UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 13

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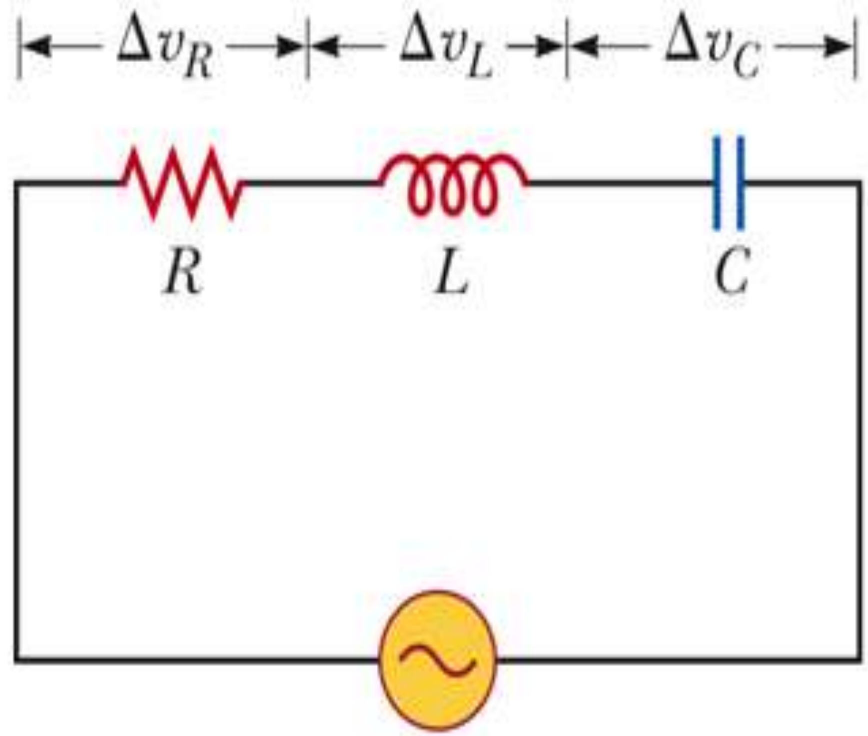
The *RLC* Series Circuit

The resistor, inductor, and capacitor can be combined in a circuit.

The current and the voltage in the circuit vary sinusoidally with time.



The *RLC* Series Circuit, cont.

The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.

The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.

- φ is the **phase angle** between the current and the applied voltage.

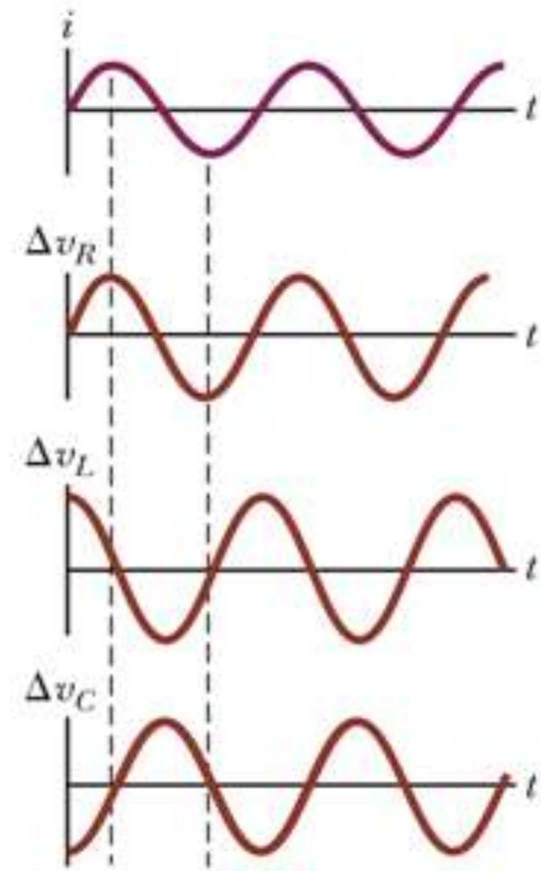
Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.

i and v Phase Relationships – Graphical View

The instantaneous voltage across the resistor is in phase with the current.

The instantaneous voltage across the inductor leads the current by 90° .

The instantaneous voltage across the capacitor lags the current by 90° .



i and *v* Phase Relationships – Equations

The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$.

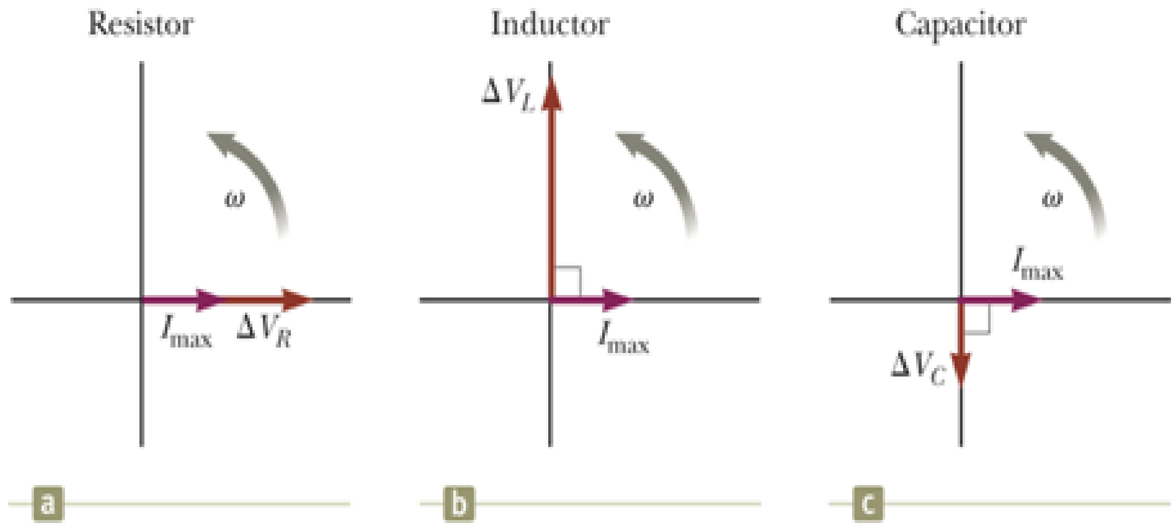
ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_{L.}$

ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_{C.}$

The sum of these voltages must equal the voltage from the AC source.

Because of the different phase relationships with the current, they cannot be added directly.

Phasor Diagrams



To account for the different phases of the voltage drops, vector techniques are used.

Remember the phasors are rotating vectors

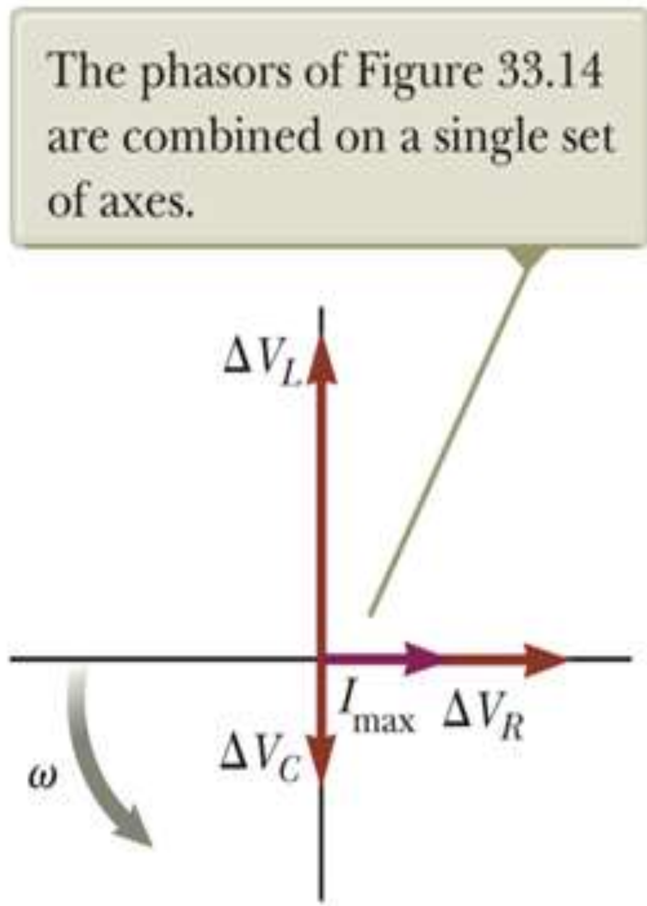
The phasors for the individual elements are shown.

Resulting Phasor Diagram

The individual phasor diagrams can be combined.

Here a single phasor I_{max} is used to represent the current in each element.

- In series, the current is the same in each element.



Vector Addition of the Phasor Diagram

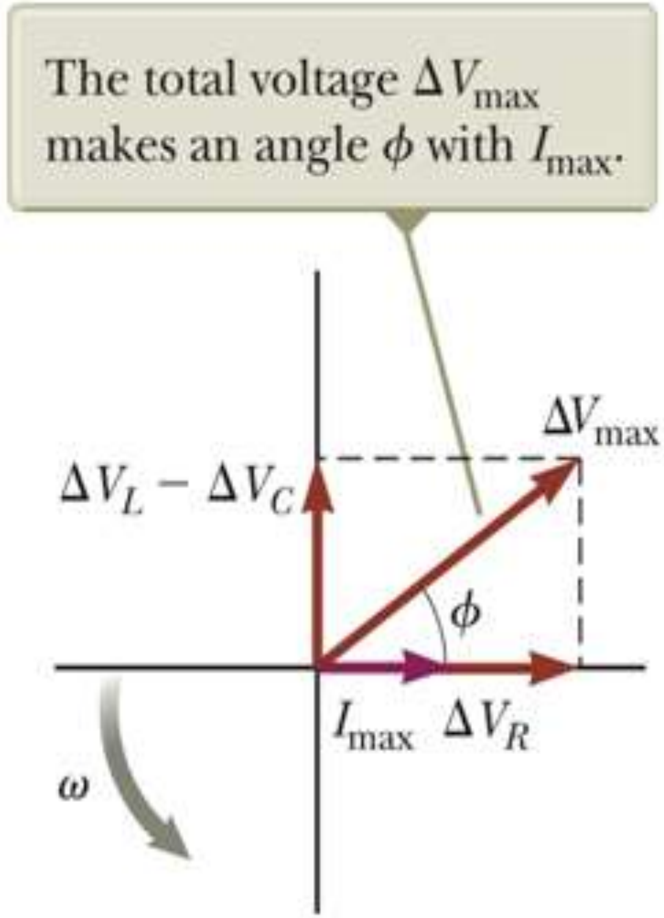
Vector addition is used to combine the voltage phasors.

ΔV_L and ΔV_C are in opposite directions, so they can be combined.

Their resultant is perpendicular to ΔV_R.

The resultant of all the individual voltages across the individual elements is Δv_{max}.

- This resultant makes an angle of φ with the current phasor I_{max}.



Total Voltage in RLC Circuits

From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{\max} R)^2 + (I_{\max} X_L - I_{\max} X_C)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Quick Quiz (Poll 1)

In a series RLC circuit, the phase difference between the current in the capacitor and the current in the inductor is?

a) 0^0

b) 90^0

c) 180^0

d) 360^0

Quick Quiz (Poll 2)

In a series RLC circuit, the phase difference between the current in the circuit and the voltage across the capacitor is?

a) 0^0

b) 90^0

c) 180^0

d) 360^0

Impedance

The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

- Impedance has units of ohms

Phase Angle

The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle can be positive or negative and determines the nature of the circuit.

Determining the Nature of the Circuit

If ϕ is positive

- $X_L > X_C$ (which occurs at high frequencies)
- The current lags the applied voltage.
- The circuit is *more inductive than capacitive.*

If ϕ is negative

- $X_L < X_C$ (which occurs at low frequencies)
- The current leads the applied voltage.
- The circuit is *more capacitive than inductive.*

If ϕ is zero

- $X_L = X_C$
- The circuit is *purely resistive.*

Power in an AC Circuit

The average power delivered by the AC source is converted to internal energy in the resistor.

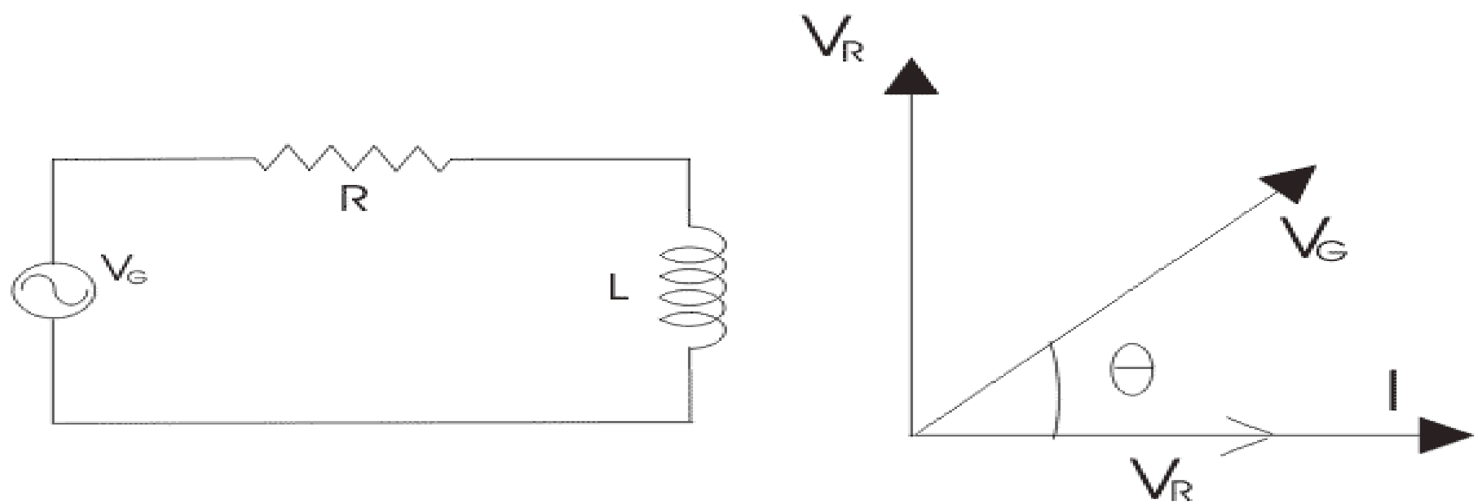
- $P_{avg} = \frac{1}{2} I_{max} \Delta V_{max} \cos \phi = I_{rms} \Delta V_{rms} \cos \phi$
- $\cos \phi$ is called the power factor of the circuit

We can also find the average power in terms of R.

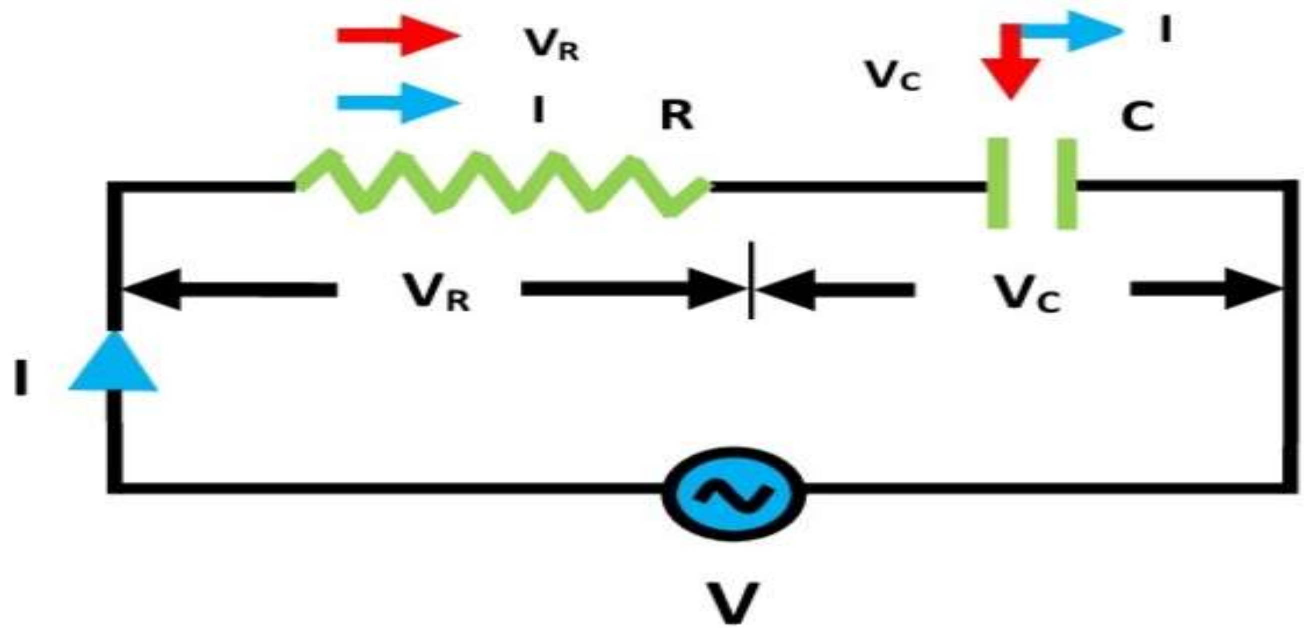
- $P_{avg} = I^2_{rms} R$

When the load is purely resistive, $\phi = 0$ and $\cos \phi = 1$

- $P_{avg} = I_{rms} \Delta V_{rms}$



(a) (b)



Resonance in an AC Circuit

Resonance occurs at the frequency ω_0 where the current has its maximum value.

- To achieve maximum current, the impedance must have a minimum value.
- This occurs when $X_L = X_C$
- Solving for the frequency gives

$$\omega_o = 1/\sqrt{LC}$$

The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.

The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.

At the resonance frequency, the current is in phase with the applied voltage.

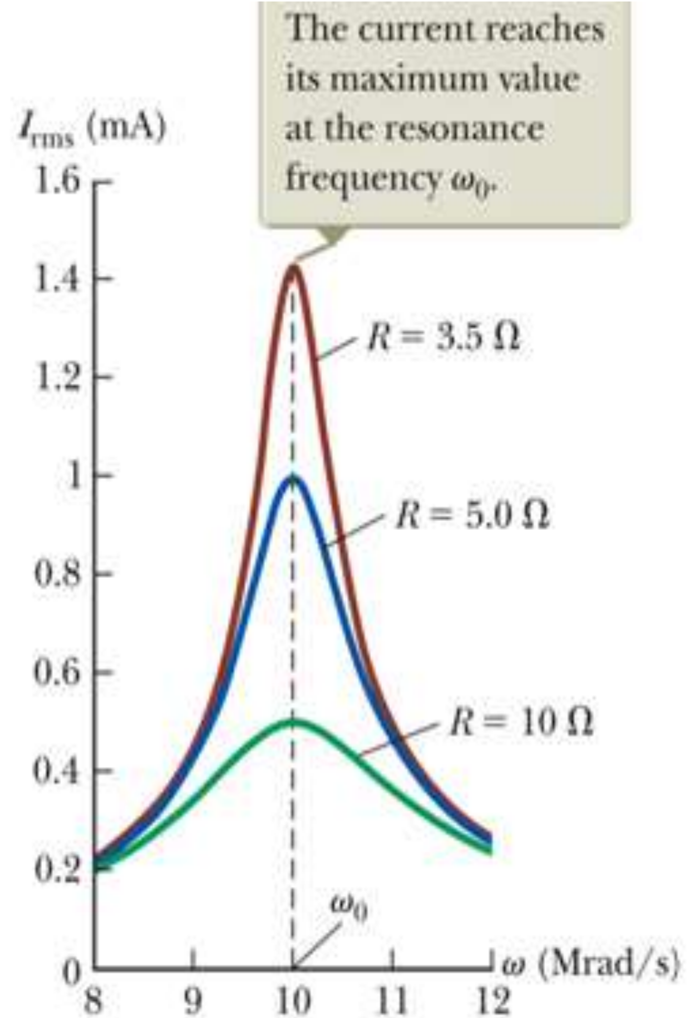
Resonance, cont.

Resonance occurs at the same frequency regardless of the value of *R*.

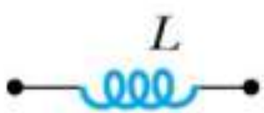
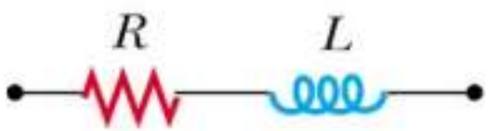
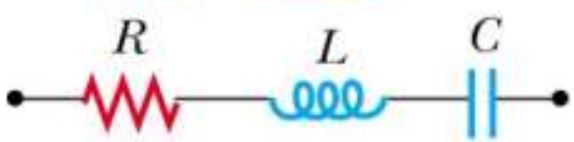
As *R* decreases, the curve becomes narrower and taller.

Theoretically, if *R* = 0 the current would be infinite at resonance.

- Real circuits always have some resistance.



Summary of Circuit Elements, Impedance, Phase Angles

	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$\tan\phi = \dfrac{X_L - X_C}{R}$
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Quick Quiz (Poll 3)

- __________ the resonant frequency, the current in the inductor lags the voltage in a series RLC circuit.
  a) Above
  b) Below
  c) Equal to
  d) Depends on the circuit