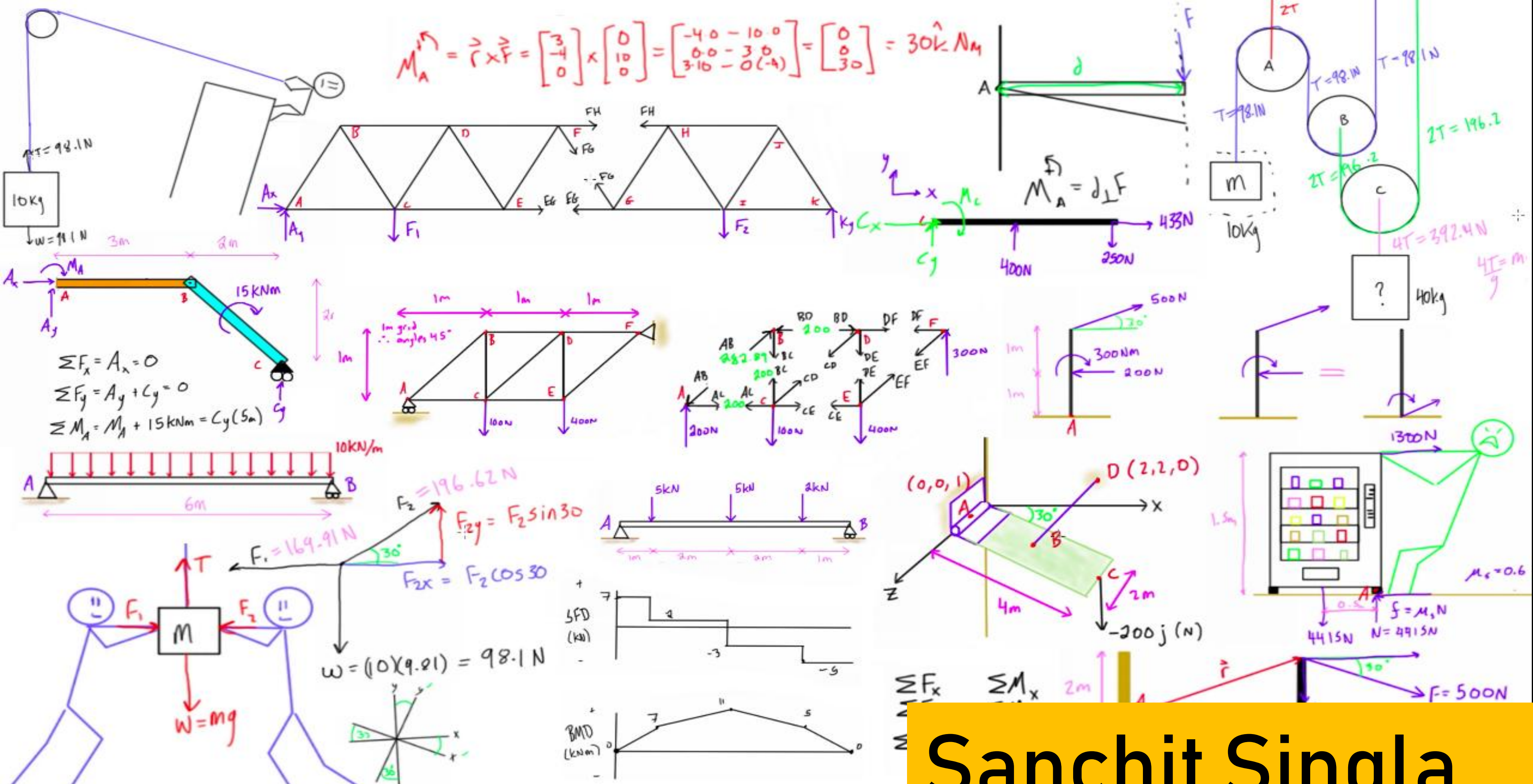

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MEC107

Basic Engineering Mechanics

Learning Outcomes

After this lecture, you will be able to

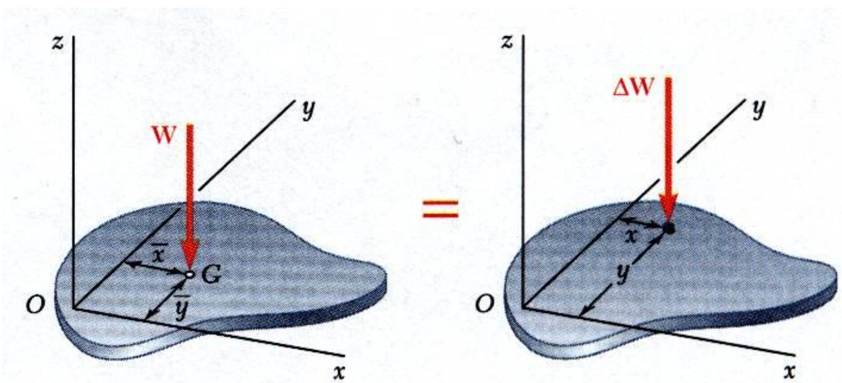
- ✓ learn about Center of Gravity.
- ✓ understand Centroids and First Moment of Areas and Line.
- ✓ know about Centroids of Common Shapes of Areas.
- ✓ understand about Centroids of Common Shapes of Lines.
- ✓ learn the procedure to find the centoids of composite plates and areas.

Introduction

- The earth exerts a gravitational force on each of the particles forming a body. These forces can be replace by a single equivalent force equal to the weight of the body and applied at the *center of gravity* for the body.
- The *centroid of an area* is analogous to the center of gravity of a body. The concept of the *first moment of an area* is used to locate the centroid.
- The Centre of Gravity (C.G.) of a body is the point through which the whole weight of the body seems to act.
- Centroid is the geometric center of the area.
- The Center of Mass of a body is the point through which the whole mass of the body is concentrated.

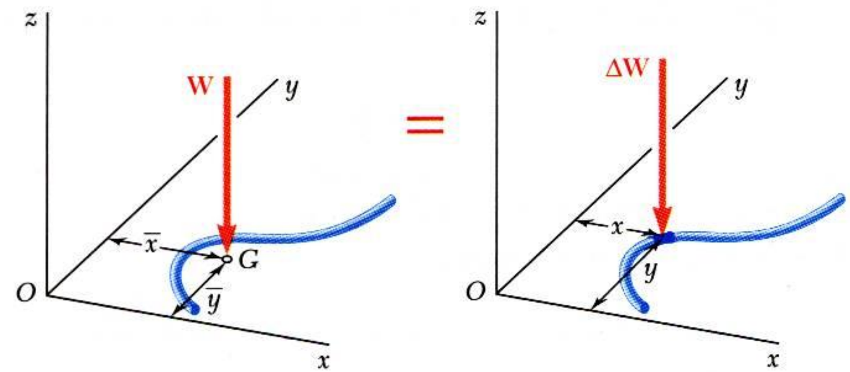
Center of Gravity of A 2D Body

- Center of gravity of a plate



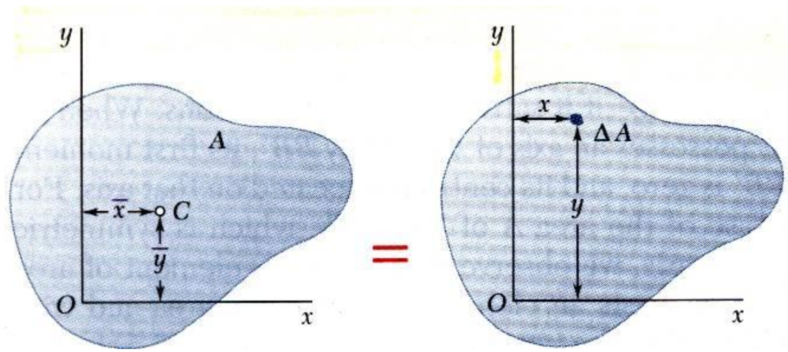
$$\sum M_y \quad \bar{x}W = \sum x\Delta W$$
$$= \int x\,dW$$
$$\sum M_y \quad \bar{y}W = \sum y\Delta W$$
$$= \int y\,dW$$

- Center of gravity of a wire



Centroids and First Moments of Areas and Lines

- Centroid of an area



$$\bar{x}W = \int x\,dW$$

$$\bar{x}(\gamma A t) = \int x(\gamma t)\,dA$$

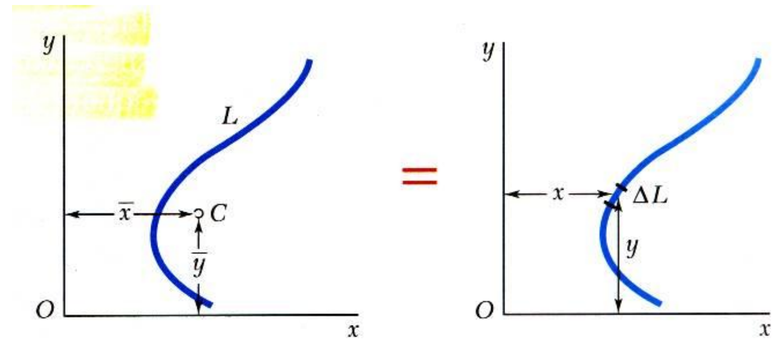
$$\bar{x}A = \int x\,dA = Q_y$$

$$= \text{first moment with respect to } y$$

$$\bar{y}A = \int y\,dA = Q_x$$

$$= \text{first moment with respect to } x$$

- Centroid of a line



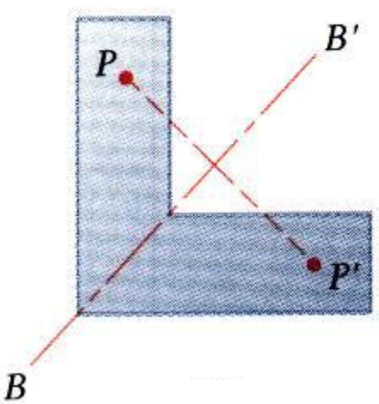
$$\bar{x}W = \int x\,dW$$

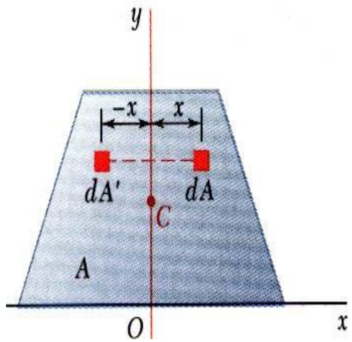
$$\bar{x}(\gamma L a) = \int x(\gamma a)\,dL$$

$$\bar{x}L = \int x\,dL$$

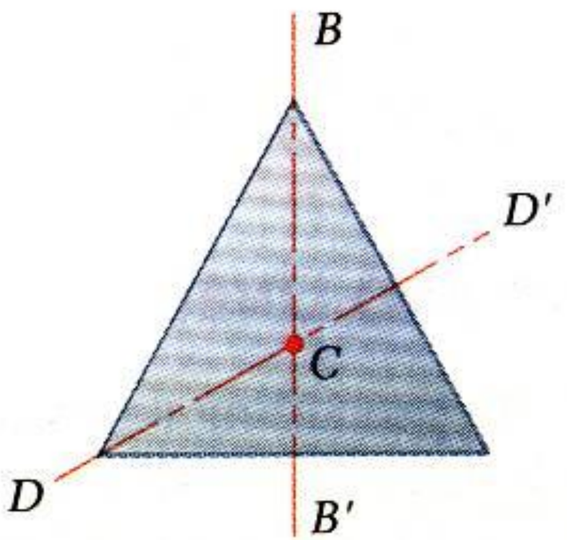
$$\bar{y}L = \int y\,dL$$

First Moments of Areas and Lines





- An area is symmetric with respect to an axis BB' if for every point P there exists a point P' such that PP' is perpendicular to BB' and is divided into two equal parts by BB'.

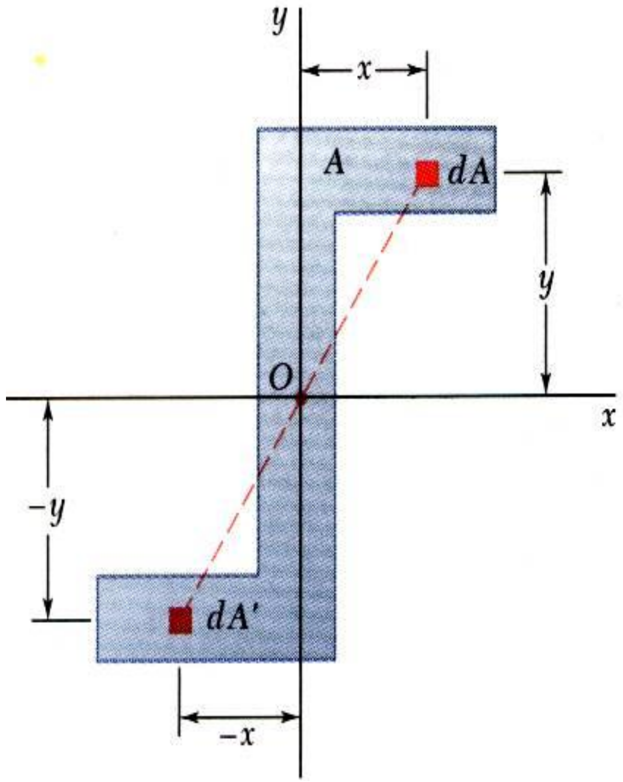


- If an area possesses a line of symmetry, its centroid lies on that axis

- The first moment of an area with respect to a line of symmetry is zero.

- If an area possesses two lines of symmetry, its centroid lies at their intersection.

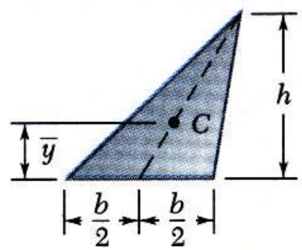
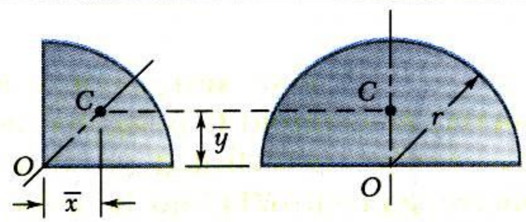
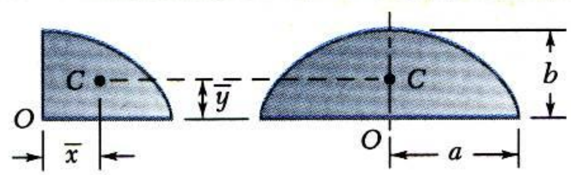
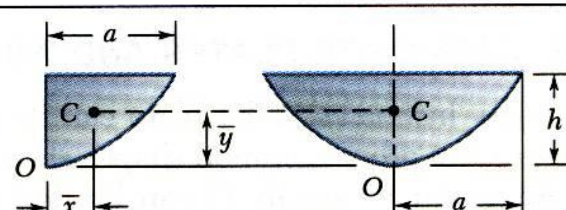
First Moments of Areas and Lines



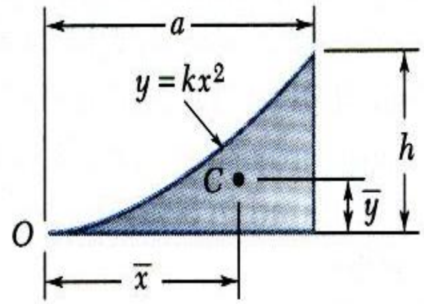
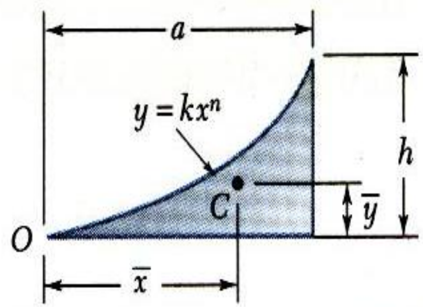
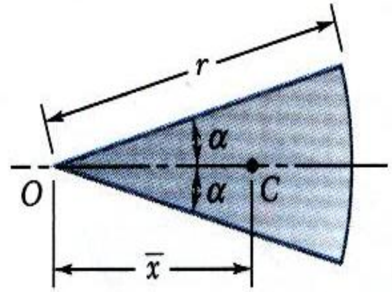
- An area is symmetric with respect to a center O if for every element dA at (x,y) there exists an area dA’ of equal area at (-x,-y).

- The centroid of the area coincides with the center of symmetry.

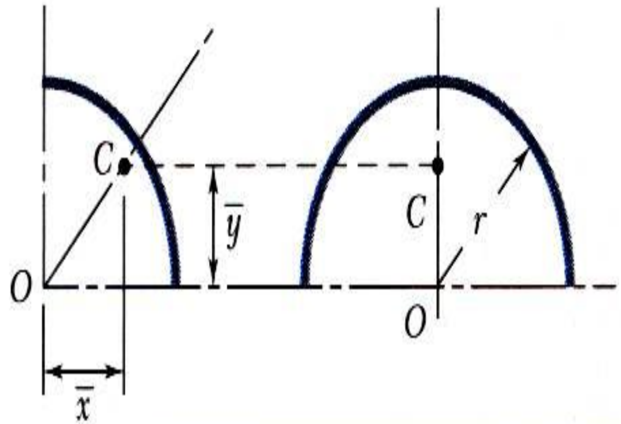
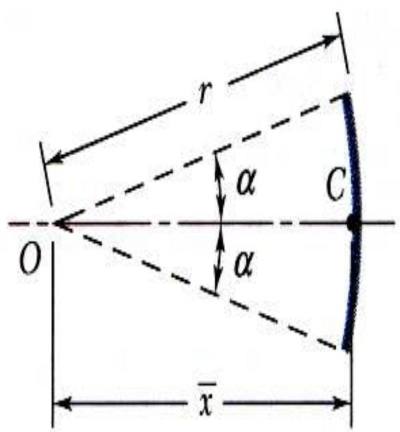
Centroids of Common Shapes of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

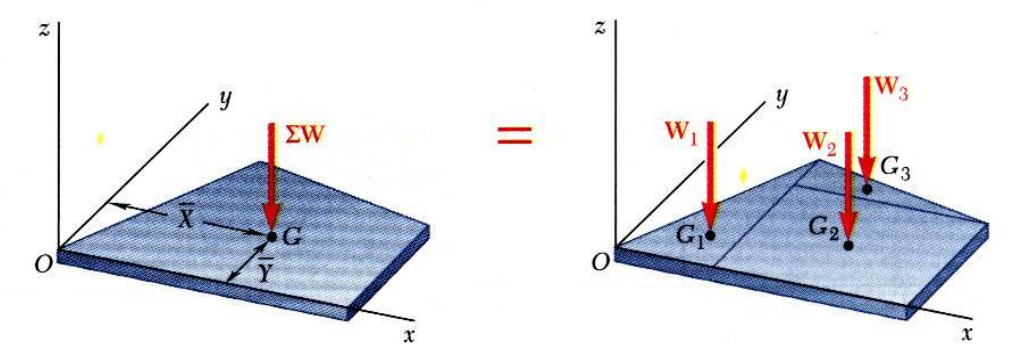
Centroids of Common Shapes of Areas

Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Centroids of Common Shapes of Areas

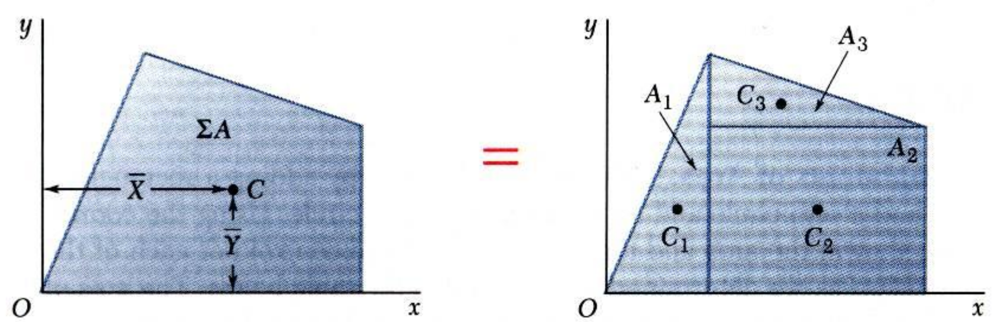
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Composite Plates and Areas



- Composite plates

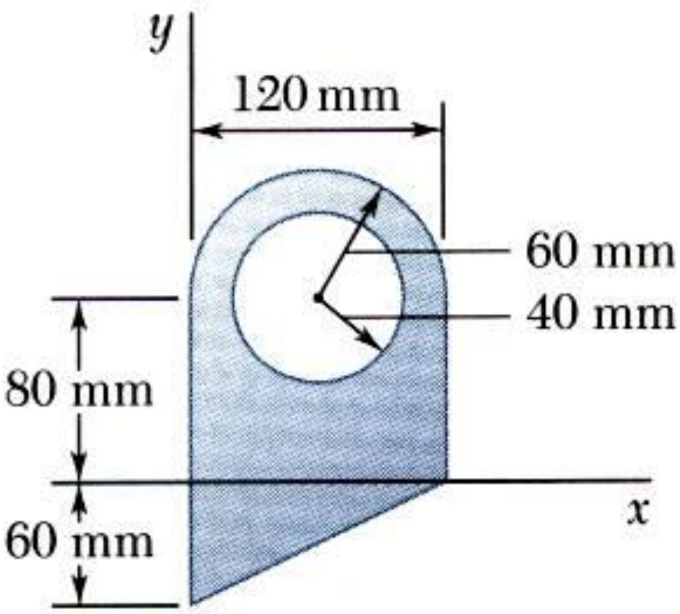
$$\bar{X}\sum W = \sum \bar{x} W$$
$$\bar{Y}\sum W = \sum \bar{y} W$$



- Composite area

$$\bar{X}\sum A = \sum \bar{x} A$$
$$\bar{Y}\sum A = \sum \bar{y} A$$

Numerical: 1

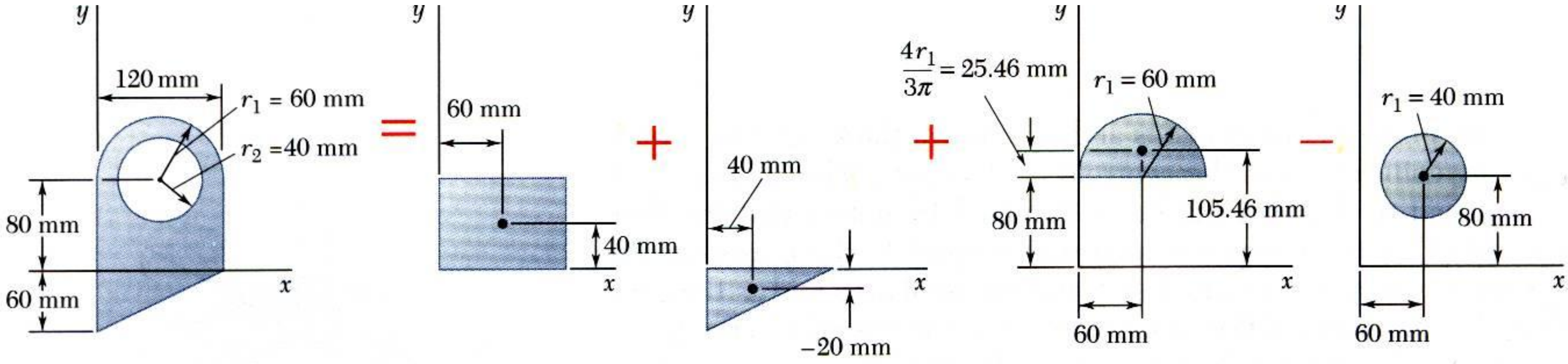


For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

<u>SOLUTION</u>:

- Divide the area into a triangle, rectangle, and semicircle with a circular cutout.
- Calculate the first moments of each area with respect to the axes.
- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.
- Compute the coordinates of the area centroid by dividing the first moments by the total area.

Numerical: 1



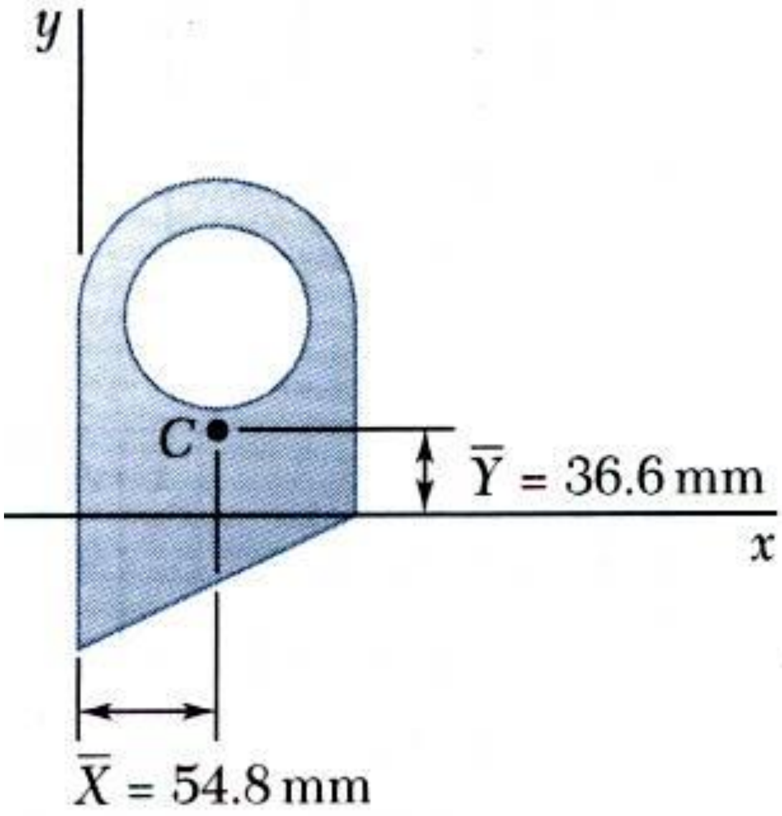
Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.

$$\boxed{\begin{aligned} Q_x &= +506.2\times10^3\,\text{mm}^3 \\ Q_y &= +757.7\times10^3\,\text{mm}^3 \end{aligned}}$$

Numerical: 1

- Compute the coordinates of the area centroid by dividing the first moments by the total area.



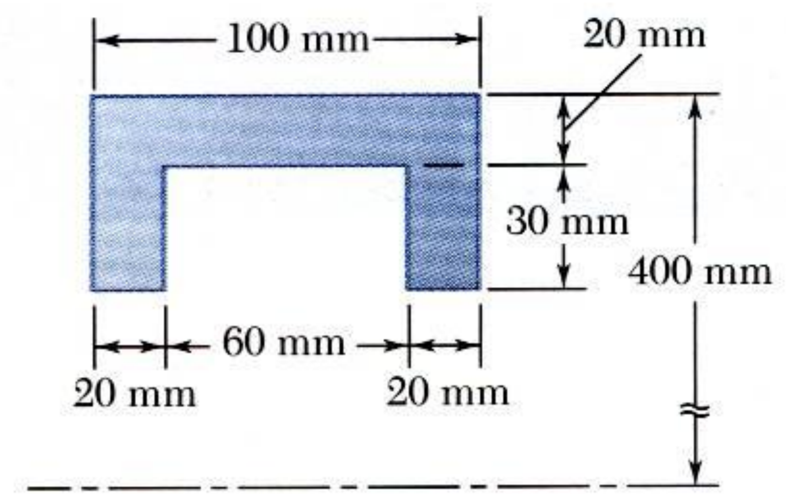
$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7 \times 10^3 \text{ mm}^3}{13.828 \times 10^3 \text{ mm}^2}$$

$$\boxed{\bar{X} = 54.8 \text{ mm}}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2 \times 10^3 \text{ mm}^3}{13.828 \times 10^3 \text{ mm}^2}$$

$$\boxed{\bar{Y} = 36.6 \text{ mm}}$$

Numerical: 2



The outside diameter of a pulley is 0.8 m, and the cross section of its rim is as shown. Knowing that the pulley is made of steel and that the density of steel is
determine the mass and weight of the rim. $\rho = 7.85\times10^{3}\ \mathrm{kg/m^{3}}$

<u>SOLUTION</u>:

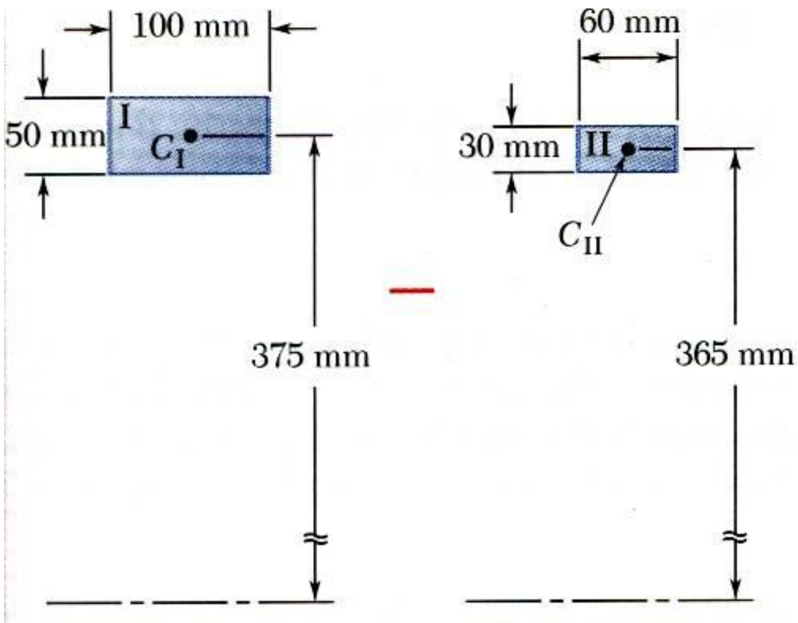
- Apply the theorem of Pappus-Guldinus to evaluate the volumes or revolution for the rectangular rim section and the inner cutout section.
- Multiply by density and acceleration to get the mass and acceleration.

Numerical: 2

SOLUTION:

- Apply the theorem of Pappus-Guldinus to evaluate the volumes or revolution for the rectangular rim section and the inner cutout section.

- Multiply by density and acceleration to get the mass and acceleration.



	Area, mm²	$\bar{y}$, mm	Distance Traveled by C, mm	Volume, mm³
I	+5000	375	$2\pi(375) = 2356$	$(5000)(2356) = 11.78 \times 10^6$
II	−1800	365	$2\pi(365) = 2293$	$(-1800)(2293) = -4.13 \times 10^6$
				Volume of rim $= 7.65 \times 10^6$

Numerical: 2

$$m = \rho V = \left(7.85\times10^{3}\ \mathrm{kg/m^{3}}\right)\left(7.65\times10^{6}\ \mathrm{mm^{3}}\right)\left(10^{-9}\ \mathrm{m^{3}/mm^{3}}\right) \qquad \boxed{m = 60.0\ \mathrm{kg}}$$

$$W = mg = \left(60.0\ \mathrm{kg}\right)\left(9.81\ \mathrm{m/s^{2}}\right) \qquad \boxed{W = 589\ \mathrm{N}}$$