CHAPTER

25 Motion of Connected Bodies

Contents

25.1. INTRODUCTION

In the previous chapter, we discussed the motion of a body, when it is subjected to some external force. But in this chapter, we shall discuss the motion of a body, when it is connected by a string to another body, which is subjected to some external force. A little consideration will show, that in this case the velocity and acceleration of both the bodies will be the same. Now in this chapter, we shall discuss the following cases :

1. Two bodies connected by a string and passing over a smooth pulley.
2. Two bodies connected by a string one of which is hanging free and the other lying on a horizontal plane.

3. Two bodies connected by a string one of which is hanging free and the other lying on an inclined plane.
4. Two bodies connected by a string and lying on two inclined planes.

25.2. MOTION OF TWO BODIES CONNECTED BY A STRING AND PASSING OVER A SMOOTH PULLEY

Consider two bodies of masses m_1 and m_2 kg respectively connected by an inextensible light string (*i.e.,* its weight is neglected) and passing over a small smooth fixed pulley as shown in Fig. 25.1.

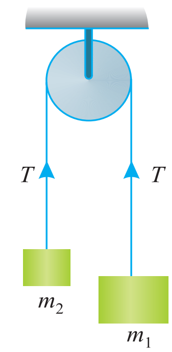

Fig. 25.1.

It may be noted that if the string is light (*i.e.* its weight is neglected) the tension will be the same throughout its length. But if the string is heavy (*i.e.,* its weight is considered) the tension will vary, depending upon the weight per unit length. Moreover, if the string is extensible the tension will also vary with the extension. It may also be noted that if the string passes over a smooth pulley, the tension will be the same on both sides. But if the string does not pass over a smooth pulley, the tension, in the two strings will also vary.

For simplicity, we shall consider light inextensible string passing over a smooth pulley, so that the tension in both the strings may be the same.

Let m_1 be greater than m_2. A little consideration will show, that the greater mass m_1 will move downwards, whereas the smaller one will move upwards. Since the string is inextensible, the upward acceleration of the mass m_2 will be equal to the downward acceleration of the mass m_1.

Let a = Acceleration of the bodies and

T = Tension in both the strings.

First of all, consider the motion of body 1 of mass m_1 kg, which is coming downwards. We know that the forces acting on it are $m_1 . g$ newtons (downwards) and T newtons (upwards). As the body is moving downwards, therefore resultant force

$$= m_1 g - T \text{ (downwards)} \qquad ...(i)$$

Since this body is moving downwards with an acceleration (a), therefore force acting on this body

$$= m_1 a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$m_1 g - T = m_1 a \qquad ...(iii)$$

Now consider the motion of body 2, of mass m_2 kg, which is moving upwards. We know that the forces acting on it are $m_2\, g$ newtons (downwards) and T newtons (upwards). As the body is moving upwards, therefore resultant force

$$= T - m_2\, g \text{ (upwards)} \qquad ...(iv)$$

Since this body is moving upwards with an acceleration (a), therefore, force acting on this body

$$= m_2\, a \qquad ...(v)$$

Equating the equations (iv) and (v),

$$T - m_2\, g = m_2\, a \qquad ...(vi)$$

Now adding equations (iv) and (v),

$$m_1\, g - m_2\, g = m_1\, a + m_2\, a$$

$$g\,(m_1 - m_2) = a\,(m_1 + m_2)$$

$$\therefore \qquad a = \frac{g(m_1 - m_2)}{m_1 + m_2}$$

From equation (*vi*) we have

$$T = m_2\, a + m_2\, g = m_2\,(a + g)$$

$$= m_2 \left[\frac{g(m_1 - m_2)}{m_1 + m_2} + g\right]$$

$$= m_2 \left[\frac{g(m_1 - m_2) + g(m_1 + m_2)}{m_1 + m_2}\right]$$

$$= \frac{m_2 . g}{m_1 + m_2}(m_1 - m_2 + m_1 + m_2)$$

$$= \frac{2\, m_1\, m_2\, g}{m_1 + m_2}$$

Example 25.1. *Two bodies of masses 45 and 30 kg are hung to the ends of a rope, passing over a frictionless pulley. With what acceleration the heavier mass comes down? What is the tension in the string?*

Solution. Given : Mass of first body (m_1) = 45 kg and mass of the second body (m_2) = 30 kg

Acceleration of the heavier mass

We know that acceleration of the heavier mass,

$$a = \frac{g(m_1 - m_2)}{m_1 + m_2} = \frac{9.8(45 - 30)}{45 + 30} = 1.96 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Tension in the string

We also know that tension in the string,

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 45 \times 30 \times 9.8}{45 + 30} = 352.8 \text{ N} \qquad \textbf{Ans.}$$

Example 25.2. *Determine the tension in the strings and accelerations of two blocks of mass 150 kg and 50 kg connected by a string and a frictionless and weightless pulley as shown in Fig. 24.2.*

Solution. Given : Mass of first block (m_1) = 150 kg and mass of second block (m_2) = 50 kg

Acceleration of the blocks

Let a = Acceleration of the blocks

T = Tension in the string in N.

First of all, consider the motion of the 150 kg block, which is coming downwards. We know that forces acting on it are $m_1.g = 150\,g$ newtons (downwards) and $2T$ newtons (upwards).

Therefore resultant force $= 150\,g - 2T$ (downwards) ...(*i*)

Since the block is moving downwards with an acceleration (a), therefore force acting on this block

$= 150\,a$ ...(*ii*)

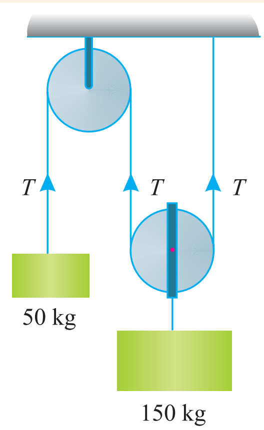

Fig. 25.2.

Equating equations (*i*) and (*ii*),

$$150\ g - 2\ T = 150\ a \qquad ...(iii)$$

Now consider the motion of 50 kg block, which is going upwards. A little consideration will show that its acceleration will be (2*a*). We know that forces acting on it are $m_2.g = 50\ g$ newtons (downwards) and T newtons upwards. Therefore resultant force

$$= T - 50\ g \text{ (upwards)} \qquad ...(iv)$$

Since the block is moving upwards with an acceleration of (2*a*) therefore force acting on this block

$$= 50 \times 2a = 100\ a \qquad ..(v)$$

Equating equations (*iv*) and (*v*),

$$T - 50\ g = 100\ a$$

Multiplying the above equation by 2,

$$2T - 100\ g = 200\ a \qquad ...(vi)$$

Adding equations (*iii*) and (*vi*),

$$50\ g = 350\ a$$

$$\therefore \qquad a = \frac{50g}{350} = \frac{50 \times 9.8}{350} = 1.4 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and acceleration of the block *B*

$$= 2a = 2 \times 1.4 = 2.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Tension in the strings

Substituting the value of *a* in equation (*iii*),

$$150g - 2T = 150 \times 1.4 = 210$$

$$\therefore \qquad 2T = 150g - 210 = 150 \times 9.8 - 210 = 1260$$

$$\text{or} \qquad T = \frac{1260}{2} = 630 \text{ N} \qquad \textbf{Ans.}$$

Example 25.3. *A system of masses connected by string, passing over pulleys A and B is shown in Fig. 25.3 below :*

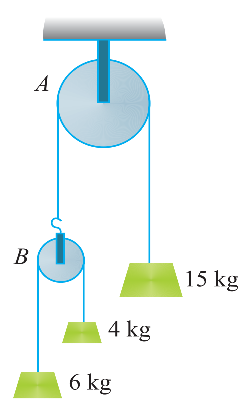

Fig. 25.3.

Find the acceleration of the three masses, assuming weightless strings and ideal conditions for pulleys.

Solution. Given : First mass $(m_1) = 15$ kg ; Second mass $(m_2) = 6$ kg and third mass $(m_3) = 4$ kg

From the system of pulleys and masses, we find that at pulley *A*, the 15 kg mass will come down with some acceleration as the total mass on the other side of the string is less than 15 kg. Similarly, at pulley *B*, the 6 kg mass will come down with some acceleration.

Let a_{15} = Acceleration of 15 kg mass

a_6 = Acceleration of 6 kg mass, and

a_4 = Acceleration of 4 kg mass.

We know that acceleration of the 15 kg mass,

$$a_{15} = \frac{g(m_1 - m_2)}{m_1 + m_2} = \frac{9.8[15-(6+4)]}{15+(6+4)} = 1.96 \text{ m/s}^2$$

Similarly,
$$a_6 = a_4 = \frac{9.8(6-4)}{6+4} = 1.96 \text{ m/s}^2$$

Now if we look at all the three masses to move simultaneously at pulleys *A* and *B*, we find that

1. The mass 15 kg will come downwards with an acceleration of 1.96 m/s^2. **Ans.**
2. The pulley *B* will go up with an acceleration of 1.96 m/s^2.
3. The 6 and 4 kg masses will go up (because the pulley *B* is going up) with an acceleration of 1.96 m/s^2. Moreover, at pulley *B*, the 6 kg mass will come down (because 4 kg mass will go up) with an acceleration of 1.96 m/s^2. Thus the net acceleration of the 6 kg mass will be zero. **Ans.**
4. At pulley *B* the 4 kg mass will go up (because the 6 kg mass will come down) with an acceleration of 1.96 m/s^2. Thus the net acceleration of the 4 kg mass will be 1.96 + 1.96 = 3.92 m/s^2. **Ans.**

25.3. MOTION OF TWO BODIES CONNECTED BY A STRING, ONE OF WHICH IS HANGING FREE AND THE OTHER LYING ON A SMOOTH HORIZONTAL PLANE

Consider two bodies of masses m_1 and m_2 kg respectively connected by a light inextensible string as shown in Fig. 25.4.

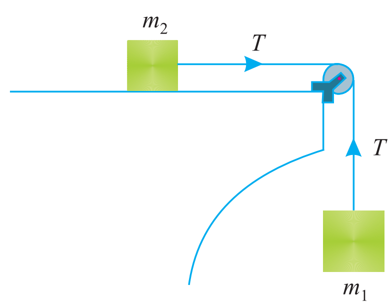

Fig. 25.4 Body lying over smooth horizontal plane.

Let the body of mass m_1 hang free and the body of mass m_2 be placed on a smooth (*i.e.,* friction between the body of mass m_2 and the horizontal plane is neglected) horizontal plane. It may be noted that the body of mass m_1 will move downwards and the body of mass m_2 along the surface of the plane.

We know that velocity and acceleration of the body of mass m_1 will be the same as that of the body of mass m_2. Since the string is inextensible, therefore, tension in both the strings will also be equal,

Let a = Acceleration of the system and

T = Tension in the strings

First of all, consider the motion of body 1 of mass m_1 downwards. We know that forces acting on it are $m_1.g$ (downwards) and T (upwards). As the body is moving downwards, therefore resultant force

$$= m_1 g - T \text{ (downwards)} \qquad ...(i)$$

Since this body is moving downwards with an accleration (a) therefore force acting on the body

$$= m_1 a \qquad ...(ii)$$

Equating the equations (i) and (ii),

$$m_1 g - T = m_1 a \qquad ...(iii)$$

Now consider the motion of body 2 of mass m_2, which is moving horizontally. We know that only force acting on it

$$= T \text{ (horizontal)} \qquad ...(iv)$$

Since this body is moving horizontally with an acceleration (a), therefore force acting on this body

$$= m_2 a \qquad ...(v)$$

Equating the equations (iv) and (v),

$$T = m_2 a \qquad ...(vi)$$

Adding equation (iii) and (vi),

$$m_1 g = m_1 a + m_2 a = a (m_1 + m_2)$$

$$\therefore \qquad a = \frac{m_1 g}{m_1 + m_2}$$

Substituting this value of a in equation (vi),

$$T = m_2 \times \frac{m_1 g}{m_1 + m_2} = \frac{m_1 m_2 g}{m_1 + m_2}$$

Example 25.4. *Find the acceleration of a solid body A of mass 10 kg, when it is being pulled by another body B of mass 5 kg along a smooth horizontal plane as shown in Fig. 25.5.*

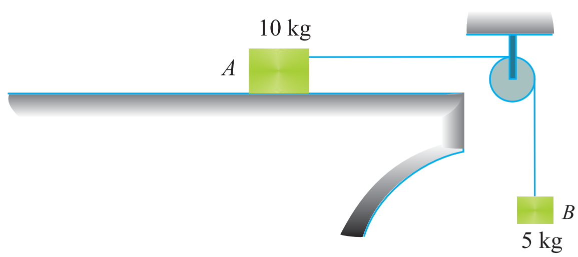

Fig. 25.5.

Also find the tension in the string, assuming the string to be inextensible. Take g = 9.8 m/s².

Solution. Given : Mass of body $A(m_2)$ = 10 kg : mass of body $B(m_1)$ = 5 kg and acceleration due to gravity (g) = 9.8 m/s^2.

Acceleration of the body A

We know that the acceleration of the body A,

$$a = \frac{m_1 g}{m_1 + m_2} = \frac{5 \times 9.8}{5 + 10} = 3.27 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Tension in the string

We know that tension in the string,

$$T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{5 \times 10 \times 9.8}{5 + 10} = 32.7 \text{ N} \qquad \textbf{Ans.}$$

25.4. MOTION OF TWO BODIES CONNECTED BY A STRING, ONE OF WHICH IS HANGING FREE AND THE OTHER LYING ON A ROUGH HORIZONTAL PLANE

Consider two bodies of masses m_1 and m_2 kg respectively, connected by a light inextensible string as shown in Fig. 25.6.

Let the body of mass m_1 hang free, and the body of mass m_2 be placed on a rough horizontal plane. Let the body of mass m_1 move downwards and the body of mass m_2 move along the surface of the plane.

We know that velocity and acceleration of the body of mass m_1 will be the same as that of mass m_2. Since the string is inextensible, therefore tensions in both the strings will also be equal.

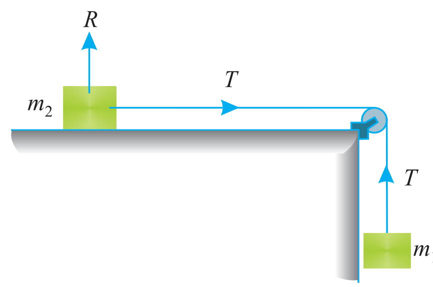

Fig. 25.6.

Let a = Acceleration of the system,

T = Tension in the string and

μ = Coefficient of friction.

We know that the normal reaction on the horizontal surface due to body of mass m_2 kg (as shown in Fig. 25.6)

$$R = m_2\, g$$

$\therefore$ Frictional force $= \mu\, R = \mu\, m_2\, g$

This frictional force will act in the opposite direction to the motion of the body of mass 2.

First of all, consider the motion of the body of mass m_1 kg, which is coming downwards. We know that forces acting on it are $m_1.g$ (downwards) and T upwards. As the body is moving downwards, therefore, resultant force

$$= m_1\, g - T \text{ (downwards)} \qquad ...(i)$$

Since this body is moving downwards with an acceleration (a), therefore force acting on this body

$$= m_1\, a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$m_1\, g - T = m_1\, a \qquad ...(iii)$$

Now consider the motion of body 2 of mass m_2 kg, which is moving horizontally. We know that the forces acting on it are T towards right and frictional force $\mu m_2 g$ towards left. As the body is moving towards right, therefore, resultant force

$$= T - \mu\, m_2\, g \qquad ...(iv)$$

Since this body is moving horizontally with an acceleration (a) therfore force acting on this body

$$= m_2\, a \qquad ...(v)$$

Equating the equations (iv) and (v),

$$T - \mu\, m_2\, g = m_2\, a \qquad ...(vi)$$

Adding equation (*iii*) and (*vi*),

$$m_1 g - \mu m_2 g = m_1 a + m_2 a$$

$$g(m_1 - \mu m_2) = a(m_1 + m_2)$$

$$\therefore \qquad a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2} \text{ m/s}^2$$

From equation (*vi*) we find that

$$T = m_2 a + \mu m_2 g = m_2 (a + \mu g)$$

Now substituting the value of a in the above equation,

$$T = m_2 \left[\frac{g(m_1 - \mu m_2)}{m_1 + m_2} + \mu g \right]$$

$$= m_2 g \left[\frac{m_1 - \mu m_2 + \mu m_1 + \mu m_2}{m_1 + m_2} \right]$$

$$= \frac{m_1 m_2 g (1 + \mu)}{m_1 + m_2}$$

Note. For smooth surface, if we substitute the value of $\mu = 0$ in the above equations for a and T, the relations obtained will be the same as we derived in the last article.

Example 25.5. *Two blocks shown in Fig. 25.7, have masses A = 20 kg and B = 10 kg and the coefficient of friction between the block A and the horizontal plane, μ = 0.25.*

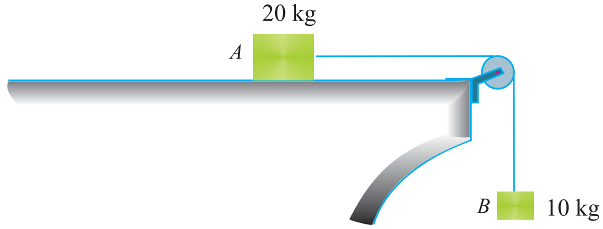

Fig. 25.7.

If the system is released, from rest, and the block B falls through a vertical distance of 1m, what is the velocity acquired by it? Neglect the friction in the pulley and the extension of the string.

Solution. Given : Mass of block A (m_2) = 20 kg ; Mass of block B (m_1) = 10 kg ; Coefficient of friction between block A and horizontal plane (μ) = 0.25 ; Initial velocity (u) = 0 (because the system is released from rest) and vertical distance (s) = 1 m

Let $\qquad v$ = Final velocity of the block A.

We know that acceleration of the block A,

$$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2} = \frac{9.8(10 - 0.25 \times 20)}{10 + 20} = 1.63 \text{ m/s}^2$$

and $\qquad v^2 = u^2 + 2as = 0 + 2 \times 1.63 \times 1 = 3.26$

$\therefore \qquad v = 1.81$ m/s **Ans.**

Example 25.6. *A block of wood A of mass 10 kg is held on a rough horizontal table. An elastic string connected to the block passes over a smooth pulley at the end of the table and then under a second smooth pulley carrying a body B of mass 5 kg as shown in Fig. 25.8.*

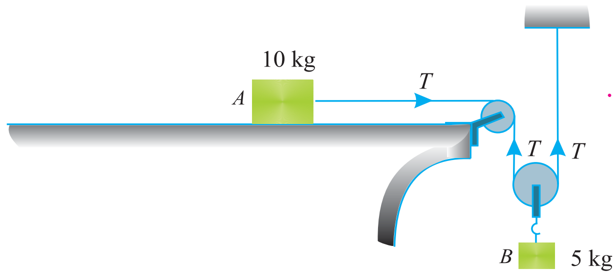

Fig. 25.8.

The other end of the string is fixed to a point above the second pulley. When the 10 kg block is released, it moves with an acceleration of g/9. Determine the value of coefficient of friction between the block and the table.

Solution. Given : Mass of block A $(m_2) = 10$ kg ; Mass of body B $(m_1) = 5$ kg and accelaration of block $A = \dfrac{g}{9}$.

Let T = Tension in the string in N, and

μ = Coefficient of friction between block and table.

We know that the normal reaction on the horizontal surface due to body of mass 10 kg

$$R = 10\,g$$

$\therefore$ Frictional force $= \mu R = \mu \times 10\,g = 10\,\mu g$

First of all consider the motion of block A, which is moving horizontally. We know that the forces acting on it are T (towards right) and frictional force $10\,\mu g$ (towards left). As the block is moving towards right, therefore resultant force

$$= T - 10\,\mu g \qquad ...(i)$$

Since the block is moving with an acceleration of (g/9) therefore force acting on it

$$= 10 \times \frac{g}{9} = \frac{10g}{9} \qquad ...(ii)$$

Equating the equations (*i*) and (*ii*)

$$T - 10\mu g = \frac{10g}{9}$$

Multiplying both sides by 2,

or $$2T - 20\mu g = \frac{20g}{9} \qquad ...(iii)$$

Now consider the motion of the block B, which is coming downwards. A little consideration will show the acceleration of this block will be half of that of the block A *i.e.* $g/18$.We know that the forces acting on it are $mg = 5\,g$ (downwards) and $2T$ (upwards). Therefore resultant force

$$= 5g - 2T \qquad ...(iv)$$

Since the block is moving with an acceleration of $(g/18)$ therefore force acting in it

$$= ma = 5 \times \frac{g}{18} = \frac{5g}{18} \qquad ...(v)$$

Equating equations (*iv*) and (*v*),

$$5g - 2T = \frac{5g}{18} \quad ...(vi)$$

Adding equations (*iii*) and (*vi*),

$$5g - 20\mu g = \frac{20g}{9} + \frac{5g}{18} = \frac{45g}{18} = 2.5\ g$$

$\therefore$ $$20\mu = 5 - 2.5 = 2.5$$

or $$\mu = \frac{2.5}{20} = 0.125 \quad \textbf{Ans.}$$

EXERCISE 25.1

1. A mass of 9 kg, while descending vertically down, drags up a mass of 6 kg by means of a string passing over a smooth pulley. Find the acceleration of the system and tension in the string. (**Ans.** 1.96 m/s^2 ; 70.6 N)
2. Two bodies of mass 3 kg and 2.5 kg are hung to the ends of a string passing over a smooth pulley. At the end of 5 seconds, the string breaks. How much higher the 2 kg mass will go? (**Ans.** 11.1 m)
3. A body of mass 4.5 kg is placed on a smooth table at a distance of 2 m from the edge. The body is connected by a light string passing over a smooth pulley. The other end of the string is connected with a body of mass 2.5 kg. Find (*i*) acceleration of the system ; and (*ii*) time that elapses before the body reaches edge of the table. (**Ans.** 2.8 m/s^2 ; 1.2 s)
4. As body of mass 4 kg lying on a rough horizontal plane is attached to one end of a string. The string passes over a smooth pulley and carries, at its other end, a body of mass 10 kg which hangs freely vertical down. If the system starts from rest, and attains an acceleration of 6 m/s^2, find the coefficient of friction. (**Ans.** 0.35)
5. Two blocks of mass 50 kg and 40 kg are connected by a light inextensible string as shown in Fig. 25.9.

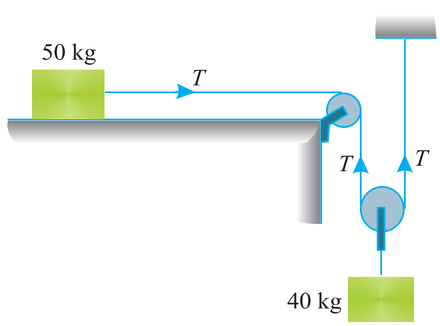

Fig. 25.9.

Find, from first principles, the acceleration of the system and tensions in the cable. Take $\mu = 0.3$. (**Ans.** 0.82 m/s^2 ; 188 N)

25.5. MOTION OF TWO BODIES CONNECTED BY A STRING, ONE OF WHICH IS HANGING FREE AND OTHER LYING ON A SMOOTH INCLINED PLANE

Consider two bodies of masses m_1 and m_2 kg respectively connected by a light inextensible string as shown in Fig. 25.10.

Let the body of mass m_1 hang free and the body of mass m_2 be placed on an inclined smooth plane (*i.e.*, the friction between the mass m_2 and the plane is neglected).

It may be noted that the body of mass m_1 will move downwards and the body of mass m_2 will move upwards along the inclined surface. A little consideration will show that the velocity and acceleration of the body of mass m_1 will be the same as that of the body of mass m_2. Since the string is inextensible, therefore tension in both the strings will also be equal.

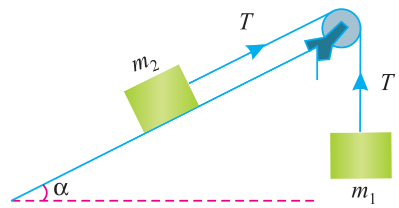

Fig. 25.10.

Let a = Acceleration of the system

T = Tension in the strings and

α = Inclination of the plane.

First of all, consider the motion of the body 1 of mass m_1 kg, which is coming downwards. We know that the forces acting, on it are $m_1 g$ (downwards) and T (upwards). As the body is moving downwards, therefore resultant force

$$= m_1\, g - T \qquad ...(i)$$

Since this body is moving downwards with an acceleration (a), therefore force acting on this body

$$= m_1\, a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$m_1\, g - T = m_1\, a \qquad ...(iii)$$

Now consider the motion of body 2 of mass m_2, which is moving upward along the inclined surface. We know that forces acting on it, along the plane, are T (upwards) and m_2.g sin α (downwards). As the body is moving upwards, therefore resultant force

$$= T - m_2\, g \sin \alpha \qquad ...(iv)$$

Since this body is moving upwards along the inclined surface with an acceleration (a), therefore force acting on this body

$$= m_2\, a \qquad ..(v)$$

Equating the equations (iv) and (v),

$$T - m_2\, g \sin \alpha = m_2\, a \qquad ...(vi)$$

Adding equations (iii) and (vi),

$$m_1\, g - m_2\, g \sin \alpha = m_1\, a + m_2\, a$$

$$g\,(m_1 - m_2 \sin \alpha) = a\,(m_1 + m_2)$$

$$a = \frac{g\left(m_1 - m_2 \sin \alpha\right)}{m_1 + m_2} \text{ m/s}^2$$

From equation (*iii*) we find that

$$T = m_1\, g - m_1\, a = m_1\, (g - a)$$

Substituting the value of a in the above equation,

$$T = m_1 \left[g - \frac{g\,(m_1 - m_2 \sin\alpha)}{m_2 + m_2} \right]$$

$$= m_1\; g \left[\frac{m_1 + m_2 - m_1 + m_2 \sin\alpha}{m_1 + m_2} \right]$$

$$= \frac{m_1 m_2\, g\,(1 + \sin\alpha)}{m_1 + m_2}$$

Note. For horizontal surface, if we substitute value of $\alpha = 0$ in the above equations for a and T, the relations obtained will be the same as derived in Art. 25.4.

Example 25.7. *A body of mass 30 kg, lying on a smooth plane inclined at 15º to the horizontal, is being pulled by a body of mass 20 kg. The 20 kg body is connected to the first body by a light inextensible string and hangs freely beyond the frictionless pulley.*

Find the acceleration, with which the body will come down.

Solution. Given : Mass of the body lying on smooth plane (m_2) = 30 kg ; Inclination of the plane with horizontal (α) = 15º and mass of the body which hangs freely beyond the pully (m_1) = 20 kg

We know that the acceleration with which the body will come down,

$$a = \frac{g\,(m_1 - m_2 \sin\alpha)}{m_1 + m_2} = \frac{9.8\,(20 - 30 \sin 15°)}{20 + 30}\ \text{m/s}^2$$

$$= \frac{9.8\,(20 - 30 \times 0.2588)}{50} = 2.4\ \text{m/s}^2 \qquad \textbf{Ans.}$$

25.6. MOTION OF TWO BODIES CONNECTED BY A STRING, ONE OF WHICH IS HANGING FREE AND THE OTHER LYING ON A ROUGH INCLINED PLANE

Consider two bodies of masses m_1 and m_2 respectively, connected by a light inextensible string as shown in Fig. 25.11.

Let the body of mass m_1 hang free and the body of mass m_2 be placed on an inclined *rough* surface. Let the body of mass m_1 move downwards and the body of mass m_2 move upwards along the inclined surface.

We know that velocity and acceleration of the body of mass m_1 will be the same, as that of the body of mass m_2. Since the string is inextensible, therefore tension in both the string will also be equal.

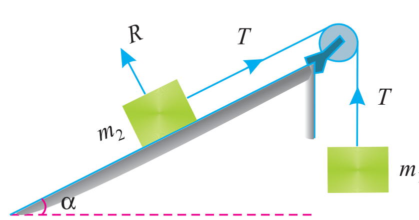

Fig. 25.11.

Let a = Acceleration of the system in m/s^2

T = Tension in the string in N,

μ = Coefficient of friction, and

α = Inclination of the plane.

We know that the normal reaction on the inclined surface due to body of mass m_2 (as shown in Fig. 25.11).

$$= m_2 \, g \cos \alpha$$

$\therefore$ Frictional force $= \mu \, m_2 \, g \cos \alpha$

This frictional force will act in the opposite direction to the motion of the body of mass 2.

First of all, consider the motion of body 1 of mass m_1, which is coming down. We know that the forces acting on it are $m_1.g$ (downwards) and T (upwards). As the body is moving downwards, therefore, resultant force

$$= m_1 g - T \qquad ...(i)$$

Since the body is moving downwards with an acceleration (a), therefore force acting on this body

$$= m_1 \, a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$m_1 \, g - T = m_1 \, a \qquad ...(iii)$$

Now consider the motion of the body 2 of mass m_2, which is moving upwards on inclined surface. We know that the forces acting on it, along the plane, are T (upwards), $m_2.g \sin \alpha$ (downwards) and force of friction $\mu.m_2.g$ (downwards). As the body is moving upwards, therefore resultant force

$$= T - m_2 \, g \sin \alpha - \mu \, m_2 \, g \cos \alpha \qquad ...(iv)$$

Since this body is moving upwards along the inclined surface with an acceleration (a) therefore force acting on this body

$$= m_2 \, a \qquad ...(v)$$

Equating the equations (iv) and (v),

$$T - m_2 \, g \sin \alpha - \mu \, m_2 \, g \cos \alpha = m_2 \, a \qquad ...(vi)$$

Adding equations (iii) and (vi),

$$m_1 \, g - m_2 \, g \sin \alpha - \mu \, m_2 \, g \cos \alpha = m_1 \, a + m_2 \, a$$

$$g \, (m_1 - m_2 \sin \alpha - \mu \, m_2 \cos \alpha) = a \, (m_1 + m_2)$$

$$a = \frac{g \left(m_1 - m_2 \sin \alpha - \mu \, m_2 \cos \alpha\right)}{m_1 + m_2}$$

From equation (iii) we find that

$$T = m_1 \, g - m_1 \, a = m_1 \, (g - a)$$

Substituting the value of a in the above equation,

$$T = m_1 \left[g - \frac{g \left(m_1 - m_2 \sin \alpha - \mu \, m_2 \cos \alpha\right)}{m_1 + m_2} \right]$$

$$= m_1 \, g \left[1 - \frac{m_1 - m_2 \sin \alpha - \mu \, m_2 \cos \alpha}{m_1 + m_2} \right]$$

$$= m_1 \, g \left[\frac{m_1 + m_2 - m_1 + m_2 \sin \alpha + \mu \, m_2 \cos \alpha}{m_1 + m_2} \right]$$

$$= \frac{m_1 \, m_2 \, g \left(1 + \sin \alpha + \mu \cos \alpha\right)}{m_1 + m_2}$$

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Notes. 1. For horizontal surface, if we substitute the value of $\alpha = 0$ in the above equations for a and T, the rotations obtained will be the same as derived in Art. 25.4.

2. For smooth surface, if we substitue the value of $\mu = 0$ in the above equations for a and T, the relations obtained will be the same as derived in the last article.

Example 25.8. *A body of mass 150 kg, rests on a rough plane inclined at 10º to the horizontal. It is pulled up the plane, from rest, by means of a light flexible rope running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a man of 80 kg at the end. If the coefficient of friction for the plane and the body is 0.2, find*

(*i*) *the tension in the rope*,

(*ii*) *the acceleration in m/s*^{2}, *with which the body moves up the plane*, *and*

(*iii*) *the distance in metres moved by the body in 4 seconds starting from rest.*

Solution. Given : Mass of the body (m_2) = 150 kg ; Inclination of plane (α) = 10º ; Mass of the man (m_1) = 80 kg and coefficient of friction (μ) = 0.2

(*i*) *Tension in the rope*

We know that tension in the rope,

$$T = \frac{m_1 m_2 g(1 + \sin\alpha + \mu\cos\alpha)}{m_1 + m_2}$$

$$= \frac{80 \times 150 \times 9.8(1 + \sin 10^\circ + 0.2\cos 10^\circ)}{80 + 150}\ \text{N}$$

$$= \frac{117\ 600\left[(1 + 0.1736) + (0.2 \times 0.9848)\right]}{230}\ \text{N}$$

$$= 700\ \text{N} \quad \textbf{Ans.}$$

(*ii*) *Acceleration*, *with which the body moves up the plane*

We also know that the acceleration with which the body moves up the plane,

$$a = \frac{g(m_1 - m_2\sin\alpha - \mu m_2\cos\alpha)}{m_1 + m_2}$$

$$= \frac{9.8(80 - 150\sin 10^\circ - 0.2 \times 150\cos 10^\circ)}{80 + 150}\ \text{m/s}^2$$

$$= \frac{9.8\,(80 - 150 \times 0.1736 - 0.2 \times 150 \times 0.9848)}{230}\ \text{m/s}^2$$

$$= 1.04\ \text{m/s}^2 \quad \textbf{Ans.}$$

(*iii*) *Distance moved by the body in* 4 *sec starting from rest*

In this case, initial velocity (u) = 0 (because starting from rest) ; Time (t) = 4 sec and acceleration (a) = 1.04 m/s^2

∴ Distance moved by the body,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 1.04(4)^2 = 8.32\ \text{m} \quad \textbf{Ans.}$$

Example 25.9. *Determine the resulting motion of the body A assuming the pulleys to be smooth and weightless as shown in Fig. 25.12.*

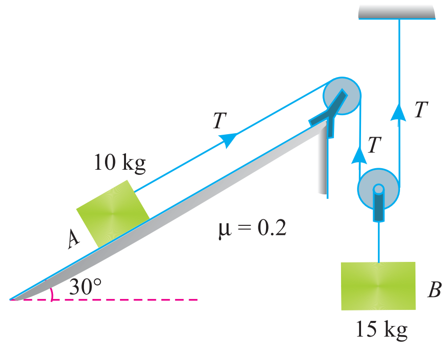

Fig. 25.12.

If the system starts from rest, determine the velocity of the body A after 10 seconds.

Solution. Given : Mass of body B $(m_1) = 15$ kg ; Mass of body A $(m_2) = 10$ kg; Inclination of plane $(\alpha) = 30°$ and coefficient of friction $(\mu) = 0.2$

Let T = Tension in the string, and

a = Acceleration of the block A.

We know that normal reaction on the inclined surface due to body A of mass 10 kg.

$$R = m_2\, g \cos \alpha = 10 \times 9.8 \times \cos 10° = 98 \times 0.9848 = 96.5 \text{ N}$$

∴ Frictional force $= \mu\, R \times 96.5 = 0.2 \times 96.5 = 19.3$ N

First of all, consider the motion of the block A, which is moving upwards. We know that the forces acting on it, along the plane, are T newtons (upwards), $m_2.g \sin \alpha$ newtons (downwards), and frictional force equal to 19.3 newtons (downwards, as the body is moving upwards). Therefore resultant force

$$= T - m_2\, g \sin \alpha - 19.3$$
$$= T - 10 \times 9.8 \sin 30° - 19.3$$
$$= T - 98 \times 0.5 - 19.3 = T - 68.3 \qquad ...(i)$$

Since the body is moving with an acceleration (a) therefore force acting on it

$$= m_2\, a = 10\, a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$T - 68.3 = 10\, a$$

Multiplying both sides by 2,

$$2T - 136.6 = 20\, a \qquad ...(iii)$$

Now consider motion of the body B, which is coming downwards. A little consideration will show that acceleration of the body B will be half the acceleration of the block A (*i.e.* $a/2$).

We know that the forces acting on it are $m_1\, g = 15 \times 9.8 = 147$ N (downwards) and $2T$ newtons (upwards). As the body is moving downwards, therefore resultant force

$$= 147 - 2T \qquad ...(iv)$$

Since the body is moving with an acceleration of $0.5\, a$ therefore force acting on it

$$= 15 \times 0.5\, a = 7.5\, a \qquad ...(v)$$

Equating equations (*iv*) and (*v*),

$$147 - 2T = 7.5\ a \qquad ...(vi)$$

Adding equations (*iii*) and (*vi*),

$$10.4 = 27.5\ a$$

or
$$a = \frac{10.4}{27.5} = 0.4 \text{ m/s}^2$$

∴ Velocity of the body *A* after 10 seconds, if the system starts from rest.

$$v = u + at = 0 + 0.4 \times 10 = 4 \text{ m/s} \quad \textbf{Ans.}$$

Example 25.10. *The system of bodies shown in Fig. 25.13 starts from rest.*

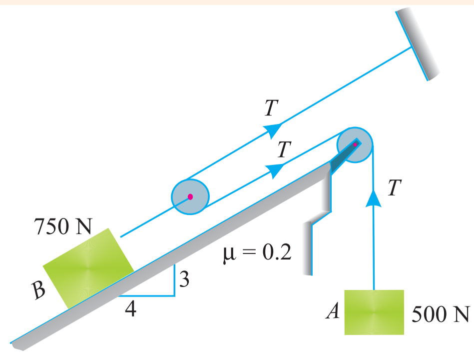

Fig. 25.13.

Determine the acceleration of body B and the tension in the string supporting body A.

Solution. Given : Weight of body *A* (W_1) = 500 N ; Weight of body *B* (W_2) = 750 N and coefficient of friction (μ) = 0.2

Acceleration of the body B

T = Tension in the strings, and

a = Acceleration of the body *B*.

From the slope of the surface, we find that

$$\tan \alpha = \frac{3}{4} = 0.75 \text{ or } \sin \alpha = 0.6 \text{ and } \cos \alpha = 0.8$$

We know that normal reaction on the inclined surface due to body of weight 750 N

$$= W_2 \cos \alpha = 750 \times 0.8 = 600 \text{ N}$$

∴ Frictional force $= \mu R = \mu \times 600 = 0.2 \times 600 = 120$ N

First of all, consider the motion of the body *B*, which is moving, upwards. We know that the forces acting on it, along the plane, are $2T$ newtons (upwards), $W_2 \sin \alpha$ newtons (downwards) and force of friction equal to 120 newtons (downwards as the body is moving upwards). Therefore resultant force

$$= 2T - W_2 \sin \alpha - 120 = 2T - 750 \times 0.6 - 120 \text{ N}$$

$$= 2T - 570 \text{ N} \qquad ...(i)$$

Since the body is moving with an acceleration (a), therefore force acting on it

$$m_2\, g = \frac{W_2}{g} \times a = \frac{750}{9.8} \times a = 76.5\ a \qquad ...(ii)$$

Equating equations (*i*) and (*ii*),

$$2T - 570 = 76.5\ a \qquad ...(iii)$$

Now consider the motion of the body *A*, which is coming downwards. A little consideration will show that the acceleration of the body *A* will be double the acceleration of the block *A* (*i.e.* $2a$). We know that the force acting on it 500 N (downwards) and *T* (upwards). As the body is moving downwards, therefore resultant force

$$= 500 - T \qquad \text{...}(iv)$$

Since the body is moving with an acceleration $2a$, therefore force acting on it

$$m_1\, 2a = \frac{W_1}{g} \times 2a = \frac{500}{9.8} \times 2a = 102a \qquad \text{...}(v)$$

Equating equations (*iv*) and (*v*),

$$500 - T = 102\, a$$

Multiplying both sides by 2,

$$1000 - 2T = 204\, a \qquad \text{...}(vi)$$

Adding equations (*iii*) and (*vi*),

$$430 = 280.5\, a$$

or
$$a = \frac{430}{280.5} = 1.5 \text{ m/s}^2$$

Tension in the string supporting body A

Substituting the value of *a* in equation (*vi*),

$$1000 - 2T = 204a = 204 \times 1.5 = 306$$

$$T = \frac{1000 - 306}{2} = 347 \text{ N} \qquad \textbf{Ans.}$$

25.7. MOTION OF TWO BODIES, CONNECTED BY A STRING AND LYING ON SMOOTH INCLINED PLANES

Consider two bodies of masses m_1 and m_2 kg respectively connected by a light inextensible string on two smooth surfaces (*i.e.* friction between the masses and the surfaces is neglected) as shown in Fig. 25.14.

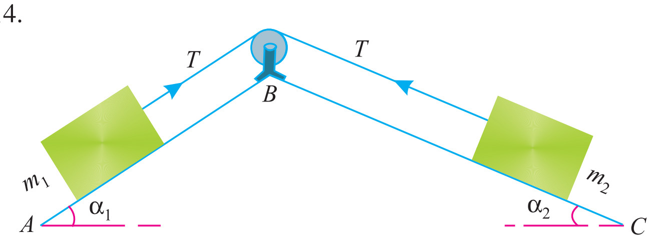

Fig. 25.14.

Let the body of mass m_1 move downwards along the inclined plane surface *AB* and the body of mass m_2 move upwards along the inclined surface *BC*. A little consideration will show, that the velocity and acceleration of the body of mass m_1 will be the same as that of the body of mass m_2. Since the string is inextensible, therefore tension in both the strings will also be equal.

Let
a = Acceleration of the system
T = Tension in the string and
α_1 and α_2 = Inclination of surfaces *AB* and *BC*

First of all, consider the motion of body 1 of mass m_1 kg which is coming down along the inclined plane *AB*. We know that forces acting on it, along the plane, are $m_1.g \sin \alpha_1$ newtons (downwards) and *T* newtons (upwards). As the body is moving downwards, therefore resultant force

$$= m_1\, g \sin \alpha_1 - T \qquad \text{...}(i)$$

Since the body is moving downwards with an acceleration (a), therefore force acting on this body

$$= m_1 \, a \qquad ...(ii)$$

Equating equations (i) and (ii),

$$m_1 \, g \sin \alpha_1 - T = m_1 \, a \qquad ...(iii)$$

Now consider the motion of the body 2 of mass m_2 which is moving upwards along the inclined plane BC. We know that the forces acting on it, along the plane, are T (upwards) and $m_2.g \sin \alpha_2$ (downwards). As the body is moving upwards, therefore resultant force

$$= T - m_2 \, g \sin \alpha_2 \qquad ...(iv)$$

Since the body is moving upwards with an acceleration (a) therefore force acting on this body

$$= m_2 \, a \qquad ...(v)$$

Equating equations (iv) and (v),

$$T - m_2 \, g \sin \alpha_2 = m_2 \, a \qquad ...(vi)$$

Adding equations (iii) and (vi)

$$m_1 \, g \sin \alpha_1 - m_2 \, g \sin \alpha_2 = m_1 \, a + m_2 \, a$$

$$g \, (m_1 \sin \alpha_1 - m_2 \sin \alpha_2) = a \, (m_1 + m_2)$$

$$\therefore \qquad a = \frac{g\left(m_1 \sin \alpha_1 - m_2 \sin \alpha_2\right)}{m_1 + m_2}$$

From equation (iii), we have

$$T = m_1 \, g \sin \alpha_1 - m_1 \, a = m_1 \, (g \sin \alpha_1 - a)$$

Substituting the value of a in above equation,

$$T = m_1 \left[g \sin \alpha_1 - \frac{g\left(m_1 \sin \alpha_1 - m_2 \sin \alpha_2\right)}{m_1 + m_2} \right]$$

$$= m_1 g \left[\frac{m_1 \sin \alpha_1 + m_2 \sin \alpha_1 - m_1 \sin \alpha_1 + m_2 \sin \alpha_2}{m_1 + m_2} \right]$$

$$= \frac{m_1 \, m_2 \, g \left(\sin \alpha_1 + \sin \alpha_2\right)}{m_1 + m_2}$$

Note. For vertical surface for the mass m_1, if we substitute the value of $\alpha_1 = 0$ and $\alpha_2 = \alpha$ in the above equations for a and T, the relations obtained will be the same as derived in Art. 25.5.

Example 25.11. *Two smooth inclined planes whose inclinations with the horizontal are 30º and 20º are placed back to back. Two bodies of mass 10 kg and 6 kg are placed on them and are connected by a light inextensible string passing over a smooth pulley as shown in Fig 25.15.*

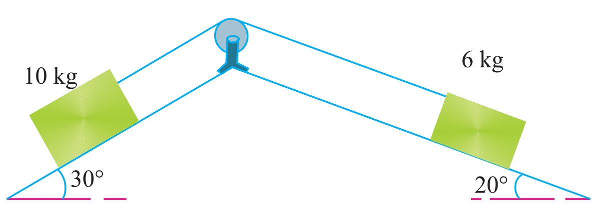

Fig. 25.15.

Find the tension in the string. Take $g = 9.8$ m/s^2.

Solution. Given : Inclination of first plane (α_1) = 30º ; Inclination of second plane (α_2) = 20º ; Mass of first body (m_1) = 10 kg and mass of second body (m_2) = 6 kg

We know that tension in the string,

$$T = \frac{m_1 m_2 g (\sin \alpha_1 + \sin \alpha_2)}{m_1 + m_2}$$

$$= \frac{10 \times 6 \times 9.8 (\sin 30º + \sin 20º)}{10 + 6} \text{ N}$$

$$= \frac{60 \times 9.8 (0.5 + 0.3420)}{16} = 31 \text{ N} \quad \textbf{Ans.}$$

Example 25.12. *Two bodies A and B are connected by a light inextensible cord as shown in Fig. 25.16.*

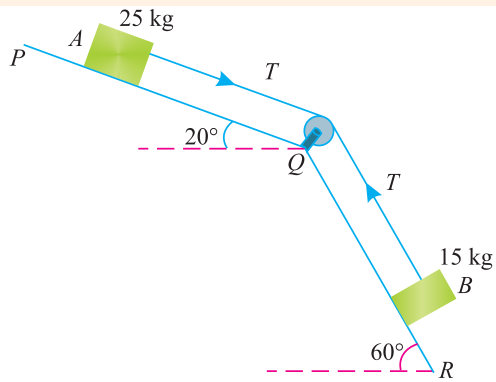

Fig. 25.16.

If both the bodies are released simultaneously, what distance do they move in 3 seconds? Neglect friction between the two bodies and the inclined surfaces.

Solution. Given : Mass of body B (m_1) = 15 kg ; Mass of body A (m_2) = 25 kg ; Inclination of plane PQ with horizontal (α_2) = 20º ; Inclination of plane QR with horizontal (α_1) = 60º ; Initial velocity (u) = 0 (because the bodies are released) and time (t) = 3 s.

Let a = Acceleration of the system, and

T = Tension in the cord.

First of all, consider the motion of body B of mass 15 kg which is coming down along the inclined surface QR. We know that forces acting on it, along the plane, QR are $m_1.g \sin \alpha_1$ newtons (downwards) and T newtons (upwards). As the body is moving downwards, therefore resultant force

$$= m_1 g \sin \alpha_1 - T = 15 \times 9{\cdot}8 \sin 60° - T$$

$$= 147 \times 0{\cdot}866 - T = 127{\cdot}3 - T \quad ...(i)$$

Since the body is moving downwards with an acceleration (a), therefore force acting on it.

$$= m_1 a = 15 a \quad ...(ii)$$

Equating equations (i) and (ii)

$$127{\cdot}3 - T = 15 a \quad ...(iii)$$

Now consider the motion of body A of mass 25 kg which is also coming down along the inclined surface PQ. We know that the forces acting on it along the plane PQ, are $m_2.g \sin \alpha_2$ (downwards) and T (again downwards). Therefore resultant force

$$= m_2 g \sin \alpha_2 + T = (25 \times 9.8 \sin 20º) + T$$

$$= 245 \times 0.342 + T = 83.8 + T \quad ...(iv)$$

Since the body is moving downwards with an acceleration (a), therefore force acting on it.

$$= m_2\, a = 25\, a \qquad ...(v)$$

Equating equations (iv) and (v)

$$83.8 + T = 25\, a \qquad ...(vi)$$

Adding equations (iii) and (vi),

$$211.1 = 40\, a$$

or
$$a = \frac{211.1}{40} = 5.3 \text{ m/s}^2$$

∴ Distance moved by the bodies in 3 seconds,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5.3(3)^2 = 23.85 \text{ m} \quad \textbf{Ans.}$$

25.8. MOTION OF TWO BODIES CONNECTED BY A STRING AND LYING ON ROUGH INCLINED PLANES

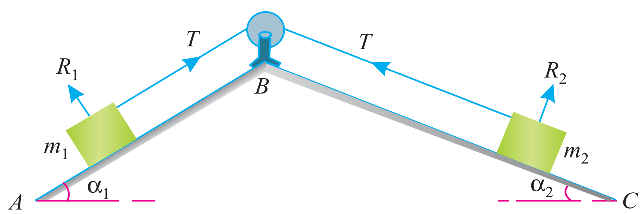

Fig. 25.17.

In this case, the friction between the bodies of masses m_1 and m_2 and the surfaces AB and BC will also be considered.

It may be noted that the force of the friction always acts in the opposite direction to the motion of a mass.

If the mass m_1 comes down and the mass m_2 goes up, the frictional force at m_1 will act upwards and the frictional force at m_2 will act downwards. Thus, while calculating the forces on any mass, the frictional force, acting in the opposite direction, should also be considered.

Example 25.13. *Two rough planes inclined at 30º and 15º to the horizontal and of the same height are placed back to back. Two bodies of masses of 15 kg and 5 kg are placed on the faces and connected by a string over the top of the planes If the coefficient of friction be 0.3 find from fundamentals the resulting acceleration.*

Solution. Inclination of first plane (α_1) = 30º ; Inclination of second plane (α_2) = 15º ; Mass of first body (m_1) = 15 kg ; Mass of second body (m_2) = 5 kg and coefficient of friction (μ) = 0.3.

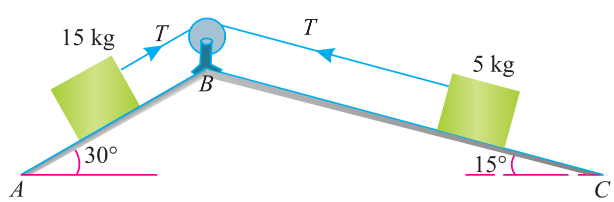

Fig. 25.18.

Let a = Acceleration of the system and

T = Tension in the string.

We know that normal reaction on the plane AB due to body of mass 15 kg.

$$R_1 = 15\, g \cos \alpha_1 = 15 \times 9.8 \cos 30^\circ$$
$$= 15 \times 9.8 \times 0.866 = 127.3 \text{ N}$$

$\therefore$ Frictional force $= \mu\, R_1 = 0.3 \times 127.3 = 38.2$ N

Similarly, normal reaction on the plane BC due to the body of mass 5 kg.

$$R_2 = 5 \text{ g} \cos \alpha_2 = 5 \times 9.8 \cos 15^\circ$$
$$= 5 \times 9.8 \times 0.9659 = 48 \text{ N}$$

$\therefore$ Frictional force $= \mu R_2 = 0.3 \times 4.8 = 14.4$ N

The frictional forces will act in the opposite directions to the motions of the two bodies.

First of all consider the motion of body 1 of mass 15 kg, which is coming down along the inclined plane AB. We know that the forces acting on it, along the plane, are $m_1.g \sin \alpha_1$ newtons (downwards), T newtons (upwards) and force of friction equal to 38.2 newton (upwards as the body is moving downwards). Therefore resultant force

$$= m_1\, g \sin \alpha_1 - T - 38.2 = (15 \times 9.8 \sin 30^\circ) - T - 38.2$$
$$= 147 \times 0.5 - T - 38.2 = 35.3 - T \quad ...(i)$$

Since the body is moving downwards with an acceleration (a), therefore force acting on it

$$= m_1\, a = 15\, a \quad ...(ii)$$

Equating the equations (i) and (ii),

$$35.3 - T = 15\, a \quad ...(iii)$$

Now consider the motion of body 2 of mass 5 kg, which is moving upwards along the inclined plane BC. We know that the forces acting on it, along the plane, are T (upwards), $m_2.g \sin \alpha_2$ (downwards) and force of friction equal to 14.4 N (downwards as the body is moving upwards). Therefore resultant force

$$= T - m_2\, g \sin \alpha_2 - 14.4 = T - (5 \times 9.8 \sin 15^\circ) - 14.4$$
$$= T - 5 \times 9.8 \times 0.2588 - 14.4 = T - 27 \quad ...(iv)$$

Since the body is moving upwards with an acceleration (a), therefore force acting on this body

$$= m_2\, a = 5\, a \quad ..(v)$$

Equating equations (iv) and (v),

$$T - 27 = 5\, a \quad ...(vi)$$

Adding equations (iii) and (vi),

$$35.3 - 27 = 15\, a + 5\, a = 20\, a$$

$$a = \frac{35.3 - 27}{20} = 0.42 \text{ m/s}^2 \textbf{ Ans.}$$

Example 25.14. *A system of bodies A,B and C in Fig. 25.19 is released from rest.*

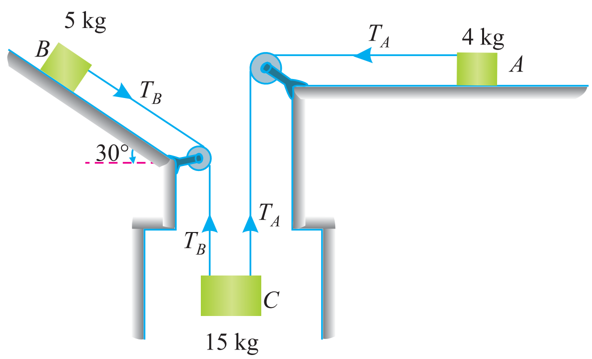

Fig. 25.19.

Find (i) acceleration of the masses and (ii) tension in the two strings. Take coefficient of friction for the contact surfaces of bodies A and B as 0.4.

Solution. Given : Mass of body A (m_A) = 4 kg ; Mass of body B (m_B) = 5 kg ; Mass of body C (m_C) = 15 kg and Coefficient of friction (μ) = 0.4

(*i*) *Acceleration of the bodies*

Let a = Acceleration of the bodies

T_A = Tension in the string connected with body A

and T_B = Tension in the string connected with body B.

We know that normal reaction on the horizontal surface due to body A,

$$R_A = m_A\, g = 4 \times 9.8 = 39.2$$

∴ Frictional force, $F_A = \mu\, R_A = 0.4 \times 39.2 = 15.68$ N

Similarly, normal reaction on the inclined surface due to body B,

$$R_B = m_B\, g \cos\alpha = 5 \times 9.8 \cos 30° = 49 \times 0.866 = 42.43$$

∴ Frictional force $F_B = \mu\, R_B = 0.4 \times 42.43 = 16.97$ N

First of all, consider the motion of body A which is moving horizontally. We know that forces acting on it are T_A (towards left) and frictional force of 15.68 N (towards right). As the body is moving towards left, therefore resultant force

$$= T_A - 15.68 \quad ...(i)$$

Since the body A is moving with an acceleration of (a) therefore force acting on it

$$= 4\, a \quad ...(ii)$$

Equating equations (*i*) and (*ii*)

$$T_A - 15.68 = 4\, a \quad ...(iii)$$

or

$$T_A = 4\, a + 15.68 \quad ...(iv)$$

Now consider the motion of the body B, which is moving downwards on inclined surface. We know that the forces acting on it, along the plane, are T_B (downwards), $m_B.g \sin\alpha$ (again downwards), frictional force equal to 16.97 N (upwards, as the block is moving downwards). Therefore resultant force

$$= T_B + m_B\, g \sin \alpha - 16.97$$

$$= T_B + 5 \times 9.8 \sin 30° - 16.97$$

$$= T_B + 49 \times 0.5 - 16.97 = T_B + 7.53 \qquad \text{...}(v)$$

Since the body is moving with an acceleration of (a), therefore force acting on it

$$= 5a \qquad \text{...}(vi)$$

Equating equations (v) and (vi),

$$T_B + 7.53 = 5\,a$$

or

$$T_B = 5\,a - 7.53 \qquad \text{...}(vii)$$

Now consider the motion of the body C, which is coming down. We know that forces acting on it are $m_C \cdot g = 15 \times 9.8 = 147$ N (downwards) and $(T_A + T_B)$ upwards. As the body is moving downwards, therefore resultant force

$$= 147 - (T_A + T_B) \qquad \text{...}(viii)$$

Since the body is moving with an acceleration (a), therefore force acting on it

$$= 15\,a \qquad \text{...}(ix)$$

Equating equations ($viii$) and (ix),

$$147 - (T_A + T_B) = 15\,a$$

Substituting the values of T_A and T_B from equations (iv) and (vii)

$$147 - [(4a + 15.68) + (5a - 7.53)] = 15a$$

$$147 - 4a - 15.68 - 5a + 7.53 = 15a$$

$$\therefore \qquad 24\,a = 138.85$$

$$a = \frac{138.85}{24} = 5.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

(ii) *Tension in the two strings*

Substituting the value of (a) in equation (iv),

$$T_A = 4a + 15.68 = (4 \times 5.8) + 15.68 = 38.88 \text{ N} \quad \textbf{Ans.}$$

Again substituting the value of (a) in equation (vii),

$$T_B = 5a - 7.53 = (5 \times 5.8) - 7.53 = 21.47 \text{ N} \quad \textbf{Ans.}$$

EXERCISE 25.2

1. Fig. 25.20 shows two masses connected by a light inextensible string passing over a smooth pulley.

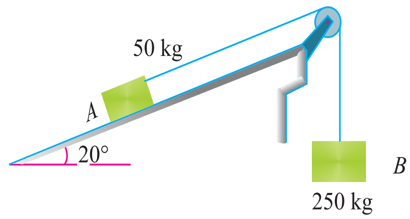

Fig. 25.20.

If the system is released from rest, with what velocity 50 kg mass will touch the floor, if initially it was 1.6 m higher than the floor level. Take coefficient of friction between 50 kg mass and inclined surface as 0.2. (**Ans.** 4.83 m/s)

2. Two bodies *A* and *B* of mass 8 kg and 10 kg are placed on two smooth inclined planes as shown in Fig. 25.21.

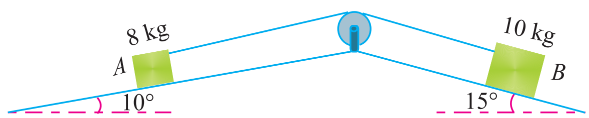

Fig. 25.21.

Find, the acceleration of the body of mass 8 kg. (**Ans.** 0.47 m/s^2)

3. Two bodies *P* and *Q* are connected by a light inextensible string as shown in Fig. 25.22.

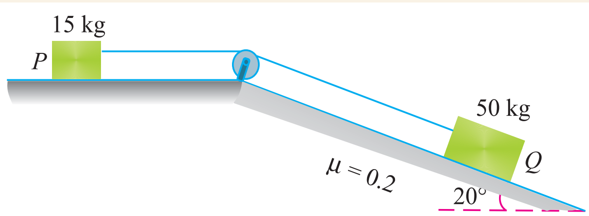

Fig. 25.22.

Find acceleration of two bodies and tension in the string. (**Ans.** 0.94 m/s^2 : 28.8)

QUESTIONS

1. Explain the reason for the tension in both the strings to be equal, when two masses are attached to its ends, and the inextensible string is made to pass over a smooth pulley.
2. Obtain a relation for the acceleration of two bodies connected by a string, when one body is hanging free and the other is lying on a smooth horizontal plane.
3. Derive an equation for the tension in the string, when one body is free and the other is lying on a rough horizontal plane.
4. Explain the procedure adopted for obtaining the relations for acceleration of two bodies and tension in the string when one of the bodies is hanging free and the other is lying on (*a*) smooth inclined plane ; and (*b*) rough inclined plane.

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5. If two bodies connected by an inextensible string are lying on two rough inclined planes, then how will you judge, as to which of the two bodies will come down?

OBJECTIVE TYPE QUESTIONS

1. Two bodies of masses 10 kg and 15 kg are hung from the ends of an extensible rope passing over a frictionless pulley. If masses of both the bodies are doubled the acceleration of the system will also be doubled.

   (*a*) Yes (*b*) No

2. Two masses of 10 kg and 5 kg are connected to two ends of a rope which is passing over a smooth pulley. The 10 kg mass is lying on a smooth horizontal plane and 5 kg mass is hanging. If the position of the two masses is interchanged, its acceleration will also change.

   (*a*) Agree (*b*) Disagree

3. If two masses are connected to the two ends of an inextensible string, passing over a pulley. One of the mass is lying on a rough horizontal plane and the other is hanging free. If the value of coefficient of friction is increased, it will increase its

   (*a*) acceleration (*b*) tension (*c*) both of them

4. Two masses of 10 kg and 15 kg are connected to two ends of an inextensible rope and passing over a smooth pulley. The 10 kg mass is lying over a rough plane, which is inclined at an angle of 25º with the horizontal. If this angle is made 30º, then

   (*a*) tension in the string will increase

   (*b*) tension in the string will decrease

   (*c*) acceleration of the system will remain the same.

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*a*)

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