UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 12

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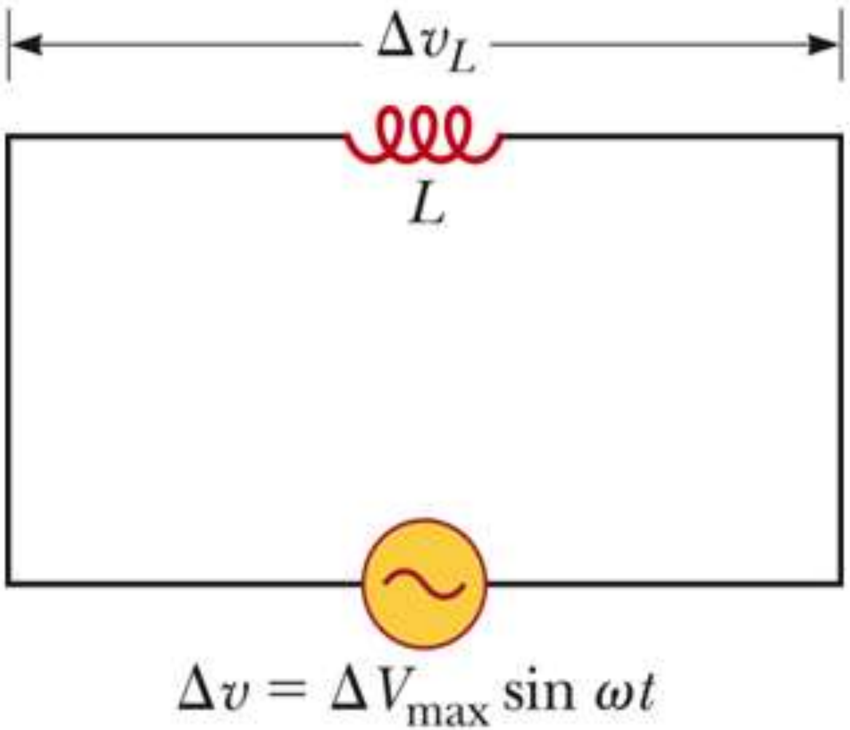
Inductors in an AC Circuit

Kirchhoff's loop rule can be applied and gives:

$\Delta v + \Delta v_L = 0$, or

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\, sin\omega t$$



Current in an Inductor

The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L}\cos \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\omega L}$$

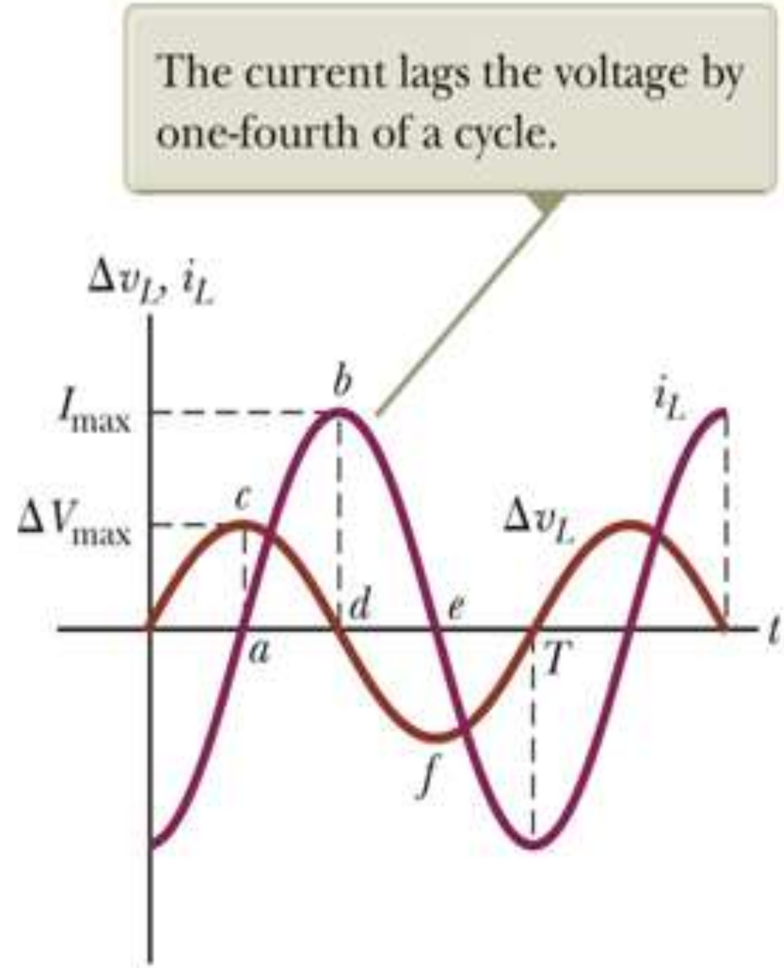
This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)$ rad = 90^o.

Phase Relationship of Inductors in an AC Circuit

The current is a maximum when the voltage across the inductor is zero.

- The current is momentarily not changing

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$).

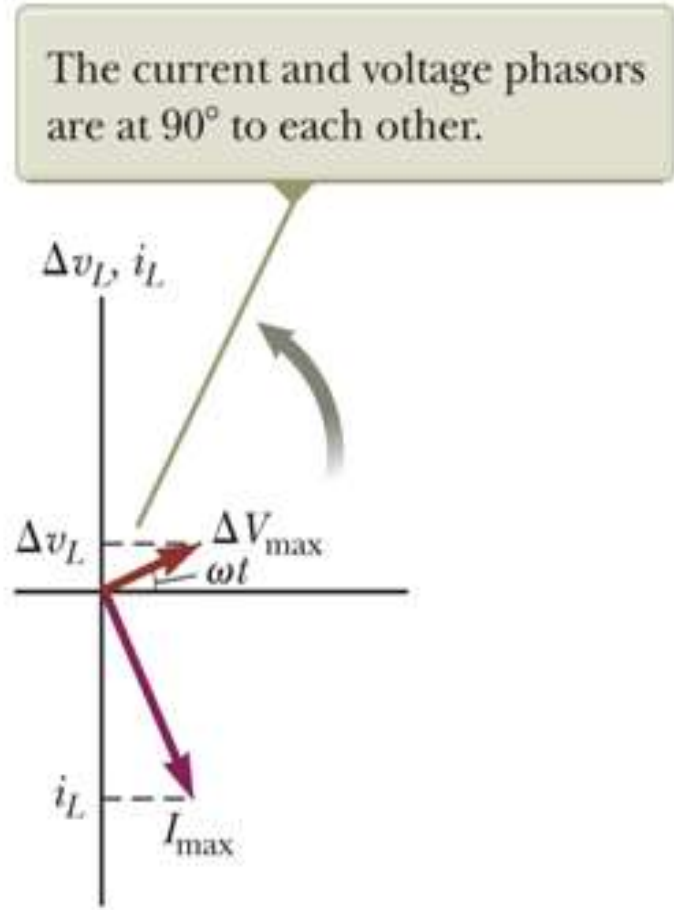


Phasor Diagram for an Inductor

The phasors are at 90° with respect to each other.

This represents the phase difference between the current and voltage.

Specifically, the current lags behind the voltage by 90°.



Inductive Reactance

The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.

Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.

The factor is the **inductive reactance** and is given by:

- $X_L = \omega L$

Inductive Reactance, cont.

Current can be expressed in terms of the inductive reactance:

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} \; or \; I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{X_L}$$

As the frequency increases, the inductive reactance increases

- This is consistent with Faraday's Law:
  - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

Voltage Across the Inductor

The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt}$$

$$= -\Delta V_{\text{max}} \sin \omega t$$

$$= -I_{\text{max}} X_L \sin \omega t$$

Quick Quiz (Poll 1)

- The two quantities are said to be in phase with each other when

a. the phase difference between two quantities is zero degree or radian

b. each of them pass through zero values at the same instant and rise in the same direction

c. each of them pass through zero values at the same instant but rises in the opposite directions

d. Both (a) or (b)

Quick Quiz (Poll 2)

- The inductive reactance of a circuit .... with the increase in supply frequency

a. increases

b. decreases

c. remains unchanged

d. unpredictable

Capacitors in an AC Circuit

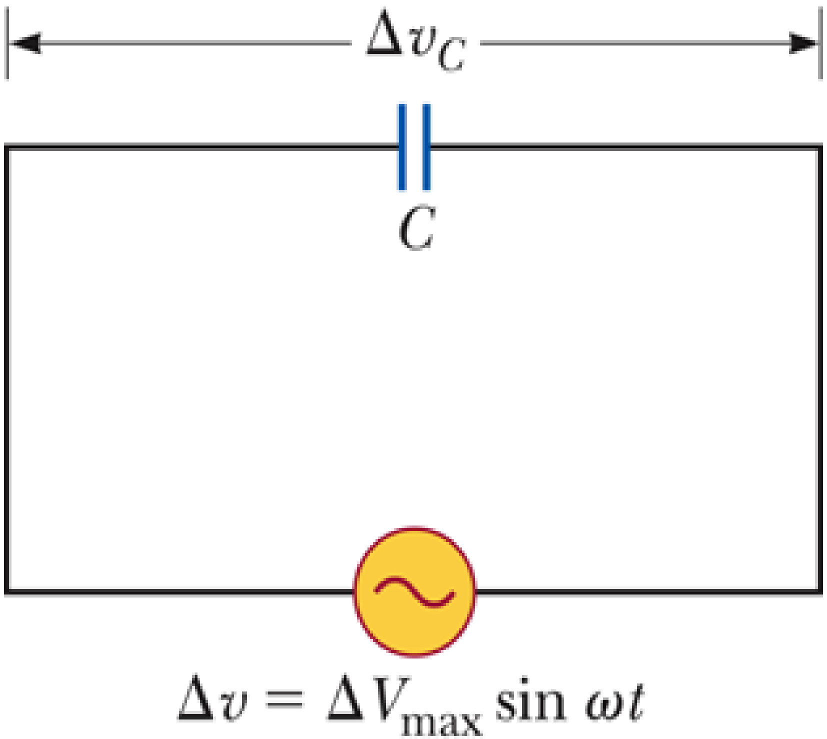
The circuit contains a capacitor and an AC source.

Kirchhoff's loop rule gives:

$\Delta v + \Delta v_c = 0$ and so

$\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$

- Δv_c is the instantaneous voltage across the capacitor.



Capacitors in an AC Circuit, cont.

The charge is $q = C\Delta V_{max} \sin \omega t$

The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

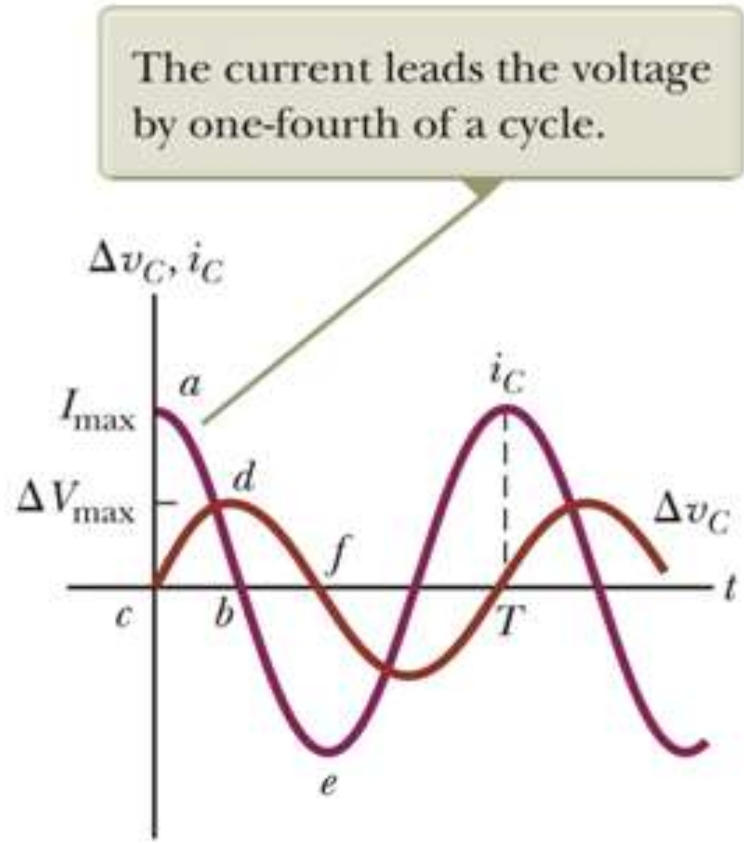
$$\text{or } i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current is $\pi/2$ rad = 90^o out of phase with the voltage

More About Capacitors in an AC Circuit

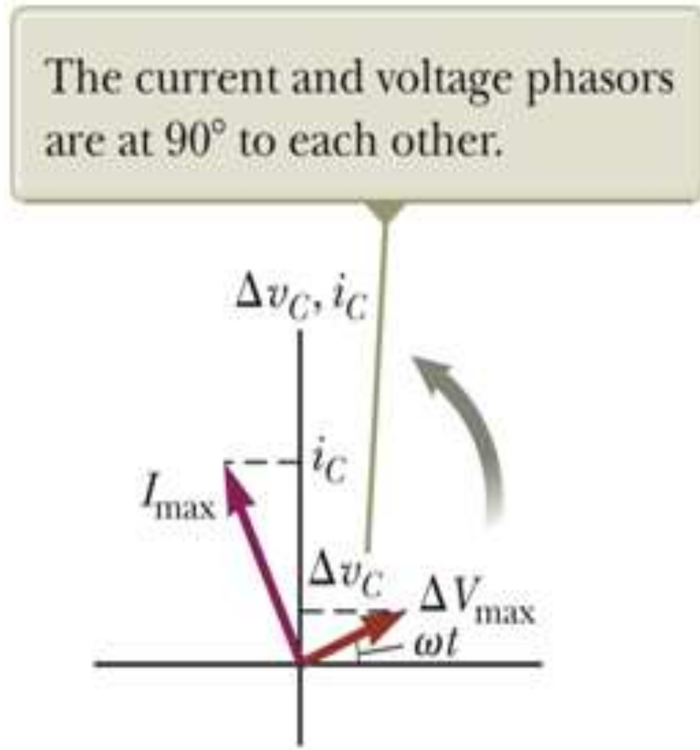
The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

The current leads the voltage by 90°.



Phasor Diagram for Capacitor

The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90º.



Capacitive Reactance

The maximum current in the circuit occurs at cos ωt = 1 which gives

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{max} = \frac{\Delta V_{max}}{X_C}$$

Voltage Across a Capacitor

The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$.

As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.

As the frequency approaches zero, X_C approaches infinity and the current approaches zero.

- This would act like a DC voltage and the capacitor would act as an open circuit.

Quick Quiz (Poll 3)

- In a pure capacitive circuit, the current will

a. lag behind the voltage by 90 degree

b. lead behind the voltage by 90 degree

c. remains in phase with voltage

d. none of the above

Quick Quiz (Poll 4)

- A phasor is a line which represents the

a. rms value and phase of an alternating quantity

b. average value and phase of an alternating quantity

c. magnitude and direction of an alternating quantity

d. none of the above