Single_NestedLoop_Complexity

Monday, August 2, 2021 3:11 PM

Time Complexity → no of key steps in terms of input size n

```
print("Hello")   # 1
for (i=1; i<n; i++)
    print(i)   #2 . n → k
print("Bye")   # 1
```

Iteration	1	2	3	4	...	k	k+1
i	1	2	3	4	...	k	k+1

Condition False

1, 2, 3, 4, .. n

---- n+1

$i = n$ → False

$k+1 = n$

$k = n-1$

$T(n) = k$

$T(n) = n-1$

$T(n) = 1 + 1 + n + n + n + 1$

$= 3n + 3$ → constant

$= O(n)$

$T(n) = O(n)$

$f(n) = 3n$

$g(n) = n$

$f(n) = O(g(n))$

$g(n) \geq c \cdot f(n)$

$2n = 5n = 6n \Rightarrow O(n)$

```
i = 1          #1
while i <= n
DO
    print(i)   #k
    i = i*2
Done
```

→ $\log_2(n)$

$(k+1) \times$

Iteration	1	2	3	4	5	...	k	k+1
variable i	1 $2^0=1$	2 $2^1=2$	4 $2^2=4$	8 $2^3=8$	16 $2^4=16$	...	2^{k-1}	2^k

k+1 → Loop condition will be false

$i = n$

$2^k = n \Rightarrow \log 2^k = \log n$

$k \log_2 2 = \log_2 n$

$\boxed{k = \log n}$

Azure

$T(n) = k$ ----(i)

$\boxed{T(n) = \log(n)}$

$i = n$

k+1

k+1 iteration will make ... loop false

i = n

while i > 1 False

Do

print(i) # k

i = i / 2

Done

k+1

k + 1 iteration will make i > 1 condition false

Anchor condition

End condition

iteration	1	2	3	4	5	...	k	k+1
i	$n = \frac{n}{2^0}$	$\frac{n}{2} = \frac{n}{2^1}$	$\frac{n}{4} = \frac{n}{2^2}$	$\frac{n}{8} = \frac{n}{2^3}$	$\frac{n}{16} = \frac{n}{2^4}$	...	$\frac{n}{2^{k-1}}$	$\frac{n}{2^k}$

$T(n) = k$

$$\boxed{T(n) = \log n}$$

$i = 1 \Rightarrow \frac{n}{2^k} = 1 \Rightarrow 2^k = n$

$\log_2 2^k = \log_2 n \Rightarrow k \cancel{\log_2 2} = \log_2 n$

$$\boxed{k = \log_2(n)}$$

i = 2

while i <= n

Do

print(i) # k

$i = i^2$

Done

Anchor

iterations	1	2	3	4	5	...	k	k+1
i	$2 = 2^{2^0}$	$4 = 2^{2^1}$	$16 = 2^{2^2}$	$256 = 2^{2^3}$	$65535 = 2^{2^4}$	...	$2^{2^{k-1}}$	2^{2^k}

$2^1 \quad 2^2 \quad 2^4 \quad 2^8 \quad 2^{16}$

$2^{2^0} \quad 2^{2^1} \quad 2^{2^2} \quad 2^{2^3} \quad 2^{2^4}$

$\boxed{i > n}$ → Anchor

$T(n) = k$

$$\boxed{T(n) = O(\log \log n)}$$

$2^{(2^k)} = n$

$\log_2 2^{(2^k)} = \log_2(n)$

$2^k \cancel{\log_2(2)} = \log_2(n)$

$\log 2^k = \log_2 \log_2 n$

$k \cancel{\log_2 2} = \log_2 \log_2 n$

$$\boxed{k = \log_2 \log_2 n}$$

$$\boxed{k = \log_2 \log_2 n}$$

Question H/W

```
i = n
WHILE i > 1
DO
    print(i)
    i = √i
Done
```

Nested loop time Complexity

Independent

```
for (i=1; i<n; i++)
    for (j=1; j<n; j=j*2)
        print(i,j)
```

Dependent

```
for (i=1; i<n; i++)
    for (j=1; j<=i; j++)
        print(i,j)
```

inner loop depends on outer loop variable

```
for (i=1; i<n; i++)
    for (j=1; j<n; j=j*2)  -> B
```

$B = \log(n)$

$A = n$

r (_-1; j<n; j=j*2)

Body

print (i,j)

B

A

$T(n) = O(n \log n)$

$T(n) = O(A \cdot B)$ → for independent loop

$= 2+2+2+2$

$= 2 \times 4$

iteration	1	2	3	...	K
i	1	2	3	...	n → n
j	log(n)	log(n)	log(n)	...	log(n)

$T(n) = \log(n) + \log(n) - \log(n) + \cdots$ log(n)

$= \log(n)(1+1+1+1+\cdots n)$

$= n\log(n)$

for (i = 2 ; i <= n^2 ; i = i + 2)

for (j = n/2 ; j <= n ; j = j * 2)

for (k = n^2 ; k >= 2 ; k = √k)

print ("Hello")

$Z = O(n^2)$

$Y = O(1)$

$X = O(\log \log n)$

$T(n) = O(X \cdot Y \cdot Z) \Rightarrow O(n^2 \cdot 1 \cdot \log \log n) \rightarrow O(n^2 \log \cdot \log(n))$

Iteration	1	2	3	4	5	...	z	z+1
i	2 2×1	4 2×2	6 2×3	8 2×4	10 2×5	...	$2*z$	$2*z+1$

$i = 2*z$ when condition will be False

$$2*z = n^2 \Rightarrow z = \frac{n^2}{2}$$

$$\rightarrow \boxed{z = O(n^2)}$$

$\frac{1}{2}$

Iteration	1	2	3	4	5	...	k	k+1
j	$n/2$	n	$2*n$	$4*n$	$8*n$	...		
	$2^{-1}*n$	$2^{0}*n$	$2^{1}*n$	$2^{2}*n$	$2^{3}*n$		$2^{k-2}*n$	

$1-2 \rightarrow -1$
$2-2 \rightarrow 0$
$3-2 \rightarrow 1$
$4-2 \rightarrow 2$

$$\boxed{2^k \cdot \cancel{n} = \cancel{n}}$$

$$2^k = 1$$

$$\log_2(2^k) = \underline{\log(1)}$$

$$\boxed{k = c}$$

$k = \underline{n}^2$; $k >= 2$; $k = (\underline{k})^{1/2}$

Iteration	1	2	3	4	5	...	k	k+1
k	n^2	n	$n^{1/2}$	$n^{1/4}$	$n^{1/8}$	...	.	
	$n^{\frac{1}{2^{-1}}}$	$n^{\frac{1}{2^0}}$	$n^{1/2^1}$	$n^{\frac{1}{2^2}}$	$n^{\frac{1}{2^3}}$		$n^{\frac{1}{2^{k-2}}}$	

$$n^{\frac{1}{2^k}} = 2 \Rightarrow \frac{1}{2^k}\log(n) = \cancel{\log_2(2)}^{\,1}$$

$$\Rightarrow 2^k = \log(n)$$

$$\Rightarrow k\,\cancel{\log_2(2)}^{\,1} = \log\log(n)$$

$$\Rightarrow \boxed{y = \log\log n}$$