# Unit-V : Vector Calculus - I.

Textbook - Advanced Engineering Mathematics
Chapter - 15 (vector Differential & Integral Calculus)

**Scalar pt. function** :- A scalar function $f(x,y,z)$ is a function defined at each pt in a certain domain D in space. Its value is real & depends only on the pt but not on any particular coordinate system being used.

$$f: \mathbb{R}^3 \to \mathbb{R}$$

$$f_1(x,y,z) = x^2+y^2+z^2$$

$$f_2(x,y,z) = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

**vector pt function** :- $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$\vec{f}(x,y,z) = f_1\hat{i}+f_2\hat{j}+f_3\hat{k}$$

$f_1, f_2$ & $f_3$ are scalar pt. function.

$$\boxed{\vec{v} = (x+y+z)\hat{i}+xyz\hat{j}+x^2y\hat{k}}$$

**Level surface** :- Let $f(x,y,z)$ be a single valued continuous scalar function defined at every point of domain D. Then $\boxed{f(x,y,z) = C}$, $C$ is any constant defines the equation of surface called level surface

Q. Find the level surface of the scalar field $f(x,y,z) = x^2+y^2+z^2$

Sol. we know that level surface is given by $f(x,y,z) = C \Rightarrow x^2+y^2+z^2 = C$.

Therefore, the level surfaces are the concentric spheres whose center at origin & radius $\sqrt{C}$.

Q. find the level surface of the scalar field. $x^2+9y^2+16z^2$.

Sol.
(a) $x^2+9y^2+16z^2 = 0$

(b) $x^2+y^2+z^2 = c$.

(c) $\frac{x^2}{c} + \frac{y^2}{9/c} + \frac{z^2}{16/c} = 1$ $c/9$ $c/16$

(d) none of these.

level surface is given by $f(x,y,z) = C$.

$\Rightarrow x^2+9y^2+16z^2 = c$

$\Rightarrow \frac{x^2}{c} + \frac{9y^2}{c} + \frac{16z^2}{c} = 1$

$\Rightarrow \frac{x^2}{c} + \frac{y^2}{c/9} + \frac{z^2}{c/16} = 1$

$$\frac{x^2}{(\sqrt{c})^2}+\frac{y^2}{(\sqrt{c/3})^2}+\frac{z^2}{(\sqrt{c/4})^2}=1$$

which is of the form $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1}$

## Parametric representation of curve/surface :-

$$\boxed{\begin{aligned} x&=f(t)\\ y&=g(t)\end{aligned}}$$

$\frac{dy}{dx}=\,??$

$x=f(t)$
$y=g(t)$
$z=h(t)$

$\int_a^b f(x)\,dx$ (a — b)

~~line integral~~

$$\boxed{\int_C f(x,y,z)\,dx}$$

$\int f(t)\,dt$

## Straight line :-

$$\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}$$

To write the eq. of line in parametric form

$$\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}=t$$

here $(x_0,y_0,z_0)$ is represents the pt through which the line passes

$\Rightarrow \boxed{\frac{x-x_0}{x_1-x_0}=t},\ \boxed{\frac{y-y_0}{y_1-y_0}=t},\ \boxed{\frac{z-z_0}{z_1-z_0}=t}$

& $\langle x_1-x_0, y_1-y_0, z_1-z_0\rangle$ represents D.R.

$\Rightarrow \boxed{x=x_0+t(x_1-x_0),\ y=y_0+t(y_1-y_0),\ z=z_0+t(z_1-z_0)}$

The vector representation of given curve is

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

2) $\hat{r} = (x_0 + t(x_1 - x_0))\hat{i} + (y_0 + t(y_1 - y_0))\hat{j} + (z_0 + t(z_1 - z_0))\hat{k}$

$\hat{r} = (x_0\hat{i} + y_0\hat{j} + z_0\hat{k}) + t((x_1 - x_0)\hat{i} + (y_1 - y_0)\hat{j} + (z_1 - z_0)\hat{k})$

$\vec{r} = \vec{a} + t\vec{b}$

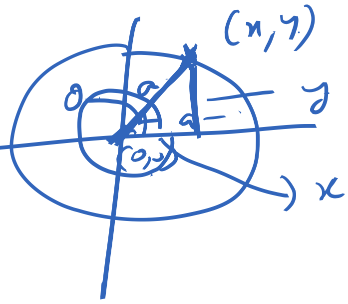
here $\vec{a} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$ represents pt through which the line passes

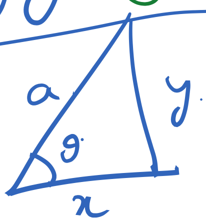
$\vec{b} = (x_1 - x_0)\hat{i} + (y_1 - y_0)\hat{j} + (z_1 - z_0)\hat{k}$ represents the direction of the line.

The eq of line in parametric form (vector form) is $\boxed{\vec{r} = \vec{a} + \vec{b}t}$

Circle : $x^2 + y^2 = a^2$

$x = a\cos\theta,\ y = a\sin\theta,\ 0 \le \theta \le 2\pi$




$\frac{x}{a} = \cos\theta \Rightarrow x = a\cos\theta$

$\frac{y}{a} = \sin\theta \Rightarrow y = a\sin\theta$

$0 \le \theta \le 2\pi$

In vector form

$\vec{r} = x\hat{i} + y\hat{j}$

$\boxed{\vec{r} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j},\ 0 \le \theta \le 2\pi}$

$x = r\cos\theta$

$y = r\sin\theta$

$0 \le r \le a$

① circle: $(x-x_0)^2+(y-y_0)^2 = a^2$

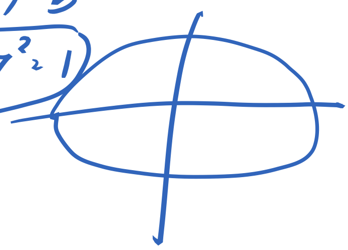
$x-x_0 = a\cos\theta$, $y-y_0 = a\sin\theta$

$x = x_0+a\cos\theta$, $y = y_0+a\sin\theta$, $0\le\theta\le 2\pi$

ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{x}{a}=X$, $\frac{y}{b}=Y$

$\Rightarrow X^2+Y^2=1$



$x^2+y^2=1$

$x=\pm\sqrt{1-y^2}$

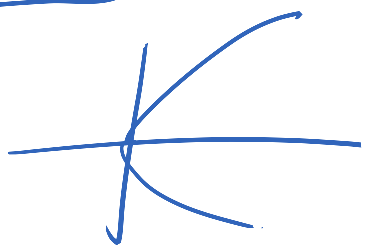
$\left(\frac{x}{a}\right)=\cos\theta$, $\left(\frac{y}{b}\right)=\sin\theta$

$\Rightarrow x=a\cos\theta$, $y=b\sin\theta$, $0\le\theta\le 2\pi$

vector form: $\vec r = x\hat i + y\hat j$

$$\boxed{\vec r = a\cos\theta\,\hat i + b\sin\theta\,\hat j}$$

Parabola: $y^2 = 4ax$



Let $y=t$, $x=\frac{y^2}{4a}=\frac{t^2}{4a}$

$\Rightarrow$ $\boxed{y=t,\ x=\frac{t^2}{4a}}$   $-\infty<t<\infty$

parametric eq of parabola in vector form.

$$\boxed{\vec r = x\hat i+y\hat j = \frac{t^2}{4a}\hat i + t\hat j}, \quad -\infty<t<\infty$$

Q. find the parametric eq. of curve.

$\boxed{x+y+z=3}$, $\boxed{y-z=0}$

Sol:- Put $z=t \Rightarrow y=t$.

$x+y+z=3 \Rightarrow x+t+t=3 \Rightarrow x=3-2t$.

parametric form is $(3-2t, t, t)$

in vector form is given by

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}=(3-2t)\hat{i}+t\hat{j}+t\hat{k}$

Q. find the parametric eq of straight line passes through the pt $(1,2,3)$ & has the direction $\hat{i}+2\hat{j}+2\hat{k}$.

Sol:- We know that parametric eq. of straight line is given by.

$\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

$\vec{r}=\vec{a}+\vec{b}t$.

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+(\hat{i}+2\hat{j}+2\hat{k})t$.

$\boxed{\vec{r}=(1+t)\hat{i}+(2+2t)\hat{j}+(3+2t)\hat{k}}$ in vector form.

$\boxed{x=1+t,\ y=2+2t,\ z=3+2t}$

a