Initial value problem :-

Q. Solve the following initial value problem $y''' - 2y'' - 5y' + 6y = 0$ with initial conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$.

Sol :- operator form of Differential eq is

$$(D^3 - 2D^2 - 5D + 6)y = 0, \quad D = \frac{d}{dx}$$

Let $y = e^{mx}$ be the soln.

A.E. $m^3 - 2m^2 - 5m + 6 = 0$

$6 = (1, 2, 3)$ $(1, 1, 6)$ ✗

$1, -2, 3$

$1 - 2 - 5 + 6 = 0$

$\Rightarrow m = 1$ is one of root of A.E.

	m^3	m^2	m	c
1	1	−2	−5	6
	↓	1	−1	−6
	1	−1	−6	0

depressed eq. $m^2 - m - 6 = 0$

$m^2 - 3m + 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$\Rightarrow (m-3)(m+2) = 0 \Rightarrow m = -2, 3.$

roots of A.E are $1, -2, 3$.

roots of A.E are $1, -2$ & 3.

general sol is $y = C_1 e^x + C_2 e^{-2x} + C_3 e^{3x}$ — (1)

$y' = C_1 e^x - 2C_2 e^{-2x} + 3C_3 e^{3x}$ — (2)

$y'' = C_1 e^x + 4C_2 e^{-2x} + 9C_3 e^{3x}$ — (3)

$y(0) = 0$, $y'(0) = 0$ & $y''(0) = 1$

when $x = 0, y = 0$.

(1) $\Rightarrow 0 = C_1 + C_2 + C_3$

$y'(0) = 0$ when $x = 0, y' = 0$

(2) $\Rightarrow 0 = C_1 - 2C_2 + 3C_3$.

$y''(0) = 1$, when $x = 0$, $y'' = 1$.

(3) $\Rightarrow 1 = C_1 + 4C_2 + 9C_3$

$$C_1 + C_2 + C_3 = 0$$
$$C_1 - 2C_2 + 3C_3 = 0$$
$$C_1 + 4C_2 + 9C_3 = 1$$

$AX = B$.

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -2 & 3 & 0 \\ 1 & 4 & 9 & 1 \end{array}\right]$$

$R_2 - R_1$, $R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 3 & 8 & 1 \end{array}\right]$$

operate $R_3 + R_2$.

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & 10 & 1 \end{array}\right] \Rightarrow \begin{array}{r} C_1 + C_2 + C_3 = 0 \\ -3C_2 + 2C_3 = 0 \\ 10C_3 = 1 \end{array}$$

$10C_3 = 1$

2) $C_3 = \frac{1}{10}$, $3C_2 = 2C_3$

$C_2 = \frac{2}{3}C_3 \therefore \frac{2}{3 \times 10} = \frac{1}{15}$

$C_1 + C_2 + C_3 = 0 \Rightarrow C_1 = -C_2 - C_3$

$= -\frac{1}{15} - \frac{1}{10} = -\left(\frac{2+3}{30}\right)$

$\boxed{C_1 = -\frac{5}{30} = \frac{-1}{6}}$

$$y = -\frac{1}{6}e^{x} + \frac{1}{15}e^{-2x} + \frac{1}{10}e^{3x}$$