UNIT-2:

K-Map

SOP-POS

Boolean function is an algebraic form of Boolean expression

Sum-of-Products (SOP) - variables are operated by AND (product) are OR(sum) together
Product-of-sums (POS) - variables are operated by OR (sum) are AND (product) together

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

SOP Expression

Write AND term for each input combination produces HIGH
Write the input variables for 1 and compliment for 0.
OR the AND terms to obtain the output function.

F(SOP) = A'BC + AB'C + ABC ' + ABC

POS Expression

Write OR term for each input combination produces LOW
Write the input variables for 0 and complement for 1
AND the OR terms to obtain the output function

F(POS)= F = (A + B + C) (A + B + C ') (A + B' + C) (A' + B + C)

POS is compliment of SOP

Min Term –Max Term

Variables			Min terms	Max terms
A	B	C	m_i	M_i
0	0	0	A' B' C' = m 0	A + B + C = M 0
0	0	1	A' B' C = m 1	A + B + C' = M 1
0	1	0	A' B C' = m 2	A + B' + C = M 2
0	1	1	A' B C = m 3	A + B' + C' = M 3
1	0	0	A B' C' = m 4	A' + B + C = M 4
1	0	1	A B' C = m 5	A' + B + C' = M 5
1	1	0	A B C' = m 6	A' + B' + C = M 6
1	1	1	A B C = m 7	A' + B' + C' = M 7

Write SOP expression for minterm $F(A,B,C) = \sum m(1,2,3)$

In binary 01 10 11

$$= \overline{A}B + A\overline{B} + AB$$

Write POS expression for minterm $F(A,B,C) = \pi M(1,2,3)$

In binary = 01 10 11

$$(A+\overline{B}).(\overline{A}+B).(\overline{A}+\overline{B})$$

$$\textit{Write SOP expression for minterm } F(A,B) = \sum m(1,2,3)$$

$$\textit{In binary} \quad 01 \quad 10 \quad 11$$

$$= \overline{A}B + A\overline{B} + AB$$

$$\textit{Write SOP expression for } F(A,B,C) = \sum m(2,4,6,7)$$

$$\textit{In binary} \quad 010 \quad 100 \quad 110 \quad 111$$

$$\overline{A}B\overline{C} + A\overline{B}\overline{C} + AB\overline{C} + ABC$$

$$\textit{Write SOP expression for } F(A,B,C) = \sum m(0,1,3,5)$$

$$\textit{Write SOP expression for } F(A,B,C,D) = \sum m(2,4,5,7,8,9,10,13,15)$$

SOP-POS Conversion

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\overline{C} + \overline{A}\,BC + A\overline{B}\,C + ABC$$

  - The evaluation is as follows:

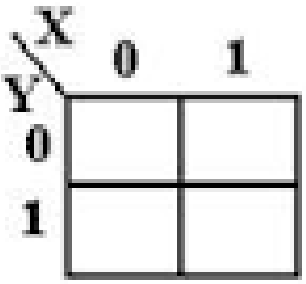
$$000 + 010 + 011 + 101 + 111$$

  - There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

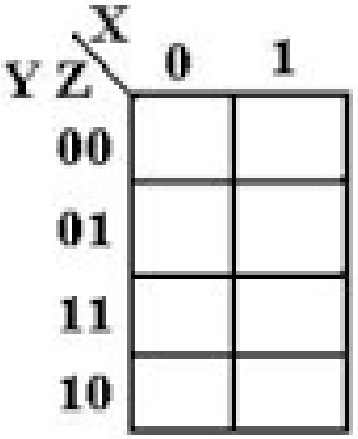
$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

K-MAP

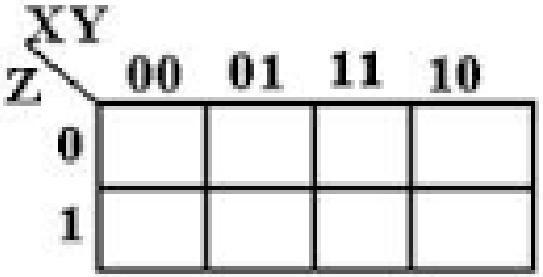
Karnaugh map is a tool for simplification of Boolean algebra.
K-Map diagram is made up of squares.
K-map is a graphical representation of SOP (Minterm).
K-Map extensively reduce the calculation and provides best minimized solution.
K-map solve the expression with grouping of neighbor cells.



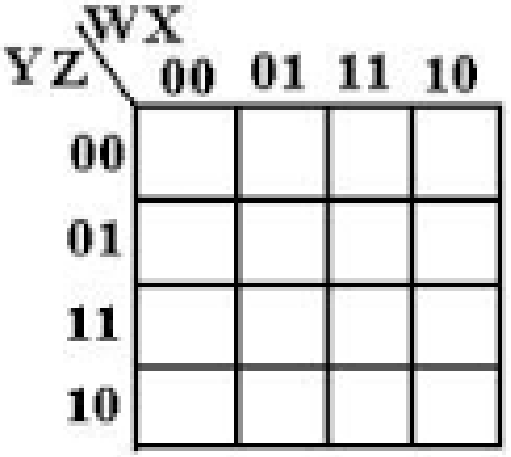
2 variable



3 variable



3 variable



4 variable

Example on Mapping

- Mapping a standard SOP expression
- Mapping a non standard SOP expression

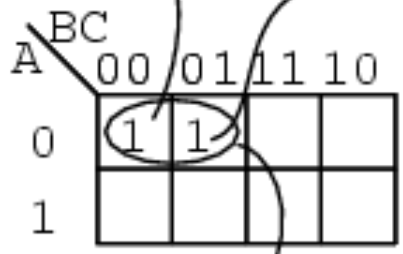
Kmap Simplification Rule

1) Construct kmap and place 1's in the squares according to the truth table.
2) Groupings can contain only 1s
3) Groups can be formed only at right angles; diagonal groups are not allowed.
4) The number of 1's in a group must be a power of 2
5) The groups must **be made as large** as possible.
6) Groups can overlap and wrap around the sides of the Kmap.
7) Every group puts a term in the solution

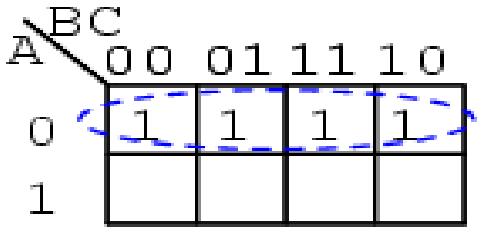
Optimized Solution
Minimum number of group
Each group covers maximum possible squares

Example on K-map

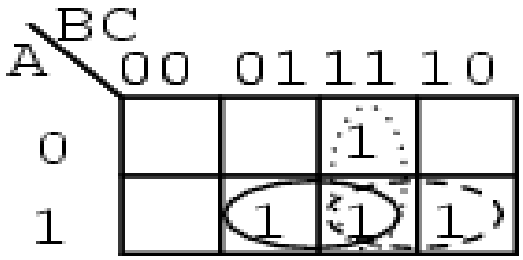
Out= $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C$



Out= $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

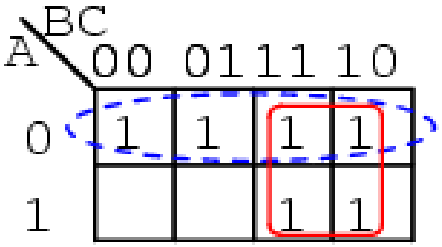


Out= $\overline{A}$



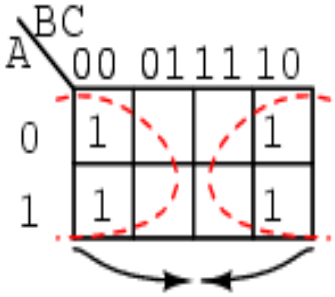
Output = AB + BC + AC

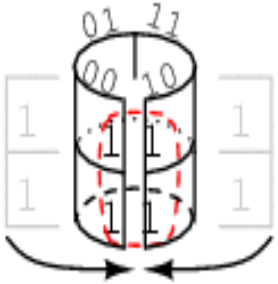
Out= $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + ABC + A\,B\,\overline{C}$



Out= $\overline{A} + B$

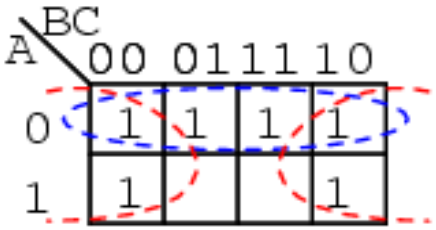
Out= $\overline{A}\,\overline{B}\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,B\,\overline{C}$





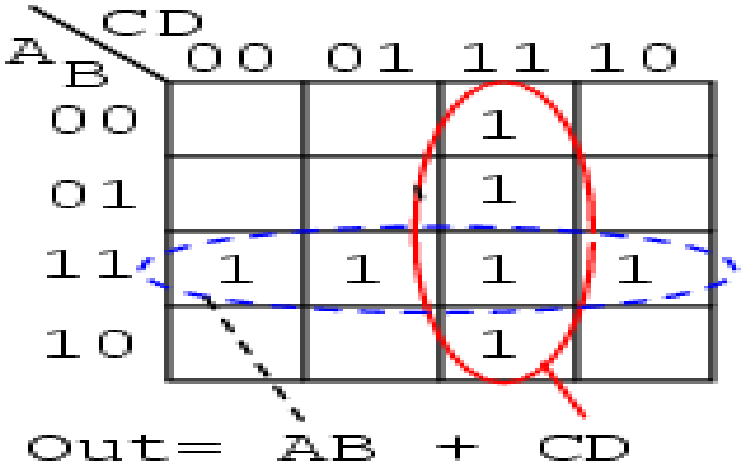
Out= $\overline{C}$

Out= $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$

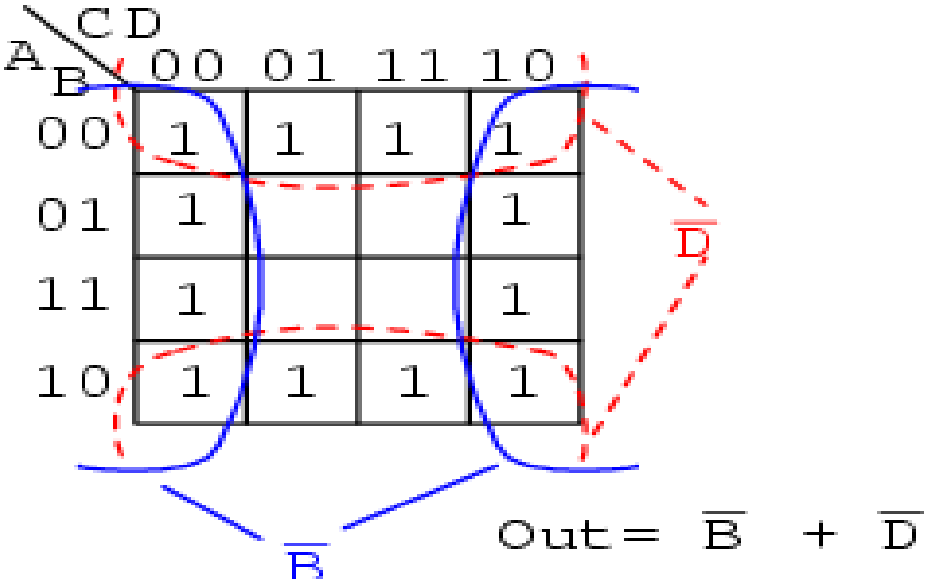


Out= $\overline{A} + \overline{C}$

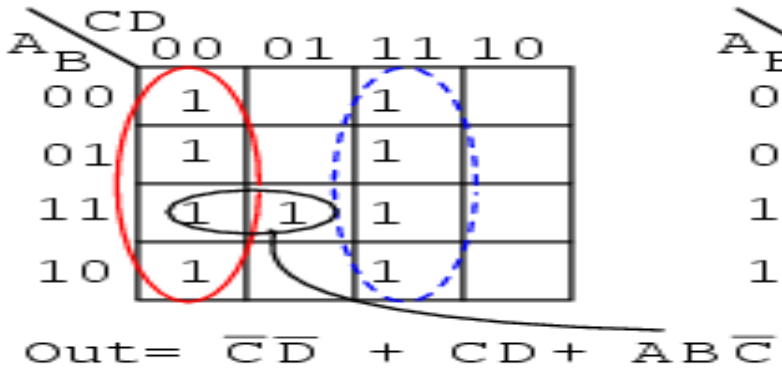
Out= $\overline{A}\overline{B}CD+\overline{A}B\overline{C}D+AB\overline{C}D+A\overline{B}\overline{C}D+AB\overline{C}\overline{D}+AB\overline{C}D+AB\overline{C}\overline{D}$

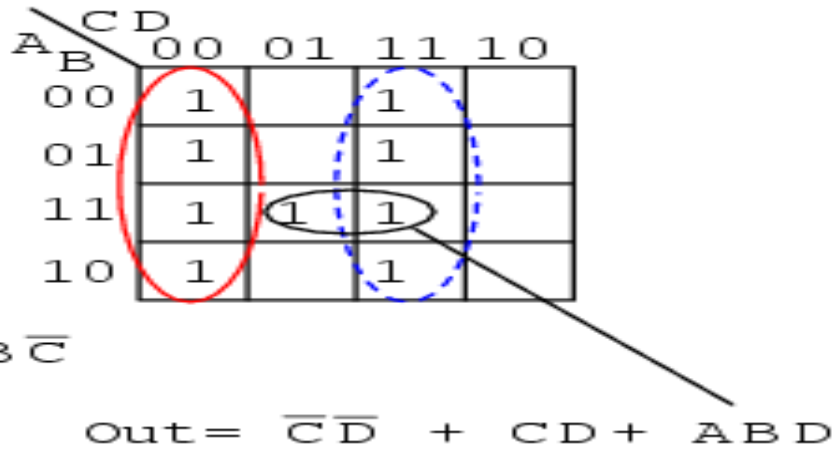


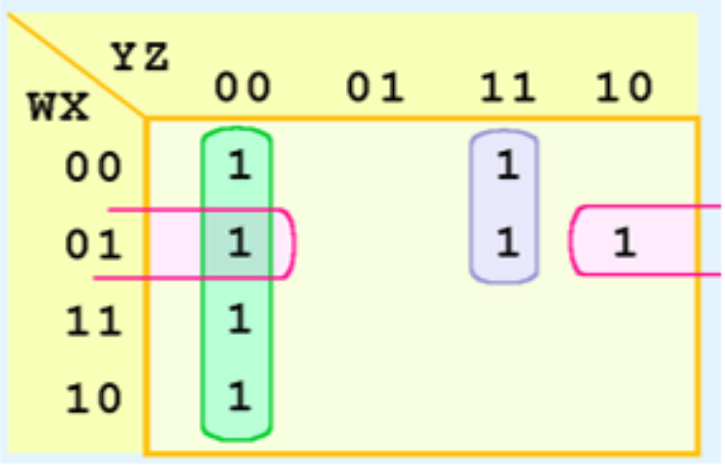
Out= $\overline{A}\overline{B}\overline{C}\overline{D}+\overline{A}\overline{B}\overline{C}D+\overline{A}\overline{B}C\overline{D}+\overline{A}\overline{B}CD$
$+B\overline{C}\overline{D}+BC\overline{D}+A\overline{B}\overline{C}\overline{D}+A\overline{B}D+A\overline{B}C\overline{D}$

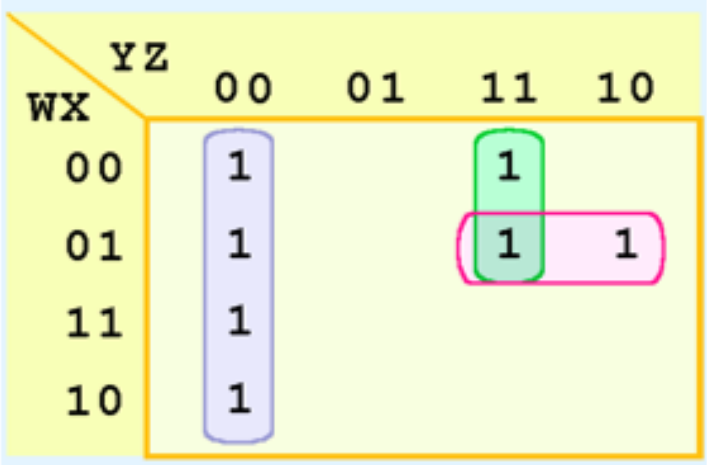


$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,C\,D + A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,D$$





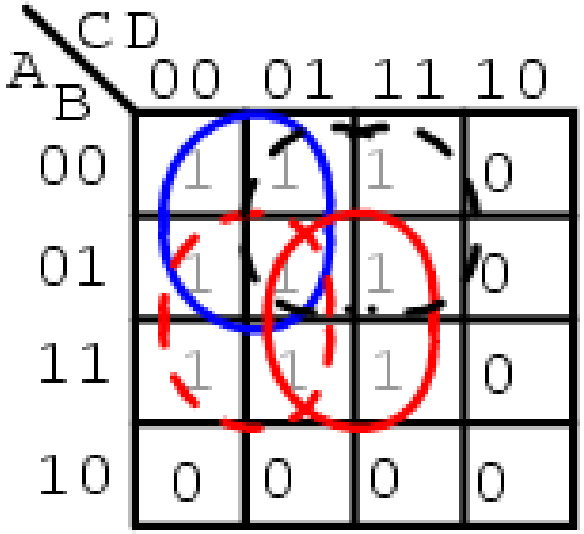




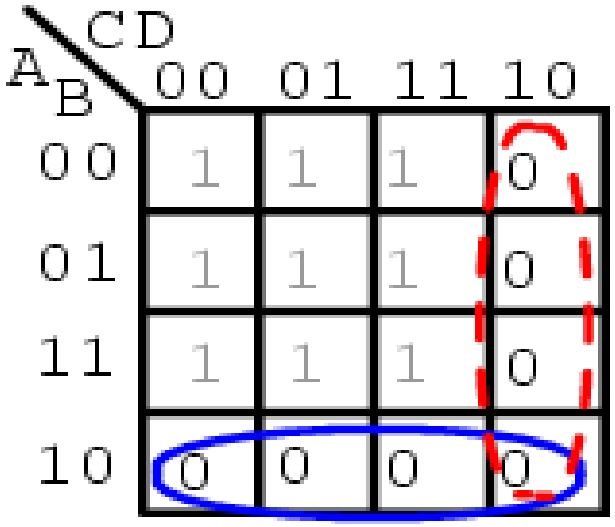
$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D + A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,D$$

$$f(A,B,C,D) = \sum_{m(0,1,3,4,5,7,12,13,15)}$$

$$f(A,B,C,D) = \prod_{M(2,6,8,9,10,11,14)}$$



$$\overline{A}\,\overline{C} + \overline{A}\,D + B\,\overline{C} + B\,D$$



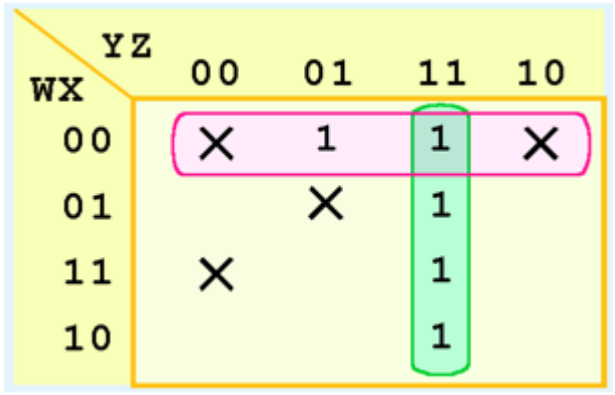
$$f(A,B,C,D) = (\overline{A}+B)\,(\overline{C}+D)$$

Example of simplification of POS expression using K-map

Kmap with Don't Care

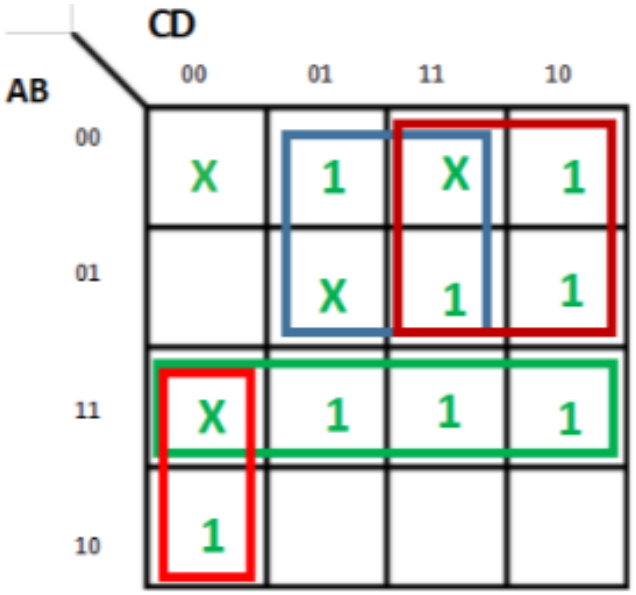
In a Kmap, a don't care condition is identified by an *X* in the cell of the minterm(s)

In simplification, we are free to include or ignore the *X*'s when creating our groups.

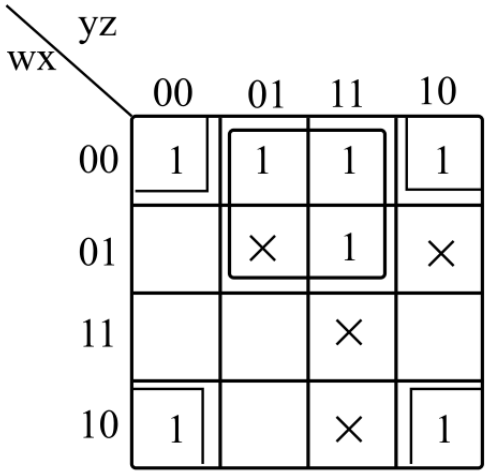


$$F(W,X,Y,Z) = \overline{W}\,\overline{Y} + YZ$$

$F(A, B, C, D) = m(1, 2, 6, 7, 8, 13, 14, 15) + d(0, 3, 5, 12)$



$$F(w,x,y,z) = \sum(0,1,2,3,7,8,10)$$
$$d(w,x,y,z) = \sum(5,6,11,15)$$



$$\overline{X}\,\overline{Z} + \overline{W}Z$$

F(ABCD)=π(0,1,3) . d(5,7)

MCQ

How many cells in a 5 variable k-map.

(a) 8

(b) 16

(c) 32

(d) 64

MCQ

Which grouping possible in 4-variable k-map.

(a) 3

(b) 5

(c) 8

(d) 10

MCQ

For a 3-variable k-map, two cell group yield a…

(a) 1- variable product

(b) 2- variable product

(c) 3- variable product

(d) value 1

MCQ

What will be out expression using K-map if F(A,B,C)=$\prod(1,3,5,7)$

(a) 1

(b) A’

(c) B’

(d) C’

MCQ

What will be out expression using K-map if F(A,B,C)= $\prod M(0,1,2,3,4,5,6,7)$

(a)0

(b) 1

(c)C’

(d) AB