# Motion Under Variable Acceleration

CHAPTER

18

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## 18.1. INTRODUCTION

In the last chapter, we have discussed the motion under constant acceleration, *i.e.*, the rate of change of velocity was constant. But in actual practice, it is seldom possible, that a body may move with a uniform velocity or uniform acceleration, at all times.

A body, which does not move with a uniform acceleration, is said to be moving with a non-uniform or variable acceleration.

In this chapter, we shall discuss the motion under variable acceleration.

## 18.2. VELOCITY AND ACCELERATION AT ANY INSTANT


**Fig 18.1.** Motion under variable acceleration.

Consider a body moving from $O$ in the direction $OX$. Let $P$ be its position at any instant as shown in Fig 18.1.

Let $s$ = Distance travelled by the body,

$t$ = Time taken by the body, in seconds, to travel this distance

$v$ = Velocity of the body, and

$a$ = Acceleration of the body.

We know that the velocity of a body, is the rate of change of its position. Mathematically :

$$v = \frac{ds}{dt} \qquad ...(i)$$

Similarly, acceleration of a body is the rate of change of its velocity. Mathematically :

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = v.\frac{dv}{ds} \qquad ...(ii)$$

## 18.3. METHODS FOR VELOCITY, ACCELERATION AND DISPLACEMENT FROM A MATHEMATICAL EQUATION

The velocity, acceleration and displacement of a body, from a mathematical expression, may be found out by either of the following two methods :

1. By differentiations and 2. By integration.

## 18.4. VELOCITY AND ACCELERATION BY DIFFERENTIATION


Motion in terms of displacement and time.

Sometimes, the given equation of motion is in terms of displacement ($s$) and time ($t$) *e.g.*,

$$s = 3t^3 + 2t^2 + 6t + 4 \qquad ...(i)$$

or $$s = 6 + 5t^2 + 6t^3 \qquad ...(ii)$$

or $$s = 2t^3 + 4t - 15 \qquad ...(iii)$$

Now differentiating both sides, of the equations, with respect to $t$,

$$\frac{ds}{dt} = 9t^2 + 4t + 6 \qquad ...(i)$$

$$\frac{ds}{dt} = 10t + 18t^2 \qquad ...(ii)$$

$$\frac{ds}{dt} = 6t^2 + 4 \qquad ...(iii)$$

The equations, so obtained by differentiation, give velocity of the body (as the velocity of a body is the rate of change of its position). Again differentiating, both sides of the above equations, with respect to $t$,

$$\frac{d^2s}{dt^2} = 18t + 4 \qquad ...(i)$$

$$\frac{d^2s}{dt^2} = 10 + 36t \qquad ...(ii)$$

$$\frac{d^2s}{dt^2} = 12t \qquad ...(iii)$$

The equations, so obtained by second differentiation, give acceleration of the body (as the acceleration of a body is the rate of change of velocity).

**Example 18.1.** *A particle, starting from rest, moves in a straight line, whose equation of motion is given by :* $s = t^3 - 2t^2 + 3$. *Find the velocity and acceleration of the particle after 5 seconds.*

**Solution.** Given : Equation of displacement : $s = t^3 - 2t^2 + 3$ ...(i)

*Velocity after 5 seconds*

Differentiating the above equation with respect to $t$,

$$\frac{ds}{dt} = 3t^2 - 4t \qquad ...(ii)$$

*i.e.*, velocity, $v = 3t^2 - 4t$ $\quad ...\left(\because \frac{ds}{dt} = \text{Velocity}\right)$

substituting $t$ equal to 5 in the above equation,

$v = 3\,(5)^2 - (4 \times 5) = 55$ m/s **Ans.**

*Acceleration after 5 seconds*

Again differentiating equation (*ii*) with respect to $t$,

$$\frac{d^2s}{dt^2} = 6t - 4 \qquad ...(iii)$$

*i.e.* acceleration, $a = 6t - 4$ $\quad ...\left(\because \frac{d^2s}{dt^2} = \text{Acceleration}\right)$

Now substituting $t$ equal to 5 in the above equation,

$a = (6 \times 5) - 4 = 26$ m/s$^2$ **Ans.**

**Example 18.2.** *A car moves along a straight line whose equation of motion is given by* $s = 12t + 3t^2 - 2t^3$, *where (s) is in metres and (t) is in seconds. calculate*

(*i*) *velocity and acceleration at start, and*

(*ii*) *acceleration, when the velocity is zero.*

**Solution.** Given : Equation of displacement : $s = 12t + 3t^2 - 2t^3$ ...(*i*)

*Velocity at start*

Differentiating the above equation with respect to $t$,

$$\frac{ds}{dt} = 12 + 6t - 6t^2 \quad \text{...}(ii)$$

*i.e.* velocity, $v = 12 + 6t - 6t^2$ $\quad \text{...}\left(\because \frac{ds}{dt} = v\right)$

Substituting $t$ equal to 0 in the above equation,

$v = 12 + 0 - 0 = 12$ m/s **Ans.**

*Acceleration at start*

Again differentiating equation (*ii*) with respect to $t$ ,

$$\frac{dv}{dt} = 6 - 12t \quad \text{...}(iii)$$

*i.e.* accleration, $a = 6 - 12t$ $\quad \text{...}\left(\because \frac{dv}{dt} = a\right)$

Now substituting $t$ equal to 0 in the above equation,

$a = 6 - 0 = 6$ m/s$^2$ **Ans.**

*Acceleration, when the velocity is zero*

Substituting equation (*ii*) equal to zero

$12 + 6t - 6t^2 = 0$

$t^2 - t - 2 = 0$ ...(Dividing by – 6)

or $t = 2$ s

It means that velocity of the car after two seconds will be zero. Now substituting the value of $t$ equal to 2 in equation (*iii*),

$a = 6 - (12 \times 2) = -18$ m/s$^2$ **Ans.**

**Example 18.3.** *The equation of motion of a particle moving in a straight line is given by :*

$$s = 18t + 3t^2 - 2t^3$$

*where (s) is in metres and (t) in seconds. Find (1) velocity and acceleration at start, (2) time, when the particle reaches its maximum velocity, and (3) maximum velocity of the particle.*

**Solution.** Given : Equation of displacement : $s = 18t + 3t^2 - 2t^3$ ...(*i*)

(1) *Velocity and acceleration at start*

Differentiating equation (*i*) with respect to $t$ ,

$$\frac{ds}{dt} = 18 + 6t - 6t^2 \quad \text{...}(ii)$$

*i.e.* velocity, $v = 18 + 6t - 6t^2$

Substituting, $t$ equal to 0 in equation (*ii*),

$v = 18 + 0 + 0 = 18$ m/s **Ans.**

Again differentiating equation (*ii*) with respect to $t$,

$$\frac{d^2s}{dt^2} = 6 - 12t \quad \text{...}(iii)$$

*i.e.* acceleration , $a = 6 - 12t$ ...(*iv*)

Substituting $t$ equal to 0 in equation (*iv*),

$$a = 6 - 0 = 6 \text{ m/s}^2 \quad \textbf{Ans.}$$

(2) *Time, when the particle reaches its maximum velocity*

For maximum velocity, let us differentiate the equation of velocity and equate it to zero. The differentiation of the equation of velocity is given by equation (*iii*).

Therefore equating the equation (*iii*) to zero,

$$6 - 12t = 0$$

or
$$t = 1/2 = 0{\cdot}5 \text{ s} \qquad \textbf{Ans.}$$

(3) *Maximum velocity of the particle*

Substituting $t$ equal to 0·5 s in equation (*ii*),

$$v = 18 + (6 \times 0{\cdot}5) - 6\,(0{\cdot}5)^2 = 19{\cdot}5 \text{ m/s} \ \textbf{Ans.}$$

## EXERCISE 18.1

1. A particle, starting from rest, moves in a straight line whose equation of motion is given by :

$$s = 3t^3 - 2t$$

where ($s$) is in metres and ($t$) in seconds. Find (*i*) velocity after 3 seconds ; (*ii*) acceleration at the end of 3 seconds ; and (*iii*) average velocity in the 4th seconds.

(**Ans.** 83 m/s ; 54 m/s$^2$ ; 114.5 m/s)

2. A car moves along a straight line, whose equation of motion is given by $s = 12t + 3t^2 - 2t^3$, where ($s$) is in metres and ($t$) in seconds. Calculate (*i*) velocity and acceleration at start ; (*ii*) acceleration when velocity is zero. (**Ans.** 12 m/s, 6 m/s$^2$ ; – 18 m/s$^2$)

3. The equation of motion of an engine is given by $s = 2t^3 - 6t^2 - 5$, where ($s$) is in metres and ($t$) in seconds. Calculate (*i*) displacement and acceleration when velocity is zero ; and (*ii*) displacement and veiocity when acceleration is zero.

(**Ans.** – 13 m ; 12 m/s$^2$ ; – 9m ; – 6 m/s)

## 18.5. VELOCITY AND DISPLACEMENT BY INTEGRATION


Motion in terms of acceleration and time.

Sometimes, the given equation of motion is in terms of acceleration ($a$) and time ($t$) *e.g.*

$$a = 4t^3 - 3t^2 + 5t + 6 \qquad ...(i)$$

$$*\frac{dv}{dt} = t^3 + 8t \qquad ...(ii)$$

$$*v\frac{dv}{ds} = 6 + 3t \qquad ...(iii)$$

$$*\frac{d^2s}{dt^2} = 4t - 8t^2 \qquad ...(iv)$$

Now integrating both sides of the above equations,

$$= \frac{t^4}{4} + \frac{8t^2}{2} + C_1 \qquad ...(i)$$

$$= 6t + \frac{3t^2}{2} + C_1 \qquad ...(ii)$$

$$= \frac{4t^2}{2} - \frac{8t^3}{3} + C_1 \qquad ...(iii)$$

where $C_1$ is the first constant of integration. The equations, so obtained, give the velocity of the body. Again integrating both sides of the above equations.

$$= \frac{t^5}{20} + \frac{8t^3}{6} + C_1t + C_2$$

$$= \frac{6t^2}{2} + \frac{3t^3}{6}\ C_1t + C_2$$

$$= \frac{4t^3}{6} - \frac{8t^4}{12} + C_1t + C_2$$

where $C_2$ is the second constant of integration. The equations, so obtained, give the displacement of the body.

It may be noticed that the method for velocity and displacement by integration is somewhat difficult, as we have to find out the values of constants of integration (*i.e.* $C_1$ and $C_2$ )

**Example 18.4.** *The motion of a particle is given by :*

$$a = t^3 - 3t^2 + 5$$

*where (a) is the acceleration in m/s² and (t) is the time in seconds. The velocity of the particle at t = 1 second is 6.25 m/sec and the displacement is 8.8 metres.*

*Calculate the displacement and velocity at t = 2 seconds.*

**Solution.** Given : Equation of acceleration : $a = t^3 - 3t^2 + 5$

Rewriting the given equation,

or $$\frac{dv}{dt} = t^3 - 3t^2 + 5 \qquad ...\left(\because\ a = \frac{dv}{dt}\right)$$

$\therefore$ $$dv = (t^3 - 3t^2 + 5)\,dt \qquad ...(i)$$

*Velocity at t = 2 seconds*

Integrating both sides of equation (*i*),

$$v = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1$$

$$= \frac{t^4}{4} - t^3 + 5t + C_1 \qquad ...(ii)$$

* These are the different forms of acceleration. A little consideration is very essential to use the proper form. As a thumb rule, if the variable acceleration is a function of *t*, then equation (*i*) or (*ii*) is used. But if it is a function of *s*, then equation (*iii*) or (*iv*) is used.

where $C_1$ is the first constant of integration. Substituting the values of $t = 1$ and $v = 6.25$ in equation (*ii*),

$$6.25 = \frac{1}{4} - 1 + 5 + C_1 = 4.25 + C_1$$

$$\therefore \qquad C_1 = 6.25 - 4.25 = 2$$

Substituting this value of $C_1$ in equation (*ii*),

$$v = \frac{t^4}{4} - t^3 + 5t + 2 \qquad ...(iii)$$

Now for velocity of the particle, substituting the value of $t = 2$ in the above equation ,

$$v = \frac{(2)^4}{4} - (2)^3 + (5 \times 2) + 2 = 8 \text{ m/s} \quad \textbf{Ans.}$$

*Displacement at t = 2 seconds*

Rewriting equation (*iii*),

$$\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2 \qquad ...\left(\because v = \frac{ds}{dt}\right)$$

$$\therefore \qquad ds = \left(\frac{t^4}{4} - t^3 + 5t + 2\right) dt \qquad ...(iv)$$

Integrating both sides of equation, (*iv*)

$$s = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2 \qquad ...(v)$$

where $C_2$ is the second constant of integration. Substituting the values of $t = 1$ and $s = 8.8$ in equation (*v*),

$$8.8 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + C_2 = 4.3 + C_2$$

$$\therefore \qquad C_2 = 8.8 - 4.3 = 4.5$$

Substituting this value of $C_2$ in equation (*v*) ,

$$s = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 4.5$$

Now for displacement of the particle, substituting the value of $t = 2$ in the above equation,

$$s = \frac{32}{20} - \frac{16}{4} + \frac{20}{2} + 4 + 4.5 = 16.1 \text{ m} \quad \textbf{Ans.}$$

**Example 18.5.** *A train, starting from rest, is uniformly accelerated. The acceleration at any instant is* $\frac{10}{v+1}$ *m/s²*, *where (v) is the velocity of the train in m/s at the instant. Find the distance, in which the train will attain a velocity of 35 km. p.h.*

**Solution.** Given : Equation of acceleration : $a = \frac{10}{v+1}$

Rewriting the given equation,

$$v \cdot \frac{dv}{ds} = \frac{10}{v+1} \qquad ...\left(\because a = v \cdot \frac{dv}{ds}\right)$$

$$\therefore \qquad v\,(v + 1)\,dv = 10\,ds \qquad ...(i)$$

$$\text{or} \qquad (v^2 + v)\,dv = 10\,ds$$

Integrating both sides of equation (*i*),

$$\frac{v^3}{3} + \frac{v^2}{2} = 10s + C_1 \quad ...(ii)$$

where $C_1$ is the first constant of integration. Substituting the values of $s = 0$ and $v = 0$ in equation (*ii*),

$$C_1 = 0$$

Substituting this value of $C_1 = 0$ in equation (*ii*),

$$\frac{v^3}{3} + \frac{v^2}{2} = 10s$$

$$\therefore \quad 2v^3 + 3v^2 = 60\,s \quad ...(iii)$$

Now for distance travelled by the train, substituting $v = 36$ km.p.h. or 10 m/s in equation (*iii*),

$$2\,(10)^3 + 3\,(10)^2 = 60\,s \text{ or } 2000 + 300 = 60\,s$$

$$s = \frac{2300}{60} = 38.3 \text{ m} \quad \textbf{Ans.}$$

**Example 18.6.** *A particle, starting from rest, moves in a straight line, whose acceleration is given by the equation :*

$$a = 10 - 0.006\,s^2$$

*where (a) is in m/s² and (s) in metres. Determine*

(*i*) *velocity of the particle,when it has travelled 50 metres.*

(*ii*) *distance travelled by the particle, when it comes to rest.*

**Solution.** Given : Equation of acceleration : $a = 10 - 0.006\ s^2$

Rewriting the given equation,

$$v\,.\frac{dv}{ds} = 10 - 0.006\,s^2 \quad ...\left(\because a = v\,.\frac{dv}{ds}\right)$$

$$\therefore \quad v.dv = (10 - 0.006\,s^2)\,ds \quad ...(i)$$

(*a*) *Velocity of the particle, when it has travelled 50 metres*

Integrating both sides, of equation (*i*),

$$\frac{v^2}{2} = 10s - \frac{0.006\,s^3}{3} + C_1 = 10s - 0.002s^3 + C_1$$

or

$$v^2 = 20s - 0.004\,s^3 + 2C_1 \quad ...(ii)$$

where $C_1$ is the first constant of integration. Substituting the values of $s = 0$ and $v = 0$ in equation (*ii*),

$$C_1 = 0$$

Substituting this value of $C_1$ in equation (*ii*),

$$v^2 = 20s - 0.004\,s^3 \quad ...(iii)$$

Now for velocity of the particle, substituting $s = 50$ m in equation (*iii*),

$$v^2 = 20\,(50) - 0.004\,(50)^3 = 1000 - 500 = 500$$

$$\therefore \quad v = \sqrt{500} = 22.36 \text{ m/s} \quad \textbf{Ans.}$$

(*b*) *Distance travelled by the particle, when it comes to rest.*

When the particle comes to rest, the velocity will be zero. Therefore substituting $v = 0$ in equation (*iii*),

$$20s - 0.004s^3 = 0 \quad \text{or} \quad s\,(20 - 0.004s^2) = 0$$

Therefore either $s = 0$ or $(20 - 0.004s^2) = 0$. A little consideration will show that when $s = 0$ the body is in its initial stage.

$$\therefore \quad 20 - 0.004s^2 = 0$$

or
$$s = \sqrt{\frac{20}{0.004}} = \sqrt{5000} = 70.7 \text{ m} \quad \textbf{Ans.}$$

**Example 18.7.** *A body moves along a straight line and its acceleration (a) which varies with time (t) is given by a = 2 – 3t. After 5 seconds , from start of observations, its velocity is observed to be 20 m/s. After 10 seconds, from start of observation, the body was at 85 metres from the origin. Determine*

(*a*) *its acceleration and velocity at the time of start*

(*b*) *distance from the origin at the start of observations,*

(*c*) *the time after start of observation in which the velocity becomes zero.*

**Solution.** Given : Equation of acceleration : $a = 2 - 3t$ ...(*i*)

(*a*) *Accelertion and velocity at the time of start*

Substituting the value of $t$ equal to 0 in the given equation (*i*),

$$a = 2 \text{ m/s}^2 \quad \textbf{Ans.}$$

Rewriting the given equation (*i*),

$$\frac{dv}{dt} = 2 - 3t \qquad \ldots\left(\because\ a = \frac{dv}{dt}\right)$$

$$\therefore \quad dv = (2 - 3t)\, dt \qquad \ldots(ii)$$

Integrating both sides of equation (*ii*) ,

$$v = 2t - \frac{3t^2}{2} + C_1 \qquad \ldots(iii)$$

where $C_1$ is the first constant of integration. Substituting the values of $t = 5$ and $v = 20$ in equation (*iii*),

$$20 = 2 \times 5 - \frac{3}{2}(5)^2 + C_1 = C_1 - 27.5$$

or
$$C_1 = 20 + 27.5 = 47.5$$

Substituting this value of $C_1$ in equation (*iii*) ,

$$v = 2t - \frac{3t^2}{2} + 47.5 \qquad \ldots(iv)$$

Now for velocity of the body at the time of start, substituting $t = 0$ in equation (*iv*),

$$v = 47.5 \text{ m/s} \quad \textbf{Ans.}$$

(*b*) *Distance from the origin at the start of observation*

Rewriting equation (*iv*),

$$\frac{ds}{dt} = 2t - \frac{3t^2}{2} + 47.5 \qquad \ldots\left(\because\ v = \frac{ds}{dt}\right)$$

$$\therefore \quad ds = \left(2t - \frac{3t^2}{2} + 47.5\right) dt$$

Integrating both sides of the above equation,

$$s = \frac{2t^2}{2} - \frac{3t^3}{6} + 47.5t + C_2 = t^2 - \frac{t^3}{2} + 47.5t + C_2 \qquad ...(v)$$

where $C_2$ is the second constant of integration. Now substituting the values of $t = 10$ and $s = 85$ in above equation,

$$85 = (10)^2 - \frac{(10)^3}{2} + 47.5 \times 10 + C_2 = 75 + C_2$$

$$\therefore \qquad C_2 = 85 - 75 = 10$$

Substituting this value of $C_2$ in equation ($v$),

$$s = t^2 - \frac{t^3}{2} + 4.75t + 10$$

Now for the distance from the origin at the time of start of observation , substituting $t$ equal to 0 in the above equation,

$s = 10$ m **Ans.**

($c$) *Time after start of observations in which the velocity becomes zero*

Substituting the value of $v$ equal to 0 in equation ($iv$),

$$0 = 2t - \frac{3t^2}{2} + 47.5$$

Multiplying both sides by – 2 and rearranging

$$3t^2 - 4t - 95 = 0$$

This is a quadratic equation in $t$,

$$\therefore \qquad t = \frac{+4 \pm \sqrt{(4)^2 + 4 \times 3 \times 95}}{2 \times 3} = 6.33 \text{ s} \qquad \textbf{Ans.}$$

## EXERCISE 18.2

**1.** The motion of a body is given by an equation :

$$a = t^2 - 2t + 2$$

where $a$ is acceleration in m/s$^2$ and $t$ is time in seconds.The velocity and displacement of the body after 1 second was $6\frac{1}{3}$ m/s and $14\frac{3}{4}$ m respectively. Find the velocity and displacement after 2 seconds. (**Ans.** $7\frac{2}{3}$ m/s ; $21\frac{2}{3}$ m )

**2.** A body starting from rest, moves along a straight line with an acceleration whose equation is given by :

$$a = 4 - \frac{t^2}{9}$$

where $a$ is in m/s$^2$ and $t$ in seconds. Find ($a$) velocity after 6 seconds, and ($b$) distance traversed in 6 seconds. (**Ans.** 16 m/s ; 60 m )

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**3.** A body starting from rest, moves in such a way that its acceleration is given by :

$$a = 3 - 0.15\,t^2$$

Find the time when the body comes to stop and distance travelled during this time.

(**Ans.** 7.75 s ; 45 m )

**4.** A car moving with a velocity of 10 m/s shows down in such a manner that the relation between velocity and time is given by :

$$v = 10 - t^2 - \frac{t^3}{2}$$

Find the distance travelled in two seconds, average velocity and average retardation of the car in these two seconds. (**Ans.** 16.67 m ; 8.33 m/s ; 4 m/s$^2$ )

## 18.6. VELOCITY, ACCELERATION AND DISPLACEMENT BY PREPARING A TABLE

Sometimes, the motion of a body is given in a tabular form containing time ($t$) and distance ($s$) or time ($t$) and acceleration ($a$) *e.g.*

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $s$ | 8 | 20 | 35 | 55 | 80 | 110 |

Or

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $a$ | 0.5 | 1.0 | 1.5 | 0.9 | 0.6 | 0 |

In such a case, the velocity, acceleration and displacement of the body may be easily found out by preparing a table, showing the other details of the motion (*i.e.,*average acceleration, increase in velocity and final velocity etc.).

**Example 18.8.** *An electric train has velocity in m/s as shown in the following table :*

| *t* | *0* | *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|---|---|
| *v* | *40* | *39* | *36* | *31* | *24* | *15* | *4* |

*Find the distance travelled by the train in the last 3 seconds.*

**Solution.** In the first sec, mean velocity of the train

$$= \frac{40 + 39}{2} = 39.5 \text{ m/s}$$

∴ Distance travelled in this sec

$$= 1 \times 39.5 = 39.5$$

Similarly, in the next sec, mean velocity of the train

$$= \frac{39 + 36}{2} = 37.5 \text{ m/s}$$

∴ Distance travelled in this sec

$$= 1 \times 37.5 = 37.5 \text{ m}$$

and total distance travelled upto the end of 2 sec

$$= 39.5 + 37.5 = 77 \text{ m}$$

Similarly, find the distances travelled by the train at the end of each sec, and prepare the table as given below :

| $t$ | $v$ | $\delta t$ $(t_2 - t_1)$ | $v_{av}$ $\left(\frac{v_2 + v_1}{2}\right)$ | *Distance travelled* in $\delta t = \delta t \times v_{av}$ | *Total distance travelled* in metres |
|---|---|---|---|---|---|
| 0 | 40 | | | | |
| | | 1 | 39.5 | 39.5 | |
| 1 | 39 | | | | 39.5 |
| | | 1 | 37.5 | 37.5 | |
| 2 | 36 | | | | 77.0 |
| | | 1 | 33.5 | 33.5 | |
| 3 | 31 | | | | 110.5 |
| | | 1 | 27.5 | 27.5 | |
| 4 | 24 | | | | 138.0 |
| | | 1 | 19.5 | 19.5 | |
| 5 | 15 | | | | 157.5 |
| | | 1 | 9.5 | 9.5 | |
| 6 | 4 | | | | 167.0 |

From the last column of the table, we find that distance travelled by the train in the last 3 seconds

= Distance travelled in 6 sec – Distance travelled in the first 3 sec

= 167.0 – 110.5 m = 56.5 m **Ans.**

**Example 18.9.** *An automobile starting from rest, moves along a straight line. Its acceleration after every 10 m distance was observed to be as below :*

| *s* | *0* | *10* | *20* | *30* | *40* | *50* |
|---|---|---|---|---|---|---|
| *a* | *2.2* | *2.4* | *2.8* | *2.0* | *1.6* | *1.0* |

*Find the velocity of the automobile at the* end of *45 metres.*

**Solution.** In the first 10 m the mean acceleration

$$= \frac{2.2 + 2.4}{2} = 2.3 \text{ m/s}^2$$

Substituting initial velocity $(u) = 0$, acceleration $(a) = 2.3$ m/s$^2$ and distance travelled $(s) = 10$ in the equation of motion *i.e.* $v^2 = u^2 + 2as$,

$$v^2 = (0)^2 + (2 \times 2.3 \times 10) = 46 \quad \text{or} \quad v = 6.78 \text{ m/s}$$

Similarly, in the next 10 m, the mean acceleration

$$= \frac{2.4 + 2.8}{2} = 2.6 \text{ m/s}^2$$

and now substituting initial velocity ($u$) = 6.78 m/s, acceleration ($a$) = 2.6 m/s$^2$ and distance travelled ($s$) = 10 m in the equation of motion *i.e.* $v^2 = u^2 + 2as$,

$$v^2 = (6.78)^2 + (2 \times 2.6 \times 10) = 98 \qquad \text{or} \qquad v = 9.9 \text{ m/s}$$

Similarly, calculate the mean accelerations and velocities of the automobile at the end of each 10 metres and prepare the table as shown below.

| $s$ | $a$ | $a_{av}$ $\frac{a_1 + a_2}{2}$ | $\delta s$ $s_1 - s_2$ | $u^2 + 2as$ | $v$ |
|---|---|---|---|---|---|
| 0 | 2.2 | | | | |
| | | 2.3 | 10 | 46 | 6.78 |
| 10 | 2.4 | | | | |
| | | 2.6 | 10 | 98 | 9.9 |
| 20 | 2.8 | | | | |
| | | 2.4 | 10 | 146 | 12.08 |
| 30 | 2.0 | | | | |
| | | 1.8 | 10 | 182 | 13.49 |
| 40 | 1.6 | | | | |
| | | 1.3 | 10 | 218 | 14.76 |
| 50 | 1.0 | | | | |

From the above table, we find that the average velocity of the automobile between 40 and 50 or 45 m of its start is 14.76 m/s **Ans.**

**Example 18.10.** *A car, starting from rest has an acceleration a in m/s$^2$ after t second from its start as given in the following table*:

| $t$ | 0 | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|
| $a$ | 10 | 9.6 | 8.4 | 0.4 | 3.6 | 0 |

*Find the speed of the car at the end of each 4 sec interval and the distance traversed.*

**Solution.** In the first 4 sec the mean acceleration

$$\frac{10 + 9.6}{2} = 9.8 \text{ m/s}^2$$

∴ Increase in velocity during these 4 sec

$$= 9.8 \times 4 = 39.2 \text{ m/s}$$

and velocity at the end of 4 sec

$$= 0 + 39.2 = 39.2 \text{ m/s}$$

∴ Average velocity in the first 4 sec

$$= \frac{0 + 39.2}{2} = 19.6 \text{ m/s}$$

and distance traversed in the first 4 sec

$$= 19.6 \times 4 = 78.4 \text{ m}$$

∴ Distance traversed up to the end of 4 sec.

$$= 0 + 78.4 = 78.4 \text{ m}$$

Similarly, in the next 4 sec the mean acceleration

$$= \frac{9.6 + 8.4}{2} = 9 \text{ m/s}^2$$

$\therefore$ Increase in velocity during these 4 sec

$$= 9 \times 4 = 36 \text{ m/s}$$

and velocity at the end of these 4 sec

$$= 39.2 + 36 = 75.2 \text{ m/s}$$

$\therefore$ Average velocity in these 4 sec

$$= \frac{39.2 + 75.2}{2} = 57.2 \text{ m/s}.$$

and distance traversed in these 4 sec

$$= 57.2 \times 4 = 228.8 \text{ m}$$

$\therefore$ Distance traversed up to the end of these 4 sec

$$= 78.4 + 228.8 = 307.2 \text{ m}$$

Similarly, calculate the mean accelerations, velocities and distances traversed by the car at the end of each 4 sec and prepare the table as shown in the following table.

| $t$ | $a$ | $a_{av}$ $\frac{a_1 + a_2}{2}$ | $\delta t$ $(t_2 - t_1)$ | *increase in v* $(a_{av} \times t)$ | $v$ | $v_{av}$ $\frac{v_1 + v_2}{2}$ | $\delta s$ $(v_{av} \times \delta t)$ | $s$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 10 | | | | 0 | | | 0 |
| | | 9.8 | 4 | 39.2 | | 19.6 | 78.4 | |
| 4 | 9.6 | | | | 39.2 | | | 78.4 |
| | | 9.0 | 4 | 36.0 | | 57.2 | 228.8 | |
| 8 | 8.4 | | | | 75.2 | | | 307.2 |
| | | 7.4 | 4 | 29.6 | | 90.0 | 360.0 | |
| 12 | 6.4 | | | | 104.8 | | | 667.2 |
| | | 5.0 | 4 | 20.0 | | 114.8 | 459.2 | |
| 16 | 3.6 | | | | 124.8 | | | 1126.4 |
| | | 1.8 | 4 | 7.2 | | 128.4 | 513.6 | |
| 20 | 6 | | | | 132.0 | | | 1640.0 |

In the above table is given the speed of car and distance traversed at the end of each 4 seconds. **Ans.**

**Example 18.11.** *A car starts from rest, and moves along a straight line. The distance covered (s) in seconds (t) from the start, were observed to be as under :*

| *t* | *0* | *5* | *10* | *15* | *20* | *25* | *30* |
|---|---|---|---|---|---|---|---|
| *s* | *0* | *20* | *100* | *230* | *330* | *380* | *400* |

*Calculate the velocity and acceleration of the car after 10 and 20 seconds from start.*

**Solution.** In the first 5 seconds, the distance covered = 20 m

$\therefore$ Average velocity after 2.5 seconds of start (or in all 5 seconds from 0 to 5)

$$= 20/5 = 4 \text{ m/s}$$

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Similarly, in the next 5 seconds, the distance covered

$$= 100 - 20 = 80 \text{ m}$$

∴ Average velocity after 7.5 seconds of start (or in all 5 seconds from 5 to 10)

$$= 80/5 = 16 \text{ m/s}$$

and rate of increase in average velocity (or acceleration ) in 5 seconds (from 2.5 to 7.5 seconds)

$$= \frac{16 - 4}{5} = 2.4 \text{ m/s}^2$$

Similarly, calculate the distances covered, average velocities and accelerations in each 5 seconds and prepare the table as shown below.

| $t$ | $s$ | $\delta t$ $(t_2 - t_1)$ | $\delta s$ $(s_2 - s_1)$ | $v = \frac{\delta s}{\delta t}$ | $v_2 - v_1$ | $a = \frac{v_2 - v_1}{t_2 - t_1}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | | | | |
| | | 5 | 20 | 4 | | |
| 5 | 20 | | | | 12 | 12/5 = 2.4 |
| | | 5 | 80 | 16 | | |
| 10 | 100 | | | | 10 | 10/5 = 2.0 |
| | | 5 | 130 | 26 | | |
| 15 | 230 | | | | – 6 | – 6/5 = – 1.2 |
| | | 5 | 100 | 20 | | |
| 20 | 330 | | | | – 10 | – 10/5 = – 2.0 |
| | | 5 | 50 | 10 | | |
| 25 | 380 | | | | – 6 | – 6/5 = – 1.2 |
| | | 5 | 20 | 4 | | |
| 30 | 400 | | | | | |

*Velocity of the car after 10 and 20 seconds from start*

From the average velocity column, we find that average velocity of the car after 7.5 seconds

$$= 16 \text{ m/s}$$

and average velocity of the car after 12.5 seconds

$$= 26 \text{ m/s}$$

∴ Average velocity of the car after 10 seconds from start

$$= \frac{16 + 26}{2} = 21 \text{ m/s} \quad \textbf{Ans.}$$

Similarly, velocity of the car after 20 seconds from start

$$= \frac{20 + 10}{2} = 15 \text{ m/s} \quad \textbf{Ans.}$$

*Acceleration of the car after 10 and 20 seconds from start*

From the acceleration (*i.e.* last) column, we find that acceleration of the car after 10 and 20 seconds from start is 2.0 m/s$^2$ and – 20 m/s$^2$ respectively. **Ans.**

## EXERCISE 18.3

**1.** A tram car, starting with an initial velocity of 7.5 m/s moves with a variable acceleration. The time-acceleration chart is given below :

| $t$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| $a$ | 0.4 | 0.9 | 1.1 | 1.8 | 1.9 | 2.3 | 2.5 | 1.9 | 1.1 |

Find the velocity of the car after 40 seconds. (**Ans.** 73.25 m/s)

**2.** Abody starts moving along a straight line with an initial velocity of 8 m/s. The acceleration in m/s$^2$ at intervals of 5 seconds were observed to be as under :

| $t$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| $a$ | 0.2 | 0.8 | 1.2 | 1.6 | 2.0 | 1.2 | 0 |

Find the distance travelled in 30 seconds from the start. (**Ans.** 723.75)

## QUESTIONS

**1.** How would you distinguish between the motion, when it is subjected to (*a*) constant acceleration, and (*b*) variable acceleration ?

**2.** Under what circumstances you would differentiate or integrate the given equation of motion of a particle to obtain, velocity, acceleration and displacement ?

**3.** If ($s$) is the distance traversed by a particle, then what does $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ represent ?

**4.** When would you prefer to prepare a table of motion of a moving body ?

## OBJECTIVE TYPE QUESTIONS

**1.** We are given an equation of displacement ($s$) in terms of time ($t$). If we differentiate it with respect to $t$, the equation so obtaned will give

(*a*) velocity (*b*) acceleration (*c*) distance traversed

**2.** The second differentiation, of the above equation will give

(*a*) velocity (*b*) acceleration (*c*) distance traversed

**3.** If we differentiate an equation in terms of acceleration and time, it will give

(*a*) velocity (*b*) distance traversed (*c*) none of these two

**4.** We are given an equation of acceleration ($a$) in terms of time ($t$). The second integration of the equation will give the velocity.

(*a*) Yes (*b*) No

**5.** Which of the following statement is wrong ?

(*a*) A body falling freely under the force of gravity is an exapmle of motion under variable acceleration.

(*b*) A bus going down the valley may have variable acceleration.

(*c*) A lift going down in a gold mine cannot have constant acceleration in the entire journey.

(*d*) In a cricket match, the ball does not move with constant acceleration.

## ANSWERS

**1.** (*a*) **2.** (*b*) **3.** (*c*) **4.** (*b*) **5.** (*a*)