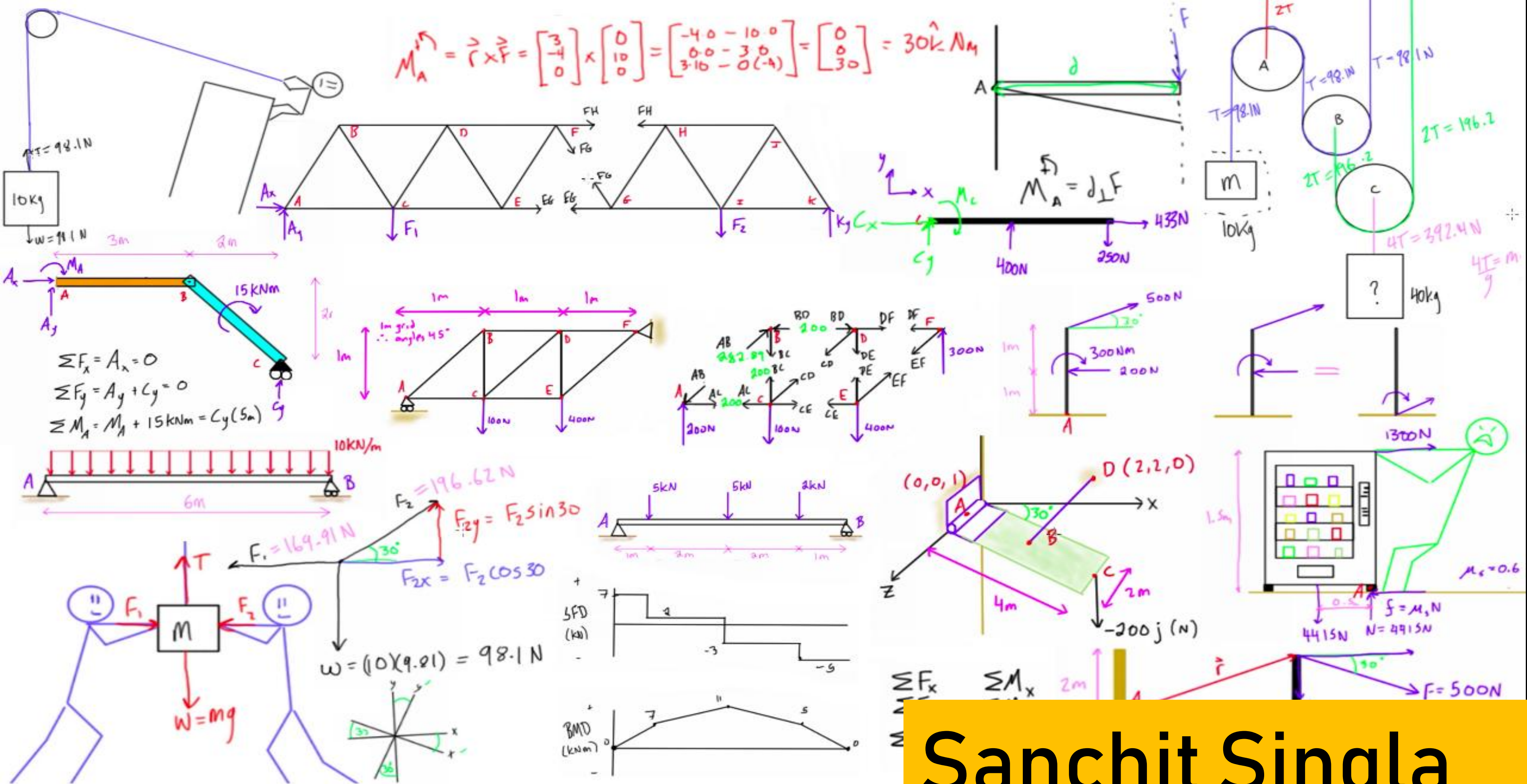


Sanchit Singla

Assistant Professor

MEC107

Basic Engineering Mechanics

Learning Outcomes

After this lecture, you will be able to

- ✓ learn about Translation Motion: Velocity and Acceleration.
- ✓ understand Rotation About a Fixed Axis: Velocity and Acceleration.
- ✓ know about Rotation About a Fixed Axis: Representatives Slab.
- ✓ understand about General Plane Motion: : Velocity and Acceleration.

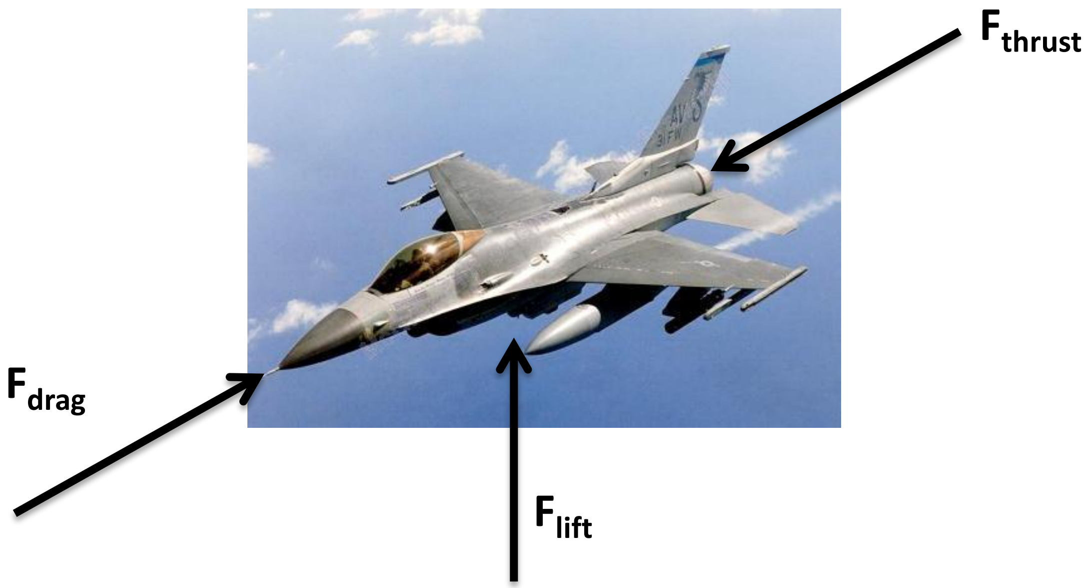
Introduction

Kinematic relationships are used to help us determine the trajectory of a golf ball, the orbital speed of a satellite, and the accelerations during acrobatic flying.

Introduction

- **Dynamics includes:**

Kinematics: study of the geometry of motion.
Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion.



Kinetics: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

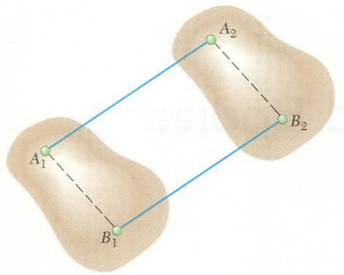
Introduction

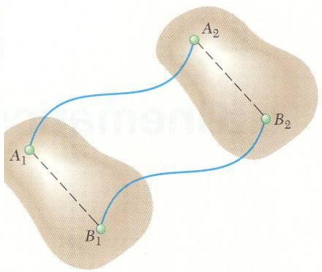
- **Particle kinetics includes:**

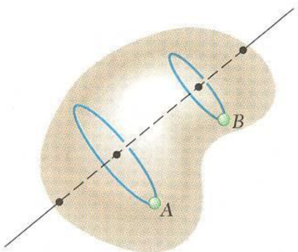
  - ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.

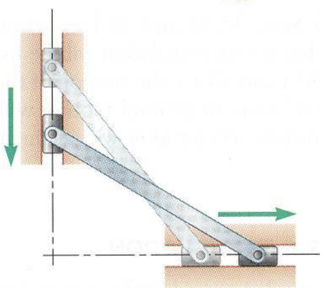
- ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

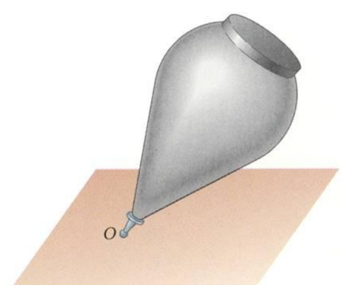
Introduction





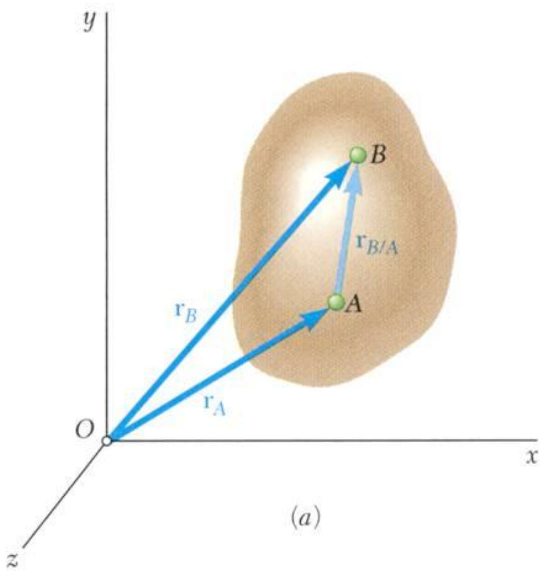


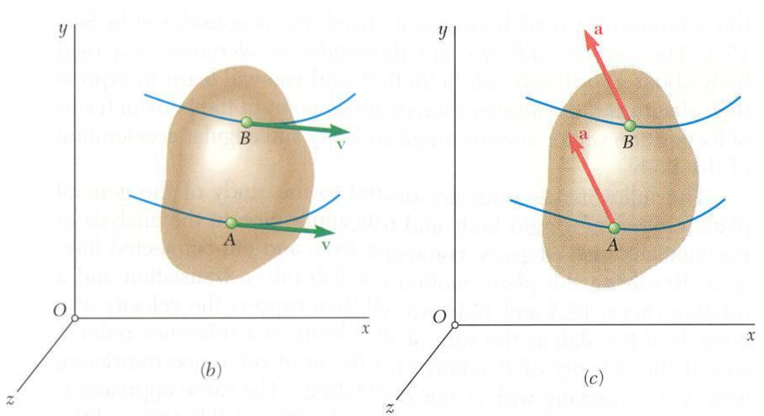




- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.
- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

Translation





- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.
- For any two particles in the body,

  $$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

  $$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

  $$\vec{v}_B = \vec{v}_A$$

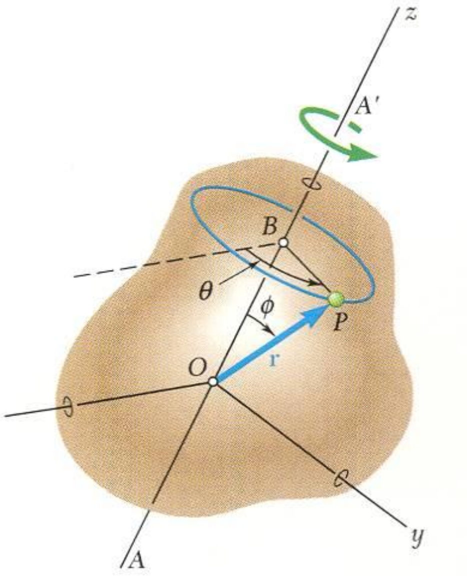
  All particles have the same velocity.
- Differentiating with respect to time again,

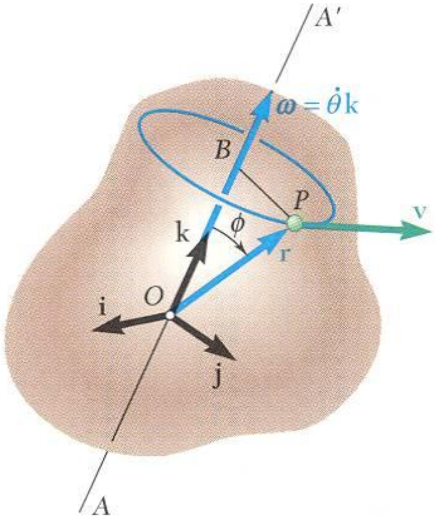
  $$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

  $$\vec{a}_B = \vec{a}_A$$

  All particles have the same acceleration.

Rotation About a Fixed Axis. Velocity





- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

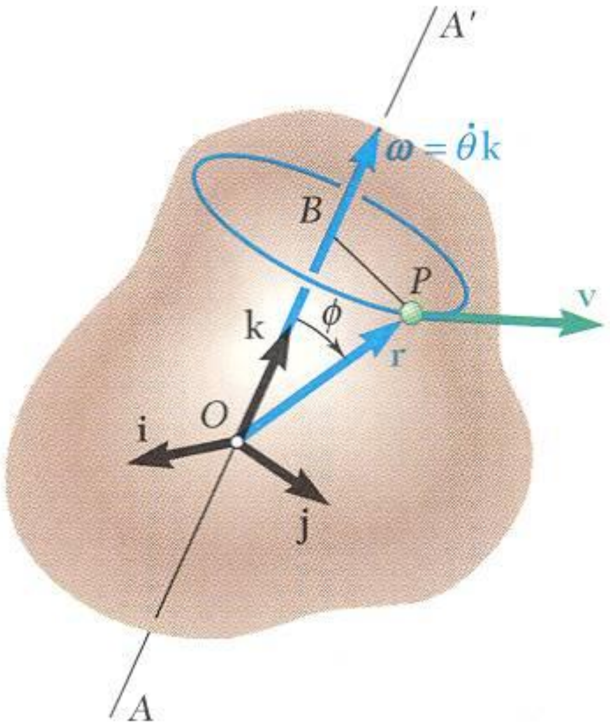
$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0} (r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

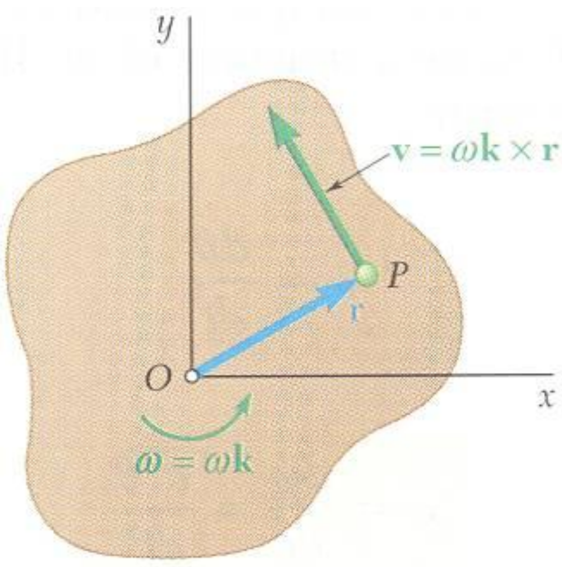
- $\dfrac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$

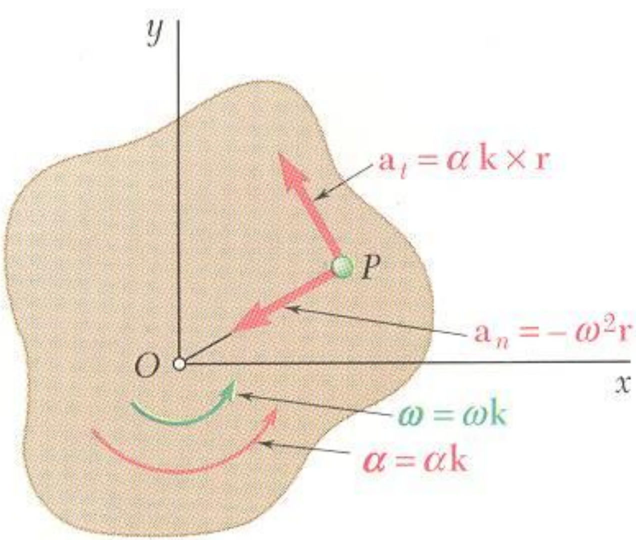
$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

- Acceleration of *P* is combination of two vectors,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$\vec{\alpha} \times \vec{r} = \text{tangential acceleration component}$$
$$\vec{\omega} \times \vec{\omega} \times \vec{r} = \text{radial acceleration component}$$

Rotation About a Fixed Axis. Representative Slab





- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point *P* of the slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega\vec{k} \times \vec{r}$$

$$v = r\omega$$

- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$

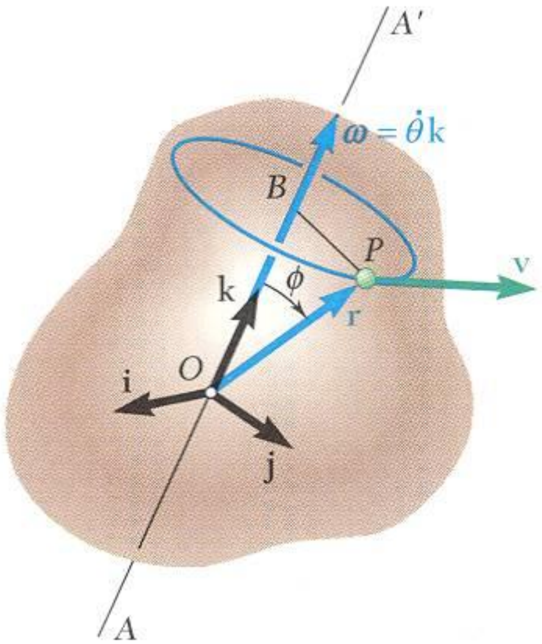
$$= \alpha\vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha\vec{k} \times \vec{r} \qquad a_t = r\alpha$$

$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \dfrac{d\theta}{dt}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$$

- *Uniform Rotation,* $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

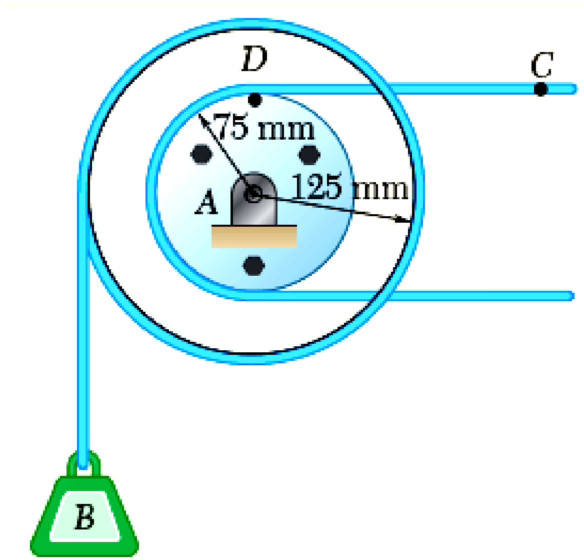
- *Uniformly Accelerated Rotation,* $\alpha =$ constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Numerical



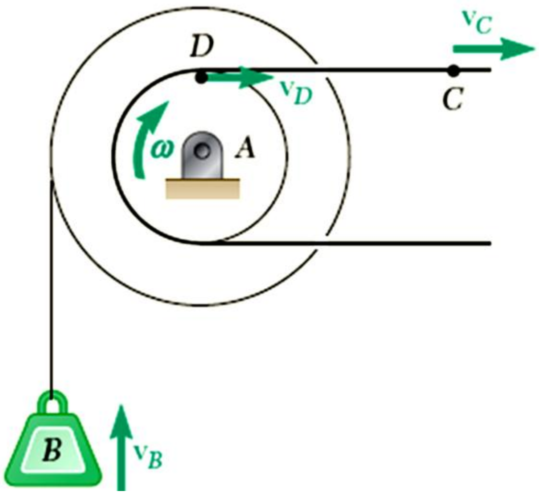
Cable *C* has a constant acceleration of 225 mm/s^2 and an initial velocity of 300 mm/s, both directed to the right.

Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load *B* after 2 s, and *(c)* the acceleration of the point *D* on the rim of the inner pulley at t = 0.

SOLUTION:

- Due to the action of the cable, the tangential velocity and acceleration of *D* are equal to the velocity and acceleration of *C*. Calculate the initial angular velocity and acceleration.
- Apply the relations for uniformly accelerated rotation to determine the velocity and angular position of the pulley after 2 s.
- Evaluate the initial tangential and normal acceleration components of *D*.

Numerical



SOLUTION:

- The tangential velocity and acceleration of *D* are equal to the velocity and acceleration of *C.*

$$(\vec{v}_D)_0 = (\vec{v}_C)_0 = 300\,\text{mm/s} \rightarrow$$
$$(v_D)_0 = r\omega_0$$
$$\omega_0 = \frac{(v_D)_0}{r} = \frac{300}{75} = 4\,\text{rad/s}$$

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$
$$(a_D)_t = r\alpha$$
$$\alpha = \frac{(a_D)_t}{r} = \frac{9}{3} = 3\,\text{rad/s}^2$$

- Apply the relations for uniformly accelerated rotation to determine velocity and angular position of pulley after 2 s.

$$\omega = \omega_0 + \alpha t = 4\,\text{rad/s} + \left(3\,\text{rad/s}^2\right)(2\,\text{s}) = 10\,\text{rad/s}$$

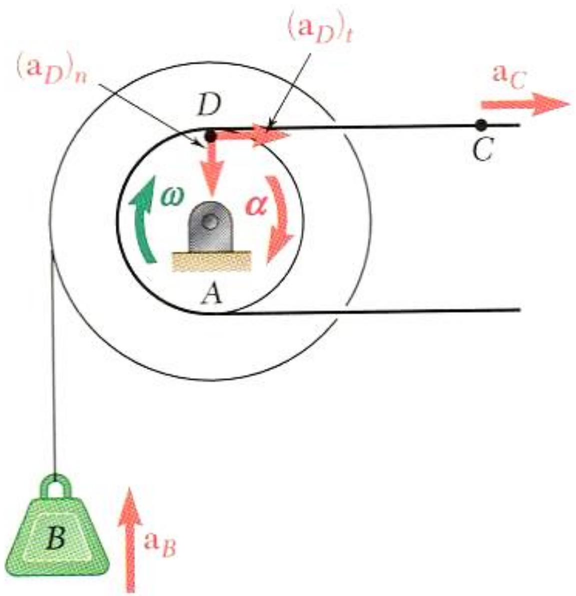
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4\,\text{rad/s})(2\,\text{s}) + \tfrac{1}{2}\left(3\,\text{rad/s}^2\right)(2\,\text{s})^2$$
$$= 14\,\text{rad}$$

$$N = (14\,\text{rad})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \text{number of revs} \qquad \boxed{N = 2.23\,\text{rev}}$$

$$v_B = r\omega = (125\,\text{mm})(10\,\text{rad/s})$$
$$\Delta y_B = r\theta = (125\,\text{mm})(14\,\text{rad})$$

$$\boxed{\begin{aligned}\vec{v}_B &= 1.25\,\text{m/s} \uparrow \\ \Delta y_B &= 1.75\,\text{m}\end{aligned}}$$

Numerical

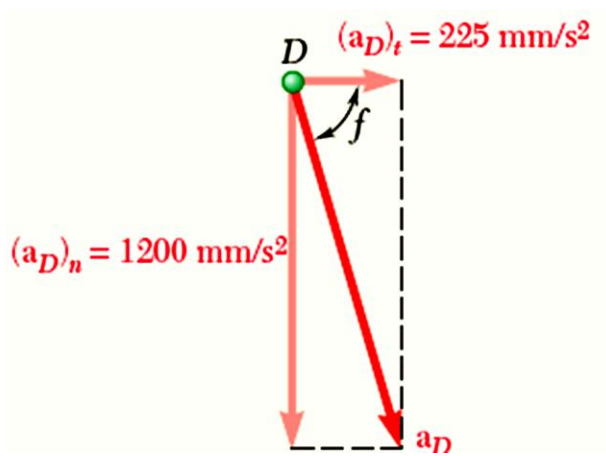


- Evaluate the initial tangential and normal acceleration components of *D*.

$$(\vec{a}_D)_t = \vec{a}_C = 225\,\text{mm/s}^2 \rightarrow$$

$$(a_D)_n = r_D\omega_0^2 = (75\text{ mm})(4\text{ rad/s})^2 = 1200\,\text{mm/s}^2$$

$$\boxed{(\vec{a}_D)_t = 225\,\text{mm/s}^2 \rightarrow \qquad (\vec{a}_D)_n = 1200\,\text{mm/s}^2 \downarrow}$$



Magnitude and direction of the total acceleration,

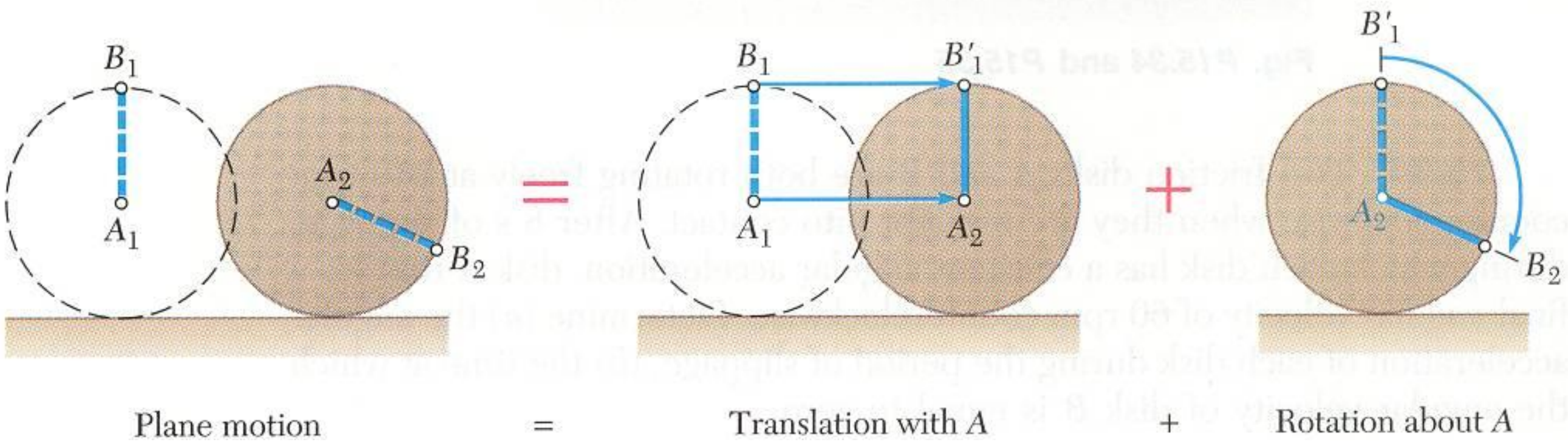
$$a_D = \sqrt{(a_D)_t^2 + (a_D)_n^2} = \sqrt{225^2 + 1200^2}$$

$$\boxed{a_D = 1220\,\text{mm/s}^2}$$

$$\tan\phi = \frac{(a_D)_n}{(a_D)_t} = \frac{1200}{225}$$

$$\boxed{\phi = 79.4^\circ}$$

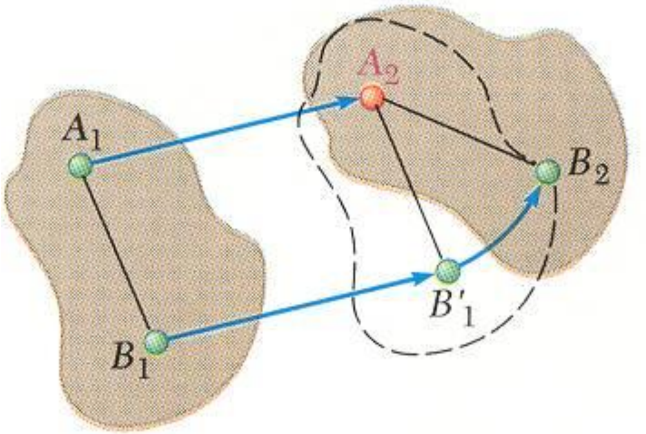
General Plane Motion



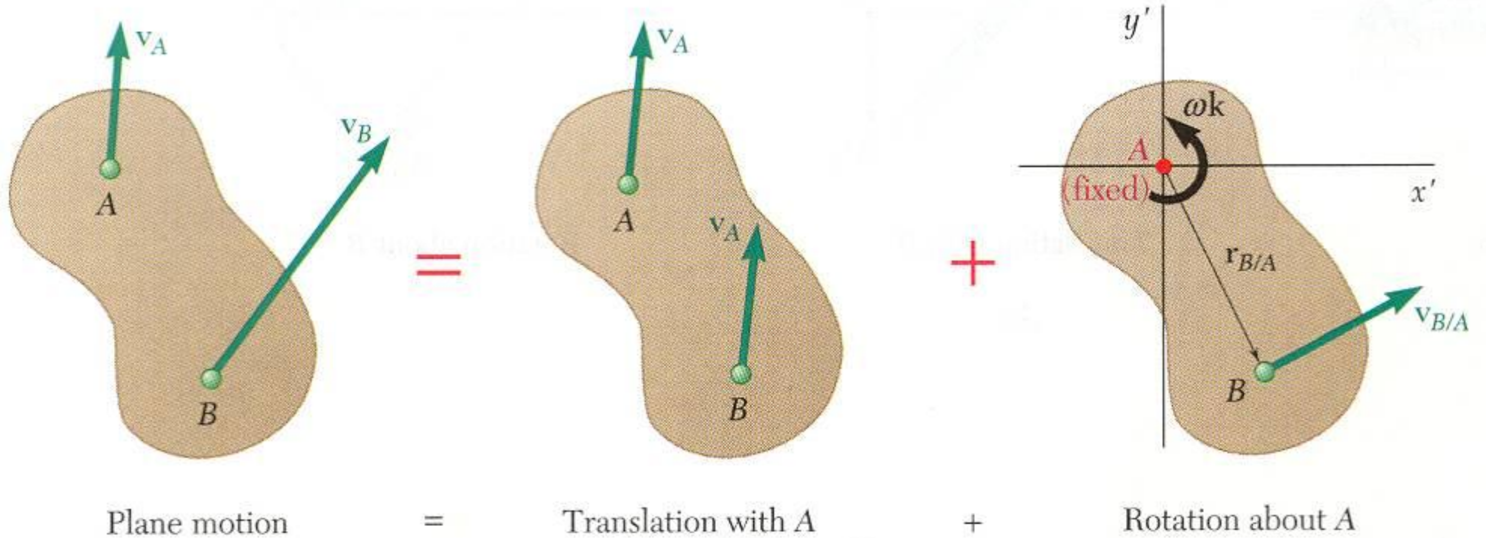
- *General plane motion* is neither a translation nor a rotation.

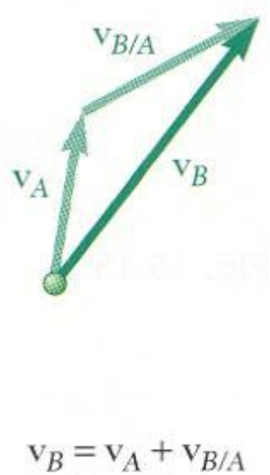
- General plane motion can be considered as the *sum* of a translation and rotation.

- Displacement of particles *A* and *B* to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2



Absolute and Relative Velocity in Plane Motion





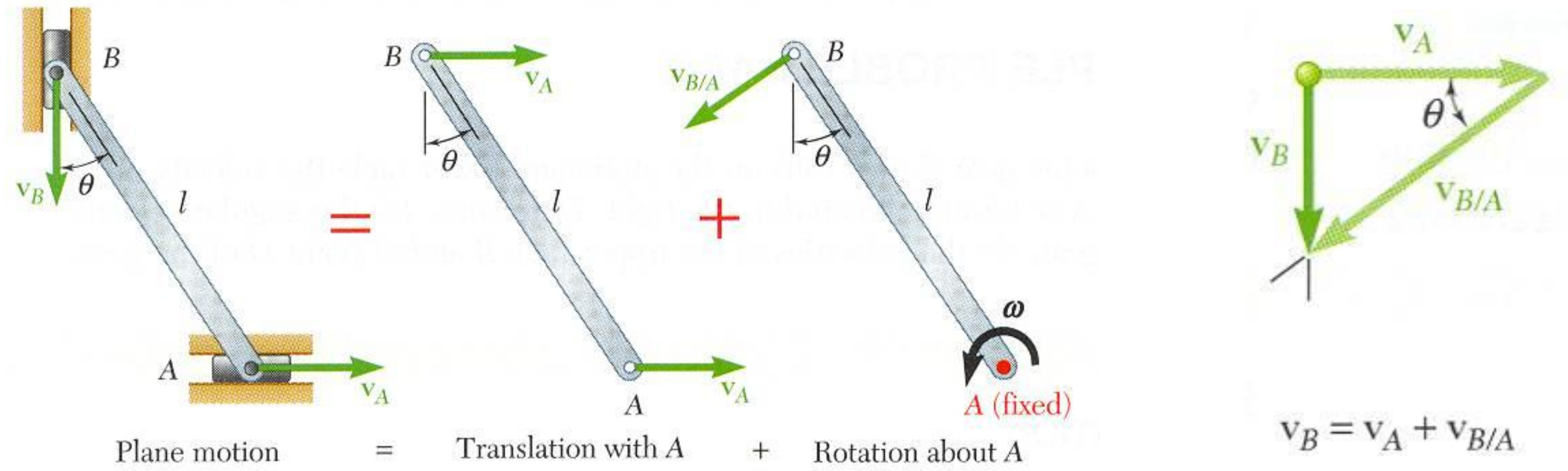
- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

Absolute and Relative Velocity in Plane Motion



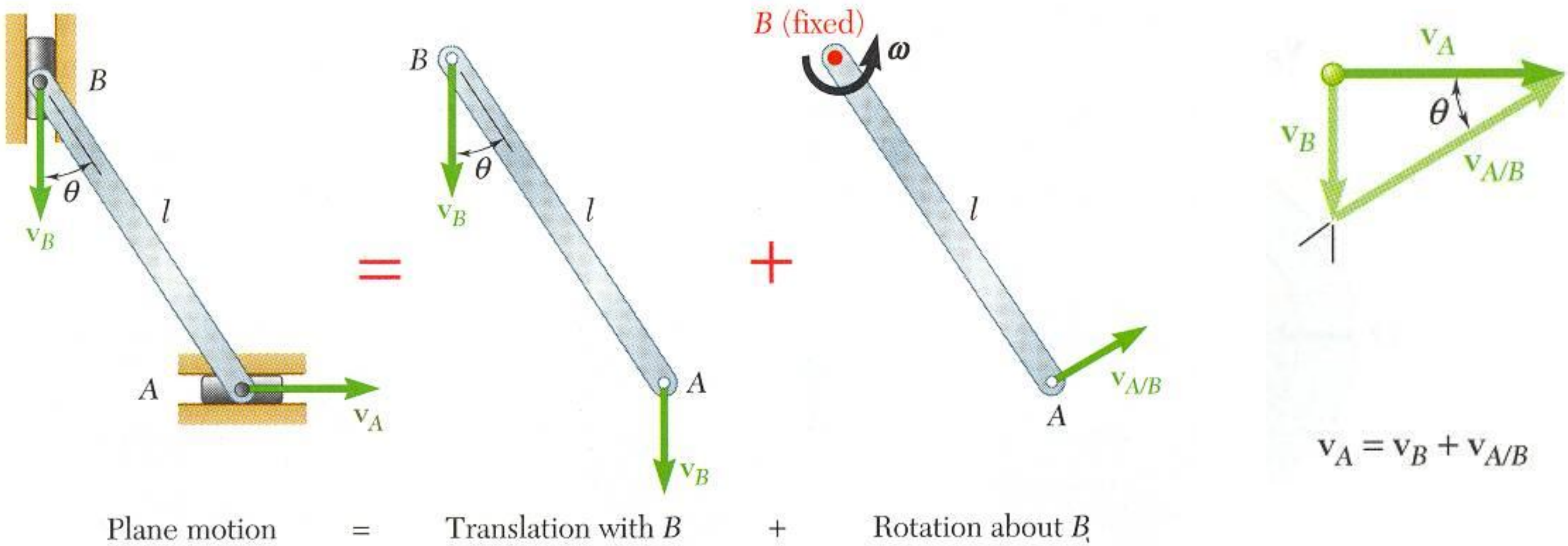
- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.

- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

$$\frac{v_B}{v_A} = \tan\theta \qquad \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

$$v_B = v_A \tan\theta \qquad \omega = \frac{v_A}{l\cos\theta}$$

Absolute and Relative Velocity in Plane Motion

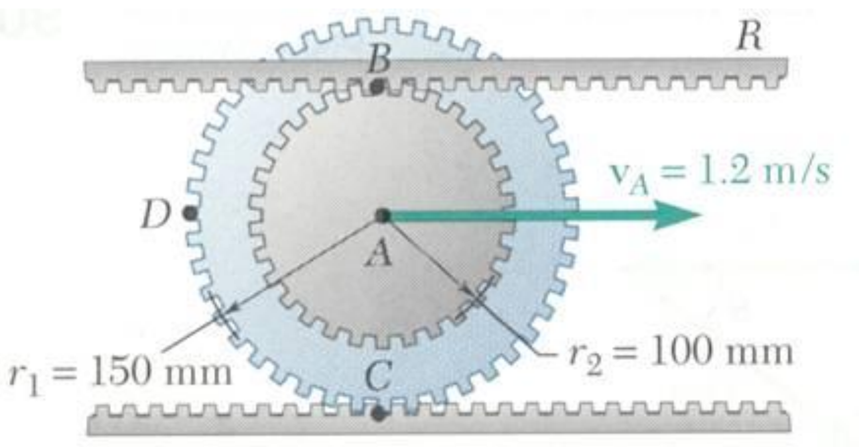


- Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.

- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.

- Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. Angular velocity is not dependent on the choice of reference point.

Numerical



The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

<u>SOLUTION</u>:

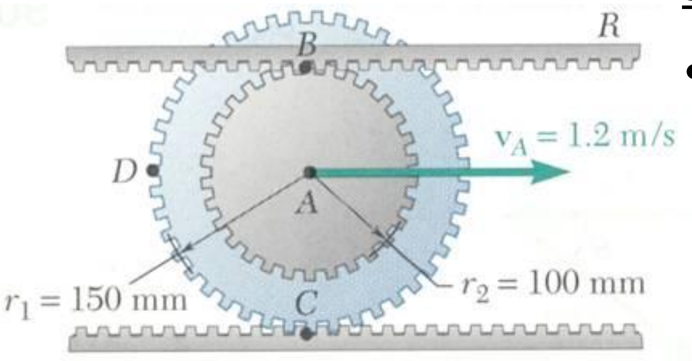
- The displacement of the gear center in one revolution is equal to the outer circumference. Relate the translational and angular displacements. Differentiate to relate the translational and angular velocities.

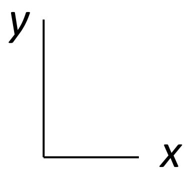
- The velocity for any point *P* on the gear may be written as

$$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega\vec{k} \times \vec{r}_{P/A}$$

Evaluate the velocities of points *B* and *D.*

Numerical





<u>SOLUTION</u>:

- The displacement of the gear center in one revolution is equal to the outer circumference.

For $x_A > 0$ (moves to right), $\omega < 0$ (rotates clockwise).

$$\frac{x_A}{2\pi r} = -\frac{\theta}{2\pi} \qquad x_A = -r_1\theta$$

Differentiate to relate the translational and angular velocities.

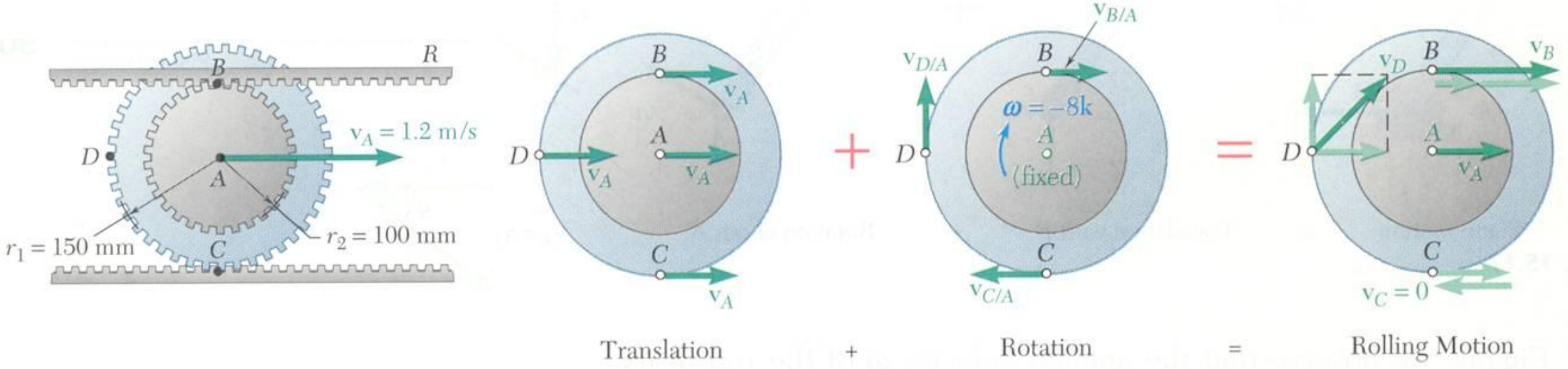
$$v_A = -r_1\omega$$

$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\,\mathrm{m}}$$

$$\boxed{\vec{\omega} = \omega\vec{k} = -(8\,\mathrm{rad/s})\vec{k}}$$

Numerical

- For any point P on the gear, $\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega\vec{k} \times \vec{r}_{P/A}$



Velocity of the upper rack is equal to velocity of point B:

$$\vec{v}_R = \vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (0.10\ \mathrm{m})\vec{j}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (0.8\,\mathrm{m/s})\vec{i}$$

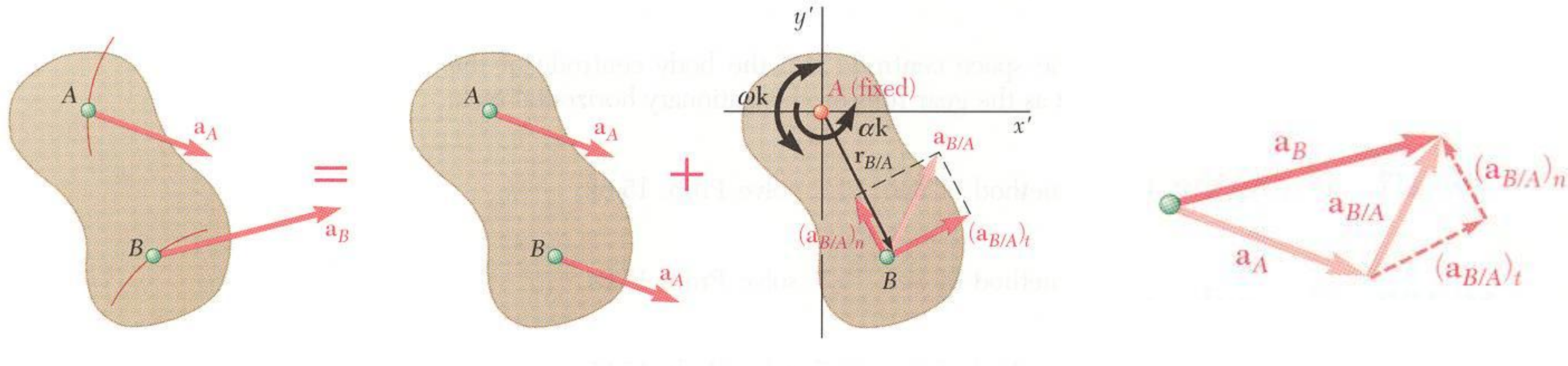
$$\boxed{\vec{v}_R = (2\,\mathrm{m/s})\vec{i}}$$

Velocity of the point D:

$$\vec{v}_D = \vec{v}_A + \omega\vec{k} \times \vec{r}_{D/A}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (-0.150\ \mathrm{m})\vec{i}$$

$$\boxed{\begin{aligned}\vec{v}_D &= (1.2\,\mathrm{m/s})\vec{i} + (1.2\,\mathrm{m/s})\vec{j}\\ v_D &= 1.697\ \mathrm{m/s}\end{aligned}}$$

Absolute and Relative Acceleration in Plane Motion



Plane motion = Translation with A + Rotation about A

- Absolute acceleration of a particle of the slab,

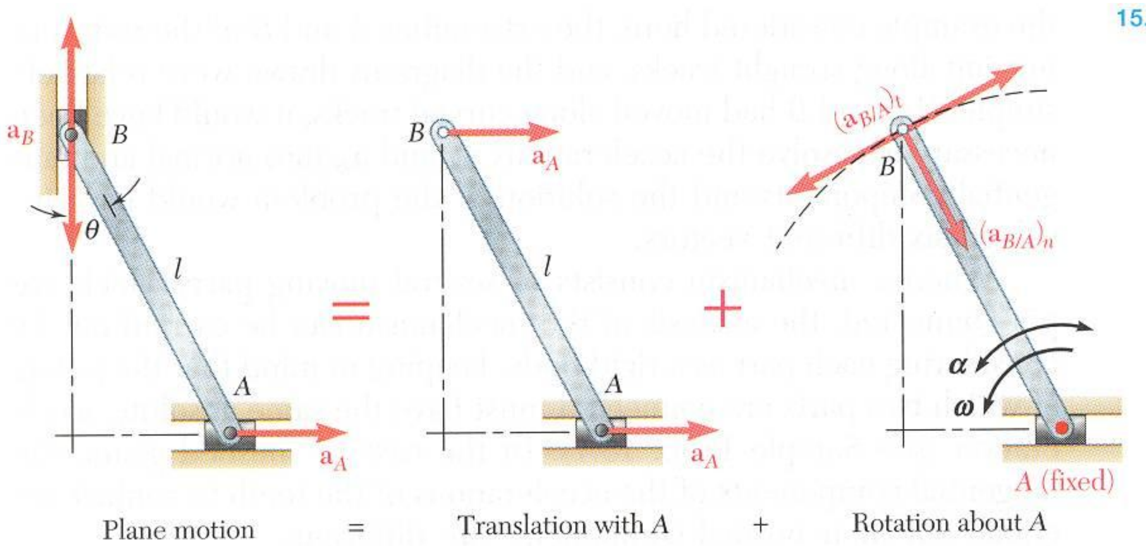
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about A includes tangential and normal components,

$$\left(\vec{a}_{B/A}\right)_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_t = r\alpha$$

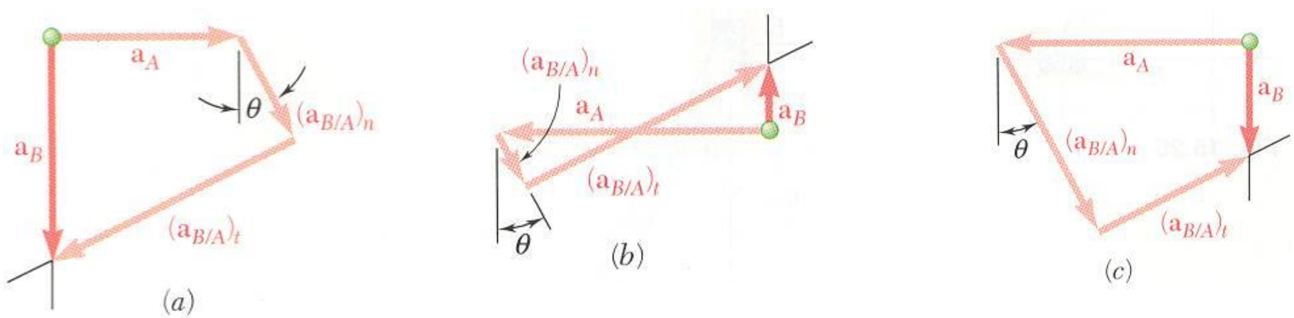
$$\left(\vec{a}_{B/A}\right)_n = -\omega^2 \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_n = r\omega^2$$

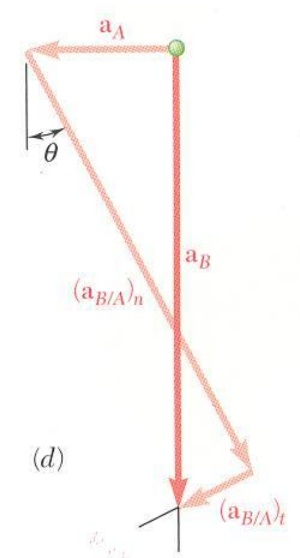
Absolute and Relative Acceleration in Plane Motion



- Given $\vec{a}_A$ and $\vec{v}_A$, determine $\vec{a}_B$ and $\vec{\alpha}$.

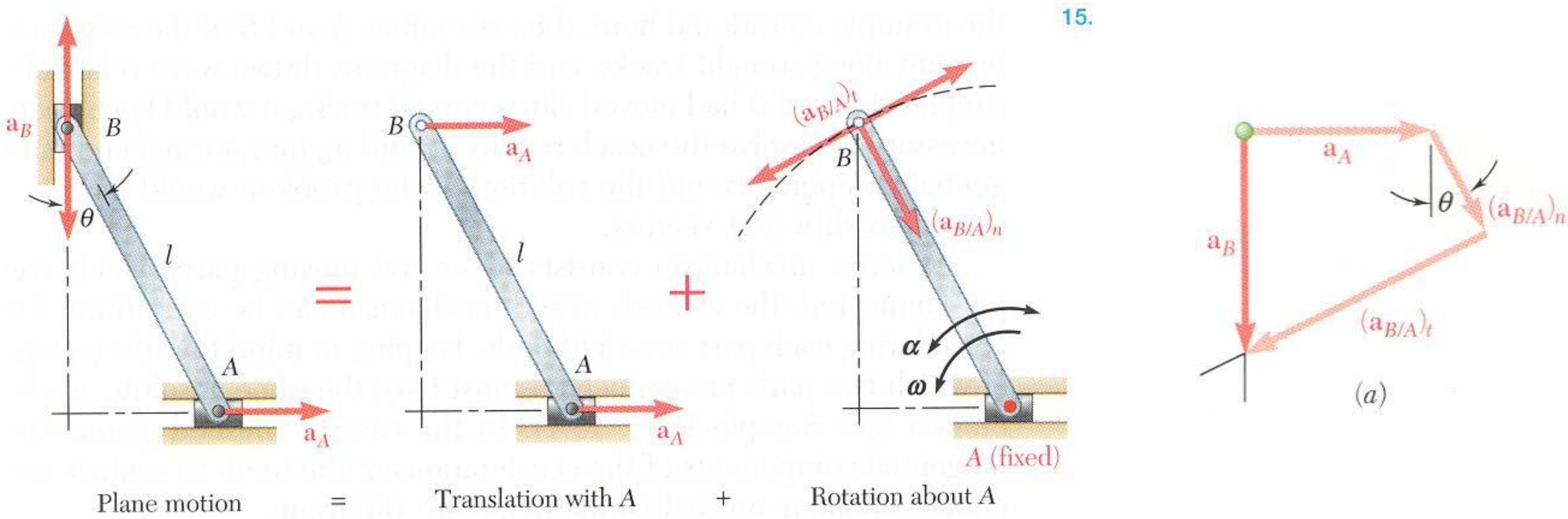
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_n + \left(\vec{a}_{B/A}\right)_t$$





- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $\left(a_{B/A}\right)_n$
  - Must also know angular velocity ω.

Absolute and Relative Acceleration in Plane Motion



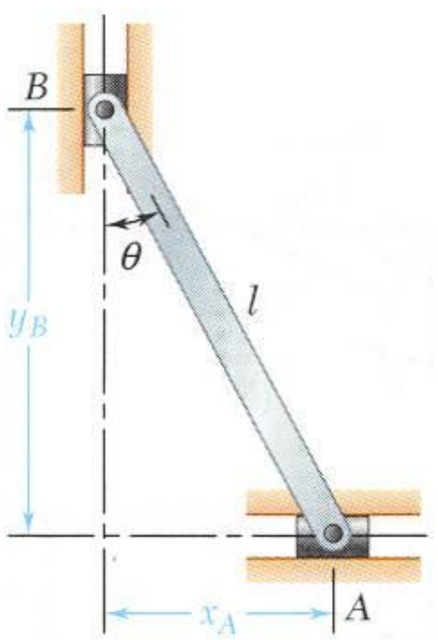
- Write $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ in terms of the two component equations,

  $\xrightarrow{+}$ *x* components: $0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$

  $+\uparrow$ *y* components: $-a_B = -l\omega^2 \cos\theta - l\alpha\sin\theta$

- Solve for a_B and α.

Analysis of Plane Motion in Terms of a Parameter



- In some cases, it is advantageous to determine the absolute velocity and acceleration of a mechanism directly.

$$x_A = l\sin\theta \qquad y_B = l\cos\theta$$

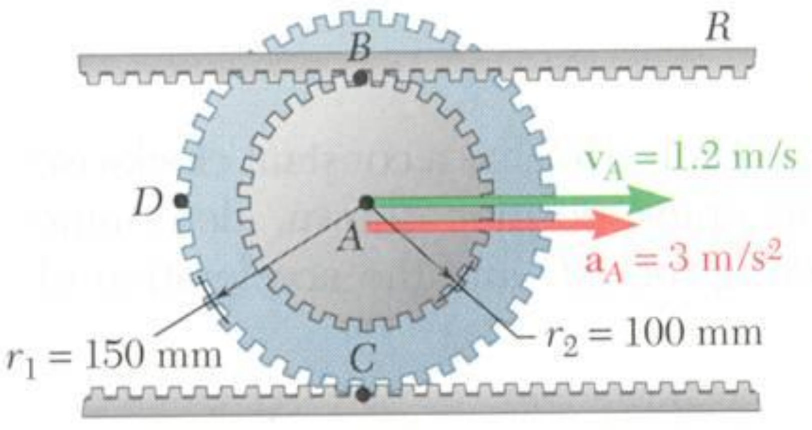
$$v_A = \dot{x}_A = l\dot{\theta}\cos\theta = l\omega\cos\theta$$

$$v_B = \dot{y}_B = -l\dot{\theta}\sin\theta = -l\omega\sin\theta$$

$$a_A = \ddot{x}_A = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta = -l\omega^2\sin\theta + l\alpha\cos\theta$$

$$a_B = \ddot{y}_B = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta = -l\omega^2\cos\theta - l\alpha\sin\theta$$

Numerical



The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s^2, respectively. The lower rack is stationary.

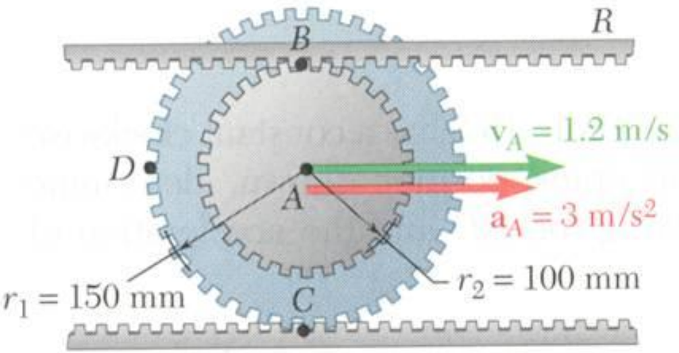
Determine *(a)* the angular acceleration of the gear, and *(b)* the acceleration of points *B*, *C*, and *D*.

SOLUTION:

- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

- The acceleration of each point on the gear is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center. The latter includes normal and tangential acceleration components.

Numerical



SOLUTION:

- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

$$x_A = -r_1\theta$$

$$v_A = -r_1\dot{\theta} = -r_1\omega$$

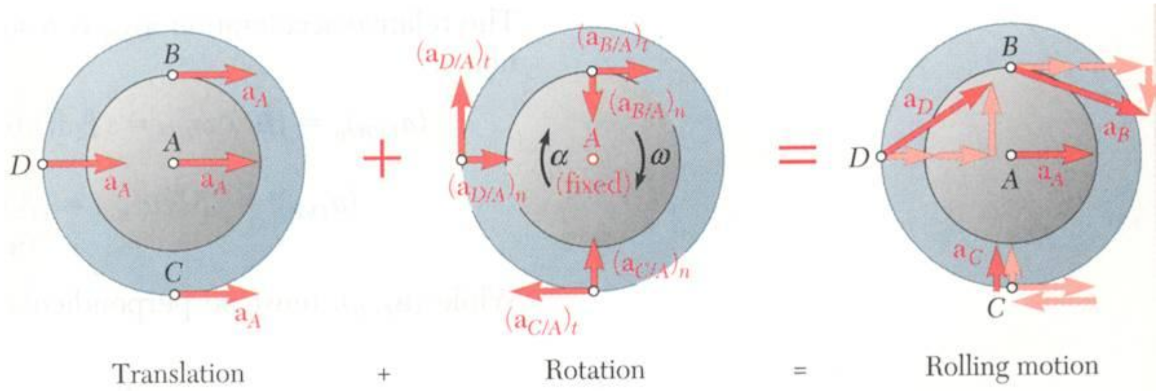
$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\ \mathrm{m}} = -8\,\mathrm{rad/s}$$

$$a_A = -r_1\ddot{\theta} = -r_1\alpha$$

$$\alpha = -\frac{a_A}{r_1} = -\frac{3\,\mathrm{m/s^2}}{0.150\ \mathrm{m}}$$

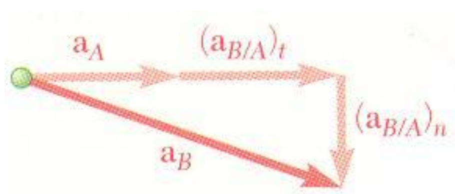
$$\boxed{\vec{\alpha} = \alpha\vec{k} = -\left(20\,\mathrm{rad/s^2}\right)\vec{k}}$$

Numerical



- The acceleration of each point is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center.

  The latter includes normal and tangential acceleration components.



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$

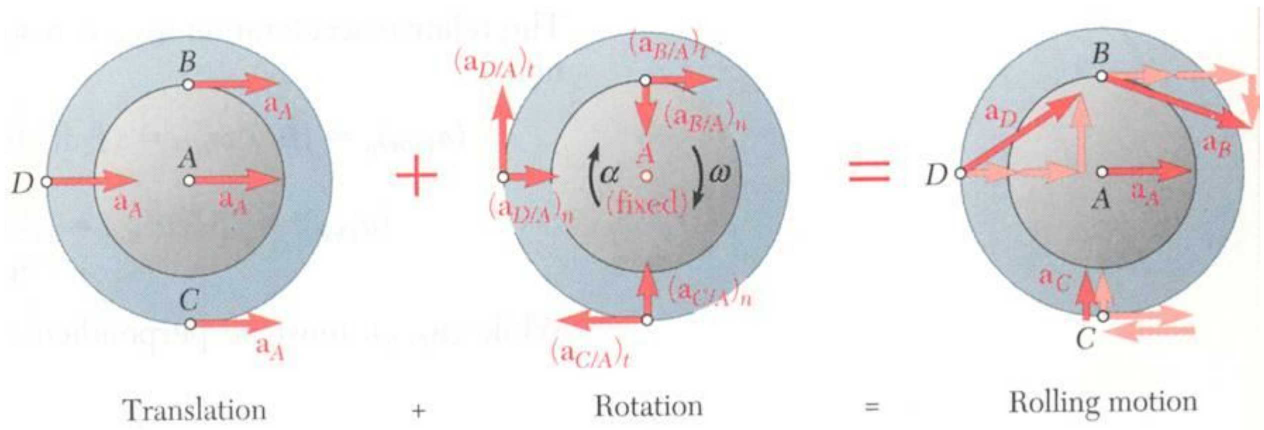
$$= \vec{a}_A + \alpha\vec{k} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

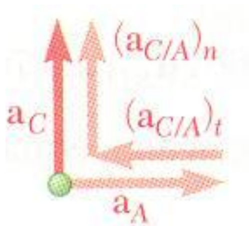
$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k} \times (0.100\,\mathrm{m})\vec{j} - (8\,\mathrm{rad/s})^2(-0.100\,\mathrm{m})\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(2\,\mathrm{m/s^2}\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j}$$

$$\boxed{\vec{a}_B = \left(5\,\mathrm{m}/s^2\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j} \qquad a_B = 8.12\,\mathrm{m/s^2}}$$

Numerical



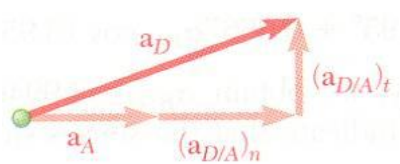


$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} = \vec{a}_A + \alpha\vec{k} \times \vec{r}_{C/A} - \omega^2 \vec{r}_{C/A}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k} \times (-0.150\,\mathrm{m})\vec{j} - (8\,\mathrm{rad/s})^2(-0.150\,\mathrm{m})\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(9.60\,\mathrm{m/s^2}\right)\vec{j}$$

$$\boxed{\vec{a}_c = \left(9.60\,\mathrm{m/s^2}\right)\vec{j}}$$



$$\vec{a}_D = \vec{a}_A + \vec{a}_{D/A} = \vec{a}_A + \alpha\vec{k} \times \vec{r}_{D/A} - \omega^2 \vec{r}_{D/A}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k} \times (-0.150\,\mathrm{m})\vec{i} - (8\,\mathrm{rad/s})^2(-0.150\,\mathrm{m})\vec{i}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(3\,\mathrm{m/s^2}\right)\vec{j} + \left(9.60\,\mathrm{m/s^2}\right)\vec{i}$$

$$\boxed{\vec{a}_D = \left(12.6\,\mathrm{m/s^2}\right)\vec{i} + \left(3\,\mathrm{m/s^2}\right)\vec{j} \qquad a_D = 12.95\,\mathrm{m/s^2}}$$