Contents

CHAPTER

21 Motion of Rotation

Contents

21.1. INTRODUCTION

Some bodies like pulley, shafts, flywheels etc., have motion of rotation (*i.e.*, angular motion) which takes place about the geometric axis of the body. The angular velocity of a body is always expressed in terms of revolutions described in one minute, *e.g.*, if at an instant the angular velocity of rotating body in N r.p.m. (*i.e.* revolutions per min) the corresponding angular velocity ω (in rad) may be found out as discussed below :

$$1 \text{ revolution/min} = 2\pi \text{ rad/min}$$

$$\therefore \quad N \text{ revolutions/min} = 2\pi N \text{ rad/min}$$

and angular velocity

$$\omega = 2\pi N \text{ rad/min} = \frac{2\pi N}{60} \text{ rad/sec}$$

21.2. IMPORTANT TERMS

The following terms, which will be frequently used in this chapter, should be clerarly understood at this stage :

1. ***Angular velocity.*** It is the rate of change of angular displacement of a body, and is expressed in r.p.m. (revolutions per minute) or in radian per second. It is, usually, denoted by ω (omega).
2. ***Angular acceleration.*** It is the rate of change of angular velocity and is expressed in radian per second per second (rad/s^2) and is usually, denoted by α. It may be constant or variable.
3. ***Angular displacement.*** It is the total angle, through which a body has rotated, and is usually denoted by θ. Mathematically, if a body is rotating with a uniform angular velocity (ω) then in t seconds, the angular displacement

$$\theta = \omega t$$

21.3. MOTION OF ROTATION UNDER CONSTANT ANGULAR ACCELERATION

Consider a particle, rotating about its axis.

Let ω_0 = Initial angular velocity,

ω = Final angular velocity,

t = Time (in seconds) taken by the particle to change its velocity from ω_0 to ω.

α = Constant angular acceleration in rad/s^2, and

θ = Total angular displacement in radians.

Since in t seconds, the angular velocity of the particle has increased steadily from ω_0 to ω at the rate of α rad/s^2, therefore

$$\omega = \omega_0 + \alpha t \qquad ...(i)$$

and average angular velocity $= \dfrac{\omega_0 + \omega}{2}$

We know that the total angular displacement,

$$\theta = \text{Average velocity} \times \text{Time} = \left(\frac{\omega_0 + \omega}{2}\right) \times t \qquad ...(ii)$$

Substituting the value of ω from equation (i),

$$\theta = \frac{\omega_0 + (\omega_0 + \alpha t)}{2} \times t = \frac{2\omega_0 + \alpha t}{2} \times t = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad ...(iii)$$

and from equation (i), we find that

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substituting this value of t in equation (ii),

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right) \times \left(\frac{\omega - \omega_0}{\alpha}\right) = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\therefore \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad ...(iv)$$

21.4. RELATION BETWEEN LINEAR MOTION AND ANGULAR MOTION

Following are the relations between the linear motion and the angular motion of a body :

S. No.	*Particulars*	*Linear motion*	*Angular motion*
1.	Initial velocity	u	ω_0
2.	Final velocity	v	ω
3.	Constant acceleration	a	α
4.	Total distance traversed	s	θ
5.	Formula for final velocity	$v = u + at$	$\omega = \omega_0 + \alpha t$
6.	Formula for distance traversed	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
7.	Formula for final velocity	$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
8.	Differential formula for velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
9.	Differential formula for acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Example. 21.1. *A flywheel starts from rest and revolves with an acceleration of 0.5 rad/sec^2. What will be its angular velocity and angular displacement after 10 seconds.*

Solution. Given : Initial angular velocity (ω_0) = 0 (becasue it starts from rest) ; Angular acceleration (α) = 0.5 rad/sec^2 and time (t) = 10 sec.

Angular velocity of the flywheel

We know that angular velocity of the flywheel,

$$\omega = \omega_0 + \alpha t = 0 + (0.5 \times 10) = 5 \text{ rad/sec} \quad \textbf{Ans.}$$

Angular displacement of the flywheel

We also know that angular displacement of the flywheel,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (0 \times 10) + \left[\frac{1}{2} \times 0.5 \times (10)^2\right] = 25 \text{ rad} \quad \textbf{Ans.}$$

Example 21.2. *A wheel increases its speed from 45 r.p.m. to 90 r.p.m. in 30 seconds. Find (a) angular acceleration of the wheel, and (b) no. of revolutions made by the wheel in these 30 seconds.*

Solution. Given : Initial angular velocity (ω_0) = 45 r.p.m. = 1.5 π rad/sec ; Final angular velocity (ω) = 90 r.p.m. = 3 π rad/sec and time (t) = 30 sec

(*a*) *Angular acceleration of the wheel*

Let α = Angular acceleration of the wheel.

We know that final angular velocity of the wheel (ω),

$$3\pi = \omega_0 + \alpha t = 1.5\,\pi + (\alpha \times 30) = 1.5\pi + 30\alpha$$

or

$$\alpha = \frac{3\pi - 1.5\pi}{30} = \frac{1.5\pi}{30} = 0.05\,\pi \text{ rad/sec}^2 \quad \textbf{Ans.}$$

Contents

(*b*) *No. of revolutions made by the wheel in 30 seconds*

We also know that total angle turned by the wheel in 30 seconds,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = [1.5\pi \times 30] + \left[\frac{1}{2} \times 0.05\ \pi(30)^2\right] = 67.5\ \pi\ \text{rad}$$

$$= \frac{67.5\pi}{2\pi} = 33.75\ \text{rev} \quad \textbf{Ans.} \qquad \text{...(1 rev} = 2\pi\text{ rad)}$$

Example 21.3. *A flywheel is making 180 r.p.m. and after 20 sec it is running at 120 r.p.m. How many revolutions will it make and what time will elapse before it stops, if the retardation is uniform ?*

Solution. Given : Initial angular velocity (ω_0) = 180 r.p.m. = 6π rad/sec ; Final angular velocity (ω) = 120 r.p.m. = 4π rad/sec and time (t) = 20 sec.

Revolutions of the wheel, before it stops

Let α = Uniform angular acceleration, and

θ = Angular displacement of the flywheel before coming to rest.

First of all, consider the angular motion of the flywheel from 180 r.p.m. to 120 r.p.m. in 20 seconds. We know that final angular velocity (ω),

$$4\pi = \omega_0 + \alpha t = 6\pi + \alpha \times 20$$

or

$$\alpha = \frac{4\pi - 6\pi}{20} = -0.1\ \pi\ \text{rad/sec}^2$$

...(Minus sign indicates retardation)

Now consider angular motion of the flywheel from 180 r.p.m. (or 6π rad/s) to zero r.p.m. (*i.e.*, coming to stop) or $\omega = 0$ with a constant acceleration of $-0.1\ \pi$ rad/s^2. We know that (ω^2).

$$0 = \omega^2_0 + 2\alpha\theta = (6\pi)^2 + 2 \times (-0.1\pi)\ \theta = 36\pi^2 - 0.2\pi\theta$$

$\therefore$

$$\theta = \frac{36\pi^2}{0.2\pi} = 180\ \pi = \frac{180\ \pi}{2\pi} = 90\ \text{rev} \quad \textbf{Ans.}$$

...(1 rev = 2π rad)

Time in which the wheel will come to rest

Let t = Time in which the wheel will come to rest.

We know that final velocity of flywheel (ω),

$$0 = \omega_0 + \alpha t = 6\pi - 0.1\ \pi\ t$$

or

$$t = \frac{6\pi}{0.1\pi} = 60\,\text{s} = 1\ \text{min} \quad \textbf{Ans.}$$

Example 21.4. *A pulley, starting from rest, is given an acceleration of 0.5 rad/s^2. What will be its speed in r.p.m. at the end of 2 minutes ? If it is uniformly retarded at the rate of 0.3 rad /s^2, in how many minutes the pulley will come to rest ?*

Solution. First of all, consider angular motion of pully from rest. In this case, initial angular velocity (ω_0) = 0 ; Acceleration (α_1) = 0.5 rad/sec^2 and time taken (t_1) = 2 minutes = 120 sec.

Angular speed of pully in r.p.m. at the end of 2 min.

We know that final angular speed of the pulley

$$\omega = \omega_0 + \alpha t_1 = 0 + (0.5 \times 120) = 60\ \text{rad/sec}$$

$$= \frac{60}{2\pi} = 9.55\ \text{r.p.s.} = 9.55 \times 60 = 573\ \text{r.p.m.} \quad \textbf{Ans.}$$

Time in which the pulley will come to rest

Let t_2 = Time in which the pulley will come to rest.

Now consider angular motion of the pulley in coming to rest. In this case, initial angular velocity (ω_0) = 60 rad/sec ; Final angular velocity (ω) = 0 and retardation (α_2) = – 0.3 rad/sec^2 (Minus sign due to retardation).

We know that final velocity of the pulley

$$0 = \omega_0 + \alpha t_2 = 60 - 0.3\, t_2$$

$$\therefore \qquad t_2 = \frac{60}{0.3} = 200 \text{ sec} \quad \textbf{Ans.}$$

Example 21.5. *A wheel rotates for 5 seconds with a constant angular acceleration and describes during this time 100 radians. It then rotates with a constant angular velocity and during the next five seconds describes 80 radians.*

Find the initial angular velocity and the angular acceleration.

Solution. Given : Time (t) = 5 sec and angular displacement (θ) = 100 rad

Initial angular velocity

Let ω_0 = Initial angular velocity in rad/s,

α = Angular acceleration in rad/s^2, and

ω = Angular velocity after 5 s in rad/s.

First of all, consider the angular motion of the wheel with constant acceleration for 5 seconds. We know that angular displacement (θ),

$$100 = \omega_0\, t + \frac{1}{2}\alpha t^2 = \omega_0 \times 5 + \frac{1}{2} \times \alpha\,(5)^2 = 5\,\omega_0 + 12.5\,\alpha$$

$$\therefore \qquad 40 = 2\omega_0 + 5\alpha \qquad \ldots(i)$$

and final velocity, $\omega = \omega_0 + \alpha t = \omega_0 + \alpha \times 5 = \omega_0 + 5\alpha$

Now consider the angular motion of the wheel with a constant angular velocityof ($\omega_0 + 5\alpha$) for 5 seconds and describe 80 radians. We know that the angular displacement,

$$80 = 5\,(\,\omega_0 + 5\alpha)$$

$$\text{or} \qquad 16 = \omega_0 + 5\alpha \qquad \ldots(ii)$$

Subtracting equation (*ii*) from (*i*),

$$24 = \omega_0 \quad \text{or} \quad \omega_0 = 24 \text{ rad/s} \quad \textbf{Ans.}$$

Angular acceleration

Substituting this value of ω_0 in equation (*ii*),

$$16 = 24 + 5\alpha \quad \text{or} \quad \alpha = \frac{16 - 24}{5} = -1.6 \text{ rad/s}^2 \quad \textbf{Ans.}$$

...(Minus sign means retardation)

Example 21.6. *A shaft is uniformly accelerated from 10 rev/s to 18 rev/s in 4 seconds. The shaft cotinues to accelerate at this rate for the next 8 seconds. Thereafter the shaft rotates with a uniform angular speed. Find the total time to complete 400 revolutions.*

Solution. Given : Initial angular velocity (ω_0) = 10 rev/s = 20 π rad/s ; Final angular velocity (ω) = 18 rev/s = 36 π rad/s ; Time taken during constant acceleration (t_1) = 4 sec ; Time taken during uniform angular velocity (t_2) = 8 sec and total angular displacement (θ) = 400 rev = 800 π rad

Let α = Angular acceleration of the shaft.

First of all, consider the motion of the shaft in the first 4 seconds. In this case, initial angular velocity (ω_0) = 20 π rad/s ; Final angular velocity (ω) = 36 rad/s.

We know that final velocity of the shaft (ω),

$$36\,\pi = \omega_0 + \alpha t_1 = 20\,\pi + (\alpha \times 4) = 20\,\pi + 4\alpha$$

$\therefore$ $$\alpha = \frac{36\pi - 20\pi}{4} = 4\pi \text{ rad/s}^2 \quad ...(i)$$

and angular displacement $$\theta_1 = \omega_0 t + \frac{1}{2}\alpha t^2 = 20\pi \times 4 + \frac{1}{2} \times 4\pi\,(4)^2 = 112\pi \text{ rad} \quad ...(ii)$$

Now consider the motion of the shaft for the next 8 seconds. In this case, initial velocity (ω_0) = 36π rad/s and angular acceleration (α) = 4π rad/s^2 as obtained in equation (*i*).

We know that final angular velocity of the shaft,

$$\omega = \omega_0 + \alpha t_2 = 36\pi + (4\pi \times 8) = 68\pi \text{ rad/s} \quad ...(iii)$$

and angular displacement, $$\theta_2 = \omega_0 t + \frac{1}{2}\alpha t^2 = 36\pi \times 8 + \frac{1}{2} \times 4\pi\,(8)^2 = 416\,\pi \text{ rad} \quad ...(iv)$$

Now consider motion of the shaft with a constant angular velocity of 68π rad/s as obtainted in equation (*iii*). We know that angular displacement of the shaft at this speed

$$= 800\,\pi - 112\,\pi - 416\,\pi = 272\,\pi \text{ rad}$$

$\therefore$ Time taken by the shaft to complete $272\,\pi$ rad

$$t_3 = \frac{272\pi}{68\pi} = 4 \text{ sec}$$

and total time to complete 400 revolutions or $800\,\pi$ rad

$$= t_1 + t_2 + t_3 = 4 + 8 + 4 = 16 \text{ sec} \qquad \textbf{Ans.}$$

Example 21.7. *A swing bridge turns through 90° in 120 seconds. The bridge is uniformly accelerated from rest for the first 40 seconds. Subsequently, it turns with a uniform angular velocity for the next 60 seconds. Now the motion of the bridge is uniformly retarded for the last 20 seconds. Find (i) angular acceleration ; (ii) maximum angular velocity ; and (iii) angular retardation of the bridge.*

Solution. Given : Angular displacement (θ) $= 90° = \dfrac{\pi}{2} = 0.5\pi$ rad ; Total time (T) = 120 s; Time for acceleration (t_1) = 40 sec; Time for uniform velocity (t_2) = 60 sec and time for retardation (t_3) = 20 sec

(*i*) *Angular acceleration of the bridge*

Let α_1 = Angular acceleration of the bridge, and

α_2 = Angular retardation of the bridge.

First of all, consider the motion of the bridge from rest in the first 40 sec. In this case, initial angular velocity (ω_1) = 0 and time (t_1) = 40 s. We know that final angular velocity of bridge,

$$\omega = \omega_o + \alpha t_1 = 0 + \alpha_1 \times 40 = 40\,\alpha_1 \quad ...(i)$$

and angular displacement, $$\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha_1 t_2^2 = 0 + \frac{1}{2}\alpha_1\,(40)^2 = 800\alpha_1 \quad ...(ii)$$

Now consider the motion of the bridge in the next 60 sec. In this case, constant angular velocity of ($40\ \alpha_1$) as obtained in equation (i) and time (t_2) = 60 sec. Therefore angular displacement during 60 sec,

$$\theta_2 = 40\ \alpha_1 \times t_2 = 40\ \alpha_1 \times 60 = 2400\ \alpha_1 \text{ rad} \qquad ...(iii)$$

Now consider the motion of the bridge in the last 20 sec. In this case, initial angular velocity (ω_0) = $40\ \alpha_1$ as obtained in equation (i) ; Final angular velocity (ω) = 0 and time (t_3) = 20 sec. We know that final angular velocity of the bridge (ω)

$$0 = \omega_0 - \alpha t_3 = 40\ \alpha_1 - \alpha_2 \times 20 \quad \text{or} \quad \alpha_2 = 2\alpha_1$$

...(Minus sign due to retardation)

and angular displacement,
$$\theta_3 = \omega_0 t_3 - \frac{1}{2}\alpha_2 t_3^2 = (40\alpha_1 \times 20) - \frac{1}{2} \times 2\alpha_1\ (20)^2$$
$$= 400\ \alpha_1 \qquad ...(iv)$$

...(Minus sign due to retardation)

We also know that total angular displacement of the bridge (θ)

$$0.5\pi = \theta_1 + \theta_2 + \theta_3 = 800\ \alpha_1 + 2400\ \alpha_1 + 400\ \alpha_1 = 3600\ \alpha_1$$

$$\therefore \qquad \alpha_1 = \frac{0.5\,\pi}{3600} = 0.436 \times 10^{-3} \text{ rad/sec}^2 \quad \textbf{Ans.}$$

(ii) *Maximum angular velocity of the bridge*

We know that maximum angular velocity of the bridge,

$$\omega = 40\ \alpha_1 = 40 \times 0.436 \times 10^{-3} = 17.44 \times 10^{-3} \text{ rad/s} \quad \textbf{Ans.}$$

(iii) *Angular retardation of the bridge*

We also know that angular retardation of the bridge,

$$\alpha_2 = 2\alpha_1 = 2 \times (0.436 \times 10^{-3}) = 0.872 \times 10^{-3} \text{ rad/sec}^2 \quad \textbf{Ans.}$$

Example 21.8. *A flywheel rotates with a constant retardation due to braking. From t = 0 to t = 10 seconds, it made 300 revolutions. At time t = 7.5 sec, its angular velocity was 40 π rad/sec. Determine (i) value of constant ratardation ; (ii) total time taken to come to rest and (iii) total revolutions made till it comes to rest.*

Solution. Given : Time interval (t) = 10 – 0 = 10 sec ; Angular displacement (θ) = 300 rev = $2\pi \times 300 = 600\ \pi$ rad ; when time (t) = 7.5 sec and angular velocity (ω) = $40\ \pi$ rad/sec

(i) *Value of constant retardation*

Let α = Constant retardation, and

ω_0 = Initial angular velocity.

First of all, consider motion of the flywheel from t = 0 to t = 10 seconds. We know that total angular displacement (θ)

$$600\,\pi = \omega_0 t - \frac{1}{2}\alpha t^2 = (\omega_0 \times 10) - \left(\frac{1}{2}\alpha\ (10)^2\right)$$
$$= 10\ \omega_0 - 50\alpha \qquad ...(i)$$

...(Minus sign due to retardation)

Now consider motion of the flywheel from t = 0 to t = 7.5 seconds. We know that final angular velocity (ω)

$$40\ \pi = \omega_0 - \alpha t = \omega_0 - \alpha \times 7.5 \qquad ...(ii)$$

...(Minus sign due to retardation)

$$\alpha = \frac{\omega_0 - 40\ \pi}{7.5} \text{ rad/sec}^2 \qquad ...(iii)$$

Substituting this value of α in equation (*i*),

$$600\,\pi = 10\,\omega_0 - 50\left(\frac{\omega_0 - 40\,\pi}{7.5}\right)$$

$$= 10\,\omega_0 - 6.67\,\omega_0 + 266.7\,\pi = 3.33\,\omega_0 + 266.7\,\pi$$

$$\therefore \qquad \omega_0 = \frac{600\,\pi - 266.7\,\pi}{3.33} = 100\,\pi \text{ rad/sec}$$

and now substituting the value of ω_0 in equation (*iii*),

$$\alpha = \frac{100\,\pi - 40\,\pi}{7.5} = 8\,\pi = 25.1 \text{ rad/sec}^2 \quad \textbf{Ans.}$$

(*ii*) *Total time taken by the flywheel to come to rest*

Now consider motion of the flywheel till it comes to rest. In this case, Initial angular velocity $(\omega_0) = 100\,\pi$ rad/sec ; Final angular velocity $(\omega) = 0$ (becasue it comes to rest) and angular acceleration $(\alpha) = 8\,\pi$ rad/sec^2

Let t = Total time taken by the flywheel to come to rest.

We know that final angular velocity of the flywheel (ω),

$$0 = \omega_0 - \alpha\, t = 100\,\pi - 8\,\pi t \qquad \text{...(Minus sign due to retardation)}$$

$$\therefore \qquad t = \frac{100\,\pi}{8\,\pi} = 12.5 \text{ sec} \quad \textbf{Ans.}$$

(*iii*) *Total revolutions made till it comes to rest*

We also know that the total revolutions made by the flywheel till it comes to rest (or in other words revolutions made in 12.5 seconds),

$$\theta = \omega_0 t - \frac{1}{2}\alpha t^2 = 100\,\pi \times 12.5 - \frac{1}{2} \times 8\,\pi\,(12.5)^2$$

...(Minus sign due to retardation)

$$= 625\,\pi \text{ rad} = \frac{625\,\pi}{2\,\pi} = 312.5 \text{ rev} \quad \textbf{Ans.}$$

EXERCISE 21.1

1. A flywheel increases its speed from 30 r.p.m. to 60 r.p.m. in 10 seconds. Calculate (*i*) the angular acceleration ; and (*ii*) no. of revolutions made by the wheel in these 10 seconds.
(**Ans.** $\pi/10$ rad/s^2 ; 7.5 rev)

2. A wheel, starting form rest, is accelerated at the rate of 5 rad/s^2 for a period of 10 seconds. It is then made to stop in the next 5 seconds by applying brakes. Find (*a*) maximum velocity attained by the wheel, and (*b*) total angle turned by the wheel.
(**Ans.** 50 rad/sec ; 375 rad)

3. A wheel is running at a constant speed of 360 r.p.m. At what constant rate, in rad/s, its motion must be retarded to bring the wheel to rest in (*i*) 2 minutes, and (*ii*) 18 revolution.
(**Ans.** 0.314 rad/s^2 ; 6.28 rad/s^2)

4. A wheel rotating about a fixed axis at 24 r.p.m. is uniformly accelerated for 70 seconds, during which time it makes 50 revolution. Find (*i*) angular velocity at the end of this interval, and (*ii*) time required for the speed to reach 150 r.p.m.
(**Ans.** – 61.4 r.p.m. ; 3 min 55.6 s)

5. A wheel rotates for 5 seconds with a constant angular acceleration and describes 80 radians. It then rotates with a constant angular velocity in the next 5 seconds and describes 100 radians. Find the initial angular velocity and angular acceleration of the wheel.
(**Ans.** 12 rad/s ; 1.6 rad/s^2)

21.5. LINEAR (OR TANGENTIAL) VELOCITY OF A ROTATING BODY

Consider a body rotating about its axis as shown in Fig. 21.1.

Let ω = Angular velocity of the body in rad/s,

r = Radius of the circular path in metres, and

v = Linear velocity of the particle on the periphery in m/s.

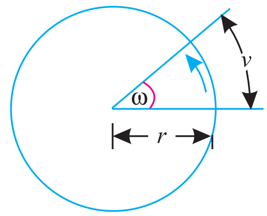

Fig. 21.1.

After one second, the particle will move v metres along the circular path and the angular displacement will be ω rad.

We know that length of arc = Radius of arc × Angle subtended in rad.

$\therefore \quad v = r\,\omega$

Example 21.9. *A wheel of 1.2 m diameter starts from rest and is accelerated at the rate of 0.8 rad/s² . Find the linear velocity of a point on its periphery after 5 seconds.*

Solution. Given : Diameter of wheel = 1.2 m or radius (r) = 0.6 m ; Initial angular velocity (ω_0) = 0 (becasue, it starts from rest) ; Angular acceleration (α) = 0.8 rad/s^2 and time (t) = 5 s

We know that angular velocity of the wheel after 5 seconds,

$$\omega = \omega_0 + \alpha t = 0 + (0.8 \times 5) = 4 \text{ rad/s}$$

$\therefore$ Linear velocity of the point on the periphery of the wheel,

$$v = r\omega = 0.6 \times 4 = 2.4 \text{ m/s} \quad \textbf{Ans.}$$

Example 21.10. *A pulley 2 m in diameter is keyed to a shaft which makes 240 r.p.m. Find the linear velocity of a particle on the periphery of the pulley.*

Solution. Given : Diameter of pulley = 2 m or radius (r) = 1 m and angular frequency (N) = 240 r.p.m.

We know that angular velocity of the pulley,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.1 \text{ rad/s} \quad \textbf{Ans.}$$

$\therefore$ Linear velocity of the particle on the periphery of the pulley,

$$v = r\omega = 1 \times 25.1 = 25.1 \text{ m/s} \quad \textbf{Ans.}$$

21.6. LINEAR (OR TANGENTIAL) ACCELERATION OF A ROTATING BODY

Consider a body rotating about its axis with a constant angular (as well as linear) acceleration. We know that linear acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) \qquad ...(i)$$

We also know that in motion of rotation, the linear velocity,

$$v = r\omega$$

Now substituting the value of v in equation (i),

$$a = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

Where α = Angular acceleration in rad/sec^2 and is equal to $d\omega/dt$.

Note. The above relation, in terms of angular acceleration may also be written as :

$$\alpha = \frac{a}{r} \qquad \textbf{Ans.}$$

Example 21.11. *A car is moving at 72 k.m.p.h., If the wheels are 75 cm diameter, find the angular velocity of the tyre about its axis. If the car comes to rest in a distance of 20 metres, under a uniform retardation, find angular retardation of the wheels.*

Solution. Given : Linear velocity (v) = 72 k.m.p.h. = 20 m/s; Diameter of wheel (d) = 75 cm or radius (r) = 37.5 m = 0.375 m and distance travelled by the car (s) = 20 m.

Angular retardation of the wheel

We know that the angular velocity of the wheel,

$$\omega = \frac{v}{r} = \frac{20}{0.375} = 53.3 \text{ rad/sec}$$

Let a = Linear retardation of the wheel.

We know that $v^2 = u^2 + 2as$

$\therefore$ $0 = (20)^2 + 2 \times a \times 20 = 400 + 40a$

or $a = -\dfrac{400}{40} = -10 \text{ m/sec}^2$ ...(Minus sign indicates retardation)

We also know that the angular retardation of the wheel,

$$\alpha = \frac{a}{r} = \frac{-10}{0.375} = -26.7 \text{ rad/sec}^2 \quad \textbf{Ans.}$$

...(Minus sign indicates retardation)

EXERCISE 21.2

1. A horizontal bar 1.5 m long and of small cross-section rotates about vertical axis through one end. It accelerates uniformly from 30 r.p.m. to 45 r.p.m. for 10 seconds. What is the linear velocity at the beginning and end of this interval ? What is the tangential component of the acceleration of the mid-point of the bar after 10 seconds.
(**Ans.** 4.71 m/s ; 7.07 m/s ; 0.118 m/s^2)
2. In a children park, a train is moving in a circular path. If the linear and angular speeds of the train are 10 m/s and 0.25 rad/s respectively, find the radius of the circular path.
(**Ans.** 40 m)
3. A motor cycle starts form rest and moves with a constant acceleration of 2.25 m/s^2. What is its angular acceleration, if the diameter of the motor cycle wheels is 750 mm.
(**Ans.** 6 rad/s^2)

21.7. MOTION OF ROTATION OF A BODY UNDER VARIABLE ANGULAR ACCELERATION

In the previous articles, we have discussed the cases of angular motion under constant acceleration. But sometimes the motion takes place under variable acceleration also. All the relations discussed in chapter 17 about the motion in a straight line under variable acceleration are applicable to the motion of rotation also.

Example 21.12. *The equation for angular displacement of a body moving on a circular path is given by :*

$$\theta = 2t^3 + 0.5$$

where θ is in rad and t in sec. Find angular velocity, displacement and acceleration after 2 sec.

Solution. Given : Equation for angular displacement $\theta = 2t^3 + 0.5$ ...(*i*)

Angular displacement after 2 seconds

Substituting $t = 2$ in equation (*i*),

$$\theta = 2\,(2)^3 + 0.5 = 16.5 \text{ rad} \quad \textbf{Ans.}$$

Angular velocity after 2 seconds

Differentiating both sides equation (*i*) with respect to *t*,

$$\frac{d\theta}{dt} = 6\,t^2 \qquad ...(ii)$$

or velocity, $\omega = 6\,t^2$ ...(*iii*)

Substituting $t = 2$ in equation (*iii*),

$$\omega = 6\,(2)^2 = 24 \text{ rad/sec} \quad \textbf{Ans.}$$

Angular acceleration after 2 seconds

Differentiating both sides of equation (*iii*) with respect to *t*,

$$\frac{d\omega}{dt} = 12\,t \text{ or Acceleration } \alpha = 12t$$

Now substituting $t = 2$ in above equation,

$$\alpha = 12 \times 2 = 24 \text{ rad/sec}^2 \quad \textbf{Ans.}$$

Example 21.13. *The equation for angular displacement of a particle, moving in a circular path (radius 200 m) is given by :*

$$\theta = 18t + 3t^2 - 2t^3$$

where θ is the angular displacement at the end of t sec. Find (i) angular velocity and acceleration at start, (ii) time when the particle reaches its maximum angular velocity; and (iii) maximum angular velocity of the particle.

Solution. Given : Equation for angular displacement $\theta = 18t + 3t^2 - 2t^3$ ...(*i*)

(*i*) *Angular velocity and acceleration at start*

Differentiating both sides of equation (*i*) with respect to *t*,

$$\frac{d\theta}{dt} = 18 + 6t - 6t^2$$

i.e. angular velocity, $\omega = 18 + 6t - 6t^2$ ...(*ii*)

Substituting $t = 0$ in equation (*ii*),

$$\omega = 18 + 0 - 0 = 18 \text{ rad/s} \quad \textbf{Ans.}$$

Differentiating both sides of equation (*ii*) with respect to *t*,

$$\frac{d\omega}{dt} = 6 - 12t$$

i.e. angular acceleration, $\alpha = 6 - 12t$ ...(*iii*)

Now substituting $t = 0$ in equation (*iii*),

$$\alpha = 6 \text{ rad/s}^2 \quad \textbf{Ans.}$$

(*ii*) *Time when the particle reaches maximum angular velocity*

For maximum angular velocity, differentiating the equation for angular velocity (*ii*) with respect to *t i.e.* equation (*iii*) and equating it to zero.

$$6 - 12t = 0 \qquad \text{or} \qquad t = \frac{6}{12} = 0.5 \text{ sec} \quad \textbf{Ans.}$$

Contents

(iii) Maximum angular velocity of the particle

The maximum angular velocity of the particle may now be found out by substituting $t = 0.5$ in equation (*ii*),

$$\omega_{max} = 18 + (6 \times 0.5) - 6\,(0.5)^2 = 19.5 \text{ rad/s} \quad \textbf{Ans.}$$

EXERCISE 21.3

1. The angular displacement of a body is given by equation $(\theta) = a + bt + ct^2$. What is the angular acceleration of the body ? (**Ans.** $2c$)
2. The relation between the angle of rotation (θ) in radians and time (t) in seconds of a rotating body is given by the equation.

$$\theta = 2t^3 + 3t^2 + 10.$$

Find displacement, angular velocity and angular acceleration after 4 seconds.

(**Ans.** 186 rad; 120 rad/sec ; 54 rad/sec^2)

QUESTIONS

1. Define motion of rotation and give three examples of it.
2. What do you understand by the term 'angular velocity' and 'angular acceleration'? Do they have any relation between them ?
3. How would you find out linear velocity of a rotating body ?
4. Obtain an equation between the linear acceleration and angular acceleration of a rotating body.

OBJECTIVE TYPE QUESTIONS

1. The angular velocity of rotating body is expressed in terms of

(*a*) revolution per minute (*b*) radians per second

(*c*) any one of the two (*d*) none of the two

2. The linear velocity of a rotating body is given by the relation

(*a*) $v = r.\omega$ (*b*) $v = r/\omega$

(*c*) $v = \omega/r$ (*d*) ω^2/r

where r = Radius of the circular path, and

ω = Angular velocity of the body in radians/s.

3. The linear acceleration of a rotating body is given by the relation

(*a*) $a = r.\alpha$ (*b*) $a = r/\alpha$

(*c*) $a = \alpha/r$ (*d*) α^2/r

where r = Radius of the circular path, and

α = Angular acceleration of the body in radians/s^2

4. If at any given instant, we know that linear velocity and acceleration of a car, we can mathematically obtain its

(*a*) angular velocity (*b*) angular acceleration

(*c*) none of the two (*d*) both of the two

5. The relationship between linear velocity and angular velocity of a cycle

(*a*) exists under all conditions

(*b*) does not exist under all conditions

(*c*) exists only when it does not slip

(*d*) exists only when it moves on horizontal plane

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*d*) **5.** (*a*)

Top