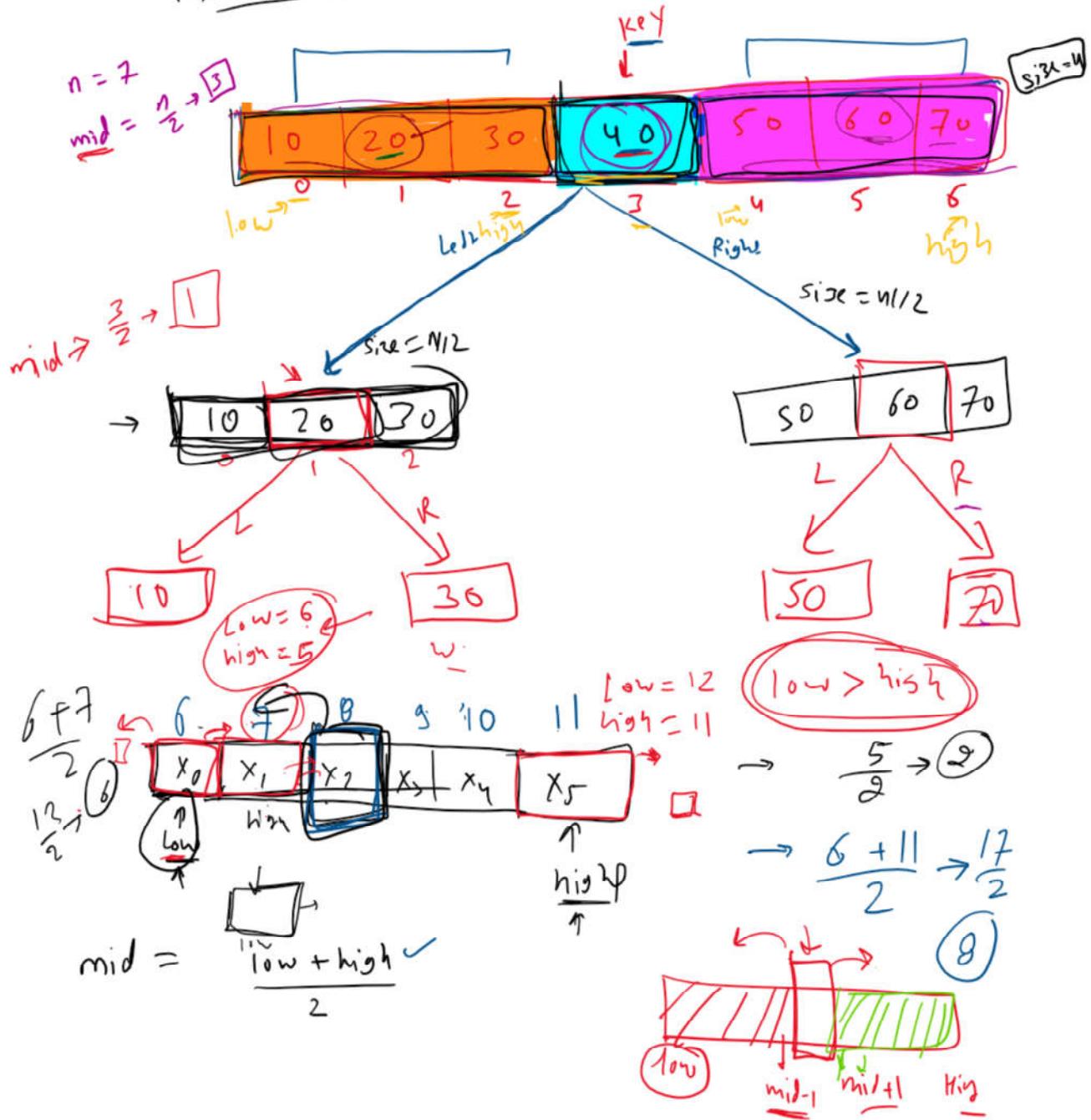


Divide & Conquer

Binary Search

↳ Break problem into similar sub problems & solve them.

(i) Sorted Array



Binary Search ($arr, \underline{low}, \underline{high}, \underline{key}$)

if $\underline{low} > \underline{high}$:
 $mid = (\underline{low} + \underline{high}) // 2$

if $arr[\underline{mid}] > \underline{key}$:

return BinarySearch($arr, \underline{low}, \underline{mid}-1, \underline{key}$)

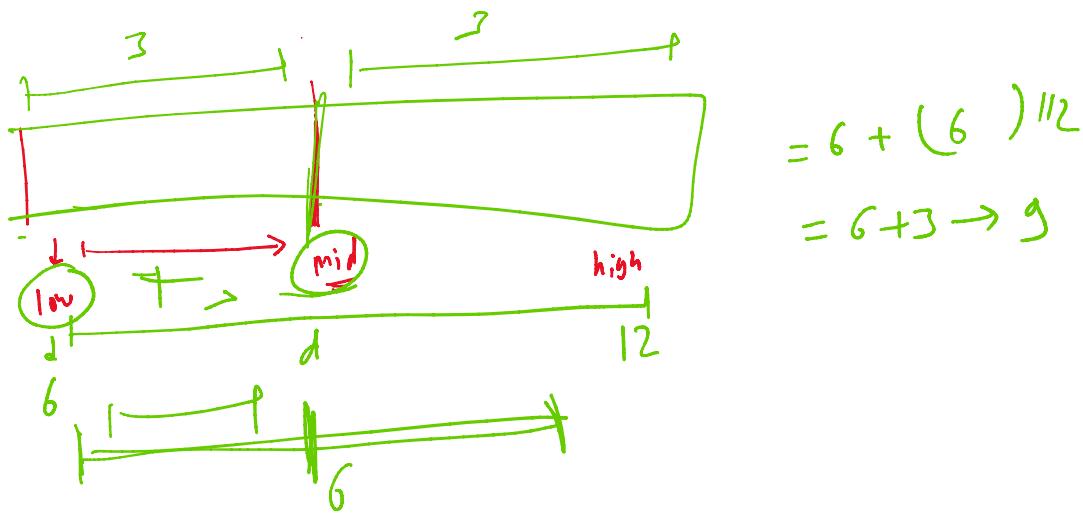


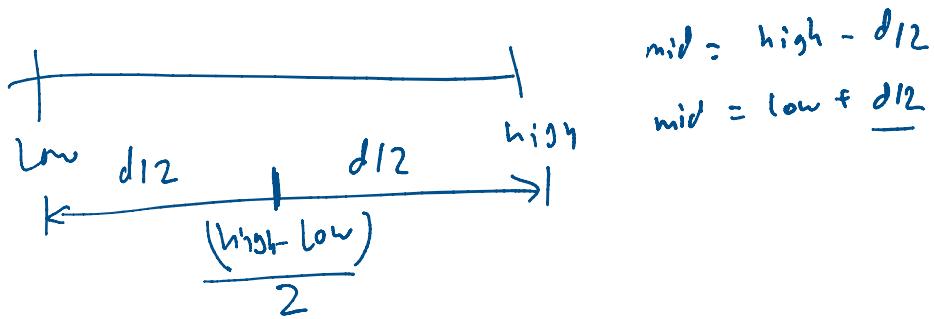

 return BinarySearch(arr, low, mid-1, key)

if arr[mid] < key:
 return BinarySearch(arr, mid+1, high, key)

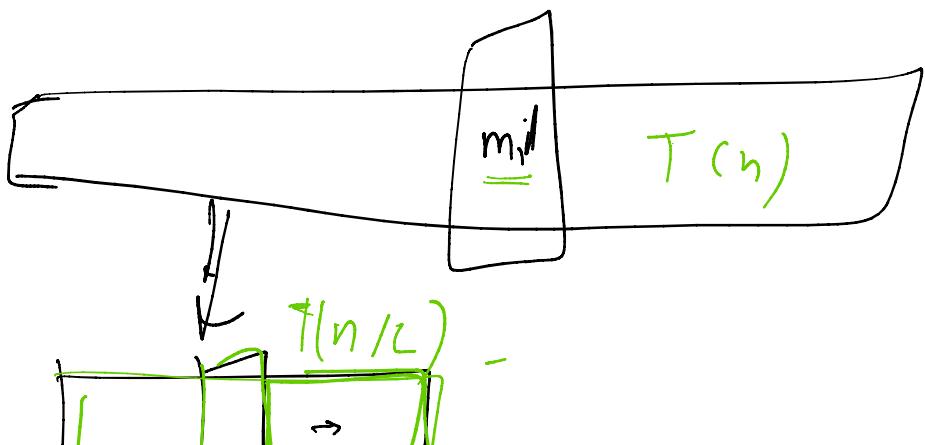
else:
return mid

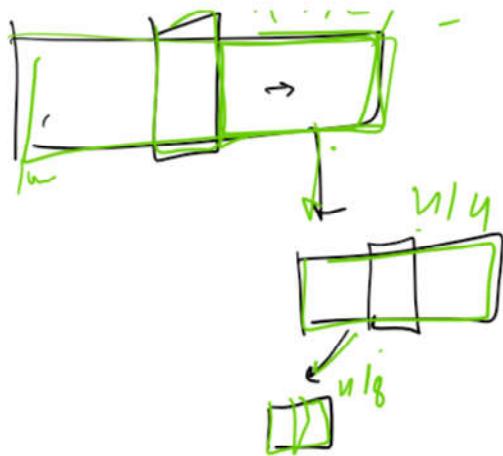
$$\text{mid} = \text{low} + \frac{(\text{high} - \text{low})}{2}$$





$$\begin{aligned}
 \text{mid} &= \text{high} - \frac{d}{2} \\
 \text{mid} &= \text{low} + \frac{d}{2}
 \end{aligned}$$

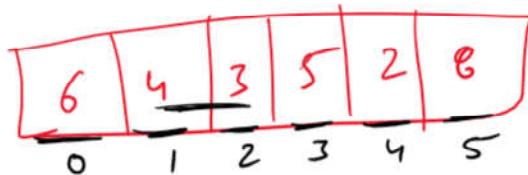




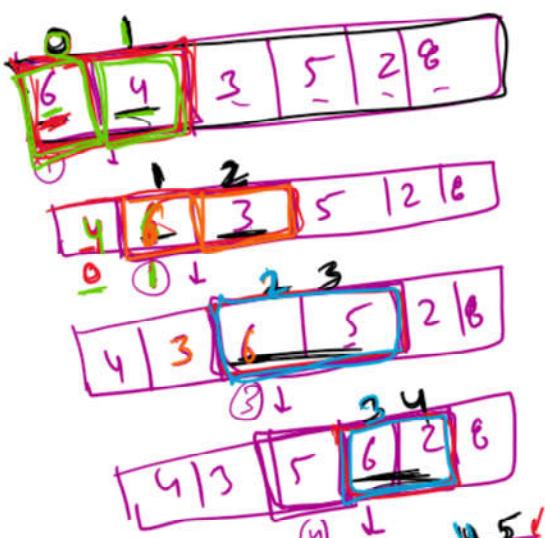
⇒ Sorting Algorithms

- ① Bubble Sort
- ② Selection Sort
- ③ Insertion Sort
- ④ Quick sort
- ⑤ merge sort

+ Bubble Sort \Rightarrow (consecutive Elements are compared)

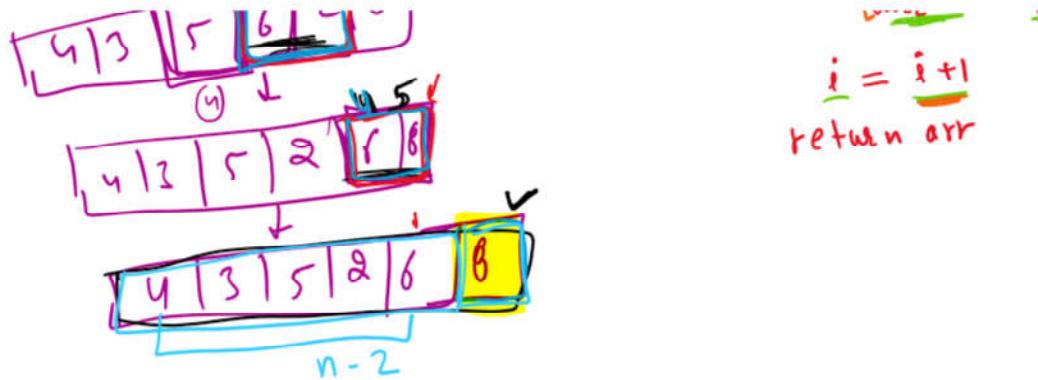


pass - 1

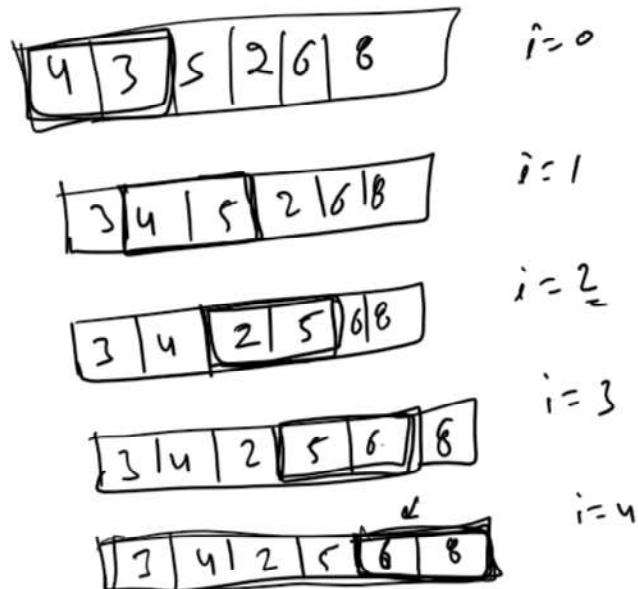


$n = 6$
 $n-2 \Rightarrow 4$ bubble-pass(arr, n)

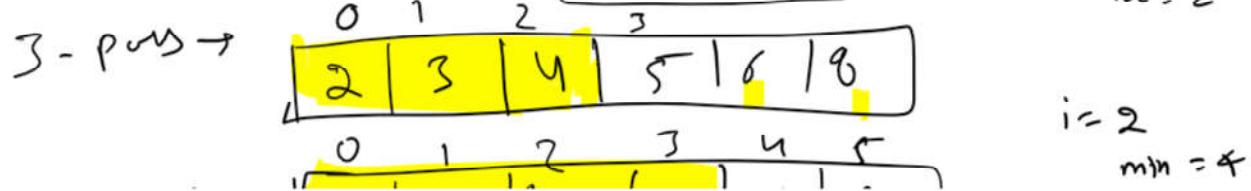
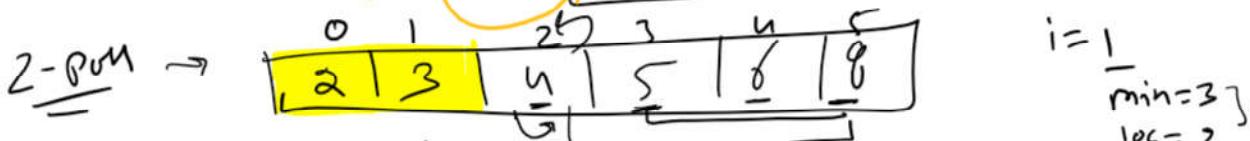
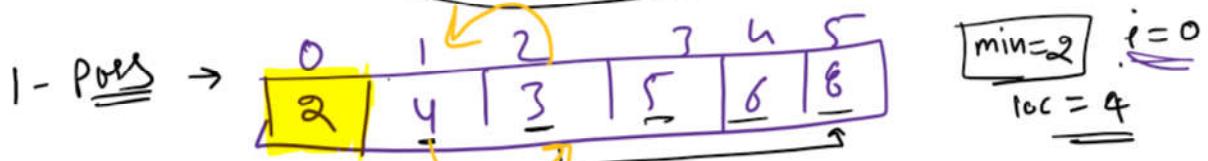
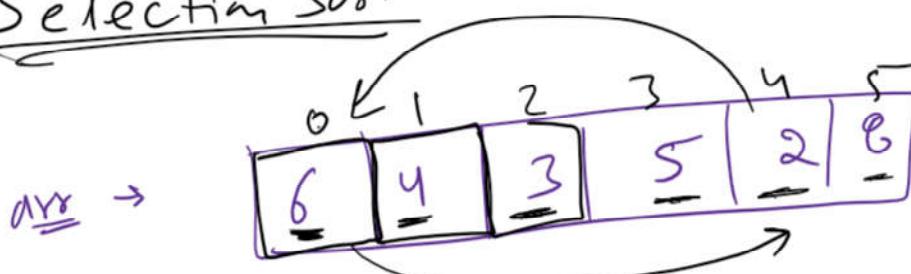
i = 0 i = 4
while i < n-2
do
 if arr[i] > arr[i+1]:
 $t = arr[i] + 6$
 $arr[i] = arr[i+1]$
 $arr[i+1] = t$
 i = i + 1



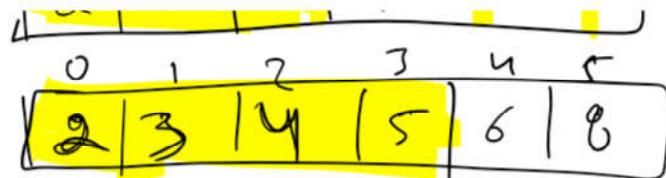
2 - pass



\Rightarrow Selection Sort



$A - \text{push}$



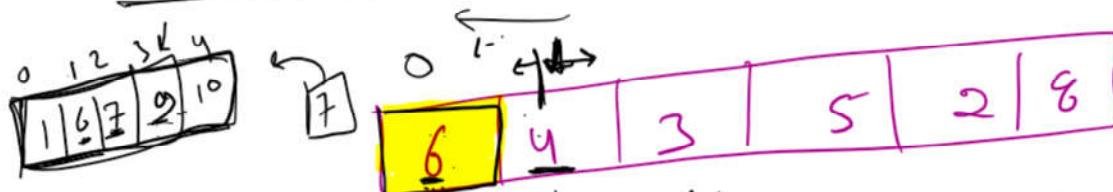
$i = 2$
 $\min = 4$
 $\text{loc} = 2$

$S - \text{push}$

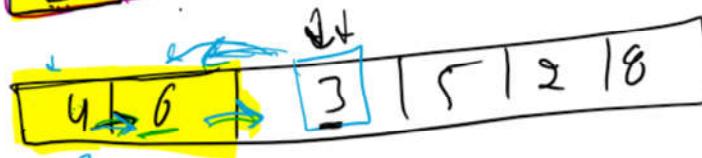


$i = 3$
 $\min = 5$
 $\text{loc} = 3$

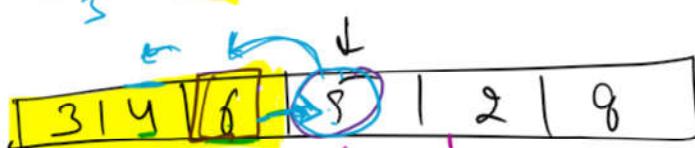
$\Rightarrow \text{Insertion Sort}$



$\text{push} \rightarrow 1$



$\text{push} \rightarrow 2$



$\text{push} \rightarrow 3$



$\text{push} \rightarrow n$



$O(1)$
 $O(n)$
 $\text{swaps} = 0$

$O(n^2)$

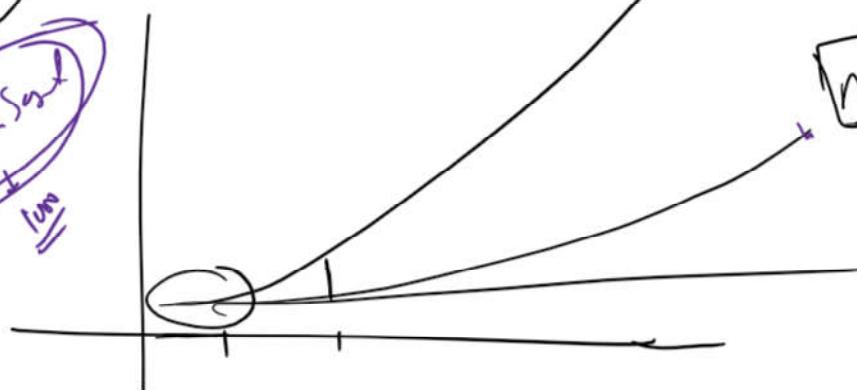
quick sort
move sort

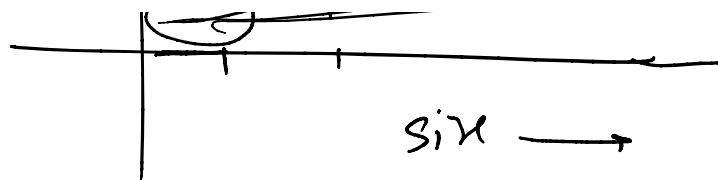
size is big

n^2

insertion sort

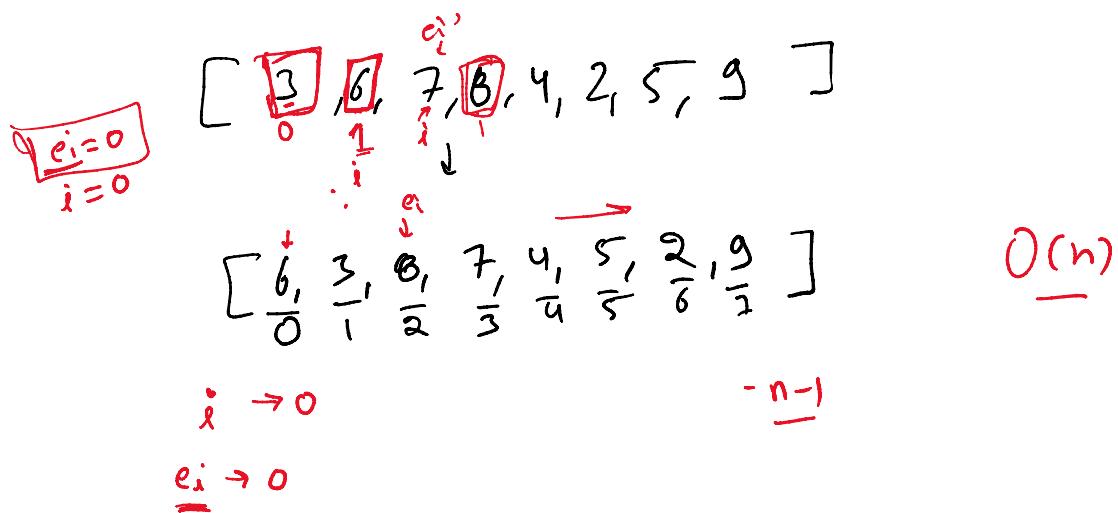
$n \log n$





Quick Sort

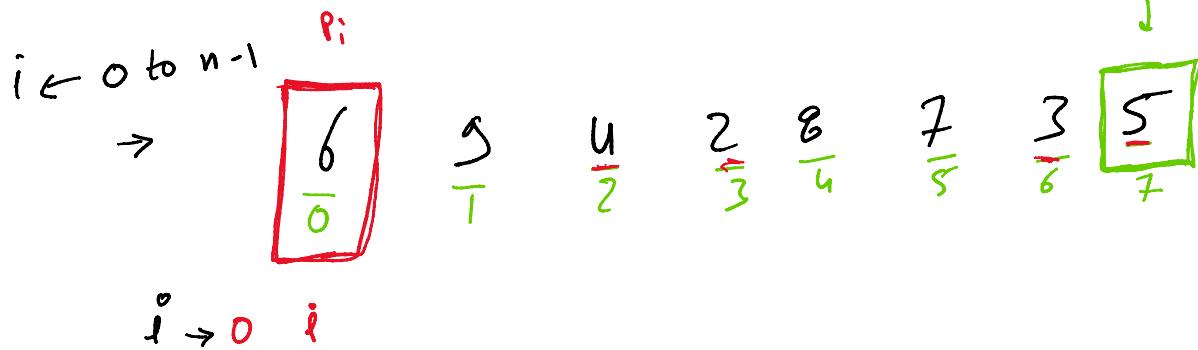
Lomuto partition scheme



Partition

\square pivot \square

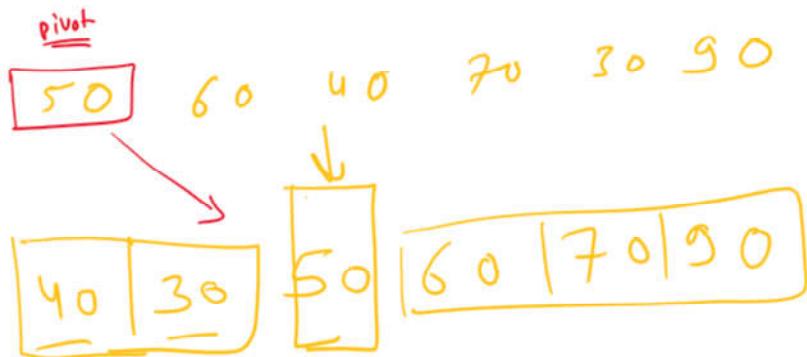
pivot
↓



$P_i \rightarrow 0$

Quick Sort → Divide & Conquer

① Partition

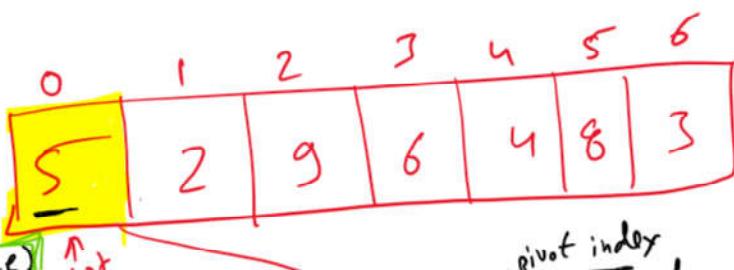


→ Divide & Conquer

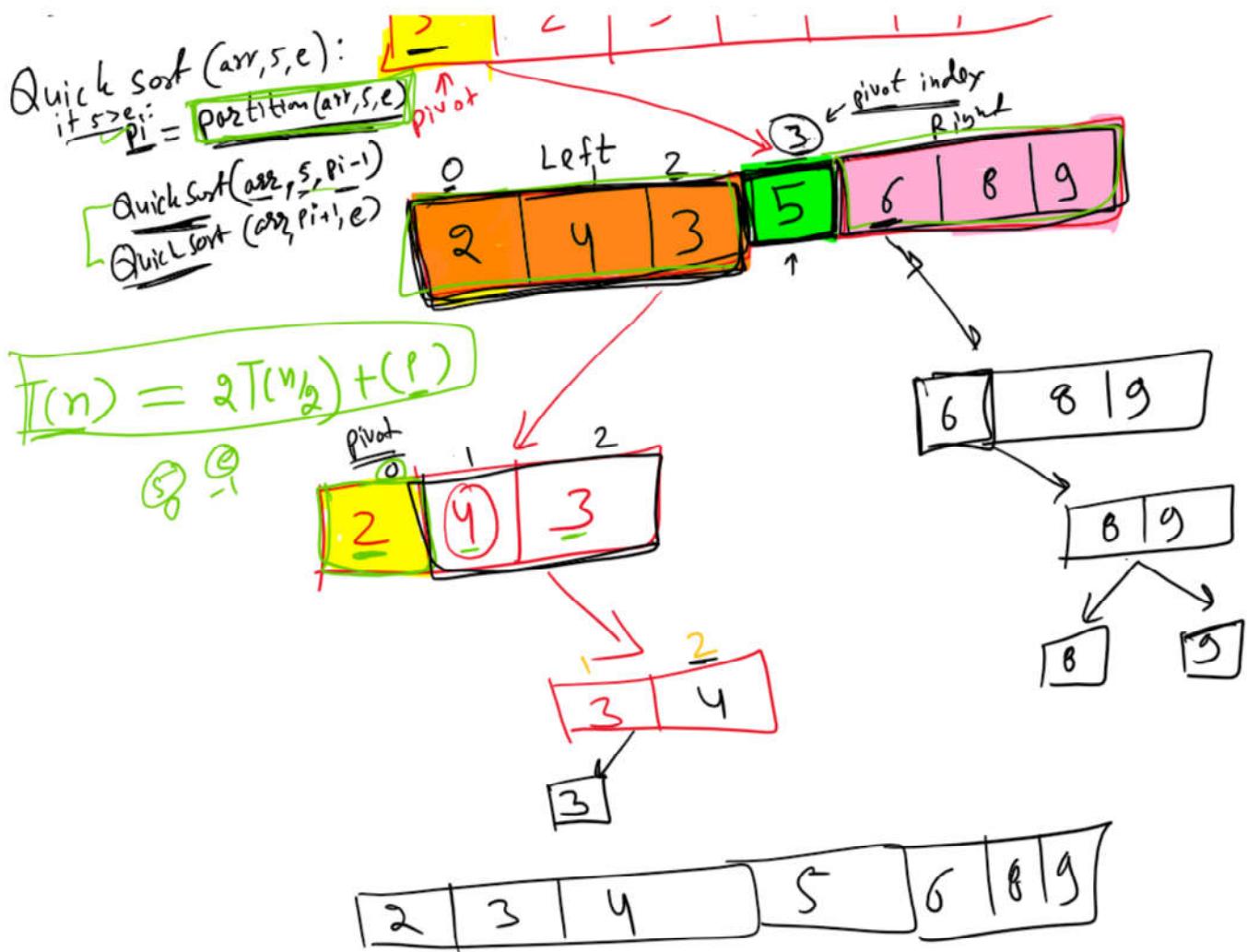
Quick Sort

first
last
mid

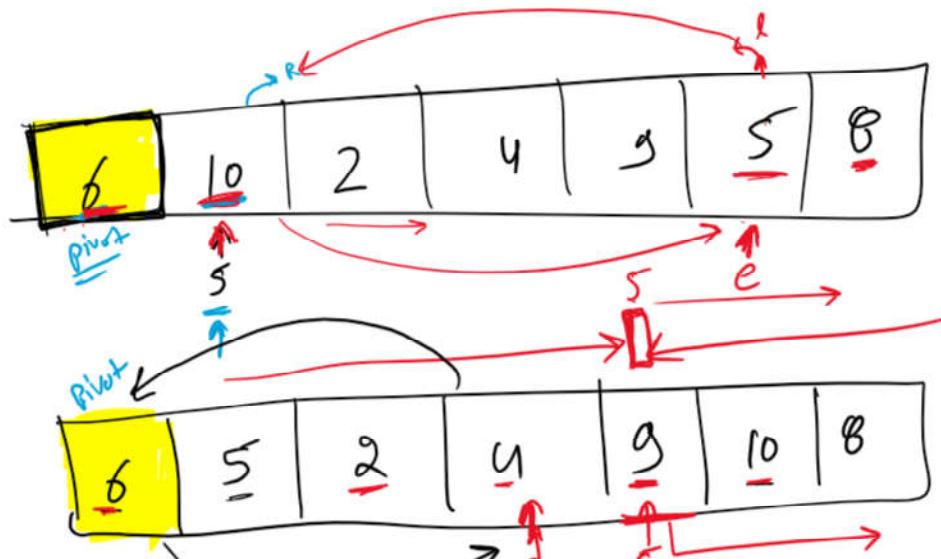
Quick sort (arr, s, e):

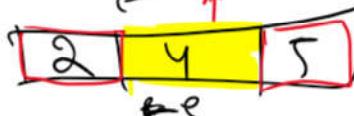
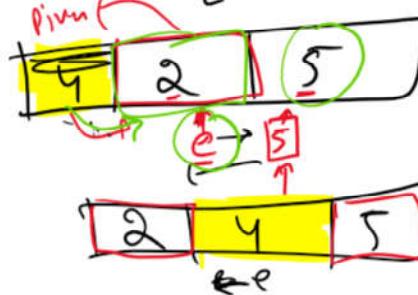
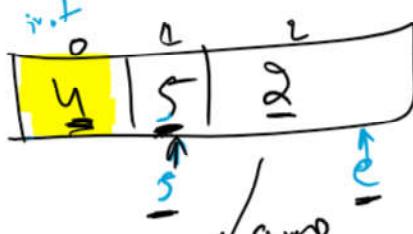
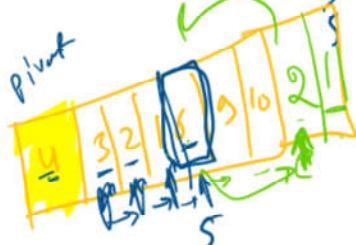
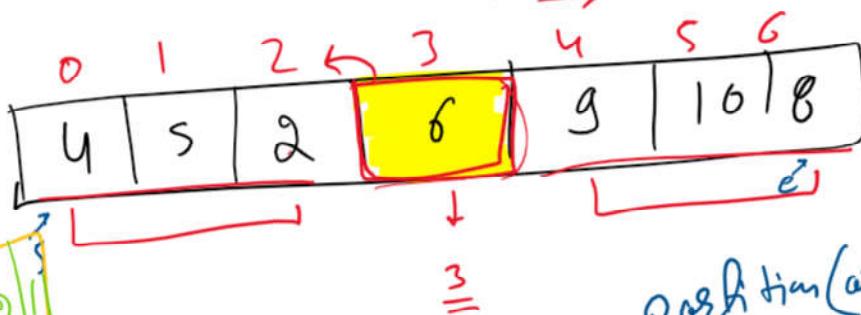
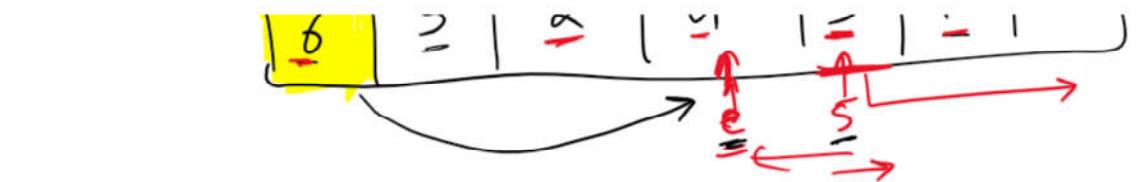


T ⇒ put pivot element to its position



→ partition Move





$\text{partition}(\text{arr}, s, e)$

$$\begin{aligned} \text{pi} &= s \\ \text{pivot} &= \text{arr}[\text{pi}] \\ s &= s+1 \end{aligned}$$

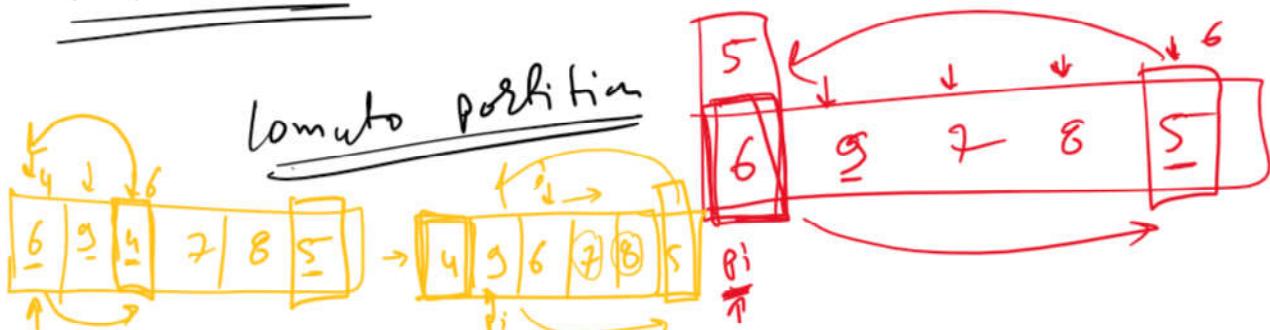
while $s \geq e$:
while $\text{arr}[s] \leq \text{arr}[\text{pi}]$:
 $s = s+1$

while $\text{arr}[e] > \text{arr}[\text{pi}]$:
 $e = e-1$

$\text{swap}(\text{arr}[s], \text{arr}[e])$

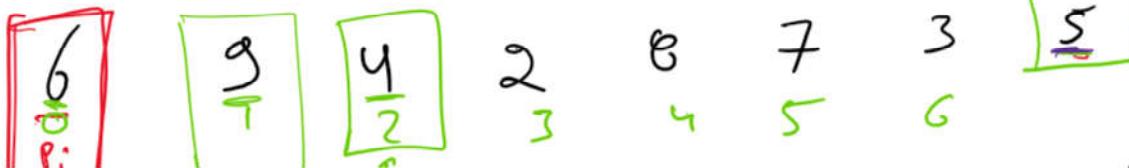
$\text{swap}(\text{arr}[\text{pi}], \text{arr}[e])$

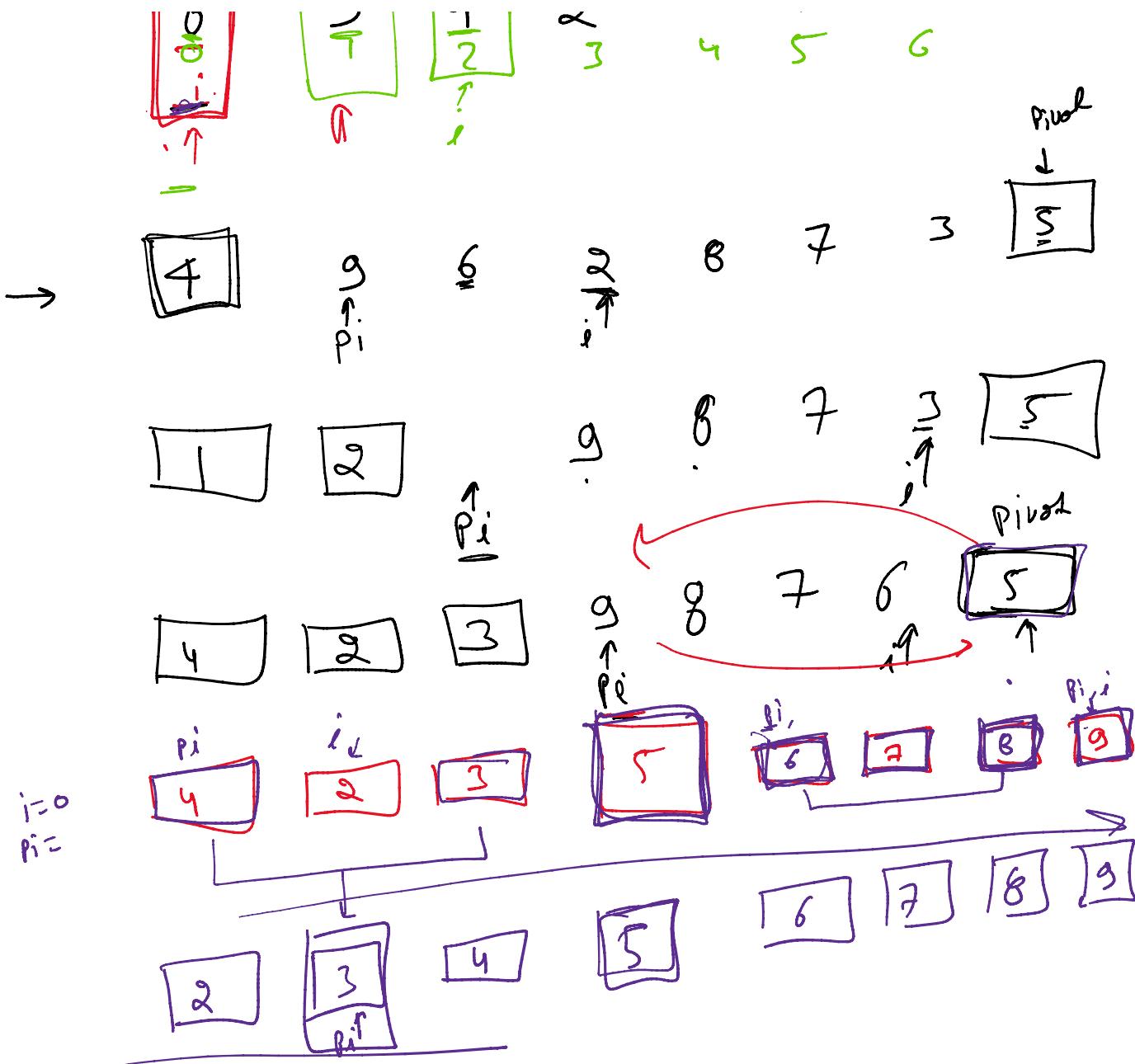
→ Quick Sort



pi → update pi if you find an element which is less than pivot elem

plot





Quick Sort(arr , start, end)

if $\text{start} < \text{end}$:

$\rightarrow \text{pi} = \underline{\text{partition}}(\text{arr}, \text{start}, \text{end})$

$\rightarrow \underline{\text{QuickSort}}(\text{arr}, \text{start}, \underline{\text{pi}-1})$

$\rightarrow \underline{\text{QuickSort}}(\text{arr}, \underline{\text{pi}+1}, \text{end})$

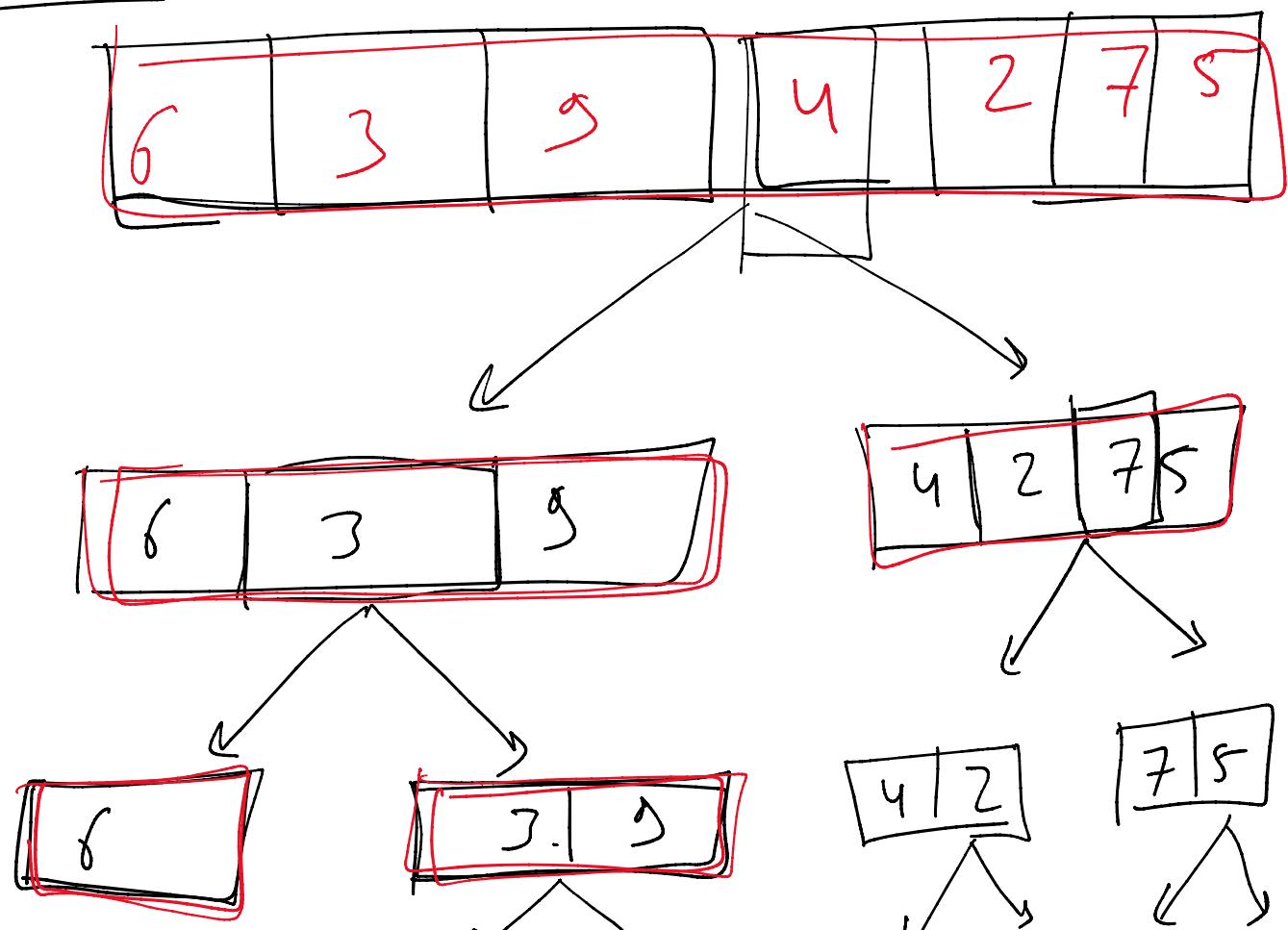
$$T(n) = \underline{2T(n/2)} + \underline{O(n)} \quad \text{--- (1)}$$

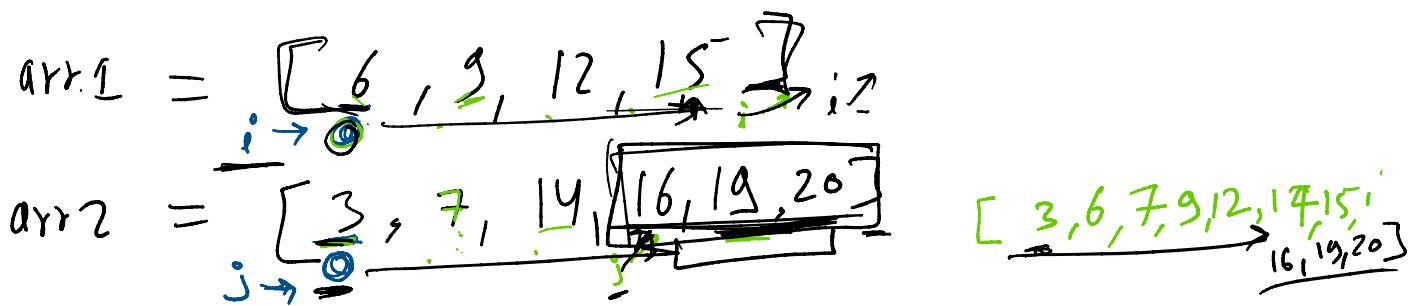
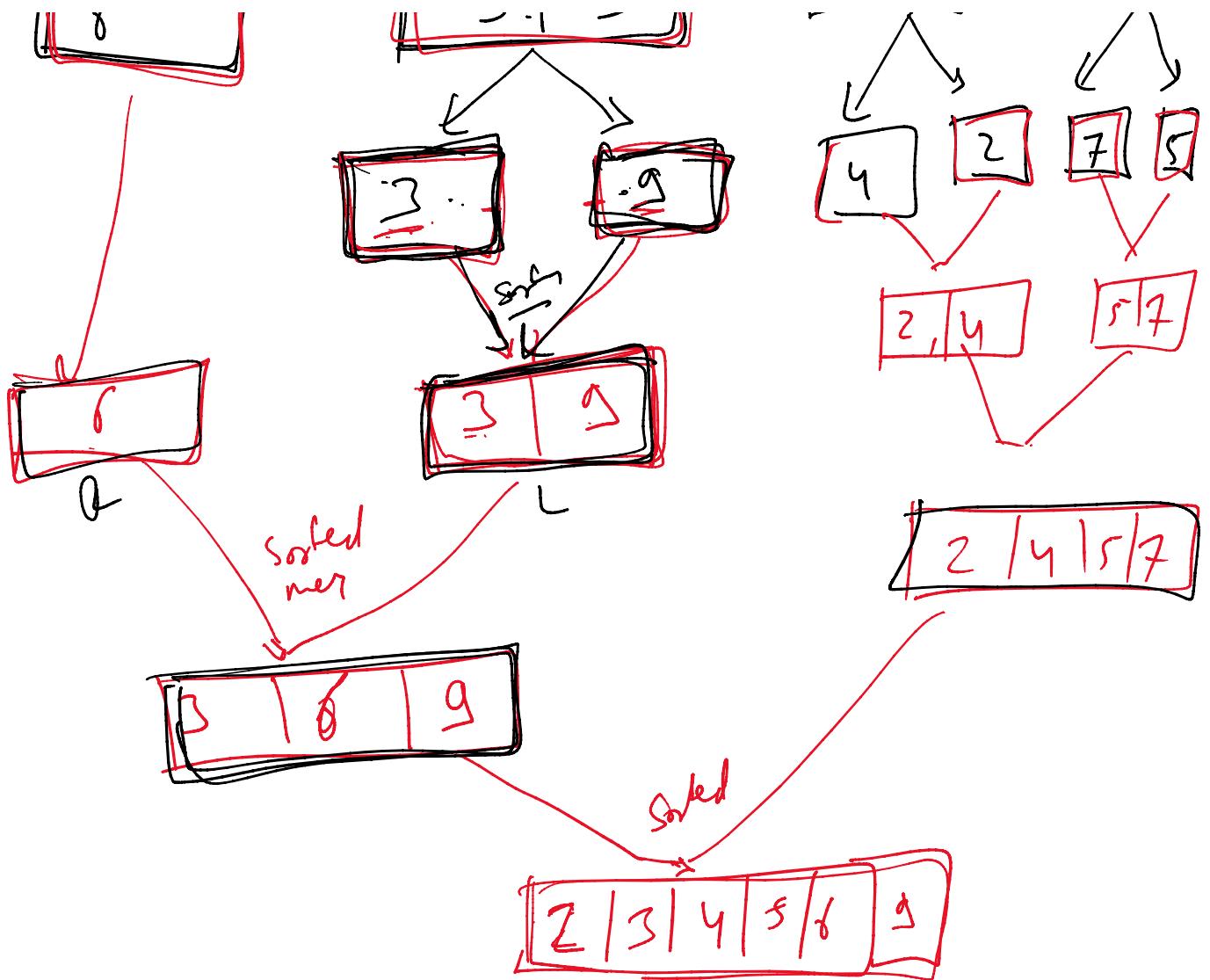
$$T(n/2) = 2T(n/4) + O(n/2) \quad \text{--- (ii)}$$

$$T(n/4) = 2T(n/8) + O(n/4) \quad \text{--- (iii)}$$

$$T(n/2^k) = 2T\left(\frac{n}{2^{k+1}}\right) + O\left(\frac{n}{2^k}\right)$$

Merge Sort





Sorted Merge (arr1, arr2)

$$i = 0$$

$$j = 0$$

$$\underline{arr} = []$$

$\dots \text{while } i < \text{len}(\text{arr1}) \text{ and } j < \text{len}(\text{arr2}):$

arr = []
while i < len(arr1) and j < len(arr2):

 if arr1[i] < arr2[j]:
 arr.append(arr1[i])

 i += 1

 else:
 arr.append(arr2[j])

 j += 1

 while i < len(arr1):
 arr.append(arr1[i])
 i += 1

 while j < len(arr2):

 arr.append(arr2[j])

 j += 1