05 March 2021 09:59

Q: For C.F of diff. eq.. $5x^2y'' + 3xy' + 2y = 0$ what will be trial sol. -.

(a) $y = e^{mx}$. (b) $y = e^{mz}$

(c) $y = x^m$ (d) none of these,

Let $\ln x = z$ $x = e^z$

$xy' = \theta y$, $x^2y'' = \theta(\theta-1)y$, $\theta = \frac{d}{dz}$.

$(5\theta(\theta-1) + 3\theta + 2)y = 0$

$\Rightarrow (5\theta^2 - 5\theta + 3\theta + 2)y = 0$

$\Rightarrow (5\theta^2 - 2\theta + 2)y = 0$ → $5m^2 - 2m + 2 = 0$

Let $y = e^{mz}$ be its sol,

$y = x^m$ be its sol

$5x^2(m(m-1)x^{m-2}) + 3x(mx^{m-1}) + 2x^m = 0$

$\Rightarrow x^m(5m(m-1) + 3m + 2) = 0$

$\Rightarrow x^m(5m^2 - 2m + 2) = 0$

Q1. What is P.I of. $(x^2D^2 + xD + 1)y = x^2$,

$D \equiv \frac{d}{dx}$.

~~(a)~~. $\frac{x^2}{3}$ ~~(b)~~ $\frac{x^2}{3} + \frac{x}{5}$

~~(c)~~. $\frac{x^2}{3} + \frac{x}{2} + 5$. ~~(d)~~ none of these.

Put $\ln x = z \Rightarrow x = e^z$.

$xDy = \theta y$, $x^2D^2y = \theta(\theta-1)y$

$$y_p = \frac{1}{\theta(\theta-1)+\theta+1} e^{2z}.$$

$$= \frac{1}{\theta^2+1} e^{2z}$$

$\left(\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \right)$

$$= \frac{1}{(2)^2+1} e^{2z} = \frac{1}{5} e^{2z}. \quad f(a) \neq 0$$

$$\boxed{y_p = \frac{1}{5} x^2}$$

Q2. Find the P.I of. $(x^2D^2 + 5)y = \cos(\ln x)^2$

(a) $\frac{\cos(\ln x)^2}{3}$ (b) $\cos(\ln x)^2$.

$$\frac{\quad}{3}$$

(c) $(\ln x)$ $\frac{\cos((\ln x)^2)}{5}$ (d) none of these.

Sol. $(x^2D^2+5)y = \cos((\ln x^2)^2) = \cos(2\ln x)$

Put $\ln x = z$, $x = e^z$.

$xDy = \theta y$, $x^2D^2y = \theta(\theta-1)y$, $\theta = \frac{d}{dz}$.

$(\theta(\theta-1)+5)y = \cos(2z)$

$(\theta^2-\theta+5)y = \cos 2z$.

$$y_p = \frac{1}{\theta^2-\theta+5}\cos 2z.$$

$$= \frac{1}{-2^2-\theta+5}\cos 2z.$$

$$= \frac{1}{1-\theta}\cos 2z$$

$$= \frac{1+\theta}{(1-\theta)(1+\theta)}\cos 2z$$

$$= \frac{1+\theta}{1-\theta^2}\cos 2z.$$

$$= \frac{\cos 2z - 2\sin 2z}{1-(-2)^2}$$

$$= \frac{\cos 2z - 2\sin 2z}{5}$$

$$= \frac{\cos(2\ln x) - 2\sin(2\ln x)}{5}$$

$$= \frac{\cos(\ln x^2) - 2\sin(\ln x^2)}{5}$$