CHAPTER

17 Linear Motion

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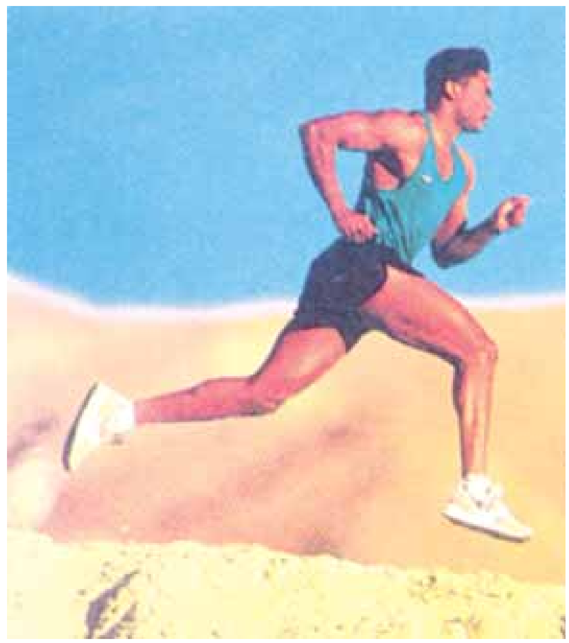

17.1. INTRODUCTION

A body is said to be at rest, if it occupies the same position with respect to its surroundings at all moments. But it is said to be in motion, if it changes its position, with respect to its surroundings.

17.2. IMPORTANT TERMS

The following terms, which will be frequently used in this chapter, should be clearly understood at this stage :

1. ***Speed.*** The speed of a body may be defined as its rate of change of displacement with respect to its surroundings. The speed of a body is irrespective of its direction and is, thus, a scalar quantity.

2. ***Velocity.*** The velocity of a body may be defined as its rate of change of displacement, with respect to its surroundings, in a particular direction. As the velocity is always expressed in particular direction, therefore it is a vector quantity.
3. ***Acceleration.*** The acceleration of a body may be defined as the rate of change of its velocity. It is said to be positive, when the velocity of a body increases with time, and negative when the velocity decreases with time. The negative acceleration is also called retardation. In general, the term acceleration is used to denote the rate at which the velocity is changing. It may be uniform or variable.
4. ***Uniform acceleration.*** If a body moves in such a way that its velocity changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration.
5. ***Variable acceleration.*** If a body moves in such a way, that its velocity changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration.
6. ***Distance traversed.*** It is the total distance moved by a body. Mathematically, if body is moving with a uniform velocity (v), then in (t) seconds, the distance traversed

$$s = vt$$

In this chapter, we shall discuss the motion under uniform acceleration only.

17.3. MOTION UNDER UNIFORM ACCELERATION

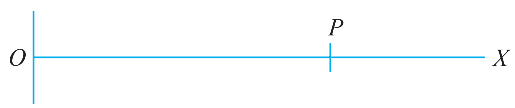

Fig. 17.1. Motion under uniform acceleration.

Consider *linear motion of a particle starting from O and moving along OX with a uniform acceleration as shown in Fig. 17.1. Let P be its position after t seconds.

Let u = Initial velocity,

v = Final velocity,

t = Time (in seconds) taken by the particle to change its velocity from u to v.

a = Uniform positive acceleration, and

s = Distance travelled in t seconds.

Since in t seconds, the velocity of the particle has increased steadily from (u) to (v) at the rate of a, therefore total increase in velocity

$$= a\,t$$

$$\therefore \quad v = u + a\,t \qquad ...(i)$$

and average velocity $= \left(\dfrac{u+v}{2}\right)$

We know that distance travelled by the particle,

$$s = \text{Average velocity} \times \text{Time}$$

$$= \left(\frac{u+v}{2}\right) \times t \qquad ...(ii)$$

* The term 'linear motion' is somtimes defined as a motion of a body which moves in such a way that all its particles move in parallel lines.

Substituting the value of v from equation (i),

$$s = \left(\frac{u + u + at}{2}\right) \times t = ut + \frac{1}{2}at^2 \qquad ...(iii)$$

From equation (i), ($i.e.$ $v = u + at$) we find that

$$t = \frac{v - u}{a}$$

Now substituting this vlaue of t in equation (ii),

$$s = \left(\frac{u + v}{2}\right) \times \left(\frac{v - u}{a}\right) = \frac{v^2 - u^2}{2a}$$

or $\qquad 2as = v^2 - u^2$

$\therefore \qquad v^2 = u^2 + 2as$

Example 17.1. *A car starting from rest is accelerated at the rate of 0.4 m/s². Find the distance covered by the car in 20 seconds.*

Solution. Given : Initial velocity (u) = 0 (because, it starts from rest) ; Acceleration (a) = 0.4 m/s^2 and time taken (t) = 20 s

We know that the distance covered by the car,

$$s = ut + \frac{1}{2}at^2 = (0 \times 20) + \frac{1}{2} \times 0.4 \times (20)^2 \text{ m} = 80 \text{ m} \quad \textbf{Ans.}$$

Example 17.2. *A train travelling at 27 km.p.h is accelerated at the rate of 0.5 m/s². What is the distance travelled by the train in 12 seconds ?*

Solution. Given : Initial velocity (u) = 27 km.p.h. = 7.5 m/s ; Accceleration (a) = 0.5 m/s^2 and time taken (t) = 12 s.

We know that distance travelled by the train,

$$s = ut + \frac{1}{2}at^2 = (7.5 \times 12) + \frac{1}{2} \times 0.5 \times (12)^2 \text{ m}$$

$$= 90 + 36 = 126 \text{ m} \quad \textbf{Ans.}$$

Example 17.3. *A scooter starts from rest and moves with a constant acceleration of 1.2 m/s². Determine its velocity, after it has travelled for 60 meters.*

Solution. Given : Initial velocity (u) = 0 (because it starts from rest) Acceleration (a) = 1.2 m/s^2 and distance travelled (s) = 60 m.

Let $\qquad v$ = Final velocity of the scooter.

We know that $\qquad v^2 = u^2 + 2as = (0)^2 + 2 \times 1.2 \times 60 = 144$

$$v = 12 \text{ m/s} = \frac{12 \times 3600}{1000} = 43.2 \text{ km.p.h.} \quad \textbf{Ans.}$$

Example 17.4. *On turning a corner, a motorist rushing at 20 m/s, finds a child on the road 50 m ahead. He instantly stops the engine and applies brakes, so as to stop the car within 10 m of the child. Calculate (i) retardation, and (ii) time required to stop the car.*

Solution. Given : Initial velocity (u) = 20 m/s ; Final velocity (v) = 0 (because the car is stopped) and distance travelled by the car (s) = 50 – 10 = 40 m (*i*) *Retardation*

Let a = Acceleration of the motorist.

We know that $v^2 = u^2 + 2as$

$$0 = (20)^2 + 2 \times a \times 40 = 400 + 80a$$

or $80a = -400$

$$\therefore \quad a = \frac{-400}{80} = -5 \text{ m/s}^2 \quad \textbf{Ans.}$$

...(Minus sign shows that the acceleration is negative *i.e.* retardation).

(*ii*) *Time required to stop the car*

Let t = Time required to stop the car in second.

We know that final velocity of the car (v),

$$0 = u + at = 20 - 5 \times t \qquad ...(\because a = -5 \text{ m/s}^2)$$

$$\therefore \quad t = \frac{20}{5} = 4 \text{ s} \quad \textbf{Ans.}$$

Example 17.5. *A motor car takes 10 seconds to cover 30 meters and12 seconds to cover 42 meters. Find the uniform acceleration of the car and its velocity at the end of 15 seconds.*

Solution. Given : When t = 10 seconds, s = 30 m and when t = 12 seconds, s = 42 m.

Uniform acceleration

Let u = Initial velocity of the car, and

a = Uniform acceleration.

We know that the distance travelled by the car in 10 seconds,

$$30 = ut + \frac{1}{2}at^2 = u \times 10 + \frac{1}{2} \times a(10)^2 = 10u + 50\,a$$

Multiplying the above equation by 6,

$$180 = 60u + 300a \qquad ...(i)$$

Similarly, distance travelled by the car in 12 seconds,

$$42 = u \times 12 + \frac{1}{2} \times a(12)^2 = 12u + 72a$$

Mulitiplying the above equation by 5,

$$210 = 60u + 360a \qquad ...(ii)$$

Subtracting equation (*i*) from (*ii*),

$$30 = 60a \quad \text{or} \quad a = \frac{30}{60} = 0.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Velocity at the end of 15 seconds

Substituting the value of a in equation (*i*)

$$180 = 60u + (300 \times 0.5) = 60u + 150$$

$$\therefore \quad u = \frac{(180 - 150)}{60} = 0.5 \text{ m/s}$$

We know that the velocity of the car after 15 seconds,

$$v = u + at = 0.5 + (0.5 \times 15) = 8 \text{ m/s} \quad \textbf{Ans.}$$

Example 17.6. *A burglar's car had a start with an acceleration of 2 m/s². A police vigilant party came after 5 seconds and continued to chase the burglar's car with a uniform velocity of 20 m/s. Find the time taken, in which the police van will overtake the burglar's car.*

Solution. Given : Acceleration of burglar's car = 2 m/s² and uniform velocity of the police party = 20 m/s.

Let t = Time taken by the police party, to overtake the burglar's car after reaching the spot.

First of all, consider the motion of the burglar's car. In this case, initial velocity (u) = 0 (because it starts from rest) ; acceleration = 2 m/s² and time taken by burglar's car to travel the distance of s m (t_1) = (t + 5) second.

We know that the distance travelled by the burglar's car,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2(t+5)^2 = (t+5)^2 \qquad ...(i)$$

Now consider the motion of the police party. In this case, uniform velocity, v = 20 m/s

∴ Distance travelled by the police party,

$$s = \text{Velocity} \times \text{Time} = 20 \times t \qquad ...(ii)$$

For the police party to overtake the burglar's car, the two distances (i) and (ii) should be equal. Therefore equating equations (i) and (ii)

$$(t+5)^2 = 20 \times t$$

or $t^2 + 25 + 10\,t = 20\,t$

or $t^2 + 25 - 10\,t = 0$

or $(t-5)^2 = 0$

$(t-5) = 0$ ...(Taking square root)

or $t = 5$ s **Ans.**

Example 17.7. *A train is uniformly accelerated and passes successive kilometre stones with velocities of 18 km.p.h. and 36 km.p.h. respectively. Calculate the velocity, when it passes the third kilometre stone. Also find the time taken for each of these two intervals of one kilometre.*

Solution. First of all, consider the motion of the train between the first and second kilometre stones. In this case, distance (s) = 1 km = 1000 m ; initial velocity (u) = 18 km.p.h. = 5 m/s ; and final velocity (v) = 36 km.p.h. = 10 m/s

Velocity with which the train passes the third km stone

Let v = Velocity with which the train passes the third km, and

a = Uniform acceleration.

We know that $v^2 = u^2 + 2as$

$$(10)^2 = (5)^2 + (2a \times 1000) = 25 + 2000\,a$$

$$\therefore \quad a = \frac{100-25}{2000} = \frac{75}{2000} = 0.0375 \text{ m/s}^2$$

Now consider the motion of the train between the second and third kilometre stones. In this case, distance (s) = 1 km = 1000 m and initial velocity (u) = 36 km.p.h. = 10 m/s.

We know that $v^2 = u^2 + 2as = (10)^2 + (2 \times 0.0375 \times 1000) = 175$

∴ $v = 13.2$ m/s = 47.5 km.p.h. **Ans.**

Time taken for each of the two intervals of one kilometre

Let t_1 = Time taken by the train to travel the first one kilometre, and

t_2 = Time taken by the train to travel the second kilometre.

We know that velocity of the train after passing the first kilometre *i.e.*, in t_1 seconds (v_1),

$$10 = u + at_1 = 5 + 0.0375\, t_1$$

$$\therefore \qquad t_1 = \frac{10 - 5}{0.0375} = 133.3 \text{ s} \quad \textbf{Ans.}$$

Similarly, velocity of the train after passing the second kilometre *i.e.* in t_2 seconds,

$$13.2 = u + at_2 = 10 + 0.0375\, t_2$$

$$\therefore \qquad t_2 = \frac{13.2 - 10}{0.0375} = 85.3 \text{ s} \quad \textbf{Ans.}$$

Example 17.8. *Two electric trains A and B leave the same station on parallel lines. The train A starts from rest with a uniform acceleration of 0.2 m/s² and attains a speed of 45 km.p.h., which is maintained constant afterwards. The train B leaves 1 minute after with a uniform acceleration of 0.4 m/s² to attain a maximum speed of 72 km.p.h., which is maintained constant afterwards. When will the train B overtake the train A ?*

Solution. Given : Initial velocity of train A (u_A) = 0 (because it starts from rest) ; Uniform acceleration of train A (a_A) = 0.2 m/s² ; Final velocity of train A (v_A) = 45 km.p.h. = 12.5 m/s ; Initial velocity of train B (u_B) = 0 (because it also starts from rest) ; Uniform acceleration of train B (a_B) = 0.4 m/s² and final velocity of train B (v_B) = 72 km.p.h. = 20 m/s

Let t_A = Time taken by the train A to attain a speed of 12.5 m/s, and

T = Time in second when the train B will overtake the train A from its start.

We know that final velocity of the train A (v_A),

$$12.5 = u_A + a_A t_A = 0 + 0.2\, t_A$$

$$\therefore \qquad t_A = \frac{12.5}{0.2} = 62.5 \text{ s}$$

and distance travelled by the train A to attain this speed,

$$s = u_A t_A + \frac{1}{2} a_A t_A^{\,2} = 0 + \frac{1}{2} \times 0.2\,(62.5)^2 = 390.6 \text{ m}$$

Similarly, final velocity of the train B (v_B),

$$20 = u_B + a_B t_B = 0 + 0.4\, t_B$$

$$\therefore \qquad t_B = \frac{20}{0.4} = 50 \text{ s}$$

and distance travelled by the train B to attain this speed,

$$s = u_B t_B + \frac{1}{2} a_B t_B^{\,2} = 0 + \frac{1}{2} \times 0.4\,(50)^2 = 500 \text{ m}$$

Now we see that the train A has travelled for (T + 60) seconds. Therefore total distance travelled by the train A during this time,

$$s_A = 390.6 + 12.5\,[(T + 60) - 62.5\,] \text{ m} \qquad \ldots(i)$$

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and total distance travelled by the Train B,

$$s_B = 500 + 20\,(T - 50)\text{ m} \qquad ...(ii)$$

For the train B to overtake the train A, the two distances (i) and (ii) should be equal. Threfore equating equations (i) and (ii),

$$390.6 + 12.5\,[(T + 60) - 62.5] = 500 + 20\,(T - 50)$$

$$12.5\,T - 31.3 = 109.4 + 20\,T - 1000$$

$$7.5\,T = 1000 - 109.4 - 31.3 = 859.3$$

$$\therefore \qquad T = \frac{859.3}{7.5} = 114.6\text{ s} \qquad \textbf{Ans.}$$

EXERCISE 17.1

1. A body starts with a velocity of 3 m/s and moves in a straight line with a constant acceleration. If its velocity at the end of 5 seconds is 5.5 m/s, find (i) the uniform acceleration, and (ii) distance travelled in 10 seconds. [**Ans.** 0.5 m/s^2 ; 55 m]
2. Two cars start off to a race with velocities (u), and (v), and travel in a straight line with a uniform accelerations of (α) and (β). If the race ends in a dead beat, prove that the length of the race is:

$$\frac{2\,(u - v)\,(u\beta - v\alpha)}{(\beta - \alpha)^2}$$

Hint. Dead beat means that the distance travelled by both the cars is the same.

3. A car starts from rest and accelerates uniformly to a speed of 72 km.p.h. over a distance of 500 m. Find acceleration of the car and time taken to attain this speed.

   If a further acceleration rises the speed to 90 km.p.h. in 10 seconds, find the new acceleration and the further distance moved. [**Ans.** 0.4 m/s^2 ; 50 sec ; 0.5 m/s^2 ; 225 m ; 62.5 m]
4. A bullet moving at the rate of 300 m/s is fired into a thick target and penetrates up to 500 mm. If it is fired into a 250 mm thick target, find the velocity of emergence. Take the resistance to be uniform in both the cases. [**Ans.** 212.1 m/s]

17.4. MOTION UNDER FORCE OF GRAVITY

It is a particular case of motion, under a constant acceleration of (g) where its *value is taken as 9.8 m/s^2. If there is a free fall under gravity, the expressions for velocity and distance travelled in terms of initial velocity, time and gravity acceleration will be :

1. $v = u + gt$
2. $s = ut + \frac{1}{2}gt^2$
3. $v^2 = u^2 + 2\,gs$

But, if the motion takes place against the force of gravity, *i.e.*, the particle is projected upwards, the corresponding equations will be :

1. $v = -\,u + gt$
2. $s = -\,ut + \frac{1}{2}gt^2$
3. $v^2 = -\,u^2 + 2\,gs$

* Strictly speaking, the value of g varies from 9.77 m/s^2 to 9.83 m/s^2 over the world. Its value, until and unless mentioned, is taken as 9.8 m/s^2.

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Notes : 1. In this case, the value of u is taken as negative due to upward motion.

2. In this case, the distances in upward direction are taken as negative, while those in the downward direction are taken as positive.

Example 17.9. *A stone is thrown upwards with a velocity of 4.9 m/s from a bridge. If it falls down in water after 2 s, then find the height of the bridge.*

Solution. Given : Initial velocity $(u) = -4.9$ m/s (*Minus sign due to upwards*) and time taken $(t) = 2$ s.

We know that height of the bridge,

$$h = ut + \frac{1}{2}gt^2 = (-4{\cdot}9 \times 2) + \frac{1}{2} \times 9{\cdot}8 \times (2)^2 \text{ m}$$

$$= -9.8 + 19.6 = 9.8 \text{ m} \quad \textbf{Ans.}$$

Example 17.10. *A packet is dropped from a balloon which is going upwards with a velocity 12 m/s. Calculate the velocity of the packet after 2 seconds.*

Solution. Given : Velocity of balloon when the packet is dropped $(u) = -12$ m/s (Minus sign due to upward motion) and time $(t) = 2$ s.

We know that when the packet is dropped its initial velocity $(u) = -12$ m/s.

$\therefore$ Velocity of packet after 2 sec.

$$v = u + gt = -12 + (9{\cdot}8 \times 2) = -12 + 19{\cdot}6 = 7{\cdot}6 \text{ m/s} \quad \textbf{Ans.}$$

Example 17.11. *A body is dropped from the top of a tall building. If it takes 2.8 seconds in falling on the ground, find the height of the building.*

Solution. Given : Initial velocity $(u) = 0$ (because it is dropped) and time taken $(t) = 2.8$ s.

We know that height of the building

$$s = ut + \frac{1}{2}gt^2 = (0 \times 2.8) + \frac{1}{2} \times 9.8 \times (2.8)^2 = 38.4 \text{ m} \quad \textbf{Ans.}$$

Example 17.12. *A stone is dropped from the top of a building, which is 65 m high. With what velocity will it hit the ground ?*

Solution. Given : Initial velocity $(u) = 0$ (because it is dropped) and height of the building $(s) = 65$ m

Let $\quad v =$ Final velocity of the stone with which it will hit the ground.

We know that $\quad v^2 = u^2 + 2gs = (0)^2 + 2 \times 9.8 \times 65 = 1274$

$\therefore \quad v = 35.7$ m/s $\quad$ **Ans.**

Example 17.13. *A body is thrown vertically upwards with a velocity of 28 m/s. Find the distance it will cover in 2 seconds.*

Solution. Given : Initial velocity $(u) = -28$ m/s (Minus sign due to upward motion) and time $(t) = 2$ s.

We know that distance covered by the body,

$$s = ut + \frac{1}{2}gt^2 = (-28 \times 2) + \frac{1}{2} \times 9.8 \times (2)^2 \text{ m}$$

$$= -56 + 19.6 = -36.4 \text{ m} \quad \textbf{Ans.}$$

....(Minus sign indicates that the body will cover the distance in upward direction)

Example 17.14. *A bullet is fired vertically upwards with a velocity of 80 m/s. To what height will the bullet rise above the point of projection ?*

Solution. Given : Initial velocity $(u) = -80$ m/s (Minus sign due to upward motion) and final velocity $(v) = 0$ (because the bullet is at maximum rise)

Let s = Height to which the bullet will rise above the point of projection.

We know that $v^2 = u^2 + 2\,gs$

$$(0)^2 = (-80)^2 + 2 \times 9.8 \times s = 6400 + 19.6\text{ s}$$

$$\therefore \quad s = \frac{6400}{-19.6} = -326.5 \text{ m} \quad \textbf{Ans.}$$

...(Minus sign indicates that the body will cover the distance in upward direction)

Example 17.15. *A body is released from a great height falls freely towards earth. Another body is released from the same height exactly one second later. Find the separation between both the bodies, after two seconds of the release of second body.*

Solution. Given : Initial velocity of both the bodies $(u) = 0$ (because they are released); Time taken by the first body $(t_1) = 3$ s and time taken by the second body $(t_2) = 3 - 1 = 2$ s (because the second body is released after 1 s of the release of first body).

We know that distance covered by the first body in 3 seconds

$$h_1 = ut_1 + \frac{1}{2}gt_1^2 = (0 \times 3) + \frac{1}{2} \times 9{\cdot}8 \times (3)^2 = 44.1 \text{ m} \qquad ...(i)$$

and the distance covered by the second body in 2 seconds

$$h_2 = ut_2 + \frac{1}{2}gt_2^2 = (0 \times 2) + \frac{1}{2} \times 9{\cdot}8 \times (2)^2 = 19.6 \text{ m} \qquad ...(ii)$$

$\therefore$ Separation between the bodies

$$= h_2 - h_1 = 44.1 - 19.6 = 24.5 \text{ m} \quad \textbf{Ans.}$$

Example 17.16. *A stone is thrown vertically upwards with a velocity of 29.4 m/s from the top of a tower 34.3 m high. Find the total time taken by the stone to reach the foot of the tower.*

Solution. Given : Initial velocity $(u) = -29.4$ m/s (Minus sign due to upward motion) and height of tower $(h) = 34.3$ m

Let t = Time taken by the stone to reach the foot of the tower

We know that height of the tower (h),

$$34.3 = ut + \frac{1}{2}gt^2 = (-29.4 \times t) + \frac{1}{2} \times 9.8t^2$$

or $\quad 4.9\,t^2 - 29.4\,t - 34.3 = 0$

$$t^2 - 6t - 7 = 0$$

This is a quadratic equation in (t).

$$\therefore \quad t = \frac{+6 \pm \sqrt{(6)^2 + (4 \times 7)}}{2} = \frac{6 \pm 8}{2} = 7 \text{ s} \quad \textbf{Ans.}$$

Example 17.17. *A stone is thrown vertically upwards, from the ground, with a velocity 49 m/s. After 2 seconds, another stone is thrown vertically upwards from the same place. If both the stone strike the ground at the same time, find the velcocity, with which the second stone was thrown upwards.*

Solution. First of all, consider the upwards motion of the first stone. In this case, initial velocity (u) = – 49 m/s (Minus sign due to upward motion) and final velocity (v) = 0 (because stone is at maximum height)

Let t = Time taken by the stone to reach maximum height.

We know that final velocity of the stone (v),

$$0 = u + gt = -49 + 9.8\,t \quad \text{...(Minus sign due to upwards motion)}$$

$$\therefore \qquad t = \frac{49}{9.8} = 5 \text{ s}$$

It means that the stone will take 5 s to reach the maximum height and another 5 s to come back to the ground.

$\therefore$ Total time of flight $= 5 + 5 = 10$ s

Now consider the motion of second stone. We know that time taken by the second stone for going upwards and coming back to the earth

$$= 10 - 2 = 8 \text{ s}$$

and time taken by the second stone to reach maximum height

$$= \frac{8}{2} = 4 \text{ s}$$

Now consider the upward motion of the second stone. We know that final velocity of the stone (v),

$$0 = u + gt = -u + 9.8 \times 4 = -u + 39.2$$

$$\therefore \qquad u = 39.2 \text{ m/s} \quad \textbf{Ans.}$$

Example 17.18. *A stone is dropped from the top of a tower 50 m high. At the same time, another stone is thrown upwards from the foot of the tower with a velocity of 25 m/s. When and where the two stones cross each other ?*

Solution. Given : Height of the tower = 50 m

Time taken by the stone to cross each other

First of all, consider downward motion of the first stone. In this case, initial velocity (u) = 0 (beacuse it is dropped)

Let t = Time taken for the stones to cross each other.

We know that distance traversed by the stone,

$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2 = 0.5\,gt^2 \quad \text{...}(i)$$

Now consider upward motion of the second stone. In this case, initial velocity = – 25 m/s (Minus sign due to upward) and distance traversed = 50 – s,

We know that the distance traversed,

$$50 - s = ut + \frac{1}{2}gt^2 = -25t + 0.5\,gt^2 \quad \text{...}(ii)$$

Adding equations (i) and (ii),

$$50 = 25\,t \quad \text{or} \quad t = \frac{50}{25} = 2 \text{ s} \quad \textbf{Ans.}$$

Point where the stones crosss each other

Substituting the value of $t = 2$ in equation (i),

$$s = 0.5 \times 9.8\,(2)^2 = 19.6 \text{ m} \quad \textbf{Ans.}$$

Example 17.19. *A stone is thrown vertically up from the top of a tower with a certain initial velocity. It reaches ground in 5.64 seconds. A second stone, thrown down from the same tower with the same initial velocity reaches ground in 3.6 seconds. Determine (i) the height of the tower, and (ii) the initial velocity of the stones.*

Solution. Given : Total time taken by first stone = 5.64 s ; Time taken by second stone = 3.6 s.

Initial velocity of the stones

Let u = Initial velocity of the stones.

First of all, consider upward motion of the first stone from the top of the tower. We know that it will first move upwards in the sky. And when its velocity becomes zero, it will start coming down. Its velocity at the top of the tower, while coming down, will be the same as that with which it was thrown upwards.

Therefore time taken by the first stone to reach maximum height and then to reach the top of the tower, from where it was thrown

$$= 5.64 - 3.6 = 2.04 \text{ s}$$

Thus time taken by the stone to reach maximum height (or in other words when its final velocity, $v = 0$)

$$= \frac{2.04}{2} = 1.02 \text{ s}$$

We know that final velocity of the stone (v),

$$0 = -u + gt = -u + (9.8 \times 1.02) = -u + 10$$

$\therefore$ $u = 10$ m/s **Ans.**

Height of the tower

Now consider downward motion of the second stone. In this case, initial velocity (u) = 10 m/s and time (t) = 3.6 s. We know that height of the tower (or distance travelled by the stone),

$$s = ut + \frac{1}{2}gt^2 = (10 \times 3.6) + \frac{1}{2} \times 9.8\,(3.6)^2 = 99.5 \text{ m} \quad \textbf{Ans.}$$

Example 17.20. *A stone is thrown up with a velocity of 20 m/s. While coming down, it strikes a glass pan, held at half the height through which it has risen and loses half of its velocity in breaking the glass. Find the velocity with which it will strike the ground.*

Solution. First of all, consider upward motion of the stone. In this case, initial velocity (u_1) = – 20 m/s (Minus sign due to upward motion) and final velocity (v_1) = 0 (because it reaches maximum height).

Let s_1 = Maximum height reached by the stone.

We know that $v_1^2 = u_1^2 + 2gs_1$

$$0 = (-20)^2 + 2 \times 9.8 \times s_1 = 400 + 19.6\, s_1$$

$$\therefore \quad s_1 = \frac{400}{-19.6} = -20.4 \text{ m} \qquad ...(i)$$

....(Minus sign indicates that height reached by the stone is in upward direction)

Now consider downward motion of the stone up to the glass pan. In this case, initial velocity (u_2) = 0 (because it starts coming down after rising to the maximum height and distance covered by the stone $(s_2) = \dfrac{20.4}{2} = 10.2$ m

Let v_2 = Final velocity of the stone, with which it strikes the pan.

We know that $v_2^2 = u_2^2 + 2\,gs_2 = 0 + (2 \times 9.8 \times 10.2) = 199.9$

$\therefore$ $v_2 = \sqrt{199.9} = 14.14$ m/s

and now consider motion of the stone after breaking the glass pan. In this case, initial velocity $(u_3) = \dfrac{14.14}{2} = 7.07$ m/s (because it looses half its velocity after striking the pan) and distance travelled by the stone $(s_3) = 20.4 - 10.2 = 10.2$ m.

Let v_3 = Final velocity of the stone, with which it strikes the ground.

We know that $v_3^2 = u_3^2 + 2\,gs_3 = (7.07)^2 + (2 \times 9.8 \times 10.2) = 249.9$

$\therefore$ $v_3 = \sqrt{249.9} = 15.8$ m/s **Ans.**

Example 17.21. *A body falling freely, under the action of gravity passes two points 10 metres apart vertically in 0.2 second. From what height, above the higher point, did it start to fall ?*

Solution. Let the body start from O and pass two points A and B 10 metres apart in 0.2 s as shown in Fig. 17.2.

First of all, consider the motion from O to A. In this case, initial velocity $(u) = 0$ (because it is falling freely) and distance travelled by body $(s) = OA = x$ m

Let t = Time taken by the body to travel from O to A.

We know that $x = ut + \dfrac{1}{2}gt^2 = 0 + \dfrac{1}{2} \times 9.8t^2 = 4.9\,t^2$ ...(*i*)

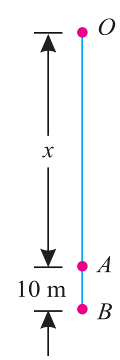

Fig. 17.2.

Now consider motion of stone from O to B. In this case, Initial velocity $(u) = 0$ (because it is falling freely) ; Distance travelled $(s_1) = OB = (x + 10)$ m and time taken $(t_1) = (t + 0.2)$ s

We know that $(x + 10) = ut_1 + \dfrac{1}{2}gt_1^2 = 0 + \dfrac{1}{2} \times 9.8(t + 0.2)^2$

$= 0 + 4.9\,(t^2 + 0.04 + 0.4\,t)$

$= 4.9\,t^2 + 0.196 + 1.96\,t$ ...(*ii*)

Subtracting equation (*i*) from (*ii*),

$10 = 0.196 + 1.96\,t$

or $t = \dfrac{10 - 0.196}{1.96} = 5$ s

Substituting the value of t in equation (*i*),

$x = 4.9\,t^2 = 4.9 \times (5)^2 = 122.5$ m **Ans.**

Alternative Method

First of all, consider the motion between A and B. In this case, distance travelled $(s) = 10$ m and time $(t) = 0.2$ s

Let u = Initial velocity of the body at A.

We know that the distance travelled (s).

$$10 = ut + \frac{1}{2}gt^2 = u \times 0.2 + \frac{1}{2} \times 9.8(0.2)^2 = 0.2\,u + 0.196$$

$\therefore$ $u = \dfrac{10 - 0.196}{0.2} = 49$ m/s

Now consider the motion from O to A. In this case, initial velocity (u) = 0 (because it is falling freely) and final velocity (v) = 49 m/s

We know that $v^2 = u^2 + 2\,gs$

$$(49)^2 = 0 + 2 \times 9.8 \times x = 1.96\,x$$

$$\therefore \quad x = \frac{(49)^2}{19.6} = 122.5 \text{ m} \quad \textbf{Ans.}$$

Example 17.22. *A stone, dropped into a well, is heard to strike the water after 4 seconds. Find the depth of well, if velocity of the sound is 350 m/s.*

Solution. First of all, consider the downward motion of the stone. In this case, initial velocity (u) = 0 (because it is dropped)

Let t = Time taken by the stone to reach the bottom of the well.

We know that depth of the well,

$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8 \times t^2 = 4.9\,t^2 \quad ...(i)$$

and time taken by the sound to reach the top

$$= \frac{\text{Depth of the well}}{\text{Velocity of sound}} = \frac{s}{350} = \frac{4.9\,t^2}{350} \quad ...(ii)$$

Since the total time taken (*i.e.* stone to reach the bottom of the well and sound to reach the top of the well) is 4 seconds, therefore

$$t + \frac{4.9\,t^2}{350} = 4$$

$$4.9\,t^2 + 350\,t = 1400$$

or $\quad 4.9\,t^2 + 350\,t - 1400 = 0$

This is a quadratic equation in t,

$$\therefore \quad t = \frac{-350 \pm \sqrt{(350)^2 + 4 \times 4.9 \times 1400}}{2 \times 4.9} = 3.8 \text{ s}$$

Now substituting the value of t in equation (i),

$$s = 4.9\,t^2 = 4.9\,(3.8)^2 = 70.8 \text{ m} \quad \textbf{Ans.}$$

Example 17.23. *A particle, falling under gravity, falls 20 metres in a certain second. Find the time required to cover next 20 metres.*

Solution. Given : Distance travelled by particle (s) = 20 m and time (t) = 1 s

Let u = Initial velocity of particle at the time of starting.

We know that distance covered by the particle in one second (s),

$$20 = ut + \frac{1}{2}gt^2 = (u \times 1) + \frac{1}{2} \times 9.8\,(1)^2 = u + 4.9$$

$$\therefore \quad u = 20 - 4.9 = 15.1 \text{ m/s}$$

and velocity of the particle after covering 20 metres (or after 1 second)

$$= u + g\,t = 15.1 + (9.8 \times 1) = 24.9 \text{ m/s}$$

Now let (t) be the time required to cover a distance of 20 metres when the particle has initial velocity of 24.9 m/s.

We know that distance covered by the particle in this time (s),

$$20 = ut + \frac{1}{2}gt^2 = 24.9t + \frac{1}{2} \times 9.8t^2 = 24.9t + 4.9t^2$$

$$4.9\,t^2 + 24.9\,t - 20 = 0$$

This is a quadratic equation in *t*.

$$\therefore \qquad t = \frac{-24.9 \pm \sqrt{(24.9)^2 - 4 \times 4.9 \times (-20)}}{2 \times 4.9} = 0.71 \text{ s} \qquad \textbf{Ans.}$$

Example 17.24. *Two particles A and B are dropped simultaneously from rest two points both 100 m above the ground. Particle A falls on the ground, while the particle B in its mid path, hits a fixed plane inclined to the horizontal as shown in the Fig. 17.3.*

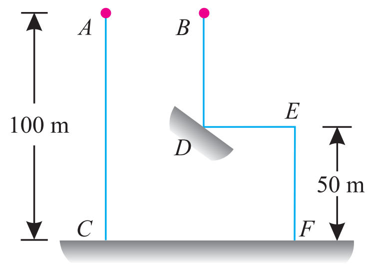

Fig. 17.3.

As a result of this impact, the direction of its velocity becomes horizontal. Compare the times of fall of the particles A and B to reach the ground.

Solution. First of all, consider motion of the stone *A* (*i.e.* from *A* to *C*). In this case, initial velocity (u_1) = 0 (because it is dropped) and distance (s_1) = 100 m

Let $\qquad t_1$ = Time taken by the particle *A* to reach *C*.

We know that distance travelled by the stone *A* (s_1)

$$100 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8\, t_1^2 = 4.9\, t_1^2$$

$$\therefore \qquad t_1 = \sqrt{\frac{100}{4.9}} = 4.52 \text{ s}$$

Now consider motion of the stone *B* (first from *B* to *D*).

In this case, initial velocity (u_2) = 0 (because it is dropped) and distance (s_2) = 50 m

Let $\qquad t_2$ = Time taken by the particle *B* to reach *D*.

We know that the distance travelled by the stone *B* (s_2)

$$50 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8\, t_2^2 = 4.9\, t_2^2$$

$$\therefore \qquad t_2 = \sqrt{\frac{50}{4.9}} = 3.19 \text{ s} \qquad \text{...}(ii)$$

Now consider motion of the stone *B* (from *D* or *E* to *F*).

Since the direction of the particle *B*, after impact at *D*, becomes horizontal, therefore its velocity in the vertical direction becomes zero. A little consideration will show that the particle *B* will take another 3.19 sec to reach from *D* to *F*. Therefore total time taken by the particle *B* to reach *F* will be *T* = 3.19 + 3.19 = 6.38 sec. Therefore ratio of the two times,

$$\frac{T}{t_1} = \frac{6.38}{4.52} = 1.41 \qquad \textbf{Ans.}$$

Example 17.25. *A cage descends in a mine shaft with an acceleration of 0.5 m/s². After the cage has travelled 25 m, a stone is dropped from the top of the shaft. Determine the* (*a*) *time taken by the stone to hit the cage, and* (*b*) *distance travelled by the cage before impact.*

Solution. First of all, consider motion of the stone. In this case, initial velocity of stone (u) = 0 (because it is dropped).

(a) *Time taken by the stone to hit the cage*

Let t = Time taken by the stone to hit the cage

We know that distance travelled by the stone before the impact,

$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8t^2 = 4.9t^2 \qquad ...(i)$$

Now consider motion of the cage for the first 25 metres.

Let t = Time taken by the cage to travel 25 m.

In this case, initial velocity of cage (u) = 0 (because it descends); Accelaration (a) = 0.5 m/s^2 and distance (s) = 25 m

We know that distance travelled by the cage (s),

$$25 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 0.5t^2 = 0.25t^2$$

$$\therefore \qquad t = \sqrt{\frac{25}{0.25}} = \sqrt{100} = 10 \text{ s}$$

It means that the cage has travelled for 10 sec, before the stone was dropped. Therefore total time taken by the cage before impact = $(10 + t)$ s.

We know that distance travelled by the cage in $(10 + t)$ s,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 0.5\,(10+t)^2 = 0{\cdot}25\,(10+t)^2 \quad ...(ii)$$

In order that the stone may hit the cage, the two distances must be equal. Therefore, equating eqution (i) and (ii),

$$4{\cdot}9\,t^2 = 0.25\,(10 + t^2) = 0.25\,(100 + t^2 + 20\,t) = 25 + 0{\cdot}25\,t^2 + 5\,t$$

or $\qquad 4{\cdot}65\,t^2 - 5\,t - 25 = 0$

This is a quadratic equation in t

$$\therefore \qquad t = \frac{5 \pm \sqrt{(5)^2 + (4 \times 4{\cdot}65 \times 25)}}{2 \times 4{\cdot}65} = 2{\cdot}92 \text{ s} \qquad \textbf{Ans.}$$

(b) *Distance travelled by the cage before impact*

Substituting the value of t in equation (ii),

$$s = 0{\cdot}25\,(10 + 2{\cdot}92)^2 = 41{\cdot}7 \text{ m} \qquad \textbf{Ans.}$$

EXERCISE 17.2

1. A stone is thrown vertically upwards with a velocity of 40 m/s. Find its position after 5 seconds. [**Ans.** 77·5 m]

2. An elevator cage is going up with a velocity of 6 m/s. When the cage was 36 m above the bottom of the shaft, a bolt gets detached from the bottom of the cage floor. Find the velocity with which the bolt strikes the bottom of the shaft and the time that elapses. [**Ans.** 27·24 m/s ; 3·39 s]

3. A stone is dropped from the top of a cliff 120 metres high. After one second, another stone is thrown down and strikes the first stone when it has just reached the foot of the cliff. Find the velocity with which the second stone was thrown. [**Ans.** 11·0 m/s]

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4. A body is projected upwards with a velocity of 30 m/s. Find (*a*) the time when its velocity will be 5 m/s ; (*b*) the time when it will be 20 metres above the point of projection and (*c*) the time when it will return to the point of projection.

[**Ans.** 2·55 s : 0·76 sec or 5·36 s ; 6·12 s]

5. A stone is dropped from the top of a tower 60 m high. Another stone is projected upwards at the same time from the foot of the tower, and meets the first stone at a height of 18 m. Find the velocity with which the second stone is projected upwards. [**Ans.** 20·48 m/s]

6. A body, falling under the force of gravity from the top of a tower, covers 5/9 height of the tower in the last second of its motion. Find the height of the tower. [**Ans.** 44·1 m]

7. A particle, starting from rest, falls 70 metres in the last second of its motion. Determine the total time taken by particle to fall, and the height from which it fell.

[**Ans.** 7·64 s ; 2·36 m]

17.5. DISTANCE TRAVELLED IN THE *n*th SECOND

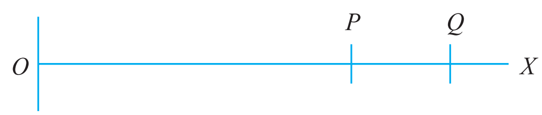


Fig. 17.4. Distance travelled in *n*th second.

Consider the motion of a particle, starting from *O* and moving along *OX* as shown in Fig. 17.4.

Let u = Initial velocity of the particle,

v = Final velocity of the particle

a = Constant positive acceleration,

s_n = Distance (OQ) travelled in n sec,

s_{n-1} = Distance (OP) travelled in $(n-1)$ sec,

s = $(s_n - s_{n-1})$ = Distane (PQ) travelled in nth sec,

n = No. of second.

Substituting the values of $t = n$ and $t = (n-1)$ in the general equation of motion,

$$s_n = un + \frac{1}{2}a(n)^2 \quad ...(i)$$

and

$$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \quad ...(ii)$$

∴ Distance travelled in the *n*th sec,

$$s = s_n - s_{n-1}$$

$$= \left[un + \frac{1}{2}a(n)^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 + 1 - 2n)$$

$$= \frac{1}{2}an^2 + u - \frac{1}{2}an^2 - \frac{1}{2}a + an$$

$$= u - \frac{1}{2}a + an = u + a\left(n - \frac{1}{2}\right) = u + \frac{a}{2}(2n-1)$$

Example 17.26. *A body was thrown vertically downwards from the top of a tower and traverses a distance of 40 metres during its 4th second of its fall. Find the initial velocity of the body.*

Solution. Given : Distance traversed (s) = 40 m ; No of second (n) = 4 and acceleration (a) = g = 9.8 m/s^2

Let u = Initial velocity of the body.

We know that distance traversed by the body in the 4th second (s),

$$40 = u + \frac{a}{2}(2n - 1) = u + \frac{9.8}{2}(2 \times 4 - 1) = u + 34.3$$

or $u = 40 - 34.3 = 5.7$ m/s **Ans.**

Alternative Method

We know that distance travelled in 3 seconds

$$s_3 = ut + \frac{1}{2}gt^2 = u \times 3 + \frac{1}{2} \times 9.8(3)^2 = 3u + 44.1 \text{ m}$$

and distance travelled in 4 seconds,

$$s_4 = ut + \frac{1}{2}gt^2 = u \times 4 + \frac{1}{2} \times 9.8(4)^2 = 4u + 78.4 \text{ m}$$

∴ Distance traversed in the 4th second

$$40 = s_4 - s_3 = (4u + 78.4) - (3u + 44.1) = u + 34.3$$

or $u = 40 - 34.3 = 5.7$ m/s **Ans.**

Example 17.27. *A particle starts from rest. Find the ratio of distances covered by it in the 3rd and 5th seconds of its motion.*

Solution. Given : Initial velocity (u) = 0 (because it starts from rest) ; Initial no. of second (n_1) = 3 and final no. of second (n_2) = 5.

We know that distance covered by the particle in the 3rd second after it starts,

$$s_3 = u + \frac{a}{2}(2n_1 - 1) = 0 + \frac{a}{2}[(2 \times 3) - 1] = \frac{5a}{2} \quad ...(i)$$

and distance covered by the particle in the 5th second after it starts,

$$s_5 = u + \frac{a}{2}(2n_2 - 1) = 0 + \frac{a}{2}[(2 \times 5) - 1] = \frac{9a}{2} \quad ...(ii)$$

∴ Ratio of distances $= s_3 : s_5 = \frac{5a}{2} : \frac{9a}{2} = 5 : 9.$ **Ans.**

Example 17.28. *A body, falling freely from rest travels in the first three seconds, a distance equal to the distance travelled by it in a certain second. Find the time of its travel for the body.*

Solution. Given : Initial velocity (u) = 0 (because it falls freely) and distance travelled in first three seconds = Distance travelled in nth second.

Let n = Time of travel for the body.

We know that distance travelled in the first 3 s

$$s = ut + \frac{1}{2}gt^2 = (0 \times 3) + \frac{1}{2} \times g \times (3)^2 = 4{\cdot}5\, g \quad ...(i)$$

and distance travelled in the nth second after it starts,

$$s_n = u + \frac{g}{2}(2n - 1) = 0 + \frac{g}{2}(2n - 1) = \frac{g}{2}(2n - 1). \quad ...(ii)$$

Since both distances are equal, therefore equating equations (i) and (ii)

$$4{\cdot}5\ g = \frac{g}{2}(2n-1) \qquad \text{or} \qquad 2n-1 = \frac{4{\cdot}5\ g}{g/2} = 9$$

or $$2n = 9 + 1 = 10 \qquad \text{or} \qquad n = 5 \text{ s} \quad \textbf{Ans.}$$

Example 17.29. *A train starts from rest with an acceleration a and describes distances s_1, s_2 and s_3 in the first, second and third seconds of its journey. Find the ratio of $s_1 : s_2 : s_3$.*

Solution. Given : Initial velocity of train (u) = 0 (because it starts from rest) ; Acceleration = a ; Distance described in 1st second = s_1; Distance described in 2nd second = s_2 and distance described in 3rd second = s_3.

We know that distance described by the train in first second,

$$s_1 = u + \frac{a}{2}(2n_1 - 1) = 0 + \frac{a}{2}[(2 \times 1) - 1] = \frac{a}{2} \qquad ...(i)$$

Similarly, distance described in second second,

$$s_2 = u + \frac{a}{2}(2n_2 - 1) = 0 + \frac{a}{2}[(2 \times 2) - 1] = \frac{3a}{2} \qquad ...(ii)$$

and distance described in third second,

$$s_3 = u + \frac{a}{2}(2n_3 - 1) = 0 + \frac{a}{2}[(2 \times 3) - 1] = \frac{5a}{2} \qquad ...(iii)$$

$\therefore$ Ratio of distances $s_1 : s_2 : s_3$

$$= \frac{a}{2} : \frac{3a}{2} : \frac{5a}{2} = 1 : 3 : 5 \quad \textbf{Ans.}$$

Example 17.30. *By what initial velocity a ball should be projected vertically upwards, so that the distance covered by it in 5th second is twice the distance it covered in its 6th second ? (Take g = 10 m/s^2)*

Solution. Given : Initial no. of second (n_1) = 5 ; Final no. of second (n_2) = 6 and acceleration due to gravity (g) = 10 m/s^2.

Let u = Initial velocity of the ball.

We know that distance covered by the ball in the 5th second after it starts

$$s_5 = -u + \frac{g}{2}(2n_1 - 1) = -u + \frac{10}{2}[(2 \times 5) - 1]$$

$$= -u + 45 \qquad ...(i)$$

...(Minus sign due to upward direction)

and distance covered by it in the 6th second after it starts

$$s_6 = -u + \frac{g}{2}(2n_2 - 1) = -u + \frac{10}{2}[(2 \times 6) - 1]$$

$$= -u + 55 \qquad ...(ii)$$

...(Minus sign due to upward direction)

Since the distance covered by the ball in the 5th second is twice the distance covered by it in the 6th second, therefore

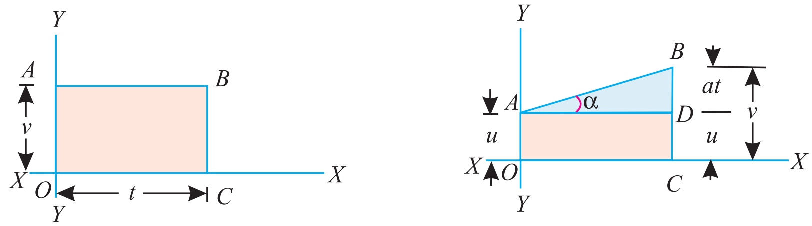
$$-u + 45 = 2(-u + 55) = -2u + 110$$

or $$u = 110 - 45 = 65 \text{ m/s} \qquad \textbf{Ans.}$$

17.6. GRAPHICAL REPRESENTATION OF VELOCITY, TIME AND DISTANCE TRAVELLED BY A BODY

The motion of body may also be represented by means of a graph. Such a graph may be drawn by plotting velocity as ordinate and the corresponding time as abscissa as shown in Fig. 17.5. (a) and (b). Here we shall discuss the following two cases :

1. *When the body is moving with a uniform velocity*



(*a*) Uniform velocity (*b*) Variable velocity

Fig. 17.5.

Consider the motion of a body, which is represented by the graph *OABC* as shown in Fig. 17.5(*a*). We know that the distance traversed by the body,

$$s = \text{Velocity} \times \text{Time}$$

Thus we see that the area of the figure *OABC* (*i.e.*, velocity × time) represents the distance traversed by the body, to some scale.

2. *When the body is moving with a variable velocity*

We know that the distance traversed by a body,

$$s = ut + \frac{1}{2}at^2$$

From the geometry of the Fig. 17.5 (*b*), we know that area of the figure *OABC*

$$= \text{Area } (OADC + ABD)$$

But area of figure $OADC = u \times t$

and area of figure $ABD = \frac{1}{2} \times t \times at = \frac{1}{2}at^2$

$\therefore$ Total area $OABC = ut + \frac{1}{2}at^2$

Thus, we see that the area of the *OABC* represents the distance traversed by the body to some scale. From the figure it is also seen

$$\tan \alpha = \frac{at}{t} = a$$

Thus, tan α represents the acceleration of the body.

Example 17.31. *A lift goes up to a height of 900 m with a constant acceleration and then the next 300 m with a constant retardation and comes to rest. Find (i) maximum velocity of the lift, if the time taken to travel is 30 seconds ; (ii) acceleration of the lift ; and (iii) retardation of the lift. Take acceleration of the lift as 1/3 of its retardation.*

Solution. Let *OAB* be the velocity-time graph, in which the ordinate *AL* represents the maximum velocity as shown in Fig. 17.6.

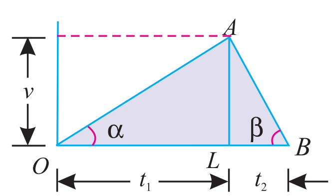


Fig. 17.6.

(*i*) *Maximum velocity of the lift*

Let
t_1 = Time of acceleration
t_2 = Time of retardation, and
v = Maximum velocity of lift.

First of all, consider motion of the lift from *O* to *A*. We know that the area of triangle *OAL* (s_1),

$$900 = \frac{1}{2} \times t_1 \times v \qquad ...(i)$$

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Now consider motion of the lift from A to B. We know that area of the triangle ALB (s_2)

$$300 = \frac{1}{2} \times t_2 \times v \qquad ...(ii)$$

Dividing equation (i) by (ii),

$$\frac{900}{300} = \frac{t_1}{t_2} \qquad \text{or} \qquad t_1 = 3t_2$$

But $\qquad t_1 + t_2 = 30 \text{ s}$

$\therefore \qquad t_1 = 22.5 \text{ s} \qquad \text{and} \qquad t_2 = 7.5 \text{ s}$

Now substituting the value of t_1 in equation (i),

$$900 = \frac{1}{2} \times 22.5 \times v = 11.25\, v$$

$$\therefore \qquad v = \frac{900}{11.25} = 80 \text{ m/s} \quad \textbf{Ans.}$$

(ii) *Acceleration of the lift*

From geometry of the figure, we find that acceleration, of the lift,

$$a_1 = \tan \alpha = \frac{AL}{OL} = \frac{80}{22.5} = 3.55 \text{ m/s}^2 \quad \textbf{Ans.}$$

(iii) *Retardation of the lift*

We also know that retardation of the lift,

$$a_2 = 3a_1 = 3 \times 3.55 = 10.65 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Example 17.32. *A train moving with a velocity of 30 km.p.h. has to slow down to 15 km.p.h. due to repairs along the road. If the distance covered during retardation be one kilometer and that covered during acceleration be half a kilometer, find the time lost in the journey.*

Solution. Let $OABCD$ be the velocity-time graph, in which AB represents the period of retardation and BC period of acceleration as shown in Fig. 17.7.

First of all, consider motion of the train from A to B. In this case, distance travelled (s_1) = 1 km; initial velocity (u_1) = 30 km. p.h. and final velocity (v_1) = 15 km.p.h.

Let $\qquad t_1$ = Time taken by the train to move from A to B.

We know that the area of the trapezium $OABE$ (s_1)

$$1 = \frac{30 + 15}{2} \times t_1 = 22.5\, t_1$$

$$\therefore \qquad t_1 = \frac{1}{22.5} \text{ hr} = 2.67 \text{ min} \qquad ...(i)$$

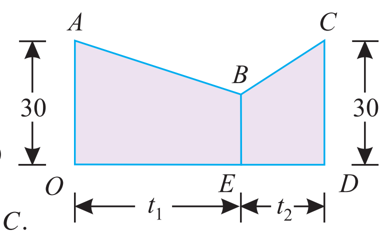

Fig. 17.7.

Now consider motion of the train from B to C. In this case, distance travelled (s_2) = 0.5 km ; Initial velocity (u_2) = 15 km.p.h. and final velocity (v_2) = 30 km.p.h.

Let $\qquad t_2$ = Time taken by the train to move from B to C.

We also know that the area of trapezium $BCDE$ (s_2),

$$\frac{1}{2} = \frac{15 + 30}{2} \times t = 22.5\, t_2$$

$$\text{or} \qquad t_2 = \frac{1}{45} \text{ hr} = 1.33 \text{ min} \qquad ...(ii)$$

$\therefore \qquad$ Total time, $t = t_1 + t_2 = 2.67 + 1.33 = 4$ min

If the train had moved uniformly with a velocity of 30 km/hr, then the time required to cover 1.5 km

$$= \frac{60}{30} \times \frac{3}{2} = 3 \text{ min} \qquad \ldots(iii)$$

∴ Time lost = 4 – 3 = 1 min **Ans.**

Example 17.33. *A cage goes down a main shaft 750 m deep, in 45 s. For the first quarter of the distance only, the speed is being uniformly accelerated and during the last quarter uniformly retarded, the acceleration and retardation being equal. Find the uniform speed of the cage, while traversing the central portion of the shaft.*

Solution. Let *OABC* represent the velocity-time graph in which *OA* represents the period of acceleration, *AB* the period of uniform velocity and *BC* the period of retardation as shown in Fig. 17.8.

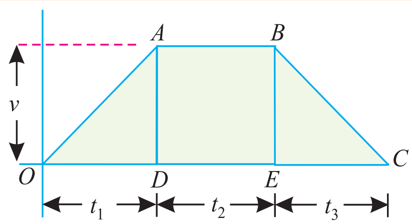

Fig. 17.8.

First of all consider motion of cage from *O* to *A*. In this case, initial velocity $(u_1) = 0$ (because it goes down from rest) and distance travelled $(s_1) = \frac{750}{4} = 187.5$ m

Let a_1 = Constant acceleration of the cage, and

v_1 = Uniform velocity of the cage (*AD* or *BE*).

We know that area of triangle *OAD* (s_1),

$$187.5 = \frac{1}{2} \times t_1 \times v_1$$

$$\therefore \quad t_1 \times v_1 = 187.5 \times 2 = 375 \qquad \ldots(i)$$

Now consider the motion of the cage from *A* to *B*. In this case, distance travelled (s_2) = 750 – (2 × 187.5) = 375 m

We also know that the area of the rectangle *ABED* (s_2),

$$375 = t_2 \times v_1 \qquad \ldots(ii)$$

From equation (*i*) and (*ii*), we find that

$$t_1 = t_2$$

Similarly $\quad t_2 = t_3$

$$\therefore \quad t_1 = t_2 = t_3$$

Since the total time taken $(t_1 + t_2 + t_3)$ is 45 seconds, therefore

$$t_1 = t_2 = t_3 = 15 \text{ s}$$

Again consider the motion of the cage from *O to A*. In this case, Initial velocity $(u_1) = 0$; distance travelled $(s_1) = 187.5$ m and time *OABC*$(t_1) = 15$ s

We know that the distance (s_1),

$$187.5 = u_1 t_1 + \frac{1}{2} a_1 t_1^2 = 0 + \frac{1}{2} a_1 (15)^2 = 112.5\, a_1$$

$$\therefore \quad a_1 = \frac{187.5}{112.5} = 1.67 \text{ m/s}^2$$

and speed of the cage while traversing the central portion of the shaft

$$v_1 = u + a_1 t_1 = 0 + 1.67 \times 15 = 25 \text{ m/s} \quad \textbf{Ans.}$$

EXERCISE 17.3

1. A car starts from rest with an acceleration of 4 m/s^2. What is the distance travelled in 8th second ? [**Ans.** 30 m]
2. A train travels between two stations 10 kilometers apart in 15 minutes. Assuming that its motion is with uniform acceleration for the first part of the journey and with uniform retardation for the rest, show that the greatest velocity of the train is 80 kilometers per hour.
3. An electric train starts from a station and comes to another station which is 5 kilometers from the first on a straight track. For three-quarters of the distance, the train is uniformly accelerated and for the remainder uniformly retarded. If the train takes 12 minutes to cover the whole journey, find (*i*) maximum speed the train attains ; (*ii*) acceleration of the train ; and (*iii*) retardation of the train. [**Ans.** 50 km.p.h. ; 333.3 km/hr^2 ; 1000 km/hr^2]
4. An electric train, travelling between two stations 1.5 km apart is uniformly accelerated for the first 10 seconds, during which period it covers 100 m. It then runs with a constant speed, until it is finally retarded uniformly in the last 50 m. Find the maximum speed of the train and the time taken to complete the journey between the two stations.

   [**Ans.** 20 m/s ; 82.5 s]
5. A passenger train passes a certain station at 60 kilometers per hour, and covers a distance of 12 kilometers with this speed and then stops at next station 15 kilometers from the first, with uniform retardation. A local train, starting from the first station covers the same distance in double this time and stops at the next station.

   Determine the maximum speed of the local train which covers a part of the distance, with uniform acceleration and the rest with uniform retardation. [**Ans.** 50 km.p.h.]

QUESTIONS

1. How would you find out, if a particular body is at rest or in motion ?
2. Distinguish clearly between speed and velocity. Give examples.
3. What do you understand by the term ‘acceleration ? Define positive acceleration and negative acceleration.
4. What is the difference between uniform acceleration and variable acceleration ?
5. How does the velocity of a moving point, that possesses a unifomly variable motion, change if acceleration is positive ? How does it change, if acceleration is negative ?
6. Prove the relationship,

$$s = ut + \frac{1}{2}at^2$$

   in a body, subjected to a uniformly accelerated motion.
7. What is the force of gravity ? How does it effect the body, when,
   (1) it is allowed to fall downwards.
   (2) it is projected upwards ?
8. Derive a relation for the distance travelled by a body in the *n*th second.
9. How would you study the motion of a body graphically ? Discuss the uses of such diagrams.

OBJECTIVE TYPE QUESTIONS

1. The relationship $s = ut + \frac{1}{2}at^2$ is applicable to bodies

(*a*) moving with any type of motion
(*b*) moving with uniform velocity
(*c*) moving with uniform acceleration
(*d*) both (*b*) and (*c*)

2. The motion under gravity is a particular case of motion under constant velocity.

(*a*) Yes (*b*) No

3. If two bodies *A* and *B* are projected upwards such that the velocity of *A* is double the velocity of *B*, then the height to which the body *A* will rise will be .... the height to which the body *B* will rise

(*a*) two times (*b*) four times (*c*) eight times

4. A lecturer told the class that if a body is projected upwards from any point, then the body while coming down at the same point will have the same velocity with which it was projected upwards. Is his statement correct?

(*a*) Agree (*b*) Disagree

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*b*) **4.** (*a*)

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