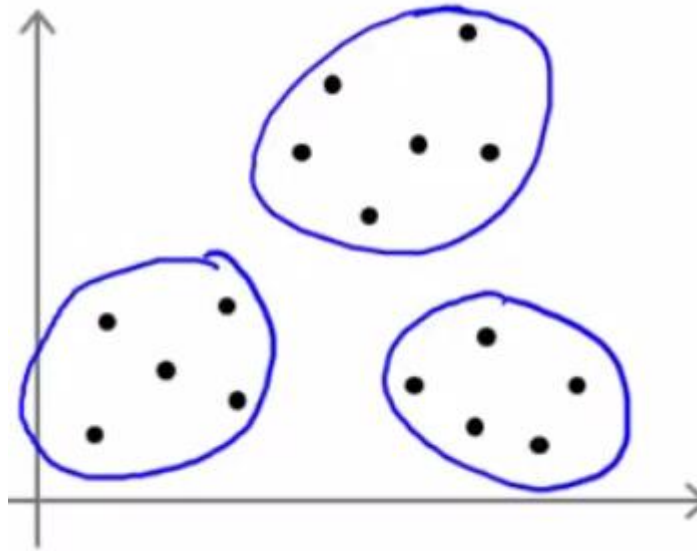
Stopping/convergence criterion

1. No (or minimum) re-assignments of data points to different clusters,
2. No (or minimum) change of centroids, or
3. Minimum decrease in the **sum of squared error** (SSE),

$$SSE = \sum_{j=1}^k \sum_{\mathbf{x} \in C_j} \text{dist}(\mathbf{x}, \mathbf{m}_j)^2 \quad (1)$$

- C_j is the j th cluster, \mathbf{m}_j is the centroid of cluster C_j (the mean vector of all the data points in C_j)

Local optima



What Is the Problem with K-Means?

- The k-means algorithm is sensitive to outliers !
- K-Medoids: Instead of taking the **mean** value of the object in a cluster as a reference point, **medoids** can be used, which is the **most centrally located** object in a cluster.

What Is the Problem with Medoids?

- More robust than k-means in the presence of noise and outliers because a medoid is less influenced by outliers or other extreme values than a mean
- Works efficiently for small data sets but does not **scale well** for large data sets.
 - $O(k(n-k)^2)$ for each iteration

where n is # of data, k is # of clusters



HOW DO WE EMPLOY DISTANCE IN A CLUSTER?

R CODE DEMO

K-means versus Hierarchical

- Flat clustering produces a single partitioning
- Flat clustering needs the number of clusters to be specified
- Flat clustering is usually more efficient run-time wise
- Hierarchical Clustering can give different partitionings depending on the level-of-resolution we are looking at
- Hierarchical clustering doesn't need the number of clusters to be specified
- Hierarchical clustering can be slow (has to make several merge/Split decisions)



ENGINEERING

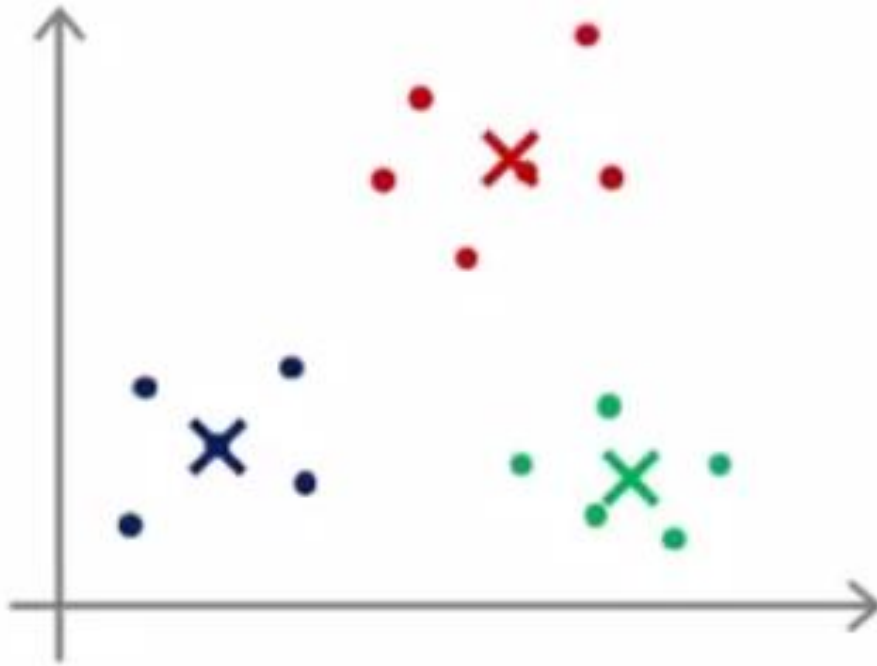


Stability Check of the Clusters

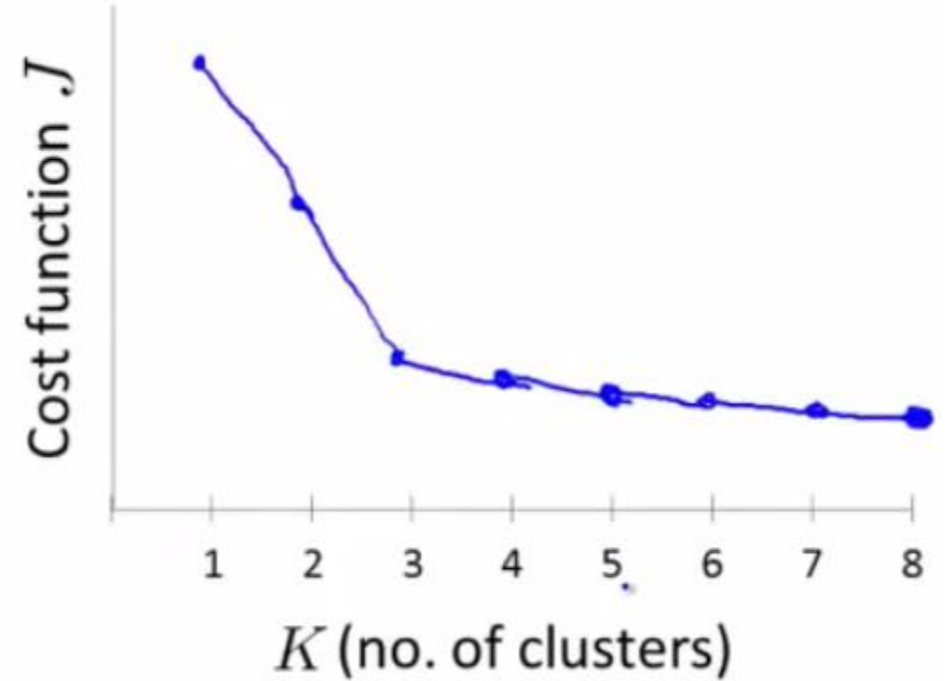


- To check the stability of the clusters take a random sample of 95% of records. Compute the clusters. If the clusters formed are very similar to the original, then the clusters are fine.

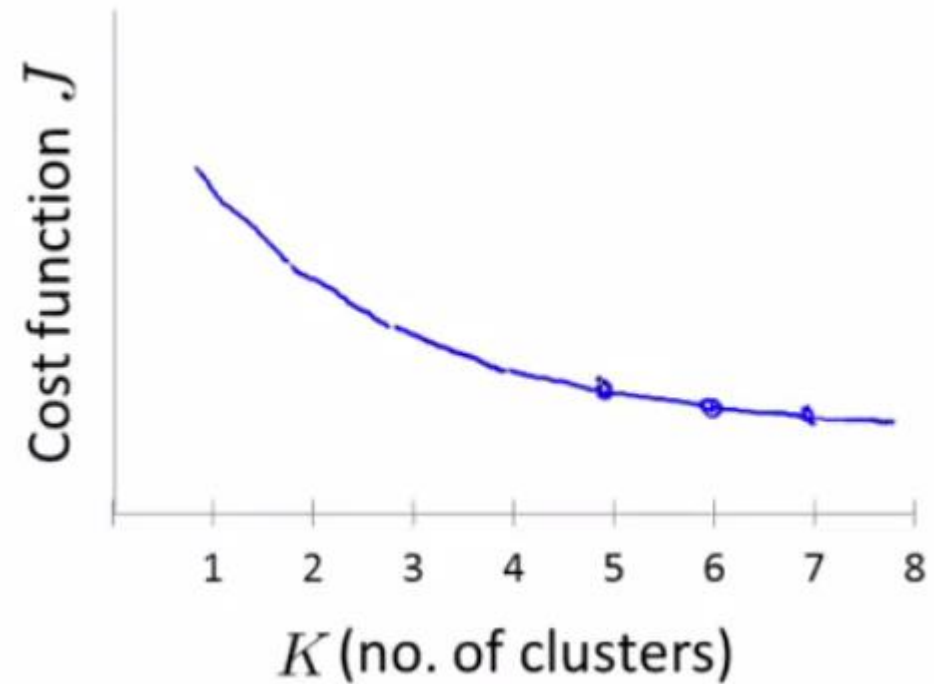
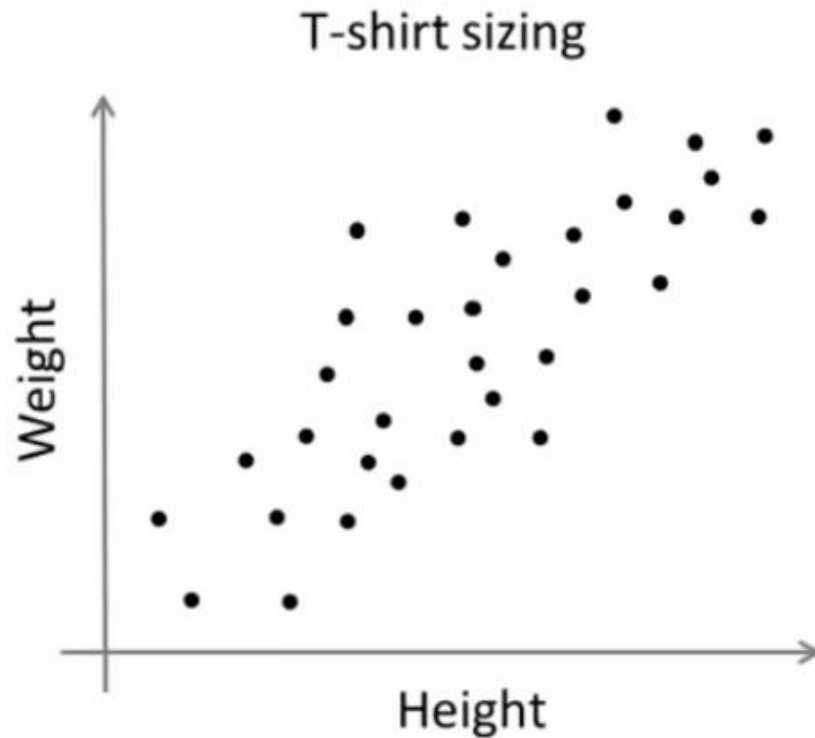
Linearly clustered data

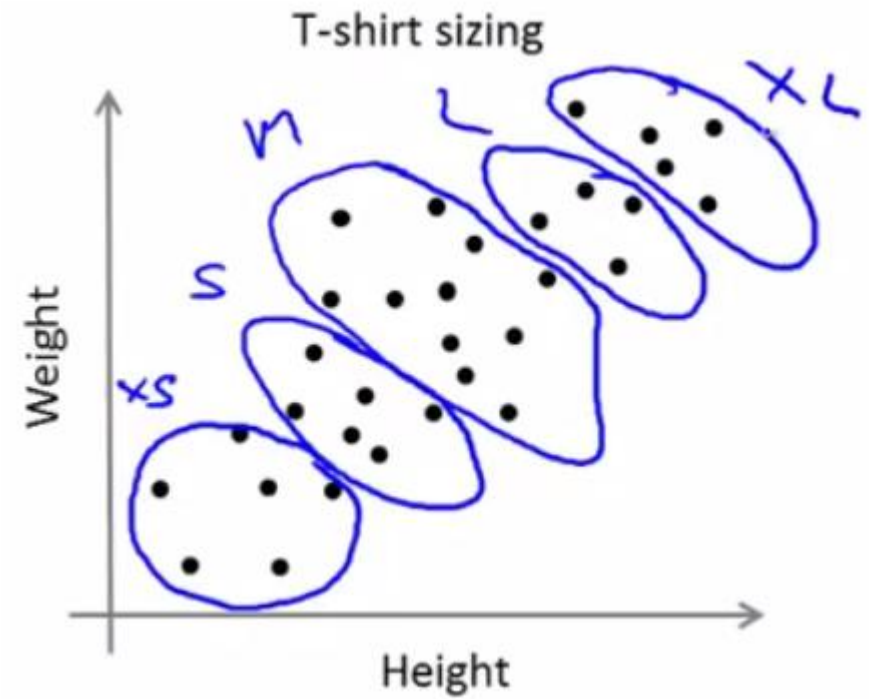
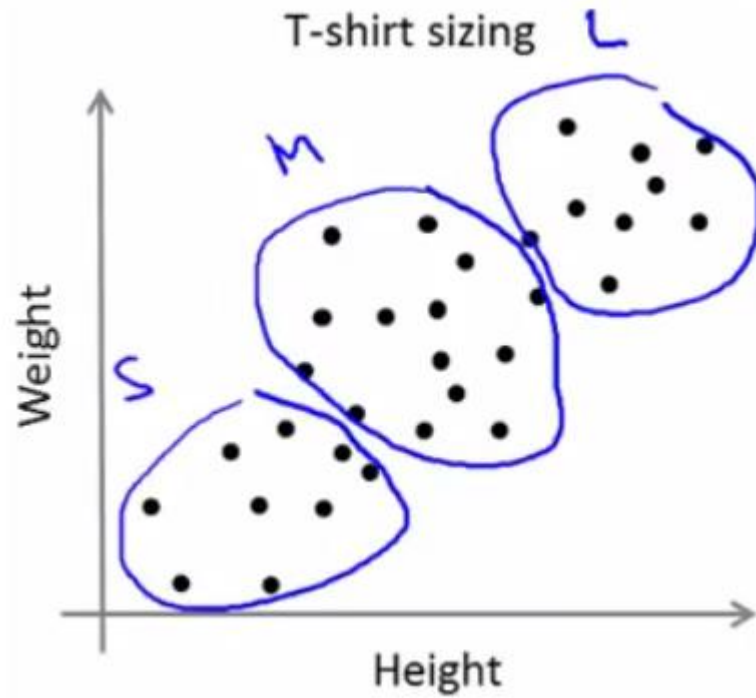


Nice



Linearly separable but merged





Linearly separable



- Run 50-500 simulations for small k (2-10). For large k (100 or so), we can do 1-5 simulations
- Pick the one that gives the best S

Clustering Process Summary

- Choose an appropriate distance metric and calculate
- Decide k either based on the elbow or business user's intuition when no elbow found
- Kernel (higher dimensions), if required
- Cluster (k-means, etc.)
- Check stability of clusters using 90% or 95% data
- Define a cluster with properties (mean, median, etc.)



Instance Based Learning

Lazy Learning

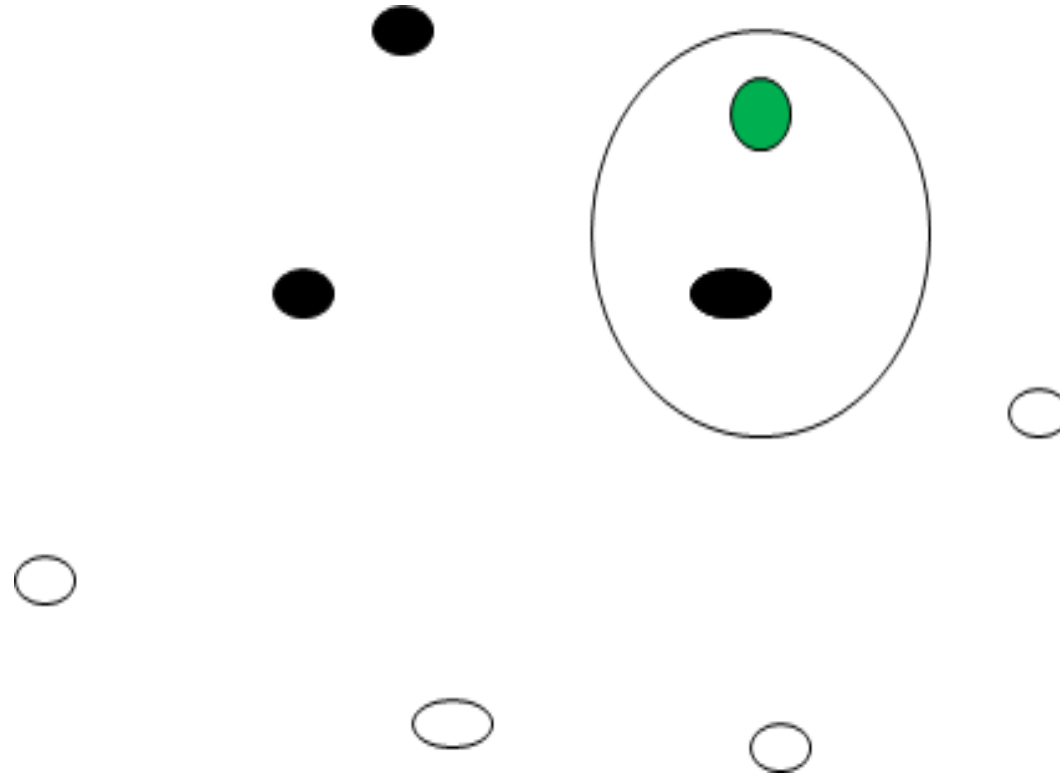


- Eager Learning
 - Explicit description of target function on the whole training set
- Instance-based / Lazy Learning
 - Learning = Storing all training instances
 - Classification = Assigning target function to a new instance

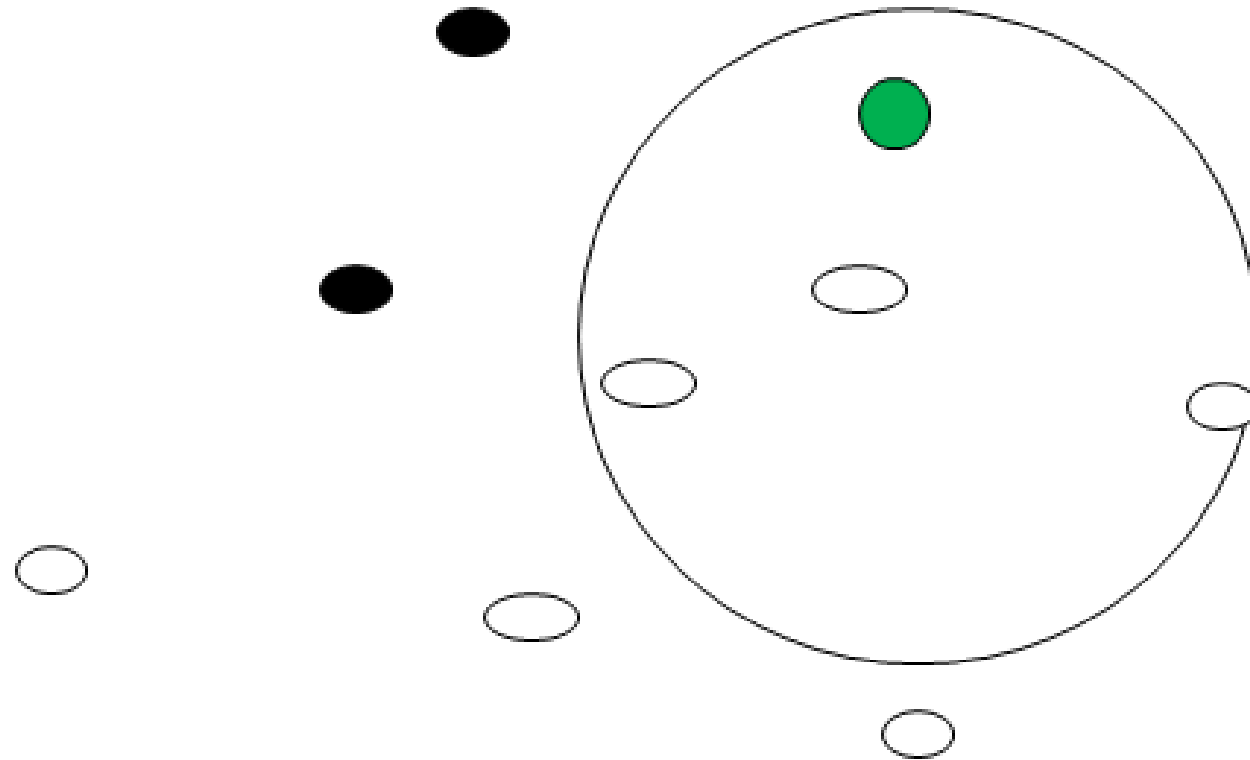


KNN

K=1



K=3



Process is simple

- Pick a number of neighbors you want to use for classification or regression (K)
- Choose a method to measure distances (same consideration as clustering)
- Keep a data set with records

Process is simple



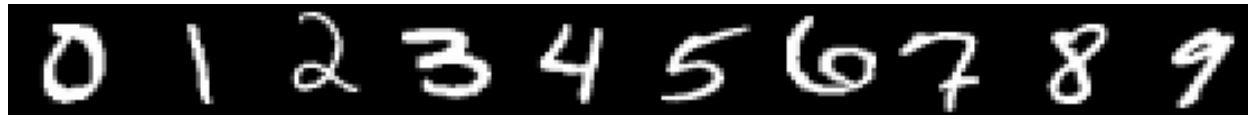
- For every new point, identify the number of nearest neighbors you picked using the method you chose
- Let them vote if it is a classification or take a mean/median for regression!

K-NN is



- Supervised
- Non parametric
- Lazy
- Local heuristic

kNN Example: Digit Recognition



- Digit Recognition
 - Handwritten digits
 - 28x28 pixel images: $d = 784$
 - 60,000 training samples
 - 10,000 test samples
- Nearest neighbour is competitive

	Test Error Rate (%)
Linear classifier (1-layer NN)	12.0
K-nearest-neighbors, Euclidean	5.0
K-nearest-neighbors, Euclidean, deskewed	2.4
K-NN, Tangent Distance, 16x16	1.1
K-NN, shape context matching	0.67
1000 RBF + linear classifier	3.6
SVM deg 4 polynomial	1.1
2-layer NN, 300 hidden units	4.7
2-layer NN, 300 HU, [deskewing]	1.6
LeNet-5, [distortions]	0.8
Boosted LeNet-4, [distortions]	0.7

K-NN



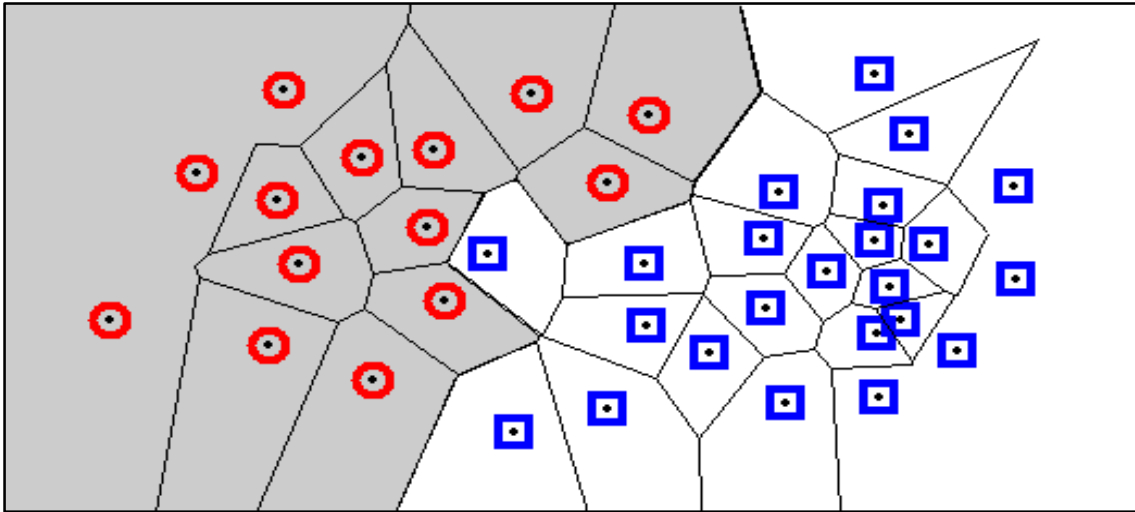
- Comes with a theoretical guarantee
- It is a Gibbs classifier. The accuracy will be bounded by 2^* Bayes optimal classifier

Advantages



- If lazy
 - Simple
 - You can draw a very complex decision surface
 - Voronoi diagrams

Decision Regions



- **A Voronoi diagram**
- Each cell contains one sample, and every location within the cell is closer to that sample than to any other sample.
- Every query point will be assigned the classification of the sample within that cell. The *decision boundary* separates the class regions based on the 1-NN decision rule.
- Knowledge of this boundary is sufficient to classify new points.

Issues with KNN and instance based techniques



- Curse of dimensionality
- Requires more memory and more time



Attributes

Records

Search process

ENGINEERING K-NN

Attributes



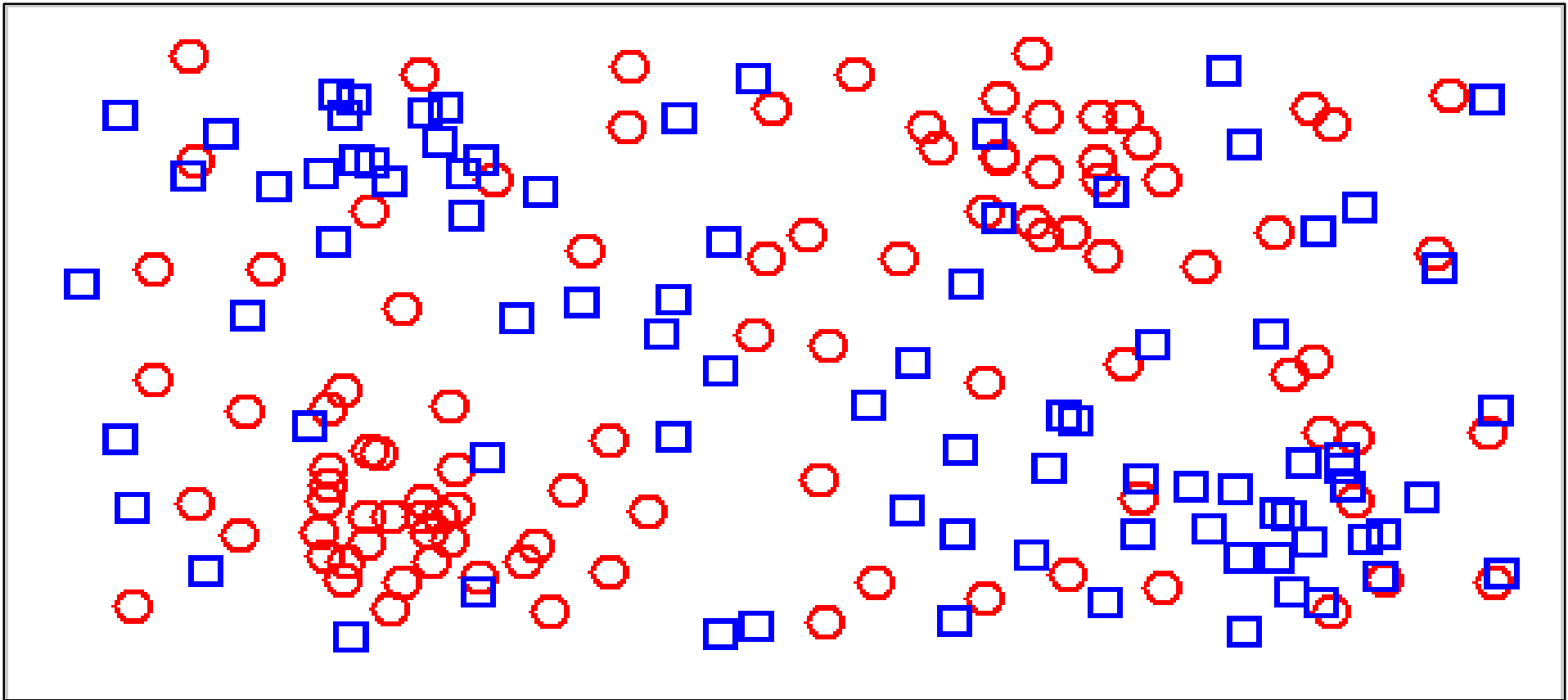
- Scaling the attributes is important
 - Attributes with larger range can dominate
 - Categorical variables and Ordinal variables need to be converted to numeric

Curse of dimensionality

- K-NN is heavily impacted as all points are at the surface and hence similar
- Reduce the dimensions
 - Correlation
 - Info gain (filter approach: We lose some that are important)
 - Wrapper methods
 - Forward selection, Backward elimination
 - Weighting attributes

Records: Outliers and overfitting

- Remove outliers



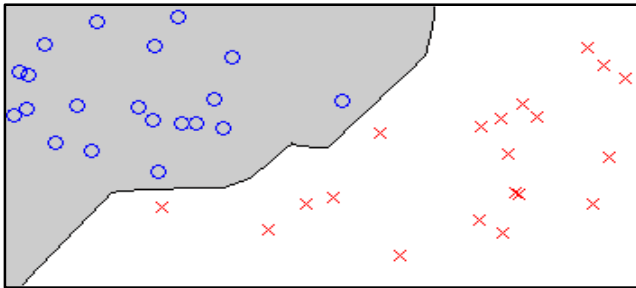


Records: Handling missing values

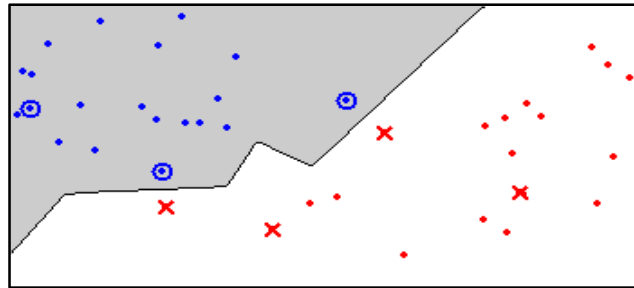
- K-NN is impacted heavily by missing values
- Imputation is one option but might be self defeating

Speeding up search

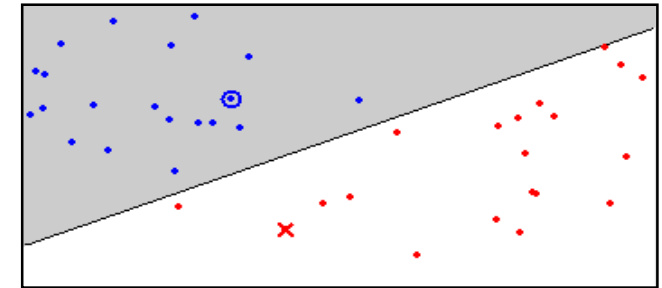
Delaunay triangulation



Original data



Condensed data



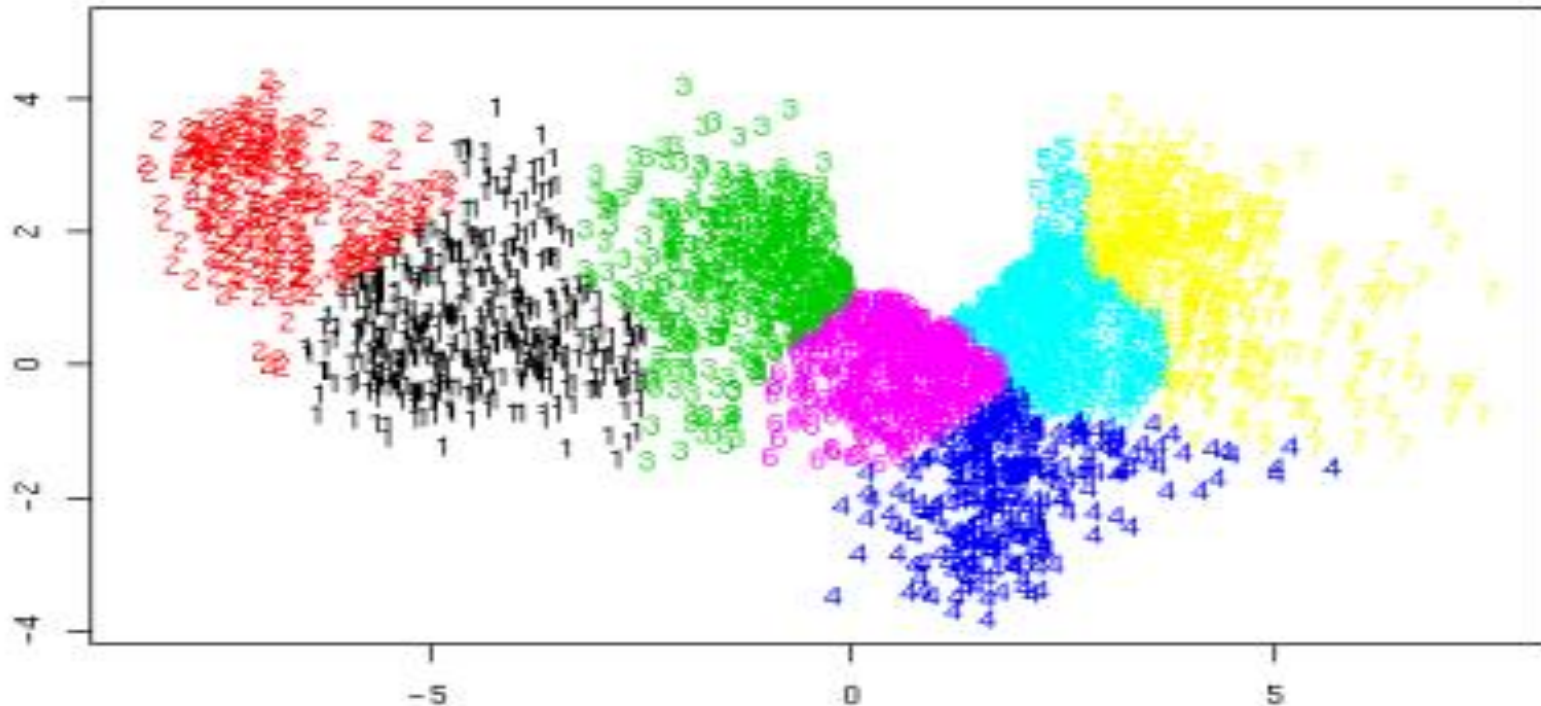
Minimum Consistent Set

Cran library: Class

Speeding up



- Clustering





COLLABORATIVE FILTERING



Collaborative filtering

- How do I recommend?
 - Association rules
 - Similarity based (collaborative filtering)
 - Model based

Collaborative filtering: primitive

Primitive version:

$$\hat{R}_{ik} = \alpha \sum_{X_j \in \mathbf{N}_i} W_{ij} R_{jk}$$

$$\alpha = (\sum |W_{ij}|)^{-1}$$

Similarity (Pearson coefficient):

$$W_{ij} = \frac{\sum_k (R_{ik} - \bar{R}_i)(R_{jk} - \bar{R}_j)}{\sqrt{\sum_k (R_{ik} - \bar{R}_i)^2 (R_{jk} - \bar{R}_j)^2}}$$

Collaborative filtering: More refined

$$\hat{R}_{ik} = \bar{R}_i + \alpha \sum_{X_j \in \mathbf{N}_i} W_{ij} (R_{jk} - \bar{R}_j)$$

Collaborative filtering



	Matrix	Star Wars	Dark knight	Rocky	Sita Aur Gita	Star Trek	Cliffhanger	A.I.	MI	X-Men
Jim	1	3	1	5	2	1			1	
Sean	2		3	2		4		5		3
John		3		4		5			3	4
Sidd	4				3		4		2	
Penny	5		2		2		5		1	
Pete		5			?		4			4

Collaborative filtering



	Matrix	Star Wars	Dark knight	Rocky	Sita Aur Gita	Star Trek	Cliffhanger	A.I.	MI	X-Men
Jim	-0.65	0.65	-0.65	1.96	0	-0.65			-0.7	
Sean	-1		-0.14	-1		0.71		1.57		-0.14
John		-1		0.24		1.434			-1	0.24
Sidd	0.783				-0.26		0.78		-1.3	
Penny	1.069		-0.53		-0.53		1.07		-1.1	
Pete		1.15			?		-0.6			-0.58

Project



- Study the papers
 - <http://cran.r-project.org/web/packages/recommenderlab/vignettes/recommenderlab.pdf>
 - <http://blog.yhathq.com/posts/recommender-system-in-r.html>
 - <http://www2.research.att.com/~volinsky/papers/ieeecocomputer.pdf>

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