MTH166
DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

Lecture #0

Course details

- LTP: – 3 1 0 [3 lectures+1 Tutorial/week]
- Text Book: **Advanced Engineering Mathematics**, by R.K. Jain, S.R.K. Iyengar, Narosa Publishing House
- Reference Book: **Higher Engineering Mathematics,** by Dr. B.S. Grewal, Khanna Publishers

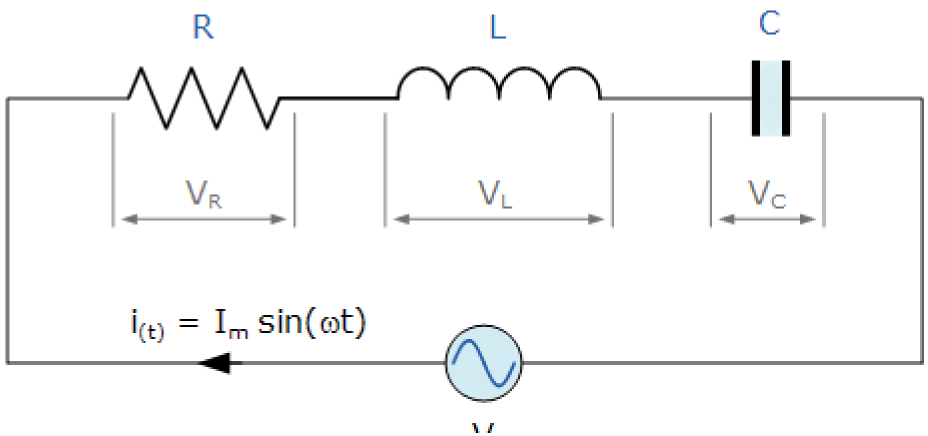
Course Assessment Model

- **Marks break up***
- Attendance 5
- CA (Two best out of three test) 25
- MTE (MCQ) 20
- ETE 50
- **Total 100**
  - Two test before MTE, one after MTE
  - MTE and ETE: MCQ to be done on OMR sheets

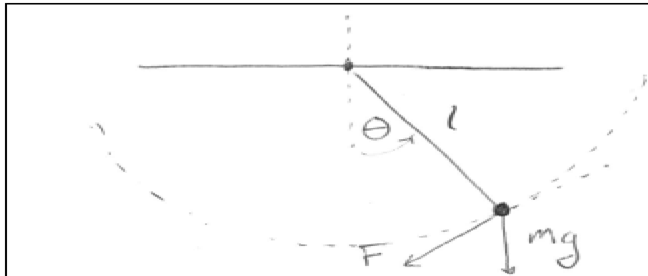
The course contents before MTE

- **Ordinary Differential equations**
  - Exact differential Equations
  - Equations reducible to exact equations
  - Equations of the first order and higher degree
  - Clairaut equation
- **Differential equations of higher order**
  - Introduction to linear differential equation, Solution of linear differential equation
  - Linear dependence and linear independence of solution
  - Method of solution of linear differential equation- Differential operator
  - Solution of second order homogeneous linear differential equation with constant coefficient
  - Solution of higher order homogenous linear differential equations with constant coefficient
- **Linear differential equations**
  - Solution of non-homogeneous linear differential equations with constant coefficients using operator method
  - Method of variation of parameters
  - Method of undetermined coefficient
  - Solution to Euler Cauchy Equation
  - Simultaneous differential equations by operator method

Why differential Equations



$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0$$



$F = -mg \sin\theta = ma$

$a = -g \sin\theta$

arc length is s

$s = l\theta$

$v = \dot{s} = l\dot{\theta}$

$a = \ddot{s} = l\ddot{\theta}$

$l\ddot{\theta} = -g \sin\theta$

$$\boxed{\ddot{\theta} = -\frac{g}{l} \sin\theta}$$

Why differential Equations

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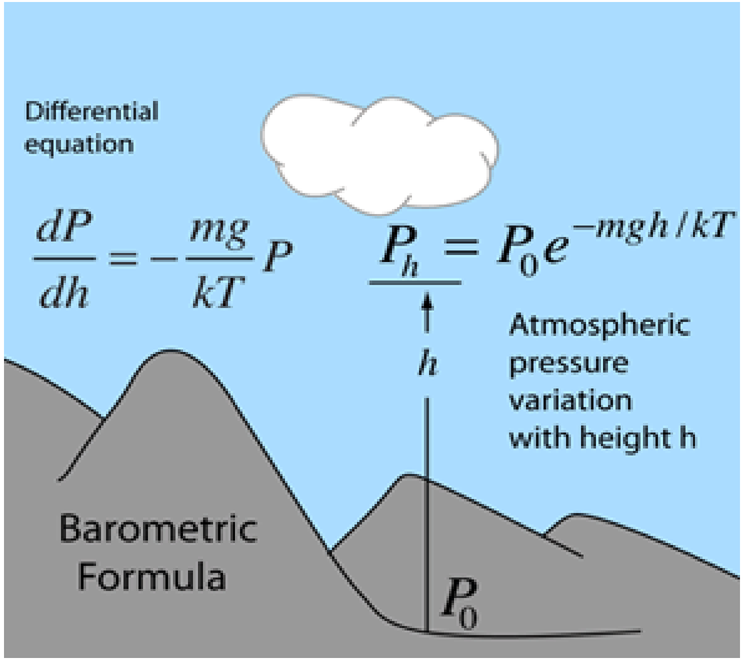
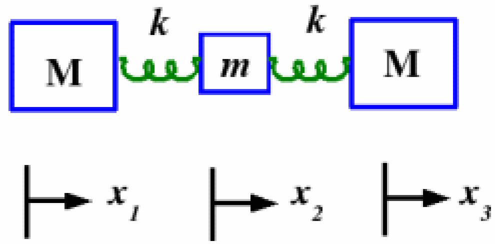
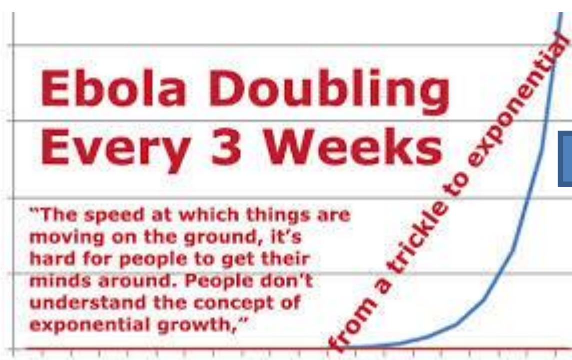




$$\frac{d^2x_1}{dt^2} = \frac{-k}{M}(x_1 - x_2) \quad \text{----------} \ (1)$$

$$\frac{d^2x_2}{dt^2} = \frac{-k}{m}(x_2 - x_1) - \frac{k}{m}(x_2 - x_3) \quad \text{----------} \ (2)$$

$$\frac{d^2x_3}{dt^2} = \frac{-k}{M}(x_3 - x_2) \quad \text{----------} \ (3)$$



$$\frac{dN}{dt} = rN$$

$$N(t) = N(0)e^{rt}$$

The course contents after MTE

- **Partial differential equation**
  - Introduction to PDE’s
  - Separation of variables
  - Solution of wave equation
  - Solution of heat equation
  - Solution of Laplace equation
- **Vector Calculus-I**

  -Limit, continuity and differentiability of vector functions

  -Length of space curve, Motion of a body or particle on a curve

  -Gradient of a scalar field and directional derivatives

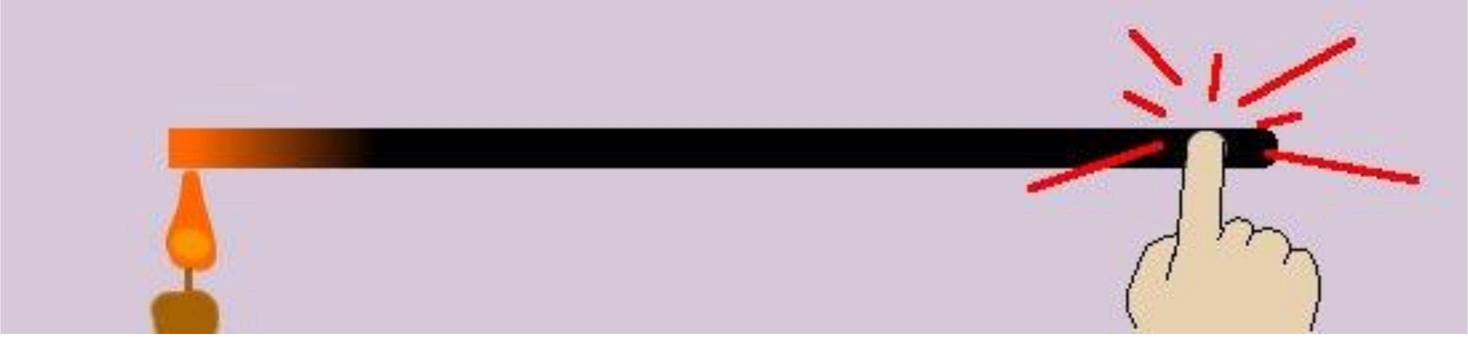
  -Divergence and curl of vector field
- **Vector Calculus-I**

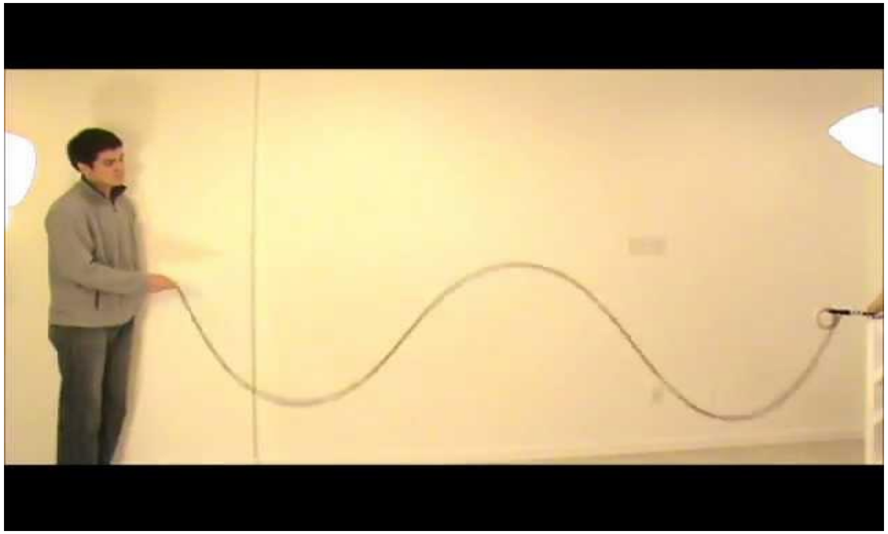
  -Line integral and Greens’ theorem

  -Surface area and Surface integral, Stokes’ theorem

  - Gauss divergence theorem

Why Partial differential equations



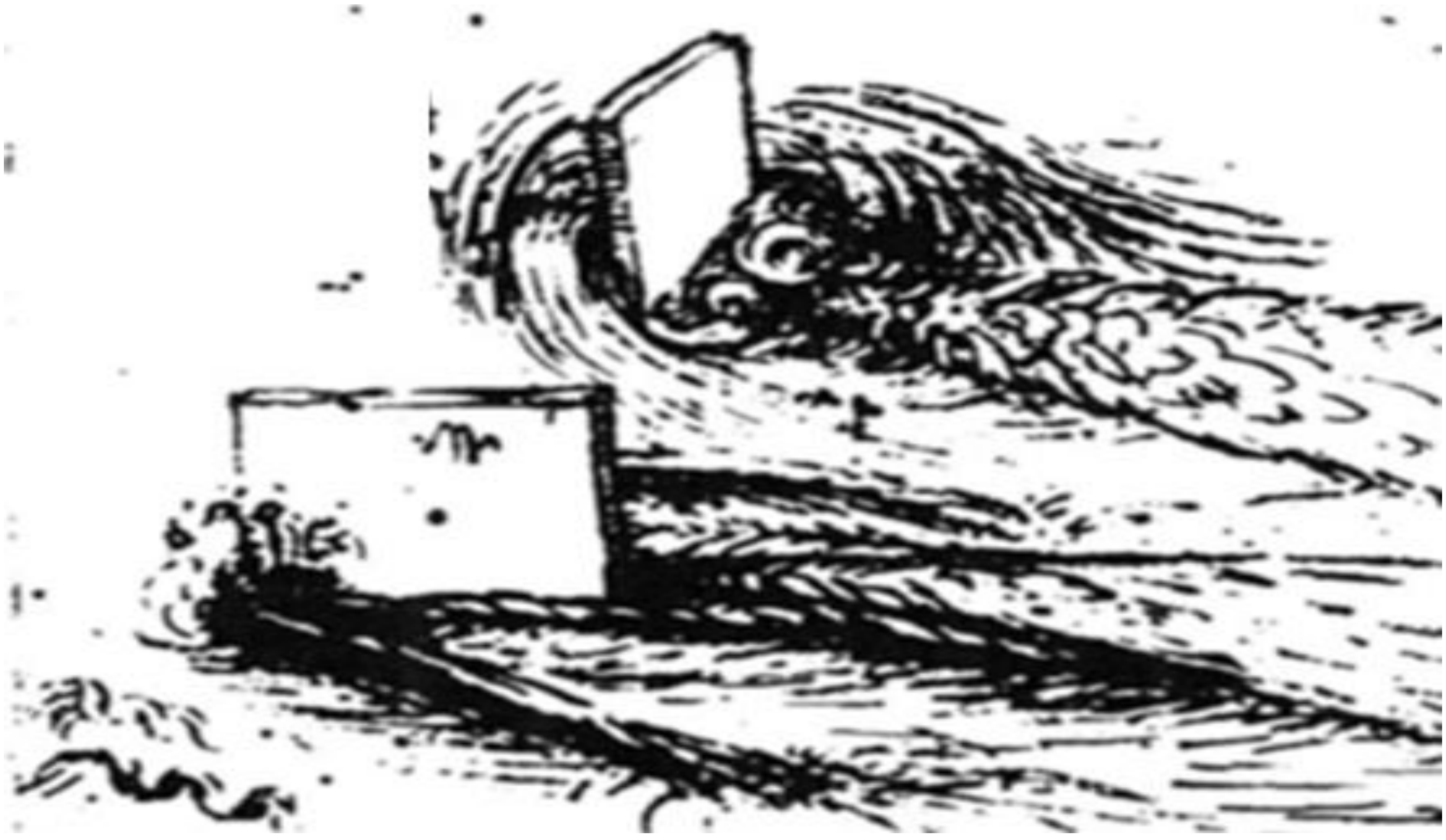


$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

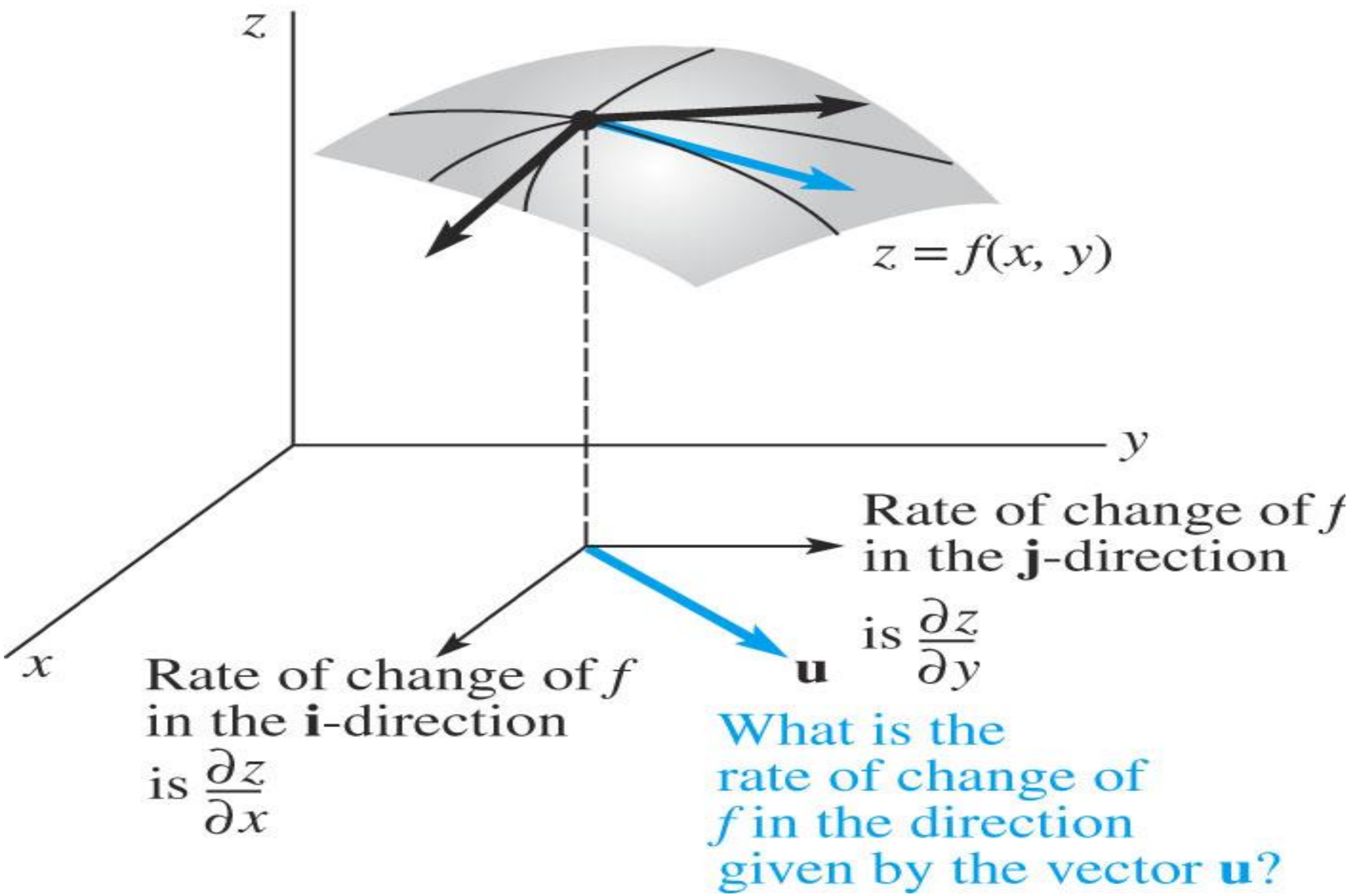
$u(x,0) = f(x)$ $\quad u(0,t) = 0$ $\quad u(L,t) = 0$

Why Vector Calculus

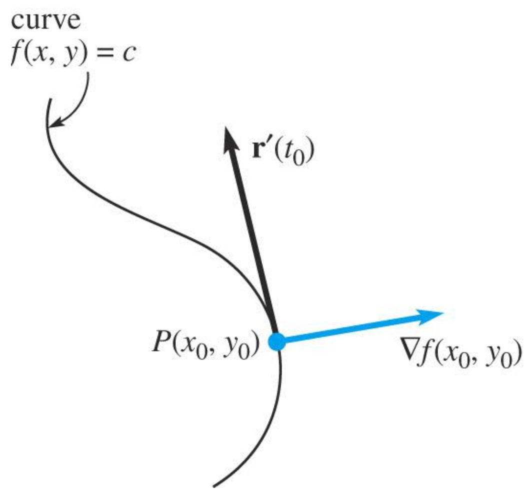
- Vector Calculus Natural to Fluid Mechanics

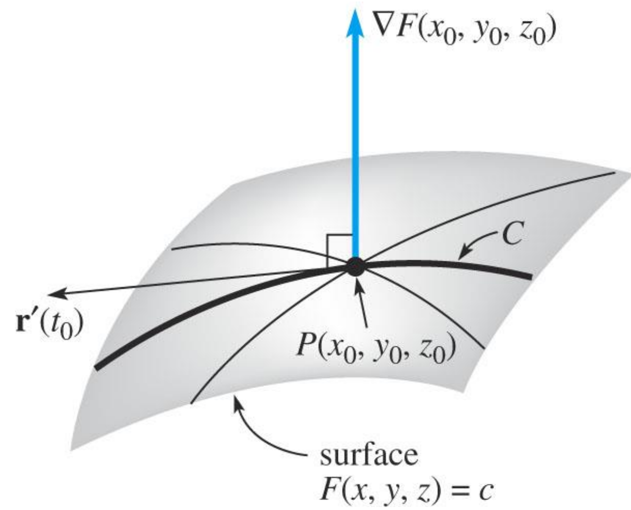


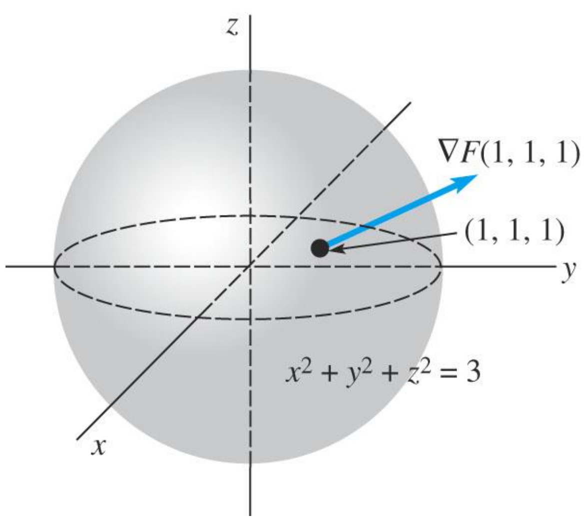
Directional Derivative

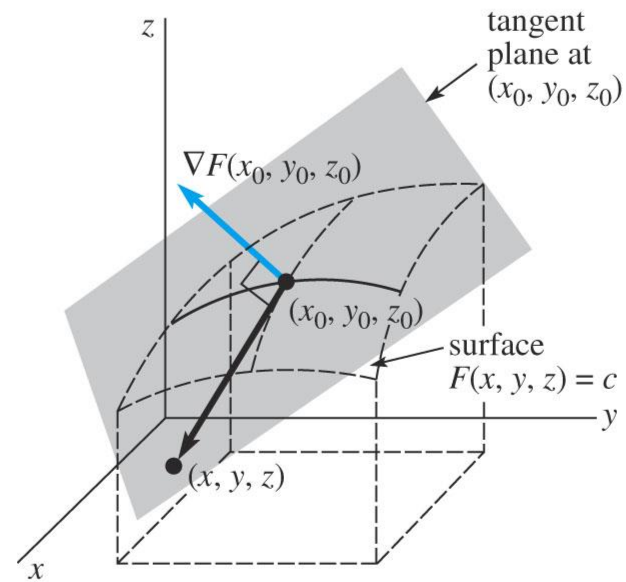


Gradient of a vector field



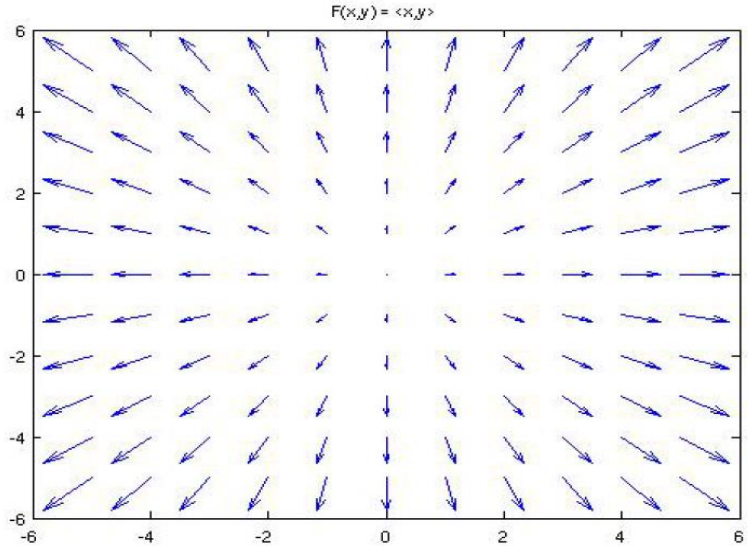


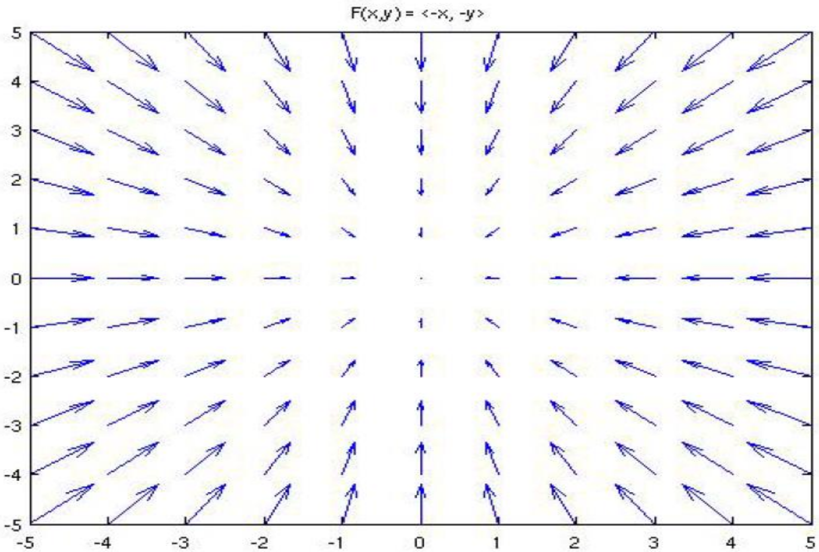




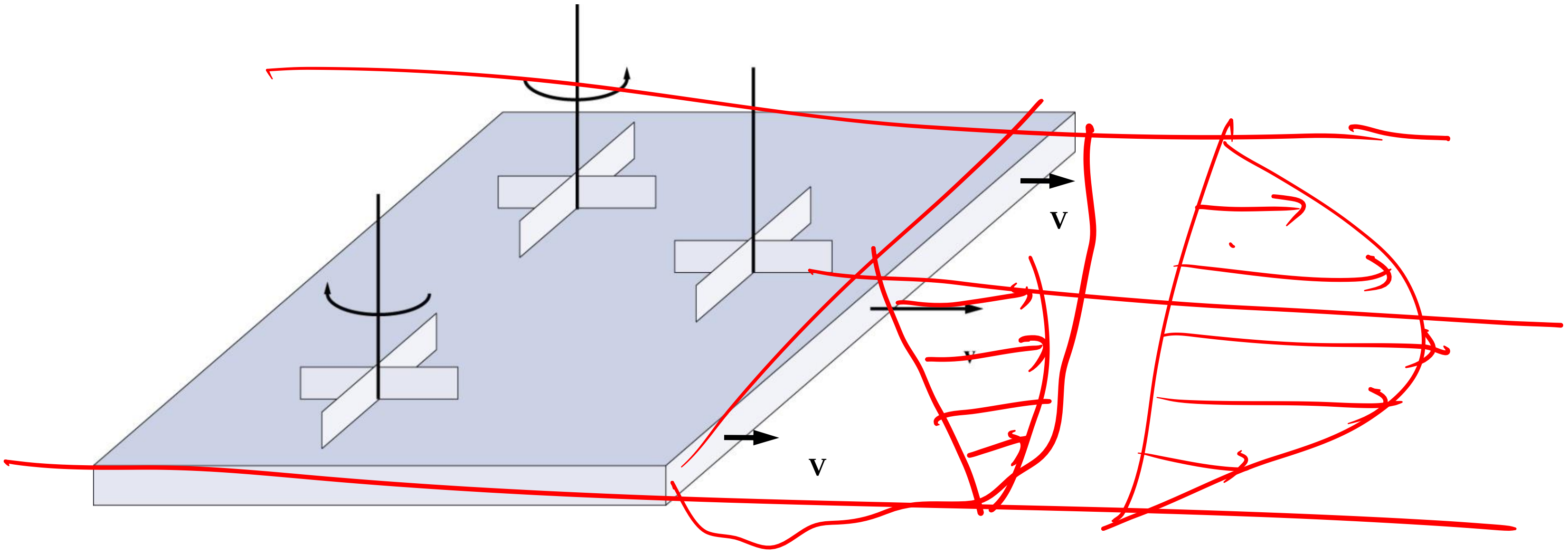
Divergence of a Vector Field

- Think of a vector field as a velocity field for a moving fluid.
- The divergence measures the expansion or contraction of the fluid.
- A vector field with constant positive or negative value of divergence.





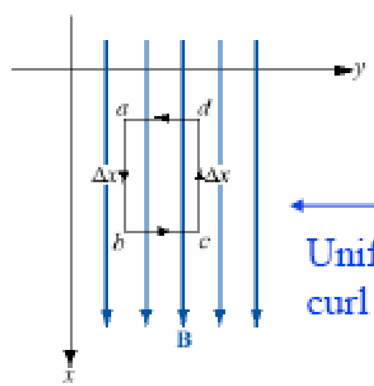
Further Use of Gradient for Human Welfare



•Assume we insert small paddle wheels in a flowing river.
•The flow is higher close to the center and slower at the edges.
•Therefore, a wheel close to the center (of a river) will not rotate since velocity of water is the same on both sides of the wheel.
•Wheels close to the edges will rotate due to difference in velocities.
•The curl operation determines the direction and the magnitude of rotation.

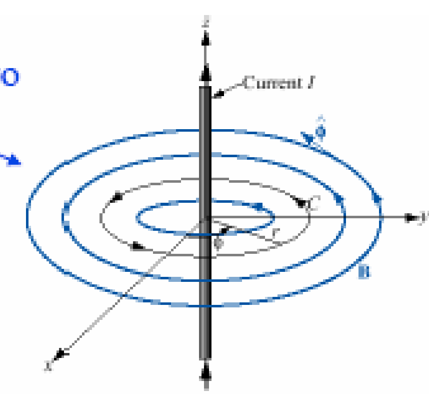
CURL OF A VECTOR (Cont'd)

The curl of the vector field is concerned with rotation of the vector field. Rotation can be used to measure the uniformity of the field, the more non uniform the field, the larger value of curl.

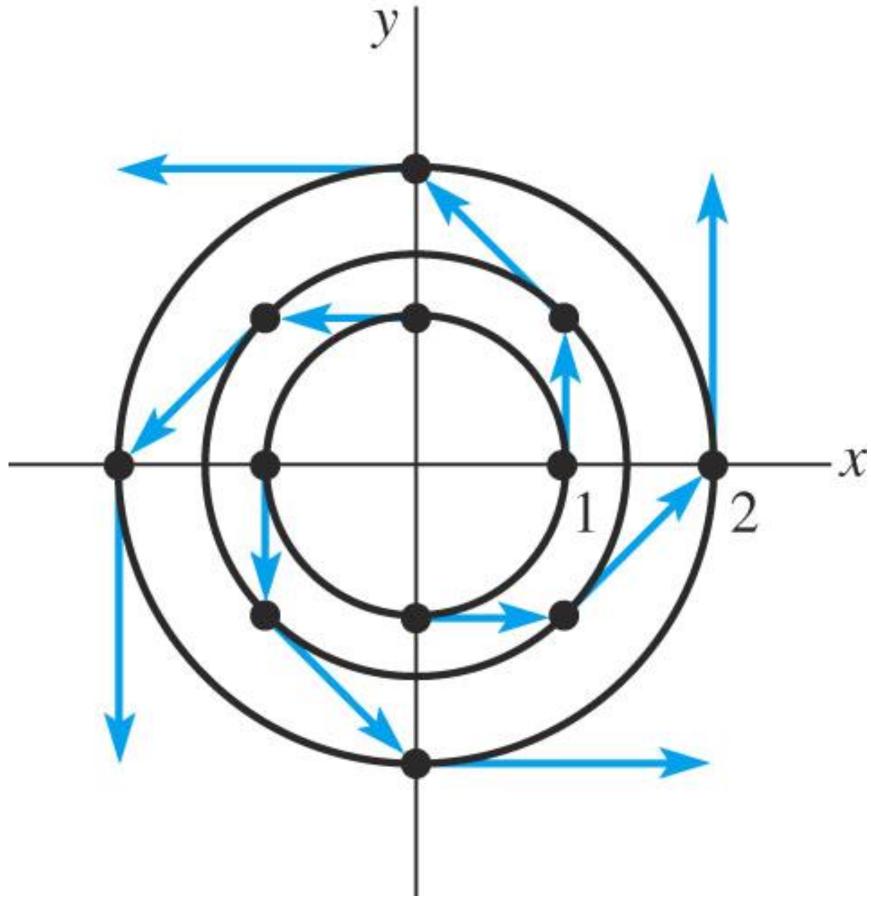


Uniform B field curl is zero.

non-uniform field, non-zero curl.



Graph the vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$

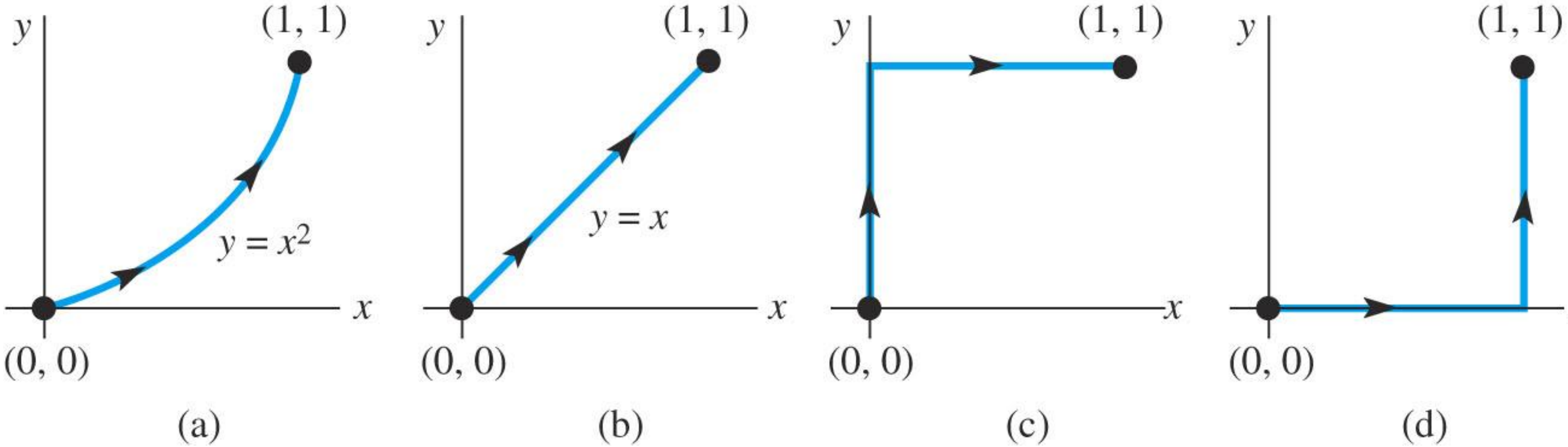


Path Independence

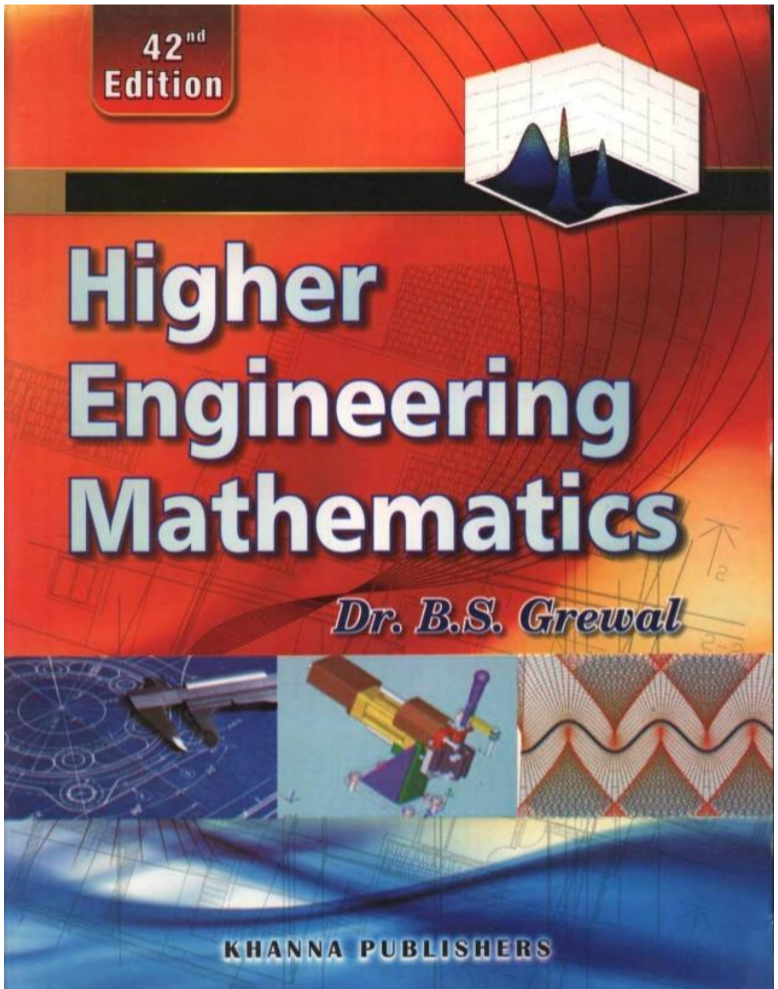
A line integral whose value is the same for ***every curve*** or ***path*** connecting *A* and *B.*

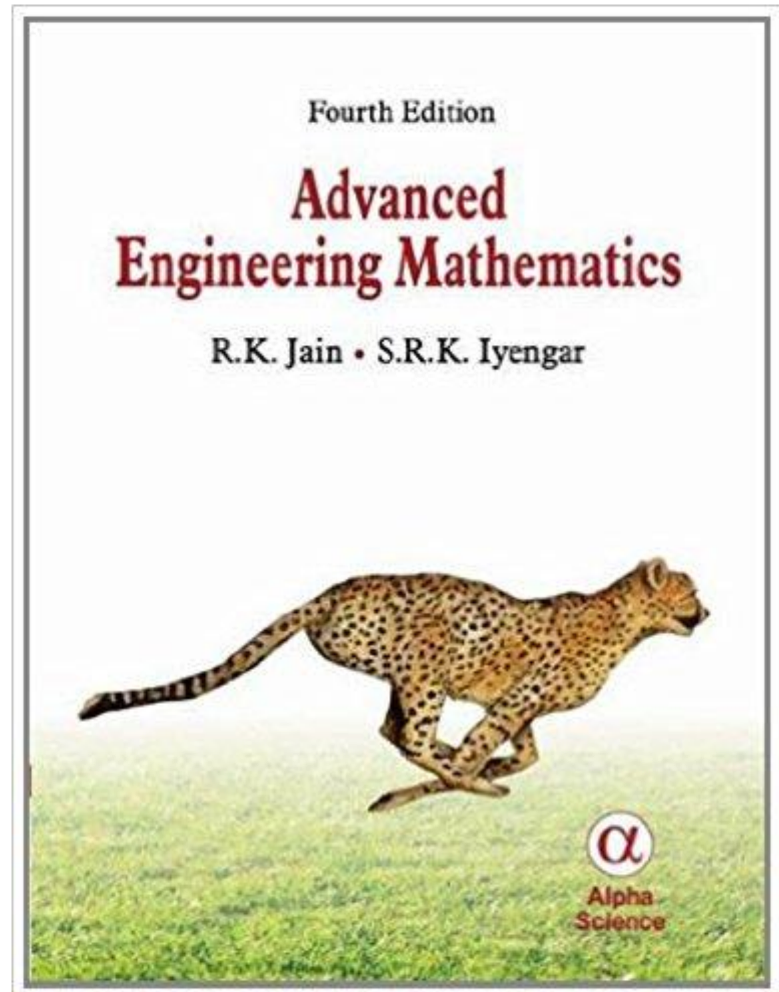
- $\int_C y\,dx + x\,dy$ has the same value on each path between (0, 0) and (1, 1) shown in Fig on next page
- Note that $\int_C y\,dx + x\,dy = 1$

Path Independence



BOOKS FOR THIS SEMESTER





Thank You !

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All THE BEST