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CHAPTER

24 Laws of Motion

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24.1. INTRODUCTION

The entire system of dynamics is based on three laws of motion, which are the fundamental laws, and were formulated by *Newton. Like other scientific laws, these are also stated in the mathematical forms which agree with actual observations.

24.2. IMPORTANT TERMS

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage :

1. ***Mass.*** It is the matter contained in a body. The units of mass are kilogram, tonne etc.

* Named after Sir Issac Newton, who enunciated these laws in 1680.

2. ***Weight.*** It is the force, by which the body is attracted towards the centre of the earth. The units of weight are the same as those of force *i.e.* N, kN etc.
3. ***Momentum.*** It is the quantity of motion possessed by a body. It is expressed mathematically as

$$\text{Momentum} = \text{Mass} \times \text{Velocity}.$$

   The units of momentum depend upon the units of mass and velocity. In S.I. units, the mass is measured in kg, and velocity in m/s, therefore the unit of momentum will be kg-m/s.
4. ***Force.*** It is a very important factor in the field of dynamics also, and may be defined as any cause which produces or tends to produce, stops or tends to stop motion. The units of force, like those of weight, are N, kN etc.
5. ***Inertia.*** It is an inherent property of a body, which offers resistance to the change of its state of rest or uniform motion.

24.3. RIGID BODY

Strictly speaking, the laws of motion, enunciated by Newton, are applicable only to the rigid bodies. Though a rigid body (or sometimes written as 'body' for the sake of simplicity) is defined in many ways by the different scientists, yet there is not much of difference between all the definitions. But the following definition of a rigid body is universally recognised.

A rigid body consists of a system of innumerable particles. If the positions of its various particles remain fixed, relative to one another (or in other words, distance between any two of its particles remain constant), it is called a solid body. It will be interesting to know that in actual practice, all the solid bodies are not perfectly rigid bodies. However, they are regarded as such, since all the solid bodies behave more or less like rigid bodies.

24.4. NEWTON'S LAWS OF MOTION

Following are the three laws of motion, which were enunciated by Newton, who is regarded as father of the Science.

1. Newton's First Law of Motion states, "*Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force.*"
2. Newton's Second Law of Motion states, "*The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts.*"
3. Newton's Third Law of Motion states, "*To every action, there is always an equal and opposite reaction.*"

24.5. NEWTON'S FIRST LAW OF MOTION

It states "*Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force.*" It is also called the law of inertia, and consists of the following two parts :

1. A body at rest continues in the same state, unless acted upon by some external force. It appears to be self-evident, as a train at rest on a level track will not move unless pulled by an engine. Similarly, a book lying on a table remains at rest, unless it is lifted or pushed.
2. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state. It cannot be exemplified because it is, practically, impossible to get rid of the forces acting on a body.

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Note. The second part of the law furnishes us with an idea about the function of a force. It also implies that a force, which is to produce a change in the rest or motion of a body must be externally impressed ; or in other words, must act from outside. A little consideration will show, that the effect of inertia is of the following two types :

1. A body at rest has a tendency to remain at rest. It is called *inertia of rest.*
2. A body in uniform motion in a straight line has a tendency to preserve its motion. It is called *inertia of motion.*

24.6. NEWTON'S SECOND LAW OF MOTION

It states, "*The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts.*" This law enables us to measure a force, and establishes the fundamental equation of dynamics. Now consider a body moving in a straight line. Let its velocity be changed while moving.

Let m = Mass of a body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration,

t = Time, in seconds required to change the velocity from u to v, and

F = Force required to change velocity from u to v in t seconds.

$\therefore$ Initial momentum $= mu$

and final momentum $= mv$

$\therefore$ Rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \qquad \ldots \left[\because \frac{v-u}{t} = a \right]$$

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the the impressed force.

$$\therefore \qquad F \propto ma = kma$$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces unit acceleration to a unit mass.

$$\therefore \qquad F = ma = \text{Mass} \times \text{Acceleration.}$$

In S.I. system of units, the unit of force is called newton briefly written as N. *A Newton may be defined as the force while acting upon a mass of 1 kg, produces an acceleration of 1 m/s^2 in the direction of which it acts.* It is also called the Law of dynamics and consists of the following two parts :

1. A body can posses acceleration only when some force is applied on it. Or in other words, if no force is applied on the body, then there will be no acceleration, and the body will continue to move with the existing uniform velocity.
2. The force applied on a body is proportional to the product of the mass of the body and the acceleration produced in it.

It will be interesting to know that first part of the above law appears to be an extension of the First Law of Motion. However, the second part is independent of the First Law of Motion.

24.7. ABSOLUTE AND GRAVITATIONAL UNITS OF FORCE

We have already discussed, that when a body of mass 1 kg is moving with an acceleration of 1 m/s^2, the force acting on the body is 1N. Therefore when the same body is moving with an acceleration of 9.8 m/s^2, the force acting on the body is 9.8 N. But we denote 1 kg mass, attracted towards the earth with an acceleration of 9.8 m/s^2 as 1 kg-wt.

$\therefore$ 1 kg-wt = 9.8 N

Similarly, 1 t-wt = 9.8 kN

The above units of force *i.e.* kg-wt and t-wt (also written as kgf and tf) are called gravitational or engineer's units of force ; whereas N or kN are absolute or scientific units of force. It is thus obvious, that the gravitational or engineer's units are '*g*' times greater than the units of force in the absolute or scientific units.

It will be interesting to know that the mass of a body, in absolute units, is *numerically equal to* the weight of the same body in gravitational units *e.g.*, consider a body whose mass,

$$m = 100 \text{ kg}$$

$\therefore$ The force, with which it will be attracted towards the centre of the earth,

$$P = ma = mg = 100 \times 9.8 = 980 \text{ N}$$

Now, as per definition, we know that the weight of a body is the force by which it is attracted towards the centre of the earth. Therefore weight of the body,

$$W = 980 \text{ N} = \frac{980}{9.8} = 100 \text{ kg-wt} \qquad ...(\because \text{ kg-wt} = 9.8 \text{ N})$$

In engineering practice, the weight of a body is of primary importance. In order to avoid inconvenience of always multiplying the force in kgf by 9.8 to determine its value in newtons, the engineers use kgf as a unit of force. To preserve the force equation (*i.e.* $P = m.a$) we take the mass of the body in metric slugs. In general, the mass of a body in kg is divided by gravitational acceleration (g) gives the mass in slugs. Mathematically 1 slug = kg/9.8. Here the following points should be clearly understood to avoid uncalled confusion :

1. If the weight of the body is given, it will be in gravitational units. Its numerical value is equal to its mass in absolute untis. *e.g.* consider a body of weight 200 kg. Then it may be written as Weight,

   $(w) = 200$ kg. wt ...(In gravitational units)

   or mass $(m) = 200$ kg ...(In absolute units)

2. Sometimes, the mass of a body is given. It is always in absolute units.
3. In the force equation (*i.e.* $P = ma$) the value of mass is taken in absolute units.

Notes. From the above discussion, it may be clearly understood that

Force = Mass × Acceleration

1. If force is in newtons, then mass is in kg (absolute units).
2. If force is in kg, then mass is in slugs (absolute units)
3. The value of acceleration is in m/s^2 in both the cases.

Example 24.1. *Determine the force, which can move a body of mass 100 kg with an acceleration of 3.5 m/s^2.*

Solution. Given : Mass of body $(m) = 100$ kg and acceleration $(a) = 3.5$ m/s^2

We know that the force, $F = ma = 100 \times 3.5 = 350$ N **Ans.**

Example 24.2. *A body has 50 kg mass on the earth. Find its weight (a) on the earth, where g = 9.8 m/s² ; (b) on the moon, where g = 1.7 m/s² and (c) on the sun, where g = 270 m/s².*

Solution. Given: Mass of body (m) = 50 kg ; Acceleration due to gravity on earth (g_e) = 9.8 m/s²;Acceleration due to gravity on moon (g_m) = 1.7 m/s² and acceleration due to gravity on sun (g_s) = 270 m/s².

(*a*) *Weight of the body on the earth*

We know that weight of the body on the earth

$$F_1 = mg_e = 50 \times 9.8 = 490 \text{ N} \quad \textbf{Ans.}$$

(*b*) *Weight of the body on the moon*

We know that weight of the body on the moon,

$$F_2 = mg_m = 50 \times 1.7 = 85 \text{ N} \quad \textbf{Ans.}$$

(*c*) *Weight of the body on the sun*

We also know that weight of the body on the sun,

$$F_3 = mg_s = 50 \times 270 = 13500 \text{ N} = 13.5 \text{ kN} \quad \textbf{Ans.}$$

Example 24.3. *A body of mass 7.5 kg is moving with a velcoity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.*

Solution. Given: Mass of body = 7.5 kg ; Velocity (u) = 1.2 m/s ; Force (F) = 15 N and time (t) = 2 s.

We know that acceleration of the body

$$a = \frac{F}{m} = \frac{15}{7.5} = 2 \text{ m/s}^2$$

∴ Velocity of the body after 2 seconds

$$v = u + at = 1.2 + (2 \times 2) = 5.2 \text{ m/s} \quad \textbf{Ans.}$$

Example 24.4. *A vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle :*

(1) when the force acts in the direction of motion, and

(2) when the force acts in the opposite direction of the motion.

Solution. Given : Mass of vehicle (m) = 500 kg ; Initial velocity (u) = 25 m/s ; Force (F) = 200 N and time (t) = 2 min = 120 s

1. *Velocity of vehicle when the force acts in the dirction of motion*

We know that acceleration of the vehicle,

$$a = \frac{F}{m} = \frac{200}{500} = 0.4 \text{ m/s}^2$$

∴ Velocity of the vehicle after 120 seconds

$$v_1 = u + at = 25 + (0{\cdot}4 \times 120) = 73 \text{ m/s} \quad \textbf{Ans.}$$

2. *Velocity of the vehicle when the force acts in the opposite direction of motion.*

We know that velcoity of the vehicle in this case after 120 seconds, (when $a = -\,0.4$ m/s²),

$$v_2 = u + at = 25 + (-\,0.4 \times 120) = -23 \text{ m/s} \quad \textbf{Ans.}$$

Minus sign means that the vehicle is moving in the reverse direction or in other words opposite to the direction in which the vehicle was moving before the force was made to act.

Example 24.5. *A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a velocity of 15 m/s. How long the body will take to stop?*

Solution. Given: Retarding force (F) = 50 N ; Mass of the body (m) = 20 kg ; Initial velocity (u) = 15 m/s and final velocity (v) = 0 (because it stops)

Let t = Time taken by the body to stop.

We know that retardation of the body

$$a = \frac{F}{m} = \frac{50}{20} = 2.5 \text{ m/s}^2$$

and final velocity of the body,

$$0 = u + at = 15 - 2.5\,t \qquad \text{...(Minus sign due to retardation)}$$

$$\therefore \qquad t = \frac{15}{2.5} = 6 \text{ s} \qquad \textbf{Ans.}$$

Example 24.6. *A car of mass 2.5 tonnes moves on a level road under the action of 1 kN propelling force. Find the time taken by the car to increase its velocity from 36 km. p.h. to 54 km.p.h.*

Solution. Given : Mass of the car (m) = 2.5 t ; Propelling force (F) = 1 kN ; Initial velocity (u) = 36 km.p.h. = 10 m/s and final velocity (v) = 54 km.p.h. = 15 m/s

Let t = Time taken by the car to increase its speed.

We know that acceleration of the car,

$$a = \frac{F}{m} = \frac{1}{2.5} = 0.4 \text{ m/s}^2$$

and final velocity of the car (v),

$$15 = u + at = 10 + 0.4\,t$$

$$t = \frac{15 - 10}{0.4} = \frac{5}{0.4} = 12.5 \text{ s} \qquad \textbf{Ans.}$$

Example 24.7. *A multiple unit electric train has 800 tonnes mass. The resistance to motion is 100 N per tonne of the train mass. If the electric motors can provide 200 kN tractive force, how long does it take to accelerate the train to a speed of 90 km/hr from rest.*

Solution. Given: Mass of electric train (m) = 800 t ; Resistance to motion = 100 N/t = 100 × 800 = 80000 N = 80 kN ; Tractive force = 200 kN ; Final velocity (v) = 90 km/hr = 25 m/s and initial velocity (u) = 0 (because it starts from rest)

Let t = Time taken by the electric train.

We know that net force available to move the train,

$$F = \text{Tractive force} - \text{Resistance to motion}$$
$$= 200 - 80 = 120 \text{ kN}$$

and acceleration of the train

$$a = \frac{F}{m} = \frac{120}{800} = 0.15 \text{ m/s}^2$$

We also know that final velocity of the body (v)

$$25 = u + at = 0 + 0.15\,t$$

or

$$t = \frac{25}{0.15} = 166.7 \text{ s} \qquad \textbf{Ans.}$$

Example 24.8. *A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising.*

Find the average resistance of the water. Neglect the resistance of air.

Solution. Given : Mass (m) = 60 kg and height of tower (s) = 20 m.

First of all, consider the motion of the man from the top of the tower to the water surface. In this case, initial velocity (u) = 0 (because the man dives) and distance covered (s) = 20 m

Let v = Final velocity of the man when he reaches the water surface.

We know that $v^2 = u^2 + 2gs = (0)^2 + 2 \times 9.8 \times 20 = 392$

$$\therefore \quad v = \sqrt{392} = 19.8 \text{ m/s}$$

Now consider motion of the man from the water surface up to the point in water from where he started rising. In this case, initial velocity (u) = 19.8 m/s ; final velocity (v) = 0 (because the man comes to rest) and distance covered (s) = 2 m

Let a = Retardation due to water resistance.

We know that $v^2 = u^2 + 2as$

$$0 = (19.8)^2 - 2a \times 2 = 392 - 4a$$

...(Minus sign due to retardation)

$$\therefore \quad a = \frac{392}{4} = 98 \text{ m/s}^2$$

and average resistance of the water,

$$F = ma = 60 \times 98 = 5880 \text{ N} \quad \textbf{Ans.}$$

Example 24.9. *At a certain instant, a body of mass 10 kg, falling freely under the force of gravity, was found to be falling at the rate of 20 m/s. What force will stop the body in (i) 2 seconds and (ii) 2 metres?*

Solution. Given : Mass of the body (m) = 10 kg ; Initial velocity (u) = 20 m/s and final velocity (v) = 0 (because, it stops)

(*i*) *Force which will stop the body in 2 seconds*

Let a = Constant retardation.

We know that final velocity of the body (v),

$$0 = u - a_1 t = 20 - 2a_1 \qquad \text{...(Minus sign due to retardation)}$$

$$\therefore \quad a_1 = \frac{20}{2} = 10 \text{ m/s}^2$$

A little consideration will show that an upward acceleration of 10 m/s^2 is required to stop the body in 2 seconds. But as the body is falling under the force of gravity (*i.e.* with an acceleration of 9.8 m/s^2) therefore, the applied force must be able to produce an acceleration of 10 + 9.8 = 19.8 m/s^2.

$\therefore$ Force required to stop the body,

$$F = ma = 10 \times 19.8 = 198 \text{ N} \quad \textbf{Ans.}$$

(*ii*) *Force which will stop the body in 2 metres*

We know that $v^2 = u^2 - 2a_2 s$ ...(Minus sign due to retardation)

$$0 = (20)^2 - 2a_2 \times 2 = 400 - 4a_2$$

or

$$a_2 = \frac{400}{4} = 100 \text{ m/s}^2$$

As discussed above, the applied force must be able to produce an acceleration of 100 + 9.8 = 109.8 m/s^2.

∴ Force required to stop the body,

$$F = ma_2 = 10 \times 109.8 = 1098 \text{ N} \textbf{ Ans.}$$

Example 24.10. *A body of mass 10 kg is moving over a smooth surface, whose equation of motion is given by the relation.*

$$s = 5t + 2t^2$$

where (s) is in metres and (t) in seconds. Find the magnitude of force responsible for the motion.

Solution. Given : Equation of motion : $s = 5t + 2t^2$

Differentiating both sides of the above equation with respect to t,

$$\frac{ds}{dt} = 5 + 4t$$

Again differentiating both sides of the above equation with respect to t,

$$\frac{d^2s}{dt^2} = 4 \quad \text{or} \quad \text{acceleration, } a = 4 \text{ m/s}^2.$$

∴ Force responsible for the motion,

$$F = ma = 10 \times 4 = 40 \text{ N} \quad \textbf{Ans.}$$

EXERCISE 24.1

1. Find the force required to give an acceleration of 1.5 m/s^2 to a body of mass 40 kg. (**Ans.** 60 N)
2. A force of 30 N acts on a body of mass 8 kg. Calculate the acceleration it can produce. (**Ans.** 3.75 m/s^2)
3. A body of mass 40 kg is moving with a constant velocity of 2.5 m/s. Now a force of 100 N is applied on the body in its direction of motion. What will be its velocity after 2 second. (**Ans.** 7.5 m/s)
4. A constant force of 100 N is applied on a body of mass 50 kg at rest. Find the distance travelled by it in 12 seconds. (**Ans.** 144 m)
5. A force of 10 N acts on a body at rest for 10 s and causes it to more 20 m during this period. What is the mass of the body? (**Ans.** 25 kg)
6. A constant force acting on a body of mass 20 kg changes its speed from 2.5 m/s to 10 m/s in 15 seconds. What is the magnitude of the force? (**Ans.** 10 N)
7. A railway coach of mass 50 tonne can exert a tractive force of 20 kN. Find the acceleration of the coach on a level track if the resistance is 150 N per tone. (**Ans.** 0.25 m/s^2)
8. An engine of mass of 25 tonnes was moving with a velocity of 72 km.p.h. The steam was shut off and brakes were applied to bring the engine to rest in 400 m. Find the uniform force exerted by the brakes. (**Ans.** 12.5 kN)

24.8. MOTION OF A LIFT

Consider a lift (elevator or cage etc.) carrying some mass and moving with a uniform acceleration

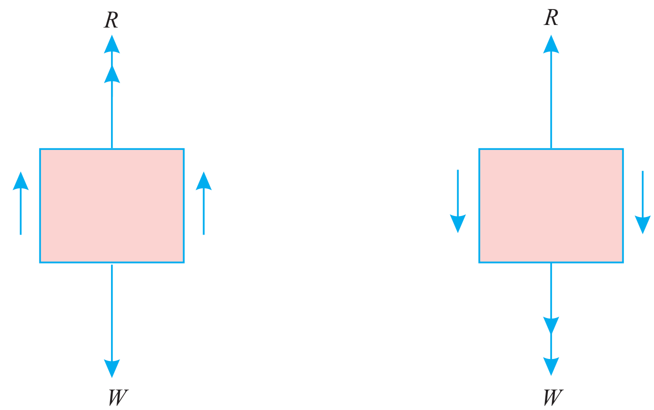
Let m = Mass carried by the lift,

a = Uniform acceleration of the lift, and

R = Reaction of the lift or tension in the cable, supporting the lift, Here we shall discuss the following two cases as shown in Fig. 24.1 (*a*) and (*b*) :

1. When the lift is moving upwards.
2. When the lift if moving downwards.

1. *When the lift is moving upwards*



(*a*) Lift moving upwards (*b*) Lift moving downwards

Fig. 24.1. Motion of a lift.

We know that downward force due to mass of the lift

$$= mg$$

and net upward force on lift, $F = R - mg$ ...(*i*)

We also know that this force = Mass × Acceleration = $m.a$ ...(*ii*)

From equations (*i*) and (*ii*),

$$R - mg = ma$$

$$\therefore \quad R = ma + mg = m\,(a + g)$$

2. *When the lift is moving downwards*

In this case, the net downward force, which is responsible for the motion of the lift.

$$= mg - R \quad ...(iii)$$

From equations (*ii*) and (*iii*),

$$ma = mg - R$$

$$\therefore \quad R = mg - ma = m\,(g - a)$$

Note. In the above cases, we have taken mass carried by the lift only. We have assumed that it includes mass of the lift also. But sometimes the example contains mass of the lift and mass carried by the lift separately.

In such a case, the mass carried by the lift (or mass of the operator etc.) will exert a pressure on the floor of the lift. Whereas tension in the cable will be given by the algebraic sum of the masses of the lift and mass carried by the lift. Mathematically. (When the lift is moving upwards), then the pressure exerted by the mass carried by the lift on its floor

$$= m_2\,(g + a)$$

and tension in the cable $= (m_1 + m_2)\,(g + a)$

where m_1 = Mass of the lift and

m_2 = Mass carried by the lift.

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Example 24.11. *A body of mass 50 kg is being lifted by a lift in an office. Find the force exerted by the body on the lift floor, when it is moving with a uniform acceleration of 1.2 m/s².*

Solution. Given : Mass of the body (m) = 50 kg and acceleration (a) = 1.2 m/s²

We know that pressure exerted by the body on the floor, when it is being lifted

$$F = m\,(g + a) = 50\,(9.8 + 1.2) = 550 \text{ N} \quad \textbf{Ans.}$$

Example 24.12. *In a factory, an elevator is required to carry a body of mass 100 kg. What will be the force exerted by the body on the floor of the lift, when (a) the lift is moving upwards with retardation of 0.8 m/s² ; (b) moving downwards with a retardation of 0.8 m/s².*

Solution. Given : Mass of the body (m) = 100 kg and acceleration (a) = – 0.8 m/s² (Minus sign due to retardation)

(*a*) *When the lift is moving upwards*

We know that force exerted by the body on the floor of the lift

$$F_1 = m\,(g + a) = 100\,(9.8 - 0.8) = 900 \text{ N} \quad \textbf{Ans.}$$

(*b*) *When the lift is moving downwards*

We also know that force exerted by the body on the floor of the lift.

$$F_2 = m\,(g - a) = 100\,(9.8 + 0.8) = 1060 \text{ N} \quad \textbf{Ans.}$$

Example 24.13. *An elevator is required to lift a body of mass 65 kg. Find the acceleration of the elevator, which could cause a force of 800 N on the floor.*

Solution. Given : Mass of the body (m) = 65 kg and Force (R) = 800 N

Let a = Acceleration of the elevator.

We know that the force caused on the floor when the elevator is going up (R),

$$800 = m\,(g + a) = 65\,(9.8 + a)$$

or

$$a = \frac{800}{65} - 9.8 = 2.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Example 24.14. *An elevator of mass 500 kg is ascending with an acceleration of 3 m/s². During this ascent, its operator whose mass is 70 kg is standing on the scale placed on the floor. What is the scale reading? What will be the total tension in the cables of the elevator during this motion?*

Solution. Given : Mass of the elevator (m_1) = 500 kg ; Acceleration (a) = 3 m/s² and mass of operator (m_2) = 70 kg

Scale Reading

We know that scale reading when the elevator is ascending,

$$R_1 = m_2\,(g + a) = 70\,(9.8 + 3) = 896 \text{ N} \quad \textbf{Ans.}$$

Total tension in the cable of the elevator

We also know that total tension in the cable of the elevator when it is ascending

$$R_2 = (m_1 + m_2)\,(g + a) = (500 + 70)\,(9.8 + 3) \text{ N}$$
$$= 7296 \text{ N} \quad \textbf{Ans.}$$

Example 24.15. *An elevator of mass 2500 kg is moving vertically downwards with a constant acceleration. Starting from rest, it travels a distance of 35 m during an interval of 10 seconds. Find the cable tension during this time. Neglecting all other resistances to motion, what are the limits of cable tension?*

Solution. Given : Mass of elevator (m) = 2500 kg ; Initial velocity (u) = 0 (because it starts from rest) ; Distance travelled (s) = 35 m and time (t) = 10 s.

Cable tension

Let a = Constant acceleration of the elevator.

We know that distance travelled by the elevator (s)

$$35 = ut + \frac{1}{2}at^2 = (0 \times 10) + \frac{1}{2}a\,(10)^2 = 50\,a$$

or $$a = \frac{35}{50} = 0.7 \text{ m/s}^2$$

∴ Tension in the cable when the elevator is moving vertically downwards,

$$R = m\,(g - a) = 2500\,(9.8 - 0.7) = 22750 \text{ N} = 22.75 \text{ kN} \quad \textbf{Ans.}$$

Limits of cable tension

A little consideration will show, that the cable tension will have two limits *i.e.* when acceleration is zero and when acceleration is maximum (*i.e.* 9.8 m/s^2).

∴ Cable tension when the elevator is moving vertically downwards with zero acceleration,

$$R = m\,(g - a) = 2500\,(9.8 - 0) = 24\,500 \text{ N} = 24.5 \text{ kN} \quad \textbf{Ans.}$$

and cable tension when the acceleration is maximum (*i.e.* 9.8 m/s^2).

$$R = m\,(g - a) = 2500\,(9.8 - 9.8) = 0 \quad \textbf{Ans.}$$

Example 24.16. *An elevator of gross mass 500 kg starts moving upwards with a constant acceleration, and acquires a velocity of 2 m/s, after travelling a distance of 3 m. Find the pull in the cables during the accelerated motion.*

If the elevator, when stopping moves with a constant deceleration from a constant velocity of 2 m/s and comes to rest in 2 s, calculate the force transmitted by a man of mass 75 kg the floor during stopping.

Solution. Given : Gross mass of elevator (m_1) = 500 kg

Pull in the cable, during accelerated motion

First of all consider motion of the elevator with a constant acceleration. In this case, initial velocity (u) = 0 (because it starts from rest) ; Final velocity acquired (v) = 2 m/s and distance travelled (s) = 3 m

Let a_1 = Constant acceleration.

We know that $v^2 = u^2 + 2a_1 s$

$$(2)^2 = 0 + 2a_1 \times 3 = 6a_1$$

or $$a_1 = \frac{4}{6} = 0.67 \text{ m/s}^2$$

∴ Force (or pull) required to produce this acceleration,

$$F_1 = m_1 a_1 = 500 \times 0.67 = 335 \text{ N}$$

and total pull in the cable, $R = (500 \times 9.8) + 335 = 5235$ N **Ans.**

Force transmitted by the man during the decelerating motion

Now consider motion of the elevator the decelaration (*i.e.* retardation). In this case, initial velocity (u) = 2m/s ; Final velocity (v) = 0 (because it comes to rest) ; time (t) = 2 s and mass of man (m_2) = 75 kg

Let a_2 = Constant deceleration *i.e.* retardation

We know that final velocity of the elevator (v)

$$0 = u + a_2 t = 2 + a_2 \times 2 = 2 + 2a_2$$

or $a_2 = -1 \text{ m/s}^2$ ...(Minus sign means retardation)

∴ Force transmitted by the man during decelerating motion,

$$R = m_2 (g - a) = 75 (9.8 - 1) = 660 \text{ N } \textbf{Ans.}$$

24.9. D'ALEMBERT'S PRINCIPLE*

It states, "*If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics.*"

We have already discussed in art. 24.6, that force acting on a body.

$$P = ma \qquad ...(i)$$

where m = mass of the body, and

a = Acceleration of the body.

The equation (i) may also be written as :

$$P - ma = 0 \qquad ...(ii)$$

It may be noted that equation (i) is the equation of *dynamics* whereas the equation (ii) is the equation of *statics*. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as *D' Alembert's principle.*

EXERCISE 24.2

1. In an office, a lift is moving upwards with an acceleration of 1.5 m/s^2. Find the pressure exerted by a body of mass 30 kg on the floor of the lift. (**Ans.** 339 N)
2. An elevator of mass 2 t is to be lifted and lowered by means of a rope. Find the tension in the rope, when the elevator is moving (i) upward with an acceleration of 2 m/s^2 and (ii) downward with an acceleration of 1.5 m/s^2. (**Ans.** 23.6 kN ; 16.6 kN)
3. A lift has an upward acceleration of 1 m/s^2. Find the pressure exerted by the man of mass 62.5 kg on the floor of the lift. If the lift had a downward acceleration of 1 m/s^2, find the pressure exerted by the man. Also find an upward acceleration of the lift, which would cause the man to exert a pressure of 750 N. (**Ans.** 675 N ; 550 N ; 2.2 m/s^2)

Example 24.17. *Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. 24.2.*

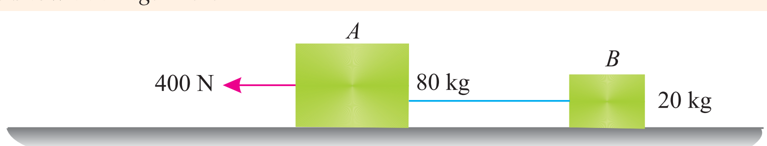


Fig. 24.2.

The coefficient of friction between the sliding surfaces of the bodies and the plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

* It is also known as the principle of kinostatics.

Solution. Given : Mass of body A (m_1) = 80 kg ; Mass of the body B (m_2) = 20 kg; Force applied on first body (P) = 400 N and coefficient of friction (μ) = 0.3

Acceleration of the two bodies

Let a = Acceleration of the bodies, and

T = Tension in the thread.

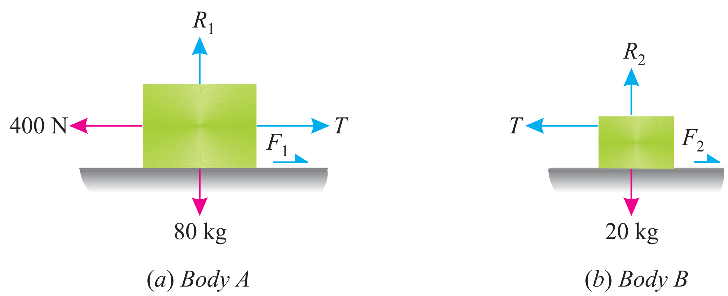


Fig. 24.3.

First of all, consider the body A. The forces acting on it are :

1. 400 N force (acting towards left)
2. Mass of the body = 80 kg (acting downwards)
3. Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)
4. Force of friction, $F_1 = \mu R_1 = 0.3 \times 784 = 235.2$ N (acting towards right)
5. Tension in the thread = T (acting towards right).

∴ Resultant horizontal force,

$$P_1 = 400 - T - F_1 = 400 - T - 235.2$$
$$= 164.8 - T \text{ (acting towards left)}$$

We know that force causing acceleration to the body A

$$= m_1 a = 80\,a$$

and according to D' Alembert's principle ($P_1 - m_1 a = 0$)

$$164.8 - T - 80\,a = 0$$

or $$T = 164.8 - 80a \qquad ...(i)$$

Now consider the body B. The forces acting on it are :

1. Tension in the thread = T (acting towards left)
2. Mass of the body = 20 kg (acting downwards)
3. Reaction $R_2 = 20 \times 9.8 = 196$ N (acting upwards)
4. Force of friction, $F_2 = \mu R_2 = 0.3 \times 196 = 58.8$ N (acting towards right)

∴ Resulting horizontal force,

$$P_2 = T - F_2 = T - 58.8$$

We know that force causing acceleration to the body B

$$= m_2 a = 20\,a$$

and according to *D'* Alembert's principle ($P_2 - m_2 a = 0$)

$$(T - 58.8) - 20\,a = 0$$

or $$T = 58.8 + 20\,a \qquad ...(ii)$$

Now equating the two values of *T* from equation (*i*) and (*ii*),

$$164.8 - 80\,a = 58.8 + 20\,a$$

$$100\,a = 106$$

or $$a = \frac{106}{100} = 1.06 \text{ m/s}^2 \quad \textbf{Ans.}$$

Tension in the thread

Substituting the value of *a* in equation (*ii*),

$$T = 58.8 + (20 \times 1.06) = 80 \text{ N} \quad \textbf{Ans.}$$

24.10. NEWTON'S THIRD LAW OF MOTION

It states "*To every action, there is always an equal and opposite reaction.*"

By *action* is meant the force, which a body exerts on another, and the *reaction* means the equal and opposite force, which the second body exerts on the first. This law, therefore, states that a force always occurs in pair. Each pair consisting of two equal opposite forces.

This law appears to be self-evident as when a bullet is fired from a gun, the bullet moves out with a great velocity, and the reaction of the bullet, in the opposite direction, gives an unpleasant shock to the man holding the gun. Similarly, when a swimmer tries to swim, he pushes the water backwards and the reaction of the water pushes the swimmer forward. Though the Newton's Third Law of Motion has a number of applications, yet recoil of gun is important from the subject point of view.

24.11. RECOIL OF GUN

According to Newton's Third Law of Motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let M = Mass of the gun,

V = Velocity of the gun with which it recoils,

m = mass of the bullet, and

v = Velocity of the bullet after explosion.

∴ Momentum of the bullet after explosion

$$= mv \quad ...(i)$$

and momentum of the gun $= MV$ ...(*ii*)

Equating the equations (*i*) and (*ii*),

$$MV = mv$$

Note. This relation is popularly known as Law of Conservation of Momentum.

Example 24.18. *A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil.*

Solution. Given : Mass of the machine gun (M) = 25 kg ; Mass of the bullet (m) = 30 g = 0.03 kg and velocity of firing (v) = 250 m/s.

Let V = Velocity with which the machine gun will recoil.

We know that $MV = mv$

$$25 \times v = 0.03 \times 250 = 7.5$$

$$\therefore \quad v = \frac{7.5}{25} = 0.3 \text{ m/s} \quad \textbf{Ans.}$$

Example 24.19. *A bullet of mass 20 g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage ; which together with the gun has mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find (a) velocity, with which the gun will recoil, (b) distance, in which it comes to rest, and (c) time taken to do so.*

Solution. Given : Mass of the bullet (m) = 20 g = 0.02 kg ; Velocity of bullet (v) = 300 m/s; Mass of the carriage with gun (M) = 100 kg and resistance to sliding (F) = 20 N

(*a*) *Velocity, with which the gun will recoil*

Let V = velocity with which the gun will recoil.

We know that $MV = mv$

$$100 \times V = 0.02 \times 300 = 6$$

$$\therefore \quad V = \frac{6}{100} = 0.06 \text{ m/s} \quad \textbf{Ans.}$$

(*b*) *Distance, in which the gun comes to rest*

Now consider motion of the gun. In this case, initial velocity (u) = 0.06 m/s and final velocity (v) = 0 (because it comes to rest)

Let a = Retardation of the gun, and

s = Distance in which the gun comes to rest.

We know that resisting force to sliding of carriage (F)

$$20 = Ma = 100\ a$$

$$\therefore \quad a = \frac{20}{100} = 0.2 \text{ m/s}^2$$

We also know that $v^2 = u^2 - 2as$ ...(Minus sign due to retardation)

$$0 = (0.06)^2 - 2 \times 0.2\ s = 0.0036 - 0.4\ s$$

$$\therefore \quad s = \frac{0.0036}{0.4} = 0.009 \text{ m} = 9 \text{ mm} \qquad \textbf{Ans.}$$

(*c*) *Time taken by the gun in coming to rest*

Let t = Time taken by the gun in coming to rest.

We know that final velocity of the gun (v),

$$0 = u + at = 0.06 - 0.2\ t \qquad \text{...(Minus sign due to retardation)}$$

$$t = \frac{0.06}{0.2} = 0.3 \text{ s} \qquad \textbf{Ans.}$$

24.12. MOTION OF A BOAT

We see that a boat boy always pushes the water back, with the help of sticks, which in turn, sets the boat in motion. It has also been experienced that if the boat is at rest and the boat boy runs on it and dives off into the water, the boat will also move backward.

The movement of the boat may be easily found out by the application of the Newton's Third Law of Motion (*i.e.,* by equating the momentum of the boat boy and the boat). Now consider a boat on which a boat boy runs and then dives off into the water.

Let M = Mass of the boat,

V = Velocity of the boat,

m = Mass of the boat boy, and

v = Velocity of the boat boy.

Momentum of the boat, after the boy jumps

$$= MV \qquad \text{...}(i)$$

and momentum of the boat boy $= mv$ ...(*ii*)

Equating equations (*i*) and (*ii*),

$$MV = mv$$

Example 24.20. *Two men, standing on a floating boat, run in succession, along its length, with a speed of 4.2 m/sec relative to the boat and dive off from the end. The weight of each man is 80 kg and that of the boat is 400 kg. If the boat was initially at rest, find the final velocity of the boat. Neglect water friction.*

Solution. Given : $v = 4.2$ m/sec ; Weight of each man, $w = 80$ kg-wt (in gravitational units) or $m = 80$ kg (in absolute units) ; Weight of boat = 400 kg-wt (in gravitational units) or $M = 400$ kg (in absolute units).

Let V = Velocity of the boat after the second man dives off the boat.

A little consideration will show, that when the first man dives off the boat, it will give some momentum to the boat as well as the second man (who is still standing on the boat). When the second man also dives off the boat, it will also give some momentum to the boat. Therefore the total momentum gained by the boat is equal to the momentum given by the first man *plus* the momentum given by the second man to the boat.

Now final momentum of the boat

$$= 400\ V \text{ kg m/s} \qquad ...(i)$$

and momentum given by the first man to the boat

$$= 80 \times 4.2 = 336 \text{ kg-m/s}$$

Similarly, momentum given by the second man to the boat

$$= 80 \times 4.2 = 336 \text{ kg-m/s}$$

We know that the final momentum of the boat

= Momentum given by the first man

+ Momentum given by the second man

$$= 336 + 336 = 672 \text{ kg-m/s} \qquad ...(ii)$$

Now equating equations (*i*) and (*ii*),

$$400\ V = 672$$

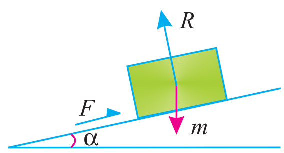
$$\therefore \qquad V = \frac{672}{400} = 1.68 \text{ m/s} \qquad \textbf{Ans.}$$

EXERCISE 24.3

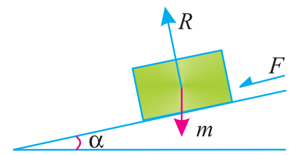
1. A bullet of 10 gm mass is fired horizontally with a velocity of 1000 m/s from a gun of mass 50 kg. Find (*a*) velocity with which the gun will recoil, and (*b*) force necessary to be ring the gun to rest in 250 mm. (**Ans.** 0.2 m/s ; 4 N)
2. A block of mass 10 kg slides over a frictionless horizontal plane with a constant velocity of 5 m/s. After some distance, the plane is inclined at an angle of 20º with the horizontal. Find the distance through which the block will slide upwards on the inclined plane before coming to rest. (**Ans** 36.5 m)

24.13. MOTION ON INCLINED PLANES

In the previous articles, we have been discussing the motion of bodies on level surface. But sometimes, the motion of body takes place down or up an inclined plane as shown in Fig. 24.4 (*a*) and (*b*).



(*a*) *Downward motion*



(*b*) *Upward motion*

Fig. 24.4. Motion on inclined plane.

Now consider a body moving downwards on an inclined plane as shown in Fig. 24.4 (*a*).

Let m = Mass of the body, and

α = Inclination of the plane.

We know that normal reaction on the inclined plane due to mass of the body.

$$R = mg \cos \alpha \qquad ...(i)$$

This component is responsible for the force of friction experienced by the body such that

$$F = \mu R$$

We also know that component of the force along the inclined plane due to mass of the body.

$$= mg \sin \alpha \qquad ...(ii)$$

This component is responsible for sliding (or moving) the body downwards. Now we can find out any detail of the motion by subtracting the force of friction (due to normal reaction) from the component along the inclined surface ($mg \sin \alpha$).

Note. If the body is moving upwards, then the component along the inclined surface is taken as an additional resistance. *i.e.* this component is added to other types to resistances.

Example 24.21. *A vehicle of mass 2 tonnes has a frictional resistance of 50 N/tonne. As one instant, the speed of this vehicle at the top of an incline was observed to be 36 km.p.h. as shown in Fig.24.5.*

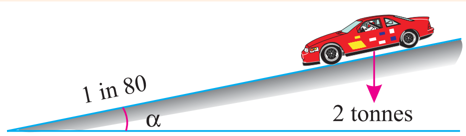


Fig. 24.5.

Find the speed of the vehicle after running down the incline for 100 seconds.

Solution. Given : Mass of vehicle (m) = 2 t = 2000 kg; Frictional resistance = 50 N/t = 50 × 2 = 100 N ; Initial velocity of vehicle (u) = 36 km.p.h. = 10 m/s ; Slope of inclination ($\sin \alpha$) = $\frac{1}{80}$ = 0.0125 and time (t) = 100 s.

Let a = Acceleration of the vehicle.

We know that force due to inclination

$$= mg \sin \alpha = 2000 \times 9.8 \times 0.0125 = 245 \text{ N}$$

∴ Net force available to move the vehicle.

$$F = \text{Force due to inclination} - \text{Frictional resistance}$$

$$= 245 - 100 = 145 \text{ N}$$

We also know that the available force (F)

$$145 = ma = 2000 \times a$$

$$\therefore \qquad \alpha = \frac{145}{2000} = 0.0725 \text{ m/s}^2$$

and speed of the train after running down the incline

$$v = u + at = 10 + (0.0725 \times 100) = 17.25 \text{ m/s} = 62.1 \text{ km.p.h.} \quad \textbf{Ans.}$$

Example 24.22. *A body of mass 200 kg is initially stationary on a 15° inclined plane. What distance along the incline must the body slide before it reaches a speed of 10 m/s? Take coefficient of friction between the body and the plane as 0.1.*

Solution. Given : Mass of the body (m) = 200 kg ; Initial velocity (u) = 0 (because, it is stationary) ; Inclination of the plane (α) = 15° ; Final velocity (v) = 10 m/s and coefficient of friction (μ) = 0.1

Let s = Distance through which the body will slide.

We know that the force responsible for sliding down the body

$$= mg \sin \alpha$$
$$= 200 \times 9.8 \sin 15° \text{ N}$$
$$= 1960 \times 0.2588 = 507.2 \text{ N}$$

and normal reaction $R = mg \cos \alpha$

$$= 200 \times 9.8 \cos 15° \text{ N}$$
$$= 1960 \times 0.9659 = 1893 \text{ N}$$

$\therefore$ Force of friction $F = \mu R = 0.1 \times 1893 = 189.3$ N

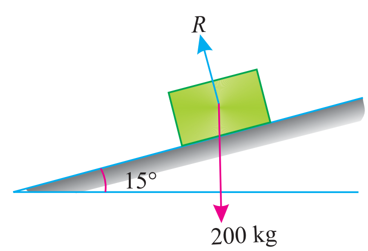


Fig. 24.6.

and net force available to move the body.

$$F = \text{Force responsible for sliding} - \text{Force of friction}$$
$$= 507.2 - 189.3 = 317.9 \text{ N}$$

We know that the available force (F)

$$317.9 = ma = 200 \times a$$

$$\therefore \qquad a = \frac{317.9}{200} = 1.59 \text{ m/s}^2$$

We also know that $v^2 = u^2 + 2as$

$$(10)^2 = (0)^2 + (2 \times 1.59 \times s) = 3.18\, s$$

$$\text{or} \qquad s = \frac{100}{3.18} = 31.4 \text{ m} \quad \textbf{Ans.}$$

Example 24.23. *A train of wagons is first pulled on a level track from A to B and then up a 5% upgrade as shown in Fig. 24.7.*

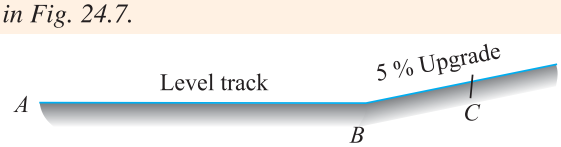


Fig. 24.7.

At some point C, the last wagon gets detached from the train, when it was travelling with a velocity of 36 km.p.h. If the detached wagon has a mass of 50 tonnes and the track resistance is 100 N per tonne, find the distance through which the wagon will travel before coming to rest.

Solution. Given : Grade = 5% or $\sin \alpha = 0.05$; Initial velocity (u) = 36 km.p.h. = 10 m/s ; Mass of the detached wagon $(m) = 50\ t$; Final velocity $(v) = 0$ (because it comes to rest) and track resistance = 100 N/t = 100 × 50 = 5000 N = 5 kN

Let s = Distance through which the wagon will travel before coming to rest, and

a = Retardation of the train.

We know that resistance to the train due to upgrade

$$= mg \sin \alpha = 50 \times 9.8 \times 0.05 = 24.5 \text{ kN}$$

∴ Total resistance to the movement of the train

$$F = \text{Resistance due to upgrade} + \text{Track resistance}$$
$$= 24.5 + 5 = 29.5 \text{ kN}$$

We know that total resistance (F)

$$29.5 = ma = 50 \times a$$

∴ $$a = \frac{29.5}{50} = 0.59 \text{ m/s}^2$$

We also know that $$v^2 = u^2 + 2as = (10)^2 - 2 \times 0.59 \times s$$

...(Minus sign due to retardation)

$$0 = 100 - 1.18\ s$$

or $$s = \frac{100}{1.18} = 84.7 \text{ m} \qquad \textbf{Ans.}$$

Example 24.24. *A truck is moving down a 10º incline when the driver applies brakes, with the result that the truck decelerates at a steady rate of 1 m/s². Investigate whether a 500 kg placed on the truck will slide or remain stationary relative to the truck. Assume the coefficient of friction between the truck surface and the load as 0.4.*

What will be the factor of safety against slipping for this load?

Solution. Given : Inclination of the plane $(\alpha) = 10º$; Acceleration $(a) = 1 \text{ m/s}^2$; Mass of the body placed on the truck (m) = 500 kg and coefficient of friction $(\mu) = 0.4$.

Stability of the load

We know that when the truck is decelerated, the body will tend to slip forward (*i.e.* downward). Therefore force caused due to deceleration,

$$P_1 = ma = 500 \times 1 = 500 \text{ N}$$

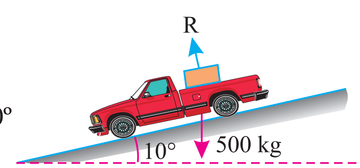


Fig. 24.8.

and component of the load along the plane

$$P_2 = mg \sin \alpha = 500 \times 9.8 \sin 10º$$
$$= 4900 \times 0.1736 \text{ N}$$
$$= 850.6 \text{ N}$$

∴ Total force, which will cause slipping,

$$= P_1 + P_2 = 500 + 850.6 = 1350.6 \text{ N}$$

We know that normal reaction of the load,

$$R = mg \cos 10º = 500 \times 9.8 \times 0.9848 = 4825.5 \text{ N}$$

and force of friction, $$F = \mu R = 0.4 \times 4825.5 = 1930.2 \text{ N}$$

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Since the force of friction (1930.2 N) is more than the force which will cause slipping (1350.6 N), therefore the load will not slip. **Ans.**

Factor of safety against slipping for this load.

We know that the factor of safety against slipping for this load

$$= \frac{\text{Force of friction}}{\text{Force causing slipping}} = \frac{1930.2}{1350.6} = 1.43 \quad \textbf{Ans.}$$

EXERCISE 24.4

1. A body of mass 500 kg, initially at rest at *A* 50 m from *B* on an 15% upgrade, is allowed to slide down as shown in Fig. 24.9.

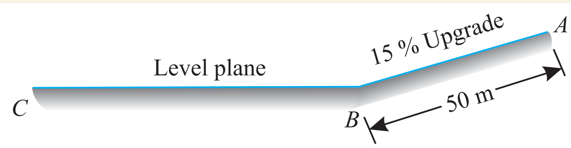

Fig. 24.9

If the coefficient of friction between the body and the plane is 0.1, find the velocity of the body at *B*. Also find the distance through which the body will travel beyond *B* on the level plane. (**Ans.** 8 m/s ; 32.65 m)

2. A locomotive of mass 200 tonnes draws a train of mass 450 tonnes. The frictional resistance is constant and equal to 75 N/t. Find the tractive force which will be required for the train to reach a speed of 72 km.p.h. in a distance of 2 km from the starting point (*i*) on a level track ; (*ii*) for going upward on an inclined plane of 1 in 240 and (*ii*) for going downward on the same inclined plane. (**Ans.** 113.75 kN ; 140.5 kN ; 87 kN)

QUESTIONS

1. State the Laws of Motion. Discuss the First Law in the light of Second Law.

2. Distinguish clearly between 'mass' and 'weight'.

3. Derive an expression for the tension in the cable supporting a lift when (*i*) it is going up, and (*ii*) it is coming down.

4. Explain the dynamic equilibrium of a rigid body in plane motion.

5. Explain clearly the term 'recoil of gun'. How will you find the velocity of the bullet?

6. How will you apply the Third Law of Motion in the case of horse pulling a cart?

OBJECTIVE TYPE QUESTIONS

1. The units of weight are the same as that of force.

(*a*) Agree (*b*) Disagree

2. The Newton's Second Law of Motion gives a relation between force, mass and

(*a*) Velocity (*b*) Acceleration (*c*) None of the two

3. Which of the following statement is wrong?

   (*a*) The matter contained in a body is called mass.

   (*b*) The force with which a body is attracted towards the centre of the earth is called weight.

   (*c*) The total motion possessed by a moving body is called impulsive force

   (*d*) none of them

4. If a lift is moving downwards with some acceleration, then tension in the cable supporting the lift is...proportional to the acceleration.

   (*a*) Directly (*b*) Indirectly

5. A science teacher told to his students that the Newton's Third Law of Motion is involved while studying the motion of rockets. Is his statement correct?

   (*a*) Yes (*b*) No

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)

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