UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 10

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AC Waveforms and Average Value

- Since ac quantities constantly change its value, we need one single numerical value that truly represents a waveform over its complete cycle.

Average Values:

- To find the average of a set of marks for example, you add them, then divide by the number of items summed.
- For waveforms, the process is conceptually the same. You can sum the instantaneous values over a full cycle, then divide by the number of points used.
- The trouble with this approach is that waveforms do not consist of discrete values.

Average in Terms of the Area Under a Curve:

$$\text{average} = \frac{\text{area under curve}}{\text{length of base}}$$

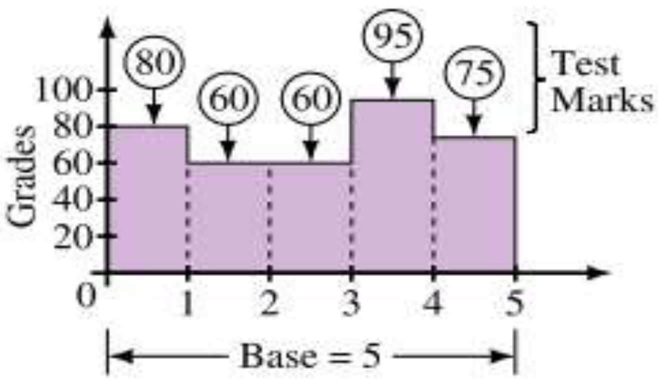


FIGURE 15–50 Determining average by area.

$$\text{average} = (80 + 60 + 60 + 95 + 75)/5 = 74$$

Or use area

$$\frac{(80 \times 1) + (60 \times 2) + (95 \times 1) + (75 \times 1)}{5} = 74$$

AC Waveforms and Average Value

- To find the average value of a waveform, divide the area under the waveform by the length of its base.
- Areas above the axis are counted as positive, while areas below the axis are counted as negative.
- This approach is valid regardless of waveshape.

- Average values are also called **dc values**, because dc meters indicate average values rather than instantaneous values.

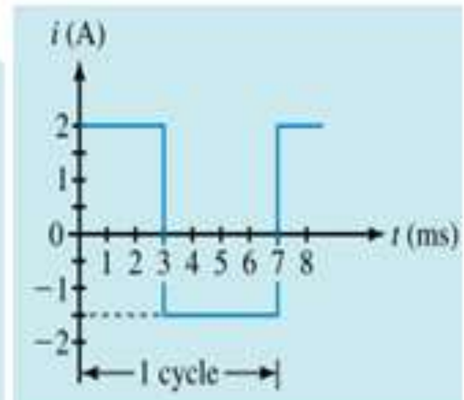
AC Waveforms and Average Value

EXAMPLE 15–23

a. Compute the average for the current waveform of Figure 15–51.

b. If the negative portion of Figure 15–51 is −3 A instead of −1.5 A, what is the average?

c. If the current is measured by a dc ammeter, what will the ammeter indicate?



Solution

a. The waveform repeats itself after 7 ms. Thus, $T = 7$ ms and the average is

$$I_{avg} = \frac{(2\text{ A} \times 3\text{ ms}) - (1.5\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \frac{6-6}{7} = 0\text{ A}$$

b. $$I_{avg} = \frac{(2\text{ A} \times 3\text{ ms}) - (3\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \frac{-6\text{ A}}{7} = -0.857\text{ A}$$

c. A dc ammeter measuring (a) will indicate zero, while for (b) it will indicate −0.857 A.

Derivation of Average Value

The average value of A.C. is the average over one complete cycle and is clearly zero, because there are alternately equal positive and negative half cycles.
Alternating current is represented as $I = I_0 \sin \omega t$

$$I_{\text{mean}} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt}$$

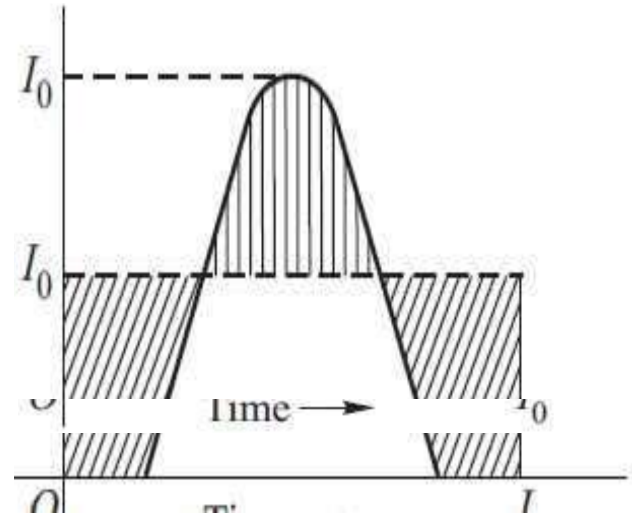
$$= \frac{I_0}{T/2} \cdot \frac{1}{\omega} \left[-\cos \omega t \right]_0^{T/2}$$

$$= \frac{2I}{T} \cdot \frac{T}{2\pi} \left[\cos 0^\circ - \cos \frac{\omega T}{2} \right]$$

$$= \frac{I_0}{\pi} \left[\cos 0^\circ - \cos \frac{\omega}{2} \cdot \frac{2\pi}{\omega} \right]$$

$$= \frac{2I_0}{\pi} = \frac{2}{\pi} \times \text{Peak value of current}$$

Similarly, $E_{\text{mean}} = \dfrac{2E_0}{\pi} = \dfrac{2}{\pi} \times \text{Peak value of voltage}$



Root Mean Square Value

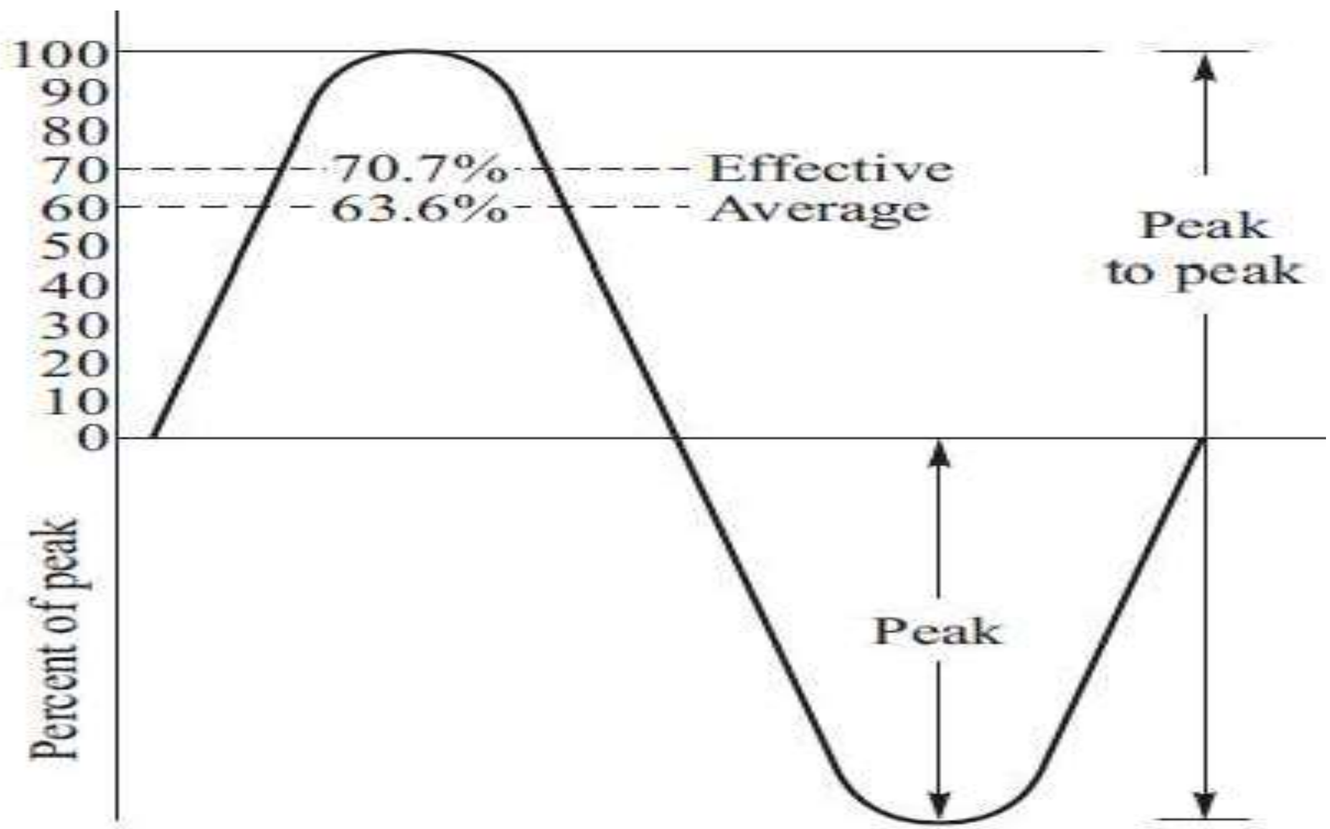
Circuit currents and voltage in A.C. circuits are generally stated as root-mean-square or rms values rather than by quoting the maximum values. The root-mean-square for a current is defined by

$$I_{rms} = \sqrt{\left(I^2\right)_{avg}}$$

That is, you take the square of the current and average it, then take the square root. When this process is carried out for a sinusoidal current

$$\left[I_m^2 \sin^2 \omega t\right]_{avg} = \frac{I_m^2}{2} \quad \text{so} \quad I_{rms} = \sqrt{\left(I^2\right)_{avg}} = \frac{I_m}{\sqrt{2}}$$

Derivation of RMS Value



Form Factor and Peak Factor

The ratio of the effective value to the average value is known as the *form factor* of a waveform of any shape (sinusoidal or nonsinusoidal). Thus,

Form factor, $$K_f = \frac{V_{rms}}{V_{av}} \tag{9.12}$$

The ***peak factor*** or ***crest factor*** or ***amplitude factor*** of a waveform is defined as the ratio of its peak (or maximum) value to its rms value. Thus,

Peak factor, $$K_p = \frac{V_m}{V_{rms}} \tag{9.13}$$

Let us calculate these two factors for ***a sinusoidal voltage waveform***,

$$K_f = \frac{V_{rms}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{0.707\,V_m}{0.637\,V_m} = \mathbf{1.11}$$

And $$K_p = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$$

Quick Quiz (Poll 1)

What is referred as the average value in AC opearation?

a) Average of all values of an alternating quantity.
b) Average of all values of the phase sequences.
c) Average of all values of the (+)ve and (-)ve half.
d) Average of all values of an alternating quantity over a complete cycle.

Quick Quiz (Poll 2)

What is form factor?

a) Average value / R.M.S. value.
b) Average value / Peak value.
c) Instantaneous value / Average value.
d) R.M.S. value / Average value.

POWER IN AC CIRCUITS

TRUE POWER:

The actual amount of power being used, or dissipated, in a circuit is called *true power*, and it is measured in watts (symbolized by the capital letter P, as always)

REACTIVE POWER:

We know that reactive loads such as inductors and capacitors dissipate zero power, yet the fact that they drop voltage and draw current gives the deceptive impression that they actually *do* dissipate power. This “phantom power” is called *reactive power*, and it is measured in a unit called *Volt-Amps-Reactive* (VAR), rather than watts. The mathematical symbol for reactive power is (unfortunately) the capital letter Q.

APPARENT POWER:

The combination of reactive power and true power is called *apparent power*, and it is the product of a circuit’s voltage and current, without reference to phase angle. Apparent power is measured in the unit of *Volt-Amps* (VA) and is symbolized by the capital letter S.

Active Power, it is the true power which is actually consumed in the circuit. We can say that it is the product of voltage and current and power factor.

Reactive power: It is the product of voltage current and sin of the phase angle.

Apparent power: It is the product of voltage and current.

$\mathbf{P}$ = true power $\quad P = I^2R \quad P = \dfrac{E^2}{R}$

Measured in units of ***Watts***

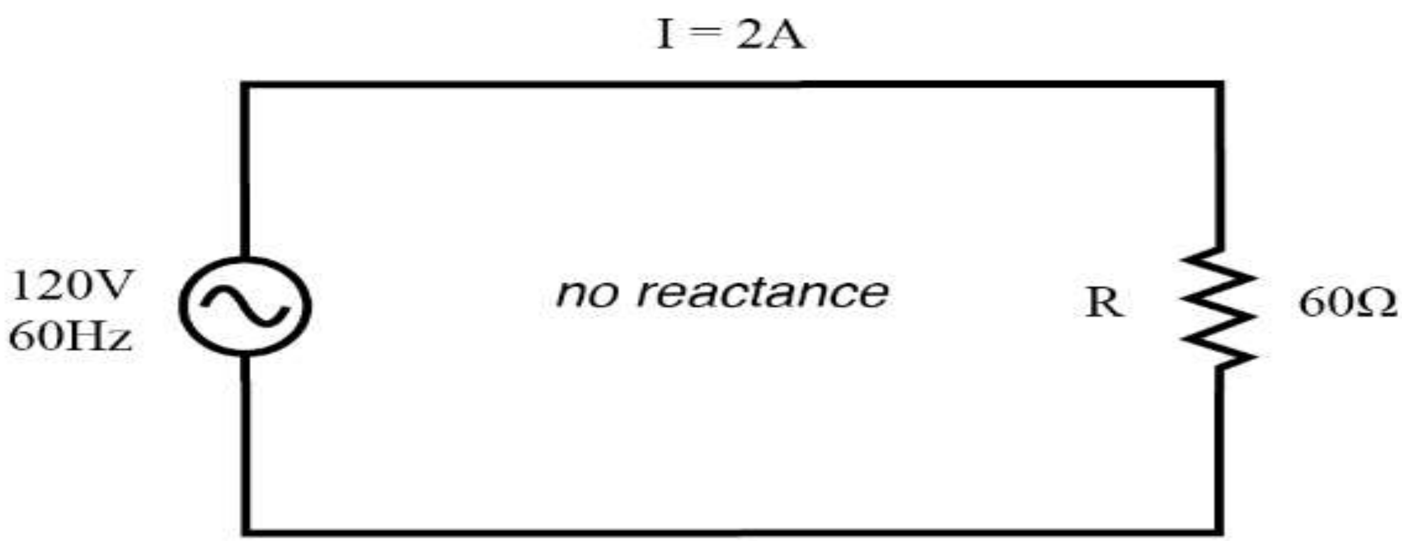
$\mathbf{Q}$ = reactive power $\quad Q = I^2X \quad Q = \dfrac{E^2}{X}$

Measured in units of ***Volt-Amps-Reactive (VAR)***

$\mathbf{S}$ = apparent power $\quad S = I^2Z \quad S = \dfrac{E^2}{Z} \quad S = IE$

Measured in units of ***Volt-Amps (VA)***

For Resistive Load

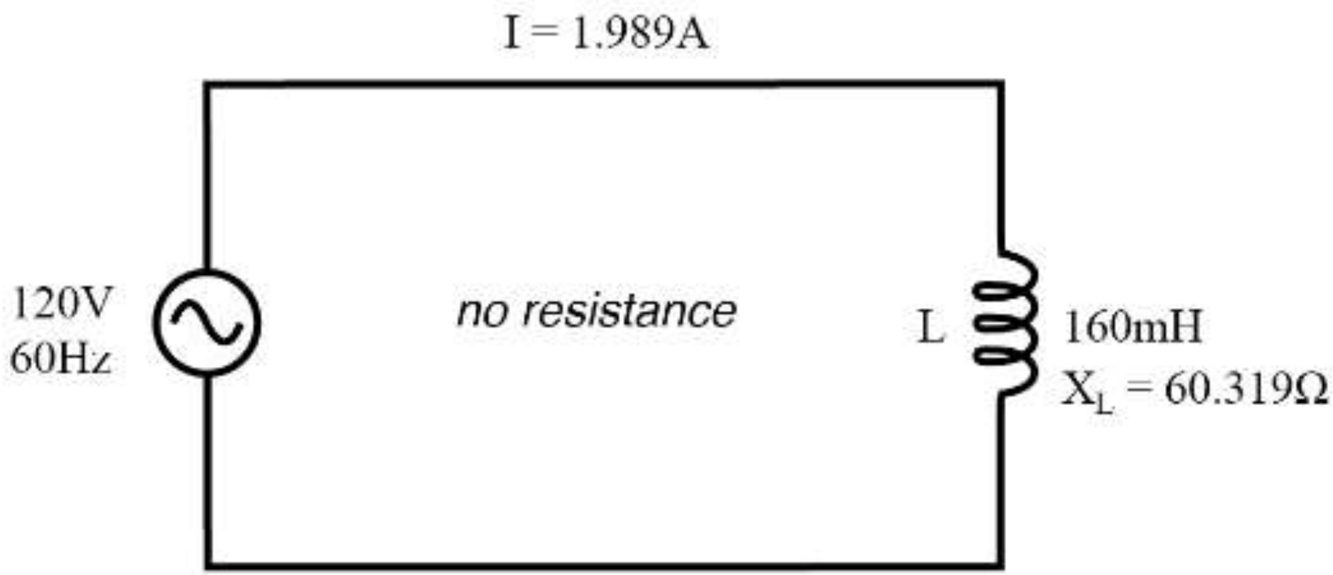


P = true power = I^2R = 240W

Q = reactive power = I^2X = 0 VAR

S = apparent power = I^2Z = 240VA

For Reactive Load

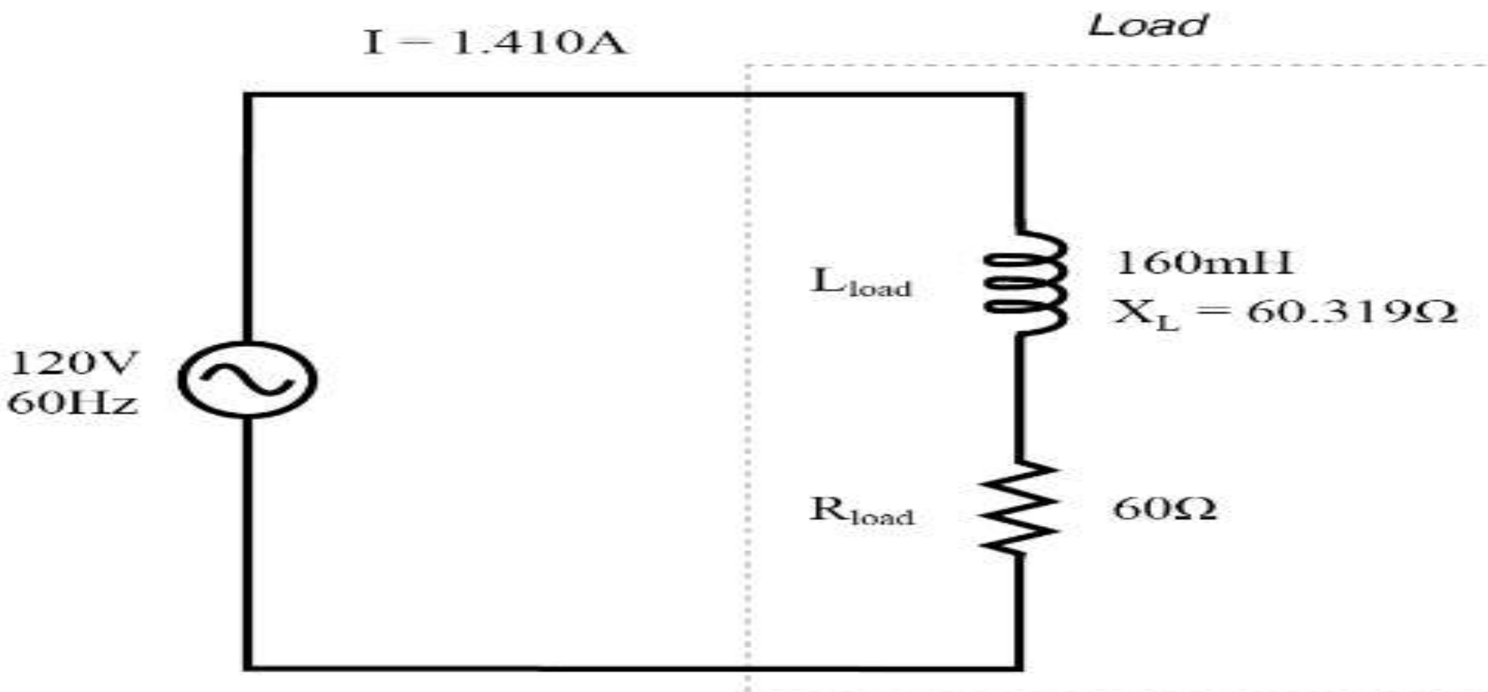


P = true power = $I^2R = 0W$

Q = reactive power = $I^2X = 238.73VAR$

S = apparent power = $I^2Z = 238.73VA$

For Resistive/Reactive Load



P = true power = I^2R = 119.365W

Q = reactive power = I^2X = 119.998VAR

S = apparent power = I^2Z =169.256VA

Example

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55 \sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1 \sin(\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Solution Here, the phase angle, $\theta = \pi/5$

The rms value of the voltage, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{55}{\sqrt{2}} = 38.89$ V

The rms value of the current, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{6.1}{\sqrt{2}} = 4.31$ A

Therefore, using Eq. 9.36, the average power is given as

$$\begin{aligned} P_{av} &= VI\cos\theta = 38.89 \times 4.31 \times \cos \pi/5 \\ &= 167.62 \times 0.809 = \mathbf{135.6\ W} \end{aligned}$$

The apparent power is

$$P_a = VI = 38.89 \times 4.31 = \mathbf{167.62\ VA}$$

Note that the apparent power is expressed in volt amperes (VA) and not in watts (W), since it is not a power in reality.

The instantaneous power at $\omega t = 0.3$ is given by Eq. 9.35, as

$$\begin{aligned} p &= VI\cos\theta - VI\cos(2\omega t - \theta) \\ &= 135.6 - 167.62 \times \cos(2 \times 0.3 - \pi/5) = \mathbf{-31.95\ VA} \end{aligned}$$

The power factor is given as

$$pf = \cos\theta = \cos \pi/5 = 0.809 = \mathbf{80.9\ \%}$$

Quick Quiz (Poll 3)

Reactive power is expressed in?

a) Watts (W)

b) Volt Amperes Reactive (VAR)

c) Volt Ampere (VA)

d) No units

Quick Quiz (Poll 4)

- Active Power is defined by

A) VI cos ϕ

B) VI sin ϕ

C) VI

D) All of the above