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Q. $2xy\,dx + (x^2+1)\,dy = 0$ — (1)

Sol :-

(a) $x^2y + y = c$. (b) $x^2y = c$.

(c) $2xy^2 + x^2 = c$. (d) none of these.

Compare eq (1) with $f\,dx + g\,dy = 0$

$f = 2xy$, $g = x^2 + 1$

$\frac{\partial f}{\partial y} = 2x$, $\frac{\partial g}{\partial x} = 2x$ $\Rightarrow$ $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$.

Sol. $\int\limits_{y\text{ constant}} f\,dx + \int\limits_{\substack{\text{Term of } g \text{ not}\\ \text{containing } x}} g\,dy = c$

$\Rightarrow \int\limits_{y\text{ constant}} 2xy\,dx + \int 1\,dy = c$.

$\Rightarrow \cancel{2}y\,\frac{x^2}{\cancel{2}} + y = c \Rightarrow \boxed{x^2y + y = c}$

$2xy\,dx + (x^2+1)\,dy = 0$.

$\Rightarrow 2xy\,dx + x^2\,dy + dy = 0$

$\Rightarrow \underbrace{y\,(2x\,dx) + x^2\,dy}\, + dy = 0$

$y\,d(x^2) + x^2\,d(y) + dy = 0$

$$y\,d(x^2) + x^2\,d(y) + dy = 0$$

$$\Rightarrow d(x^2y) + dy = 0$$

$$\Rightarrow d(x^2y + y) = 0. \Rightarrow \text{eq is exact}$$

sol. $x^2y + y = c$

Q. $\underbrace{(e^{2y}+1)\cos x\,dx + 2e^{2y}\sin x\,dy = 0}$

(a) $e^{2y}\cos x + \sin x = c.$

(b) $e^{2y}\sin x + \cos x = c.$

~~(c)~~ $(e^{2y}+1)\sin x = c.$

(d) none of these.

Compare it with $f\,dx + g\,dy = 0$

$f = (e^{2y}+1)\cos x$, $g = 2e^{2y}\sin x.$

$\frac{\partial f}{\partial y} = \cos x\,(2e^{2y}+0)$, $\frac{\partial g}{\partial x} = 2e^{2y}\cos x.$

Clearly $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} \Rightarrow$ eq. is exact

sol. $\int\limits_{y\text{ const}} f\,dx + \int\limits_{\substack{\text{term of } g \\ \text{not containing } x}} g\,dy = C$

$$\Rightarrow \int\limits_{y\text{ constant}} (e^{2y}+1)\cos x\,dx + \int 0\,dy = C$$

$$\Rightarrow (e^{2y}+1)\sin x = C$$

$$(e^{2y}+1)\cos x\,dx + 2e^{2y}\sin x\,dy = 0$$

$$(e^{2y}+1)\cos x\,dx + \sin x\,(2e^{2y}dy) = 0$$

$$(e^{2y}+1)\,d(\sin x) + \sin x\,(d(e^{2y}+1)) = 0$$

$$= d(\sin x\,(e^{2y}+1)) = 0 \Rightarrow \text{eq. is exact.}$$

sol: $\boxed{\sin x\,(e^{2y}+1) = C}$