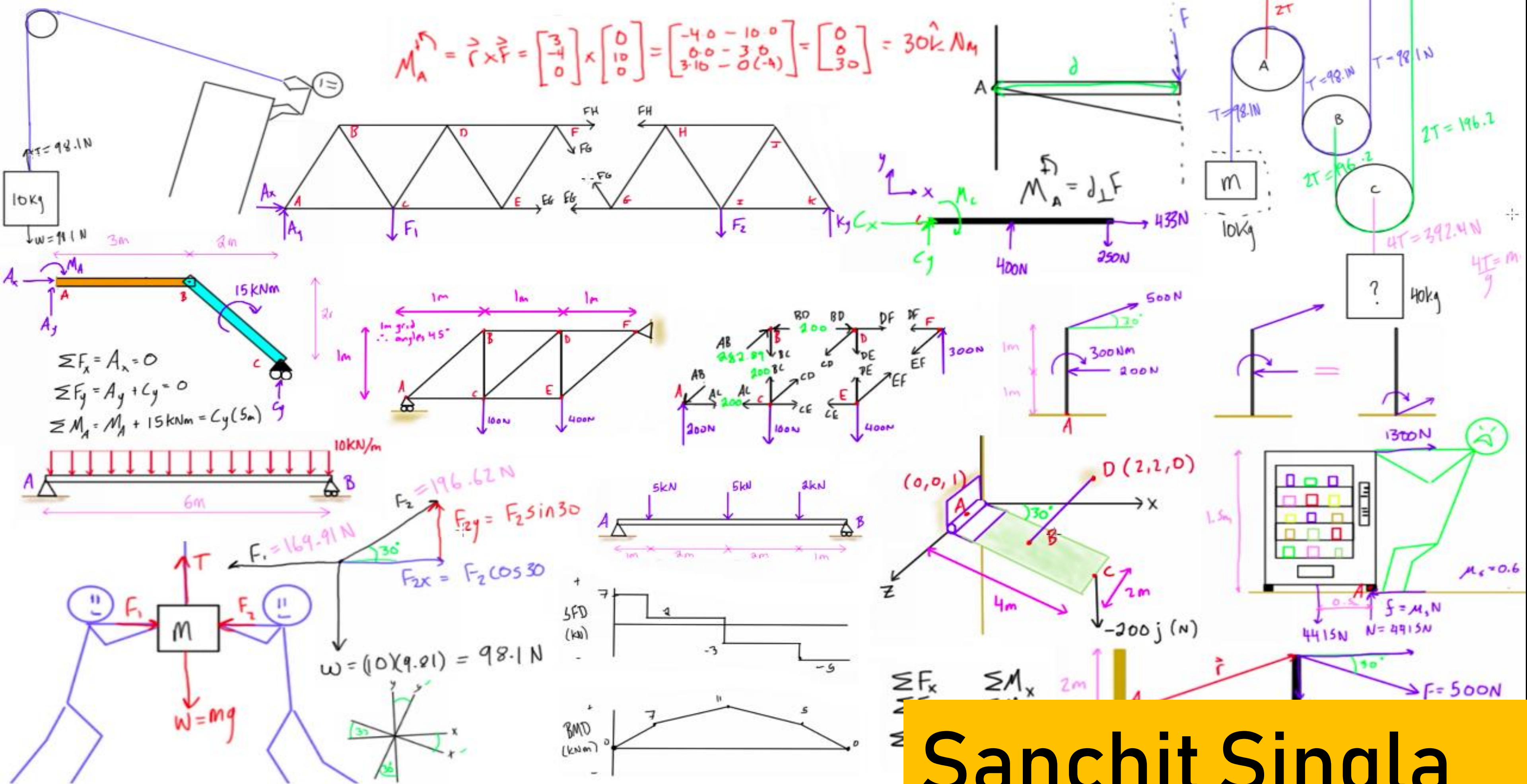

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MEC107

Basic Engineering Mechanics

Learning Outcomes

After this lecture, you will be able to

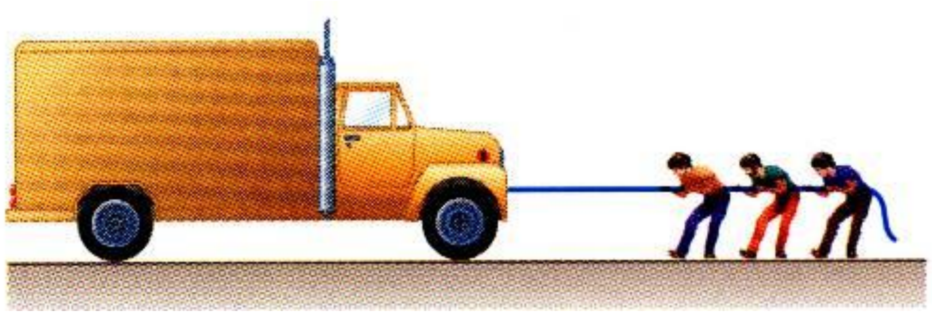
- ✓ understand about external and internal forces
- ✓ know about principle of transmissibility
- ✓ know vector product.
- ✓ learn about moment of forces about a point.
- ✓ understand about rectangle components of the MoF.

Introduction

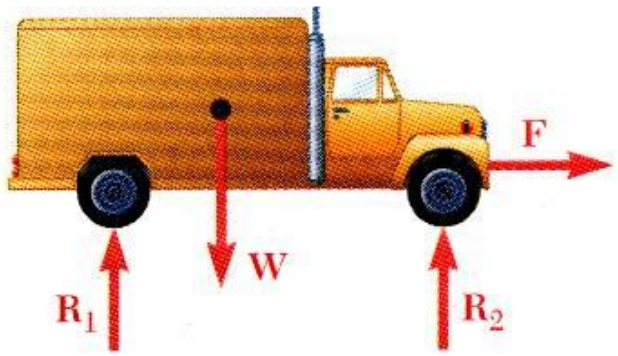
- Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered.
- Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body.
- Topics describes the effect of forces exerted on a rigid body and how to replace a given system of forces with a simpler equivalent system.
  - moment of a force about a point
  - moment of a force about an axis
  - moment due to a couple
- Any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple.

External and Internal Forces

- Forces acting on rigid bodies are divided into two groups:
    - External forces
    - Internal forces



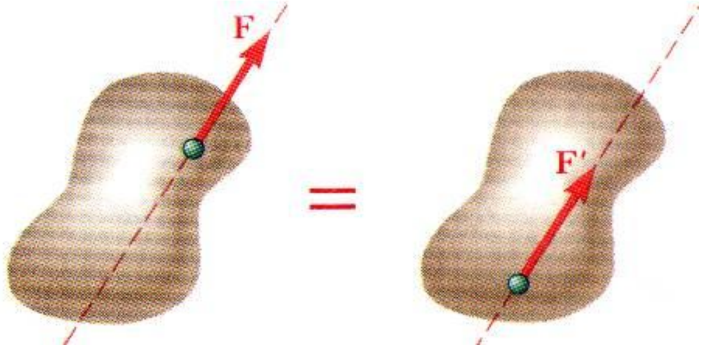
- External forces are shown in a free-body diagram.



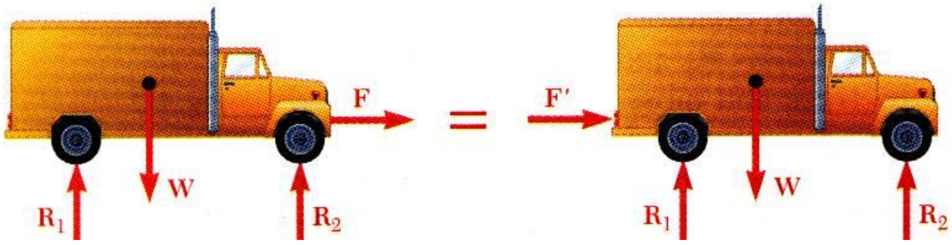
- If unopposed, each external force can impart a motion of translation or rotation, or both.

Principle of Transmissibility

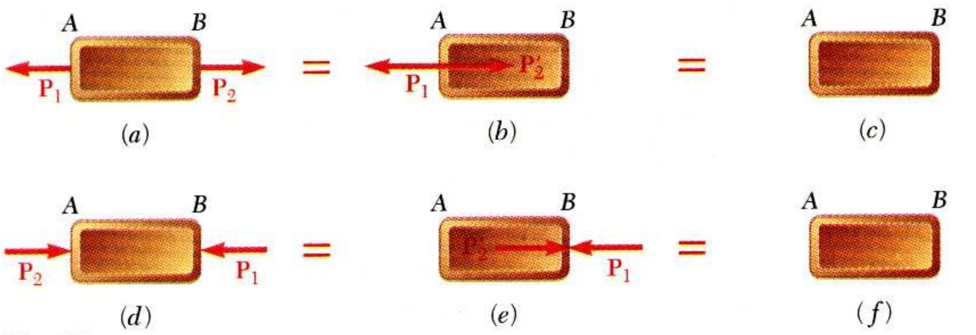
- *Principle of Transmissibility* - Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
  NOTE: **F** and **F'** are equivalent forces.



- Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck.

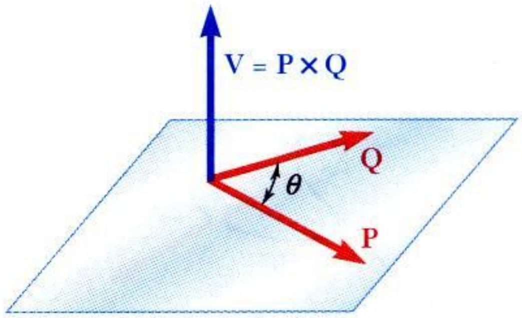


- Principle of transmissibility may not always apply in determining internal forces and deformations.



Vector Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.
- Vector product of two vectors ***P*** and ***Q*** is defined as the vector ***V*** which satisfies the following conditions:
  1. Line of action of ***V*** is perpendicular to plane containing ***P*** and ***Q***.
  2. Magnitude of ***V*** is $V = PQ \sin\theta$
  3. Direction of ***V*** is obtained from the right-hand rule.



(a)

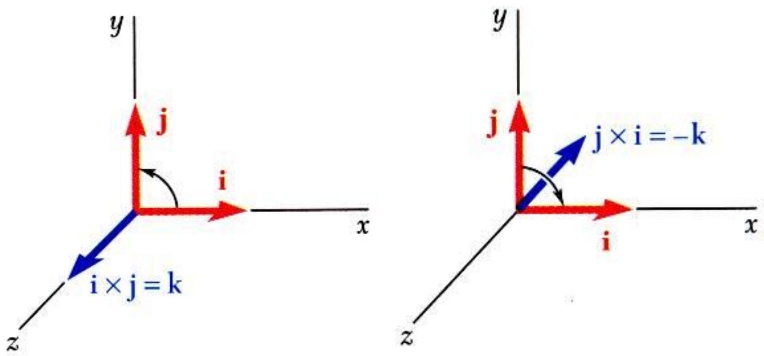
(b)

- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \times \boldsymbol{Q}_1 + \boldsymbol{P} \times \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} \neq \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

Vector Product : Rectangular Components

- Vector products of Cartesian unit vectors,

$$
\begin{array}{lll}
\vec{i} \times \vec{i} = 0 & \vec{j} \times \vec{i} = -\vec{k} & \vec{k} \times \vec{i} = \vec{j} \\
\vec{i} \times \vec{j} = \vec{k} & \vec{j} \times \vec{j} = 0 & \vec{k} \times \vec{j} = -\vec{i} \\
\vec{i} \times \vec{k} = -\vec{j} & \vec{j} \times \vec{k} = \vec{i} & \vec{k} \times \vec{k} = 0
\end{array}
$$



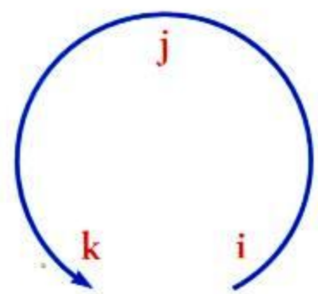
- Vector products in terms of rectangular coordinates

$$\vec{V} = \left(P_x\vec{i} + P_y\vec{j} + P_z\vec{k}\right) \times \left(Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}\right)$$

$$= \left(P_yQ_z - P_zQ_y\right)\vec{i} + \left(P_zQ_x - P_xQ_z\right)\vec{j}$$

$$+ \left(P_xQ_y - P_yQ_x\right)\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$



Moment of Force About a Point

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on it point of application.
- The *moment* of $\boldsymbol{F}$ about O is defined as

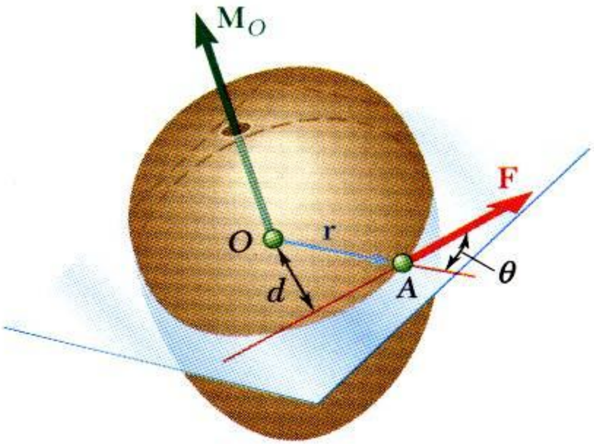
$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$$

- The moment vector $\boldsymbol{M}_O$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.
- Magnitude of $\boldsymbol{M}_O$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M}_O$.

$$M_O = rF\sin\theta = Fd$$

  The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F'}$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.

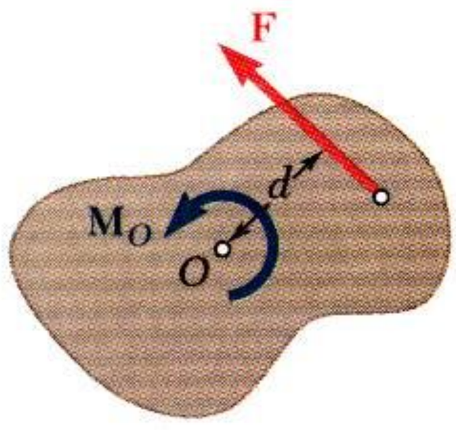


(a)

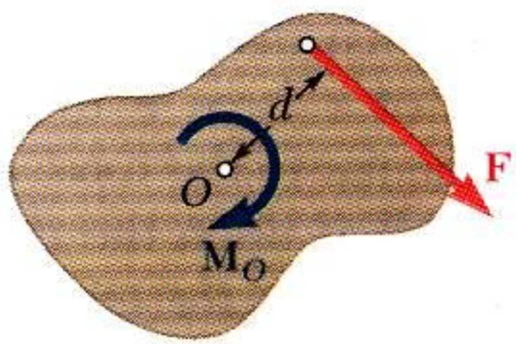
(b)

Moment of Force About a Point

- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.
- *The plane of the structure contains the point O and the force F. MO, the moment of the force about O is perpendicular to the plane.*
- *If the force tends to rotate the structure clockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive.*
- *If the force tends to rotate the structure counterclockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.*



(a) $\mathbf{M}_O = +Fd$



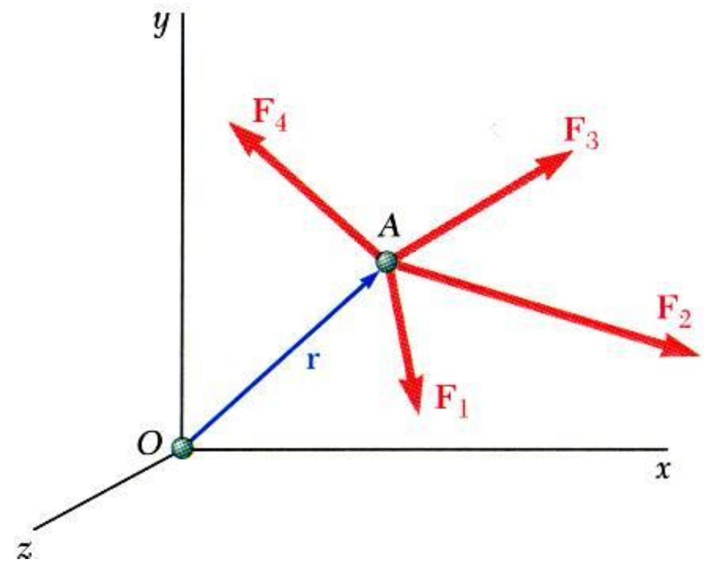
(b) $\mathbf{M}_O = -Fd$

Varignon's Theorem

- The moment about a give point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O.

$$\vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$

- Varigon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.



Rectangular Components of the Moment of a Force

The moment of ***F*** about *O*,

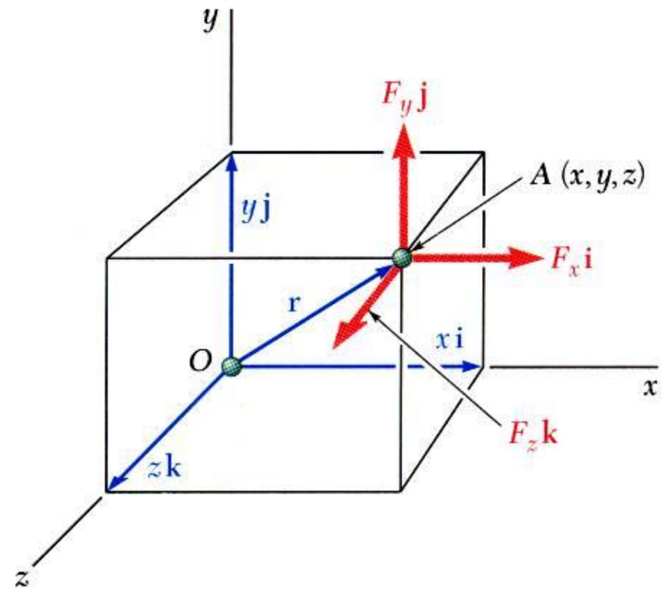
$$\vec{M}_O = \vec{r} \times \vec{F}, \qquad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)\vec{i} + \left(zF_x - xF_z\right)\vec{j} + \left(xF_y - yF_x\right)\vec{k}$$



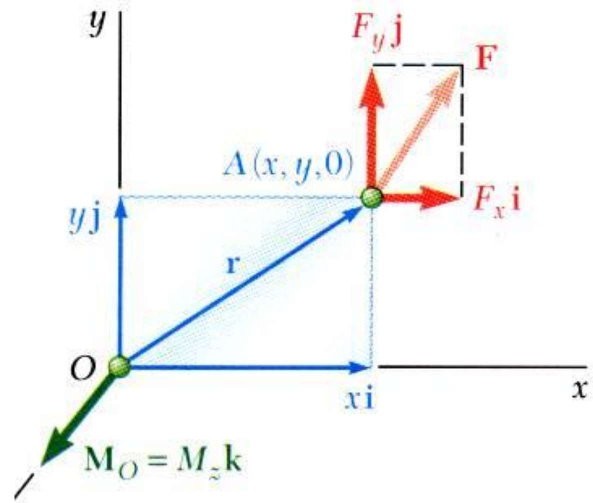
Rectangular Components of the Moment of a Force

For two-dimensional structures,

$$\vec{M}_O = \left(xF_y - yF_z\right)\vec{k}$$

$$M_O = M_Z$$

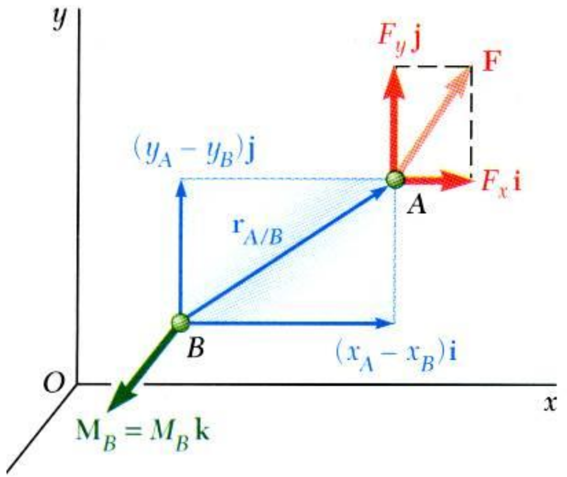
$$= xF_y - yF_z$$



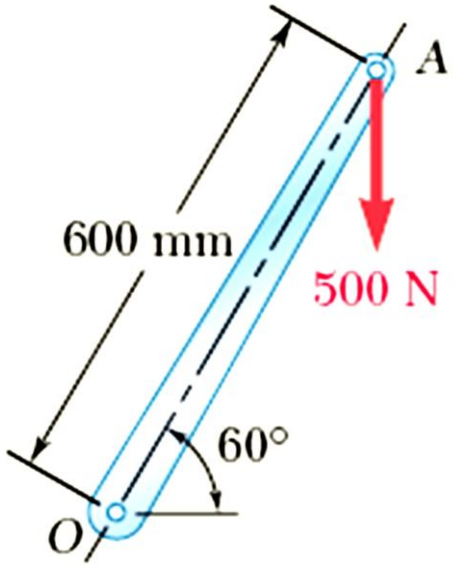
$$\vec{M}_O = \left[\left(x_A - x_B\right)F_y - \left(y_A - y_B\right)F_z\right]\vec{k}$$

$$M_O = M_Z$$

$$= \left(x_A - x_B\right)F_y - \left(y_A - y_B\right)F_z$$



Numerical



A 500-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

a) moment about *O*,

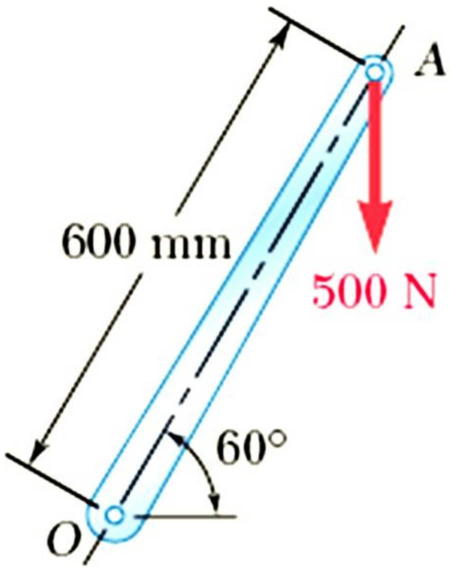
b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 1200-N vertical force to produce the same moment,

e) whether any of the forces from b, c, and d is equivalent to the original force.

Numerical



SOLUTION:

a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

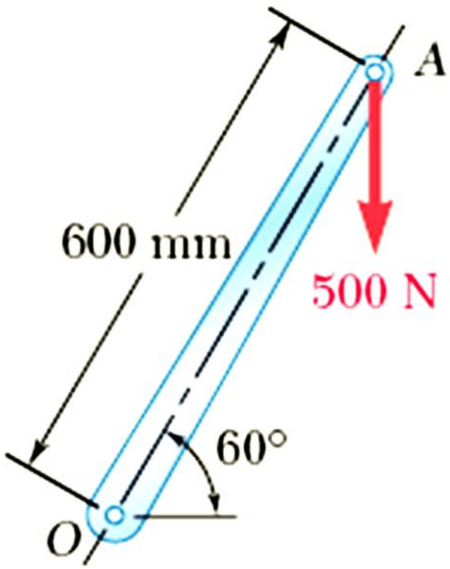
$$M_O = Fd$$

$$d = (600mm)\cos 60° = 300mm = 0.3m$$

$$M_O = (500N)(0.3\,\mathrm{m})$$

$$\boxed{M_O = 150\,\mathrm{N.m}}$$

Numerical



b) Horizontal force at *A* that produces the same moment,

$$d = (600\,\text{mm})\sin 60° = 519.6\,\text{mm} = 0.5196\,\text{m}$$

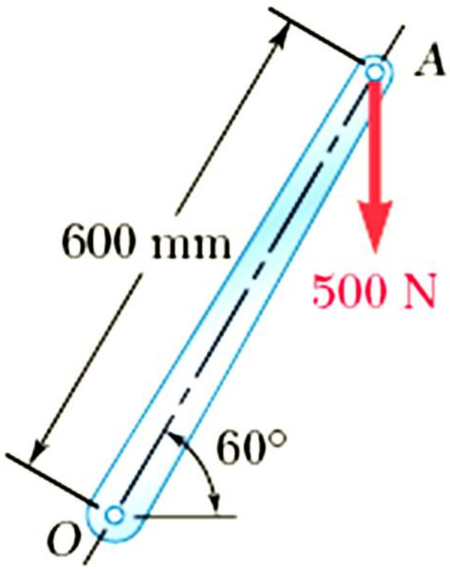
$$M_O = Fd$$

$$150\,\text{N.m} = F(0.5196m)$$

$$F = \frac{150\,\text{N.m}}{0.5196\text{m}}$$

$$\boxed{F = 288.68\text{N}}$$

Numerical



c) The smallest force A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA.

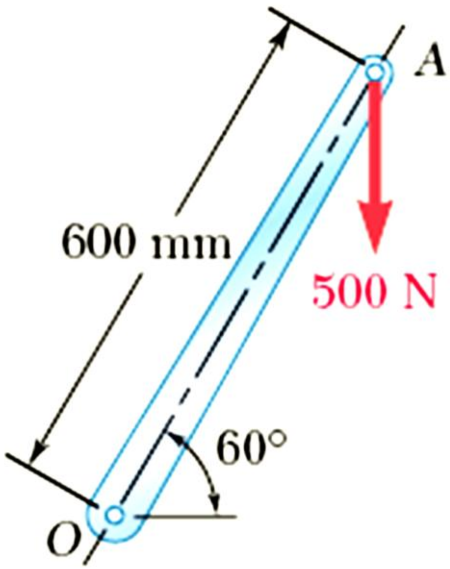
$$M_O = Fd$$

$$150\,\text{N.m} = F(0.6\,\text{m})$$

$$F = \frac{150\,\text{N.m}}{0.6\,\text{m}}$$

$$\boxed{F = 250\,\text{N}}$$

Numerical



d) To determine the point of application of a 1200 N force to produce the same moment,

$$M_O = Fd$$

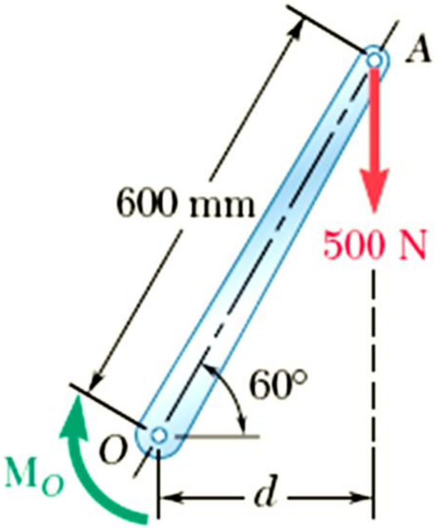
$$150\,\text{N.m} = (1200\,\text{N})d$$

$$d = \frac{150\,\text{N.m}}{1200\,\text{N}} = 125\,\text{mm}$$

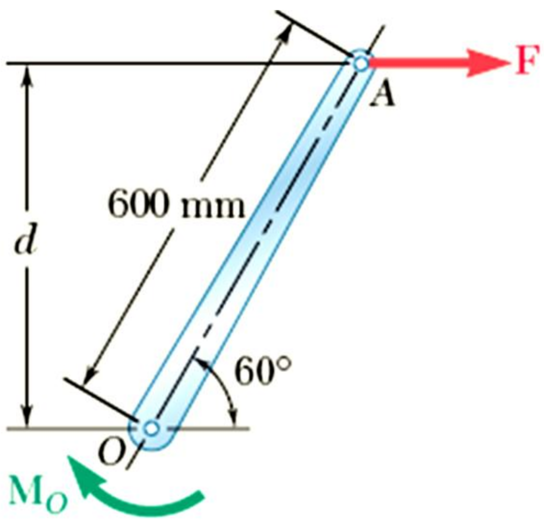
$$OB\cos 60° = 125\,\text{mm}$$

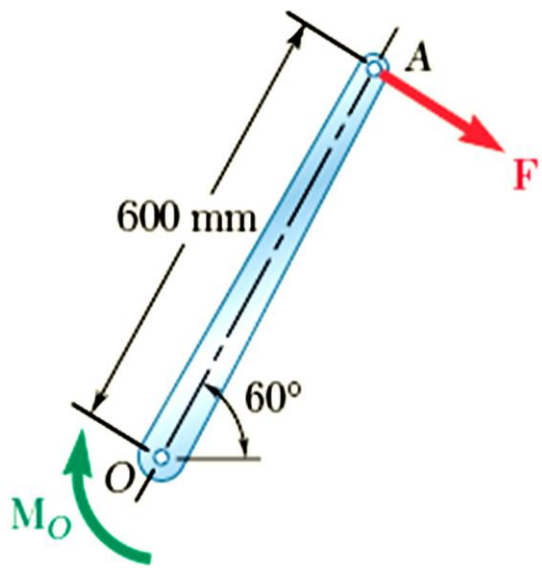
$$\boxed{OB = 250\,\text{mm}}$$

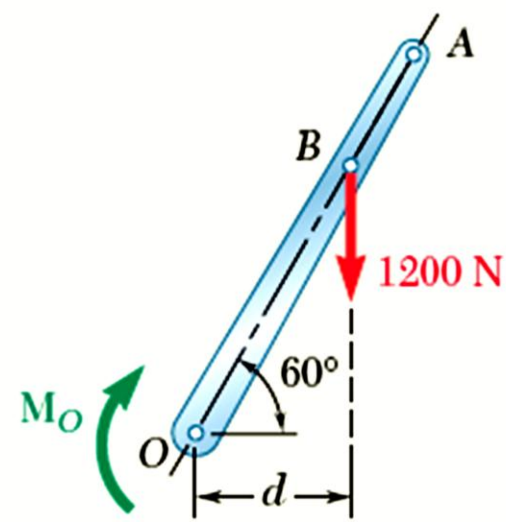
Numerical



e) Although each of the forces in parts b), c), and d) produces the same moment as the 500-N force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 500-N force.

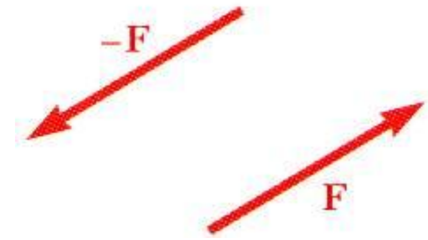






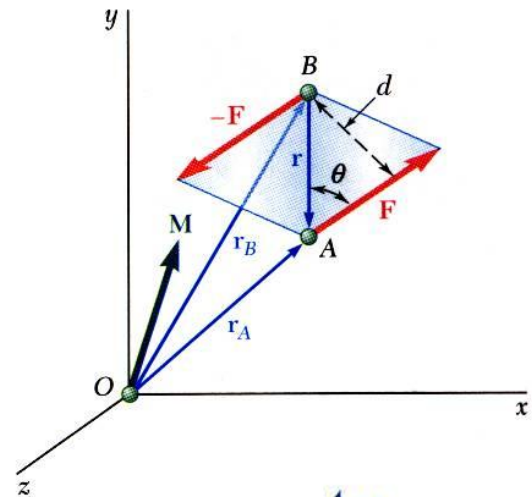
Moment of Couple

- Two forces ***F*** and ***-F*** having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.

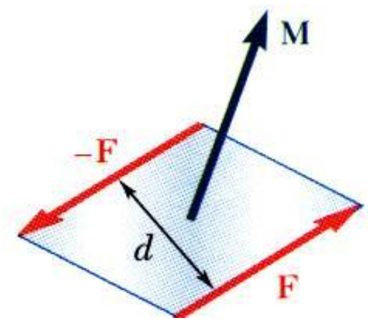


- Moment of the couple,

$$\begin{aligned}\vec{M} &= \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right) \\ &= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F} \\ &= \vec{r} \times \vec{F} \\ M &= rF\sin\theta = Fd\end{aligned}$$



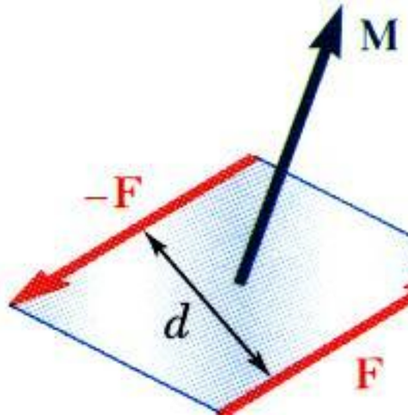
- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

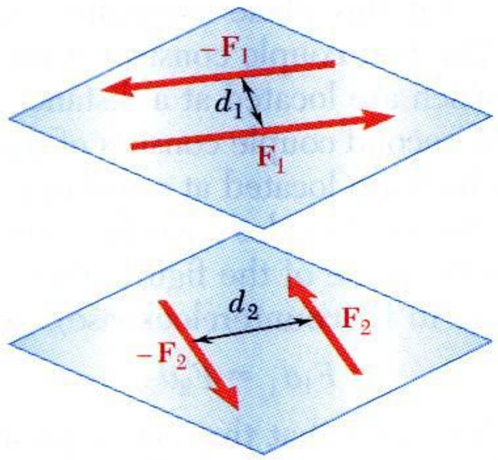


Moment of Couple

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.

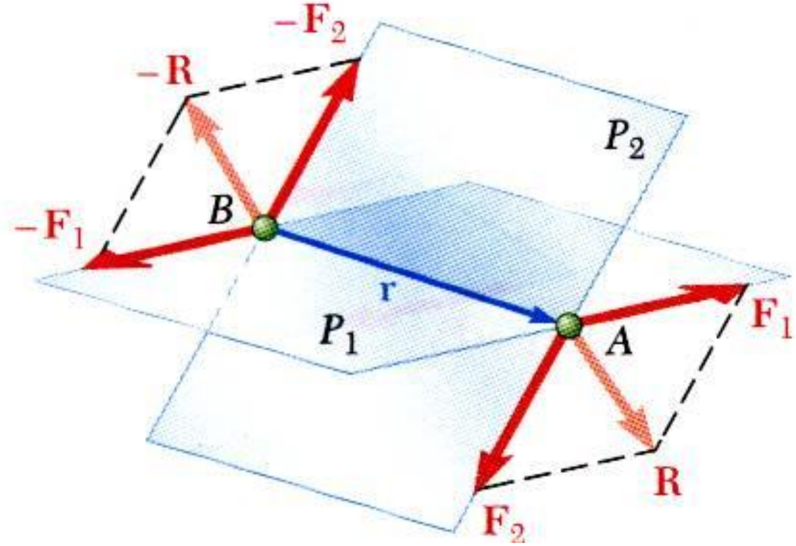




Addition of Couple

- Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$
$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$

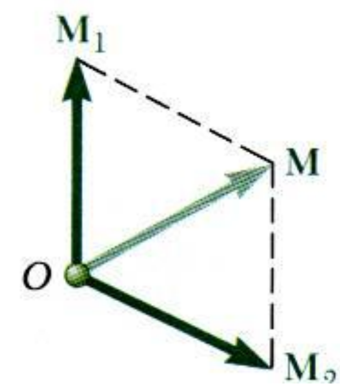


- Resultants of the vectors also form a couple

$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$$

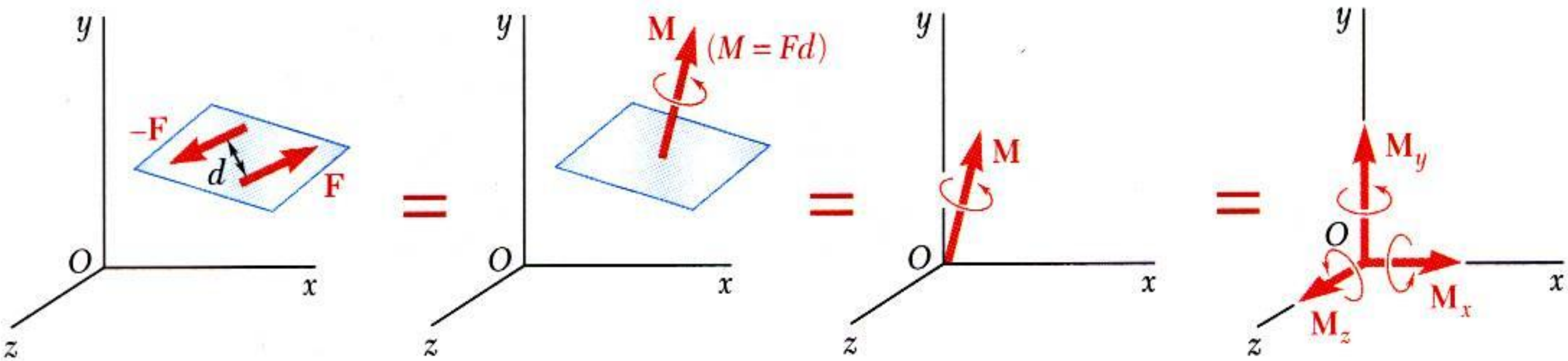
- By Varigon's theorem

$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$
$$= \vec{M}_1 + \vec{M}_2$$



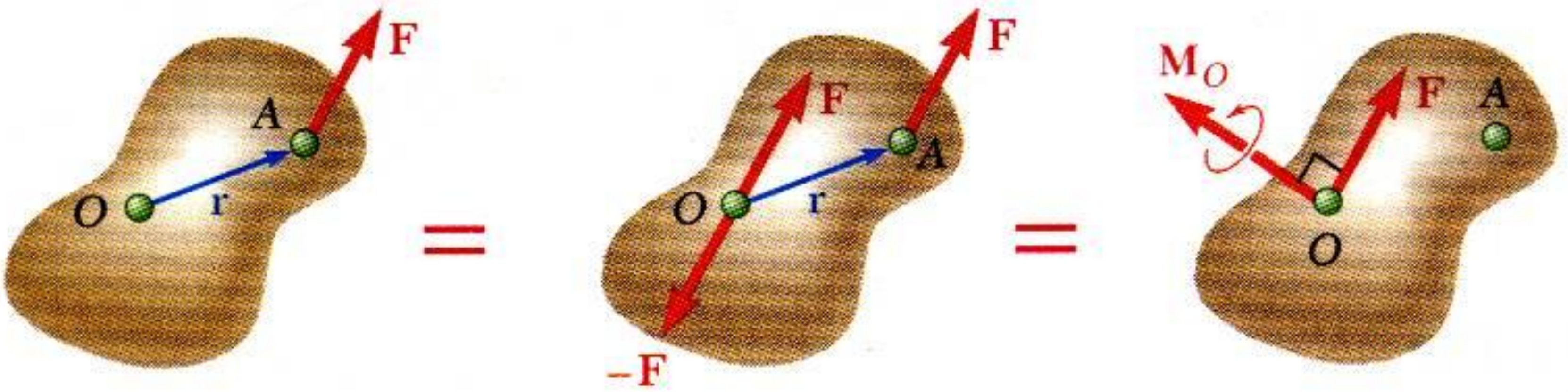
- Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Can Be Represented by Vectors



- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- *Couple vectors* obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Resolution of a Force Into a Force at O and a Couple



- Force vector ***F*** can not be simply moved to *O* without modifying its action on the body.
- Attaching equal and opposite force vectors at O produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a force-couple system.