DATA MINING
Assignment – 7
(Eigen Face Analysis)

1. Perform Eigen Face visualization of images using PCA
2. Reconstruct images using different number of principal components
3. Find minimum number of principal components needed to properly reconstruct images
4. Calculate mean square error, PSNR and SSI for each images at different number of principal components

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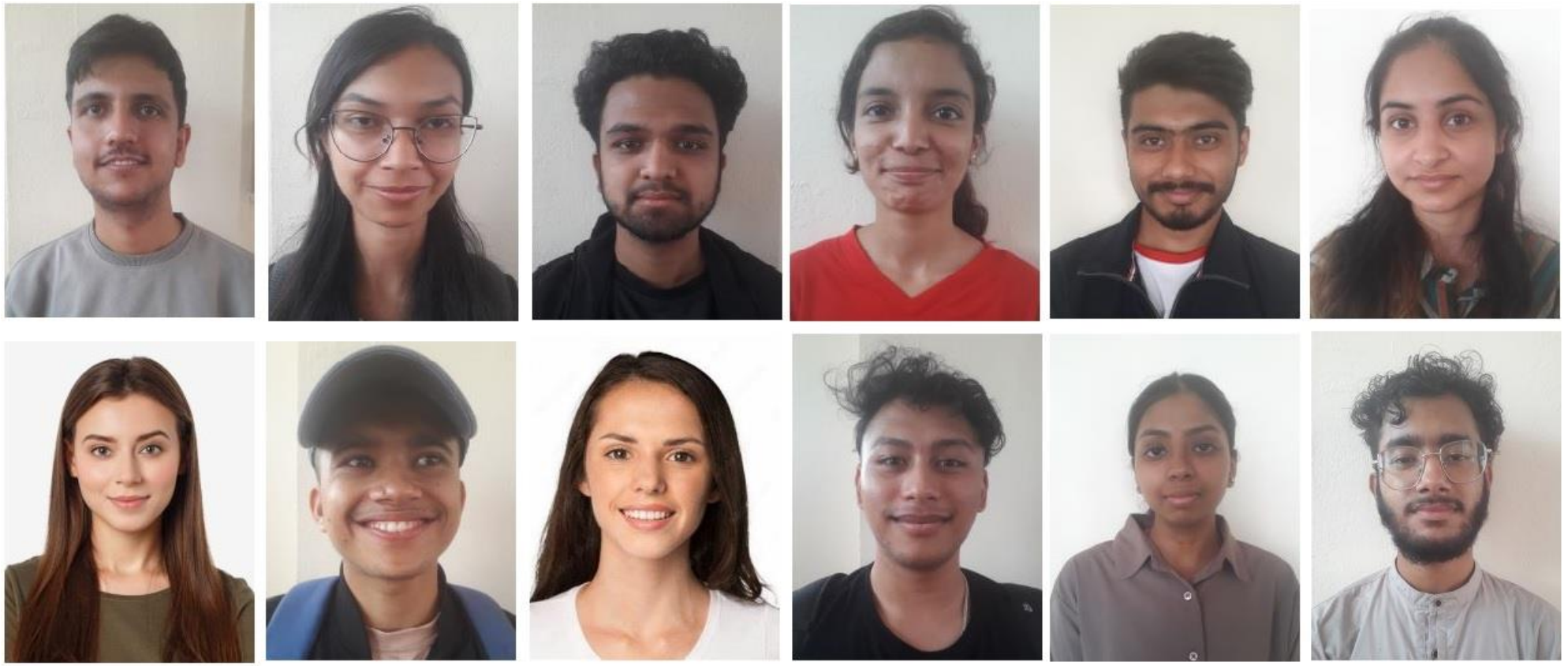
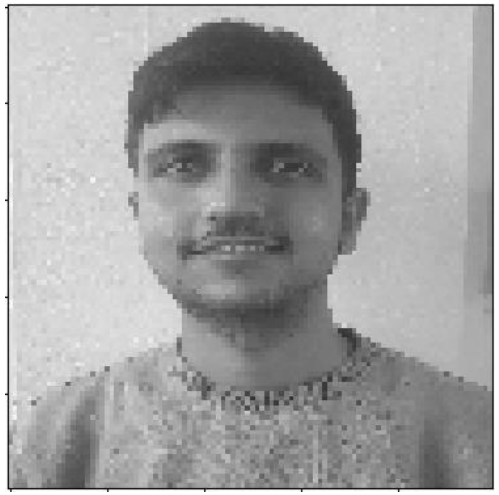
Eigen Face Analysis – [1]

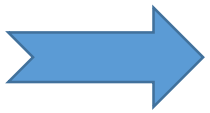
- A total of 12 images are used for Eigen Face Analysis



Eigen Face Analysis – [2]

- Each images are converted to grayscale
- The grayscaled images are resized to 100 * 100 images
- Intensity matrix of each images are obtained



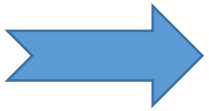


[[212 210 215 ... 171 166 168]
 [212 212 205 ... 168 169 166]
 [208 216 210 ... 170 168 166]
 ... ...
 [180 178 147 ... 136 127 122]
 [173 151 139 ... 134 141 125]
 [173 141 136 ... 143 127 121]]$_{100 * 100}$

Eigen Face Analysis – [3]

- Intensity matrix (2D) of the images are converted to row matrix (1D array)

$$\begin{bmatrix} 212 & 210 & 215 & \dots & 171 & 166 & 168 \\ 212 & 212 & 205 & \dots & 168 & 169 & 166 \\ 208 & 216 & 210 & \dots & 170 & 168 & 166 \\ \dots & & & & & & \\ 180 & 178 & 147 & \dots & 136 & 127 & 122 \\ 173 & 151 & 139 & \dots & 134 & 141 & 125 \\ 173 & 141 & 136 & \dots & 143 & 127 & 121 \end{bmatrix}_{100 * 100}$$



$$[212\ 210\ 215\ \dots\ 143\ 127\ 121]_{1 * 10000}$$

Eigen Face Analysis – [4]

- The row matrix of each images are stacked to form a matrix of size 12 * 4096

$$
\begin{bmatrix}
212 & 210 & 215 & \dots & 143 & 127 & 121 \\
202 & 195 & 194 & \dots & 23 & 23 & 24 \\
228 & 229 & 228 & \dots & 48 & 47 & 48 \\
\dots & \dots & & & & & \\
226 & 225 & 222 & \dots & 38 & 43 & 54 \\
245 & 245 & 245 & \dots & 210 & 209 & 210 \\
254 & 254 & 254 & \dots & 226 & 226 & 222
\end{bmatrix}_{12 * 10000}
$$

Eigen Face Analysis – [5]

- Calculate mean face

[229.66666667 228.5 228.33333333 ... 90.41666667 95.08333333 96.91666667] $_{1 * 10000}$

- Subtract mean face from each individual intensity matrix to obtain centered image matrix

[[-17.66666667 -18.5 -14.33333333 ... 52.58333333 31.91666667 24.08333333]
[-3.66666667 -3.5 -6.33333333 ... -52.41666667 -52.08333333 -42.91666667]
[15.33333333 16.5 16.66666667 ... 119.58333333 113.91666667 113.08333333]
... ...
[-15.66666667 -12.5 -18.33333333 ... -27.41666667 -25.08333333 -23.91666667]
[19.33333333 20.5 19.66666667 ... -20.41666667 -8.08333333 -26.91666667]
[13.33333333 12.5 17.66666667 ... -2.41666667 -1.08333333 1.08333333]] $_{12 * 10000}$

Eigen Face Analysis – [6]

- Calculate the covariance of centered image

[[291.15151515 306.45454545 317.3030303 ... 689.78787879 763.84848485 737.6969697]
[306.45454545 327. 335.63636364 ... 724.22727273 799.59090909 767.5]
[317.3030303 335.63636364 352.42424242 ... 779.57575758 847.78787879 811.93939394]
... ...
[689.78787879 724.22727273 779.57575758 ... 4871.17424242 4681.23484848 4629.31060606]
[763.84848485 799.59090909 847.78787879 ... 4681.23484848 4596.4469697 4572.28030303]
[737.6969697 767.5 811.93939394 ... 4629.31060606 4572.28030303 4667.53787879]]$_{10000 * 10000}$

Eigen Face Analysis – [7]

- Perform Eigen value decomposition to obtain Eigen value and Eigen vector of covariance matrix
- Eigen Values are

[-5.60506474e-09 -4.95808563e-09 -3.76855834e-09 ... 3.36601394e+06 4.92132931e+06 9.37569671e+06] $_{1 * 10000}$

- Eigen Vectors are

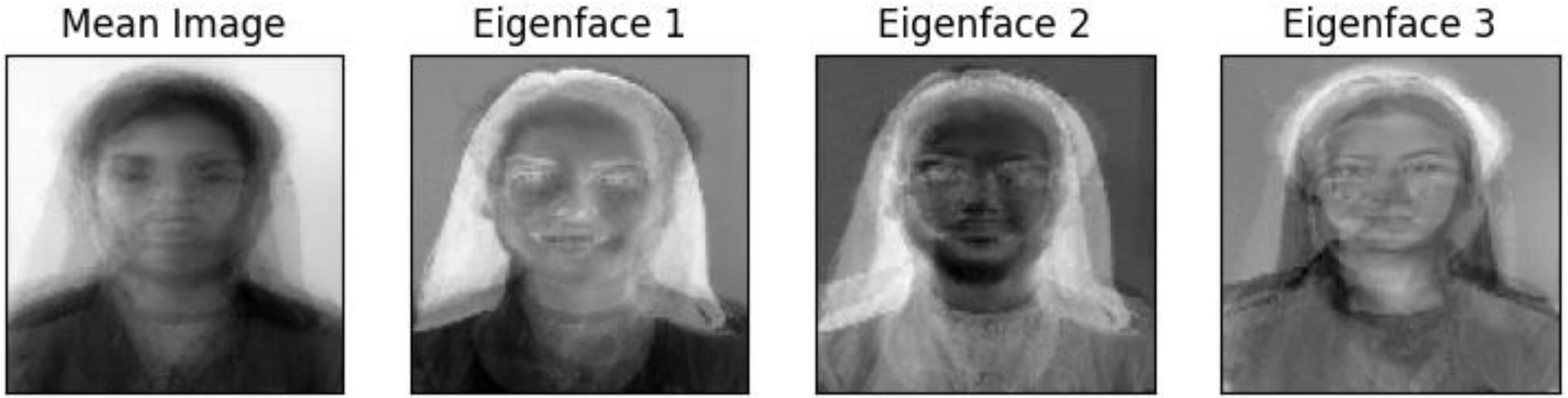
[[0. 0.00275094 0. ... 0.00662084 -0.00173115 -0.00254285]
[0.08913935 -0.35208869 0.01848966 ... 0.00701342 -0.00188465 -0.00265253]
[-0.40114349 -0.62001508 0.05351701 ... 0.00744324 -0.00133498 -0.00284689]
... ...
[-0.02194203 0.00078267 -0.03688168 ... 0.01411051 -0.007828 -0.01716053]
[0.00486178 0.01442121 0.03503409 ... 0.0150932 -0.00738104 -0.01690141]
[0.02878903 0.01854891 0.03101356 ... 0.01446582 -0.00572168 -0.01733519]] $_{10000 * 10000}$

Eigen Face Analysis – [8]

- Arrange Eigen values and Eigen vectors
- Select a subset of top '*K*' Eigen values
  - Choosing *K* = 3, 8, 10, 12
  - Images are reconstructed
  - Mean square Error, PSNR(Peak Signal-to-Noise Ratio) and SSI(Structural Similarity Index) are calculated

Eigen Face Analysis – [9]

- Project mean centered face to subspace made by the '*K*' Eigen vector to obtain image after plotting
- Top 3 Eigen face and mean image



Eigen Face Analysis – [10]

- Eigen Face Reconstruction
  - Preprocess all the facial image
  - Calculate mean face
  - Calculate covariance matrix
  - Perform Eigen value decomposition
  - Select '*K*' number of Principal Components (Eigenfaces)
  - Project the mean-centered face vector onto selected eigenfaces

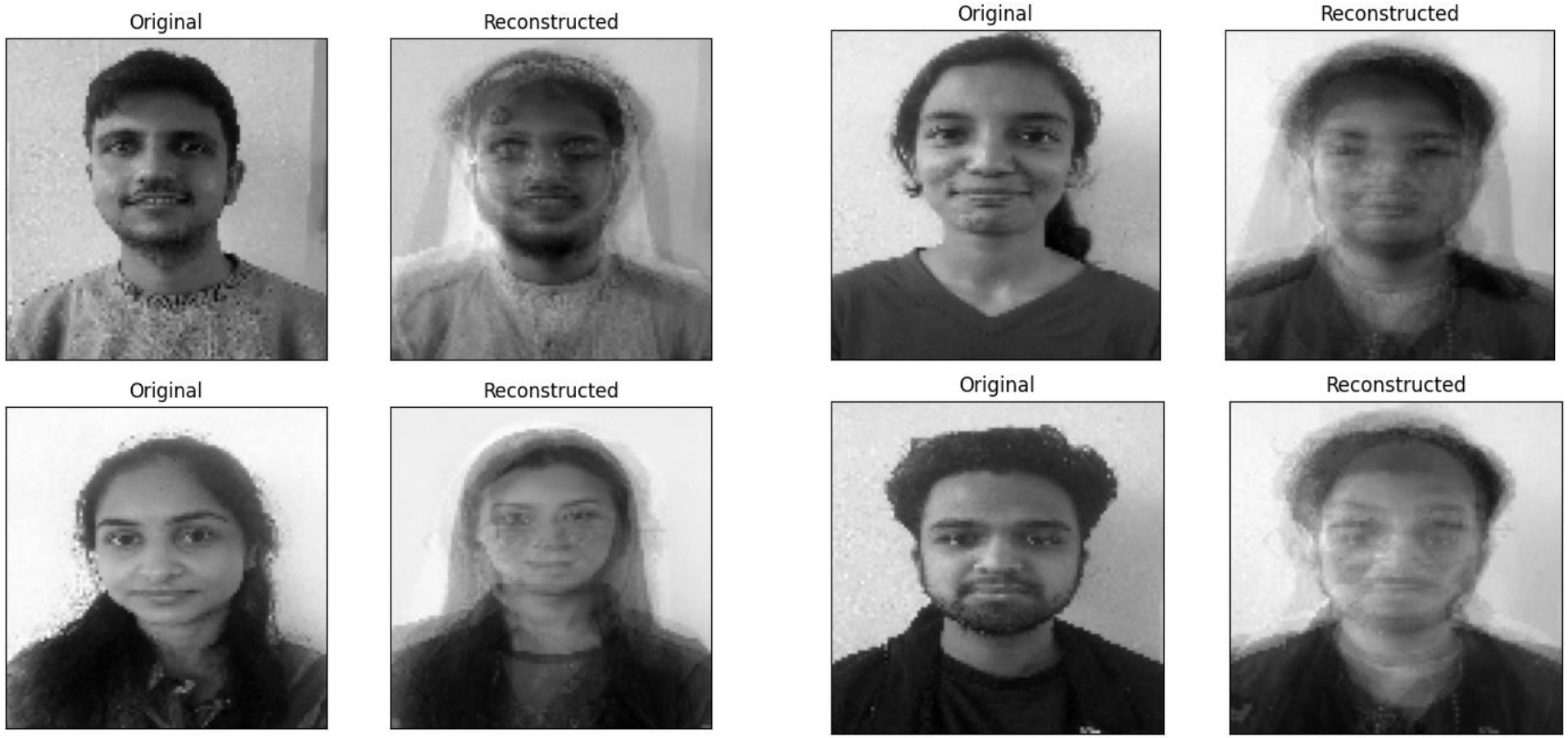
    $$Projection = \sum_{i=1}^{k} \alpha_i . u_i$$

    where α_i are the projection coefficients and u_i are the selected eigenfaces
  - Reconstruct face by projected face to mean face

    Reconstructed Face=Mean Face + Projection

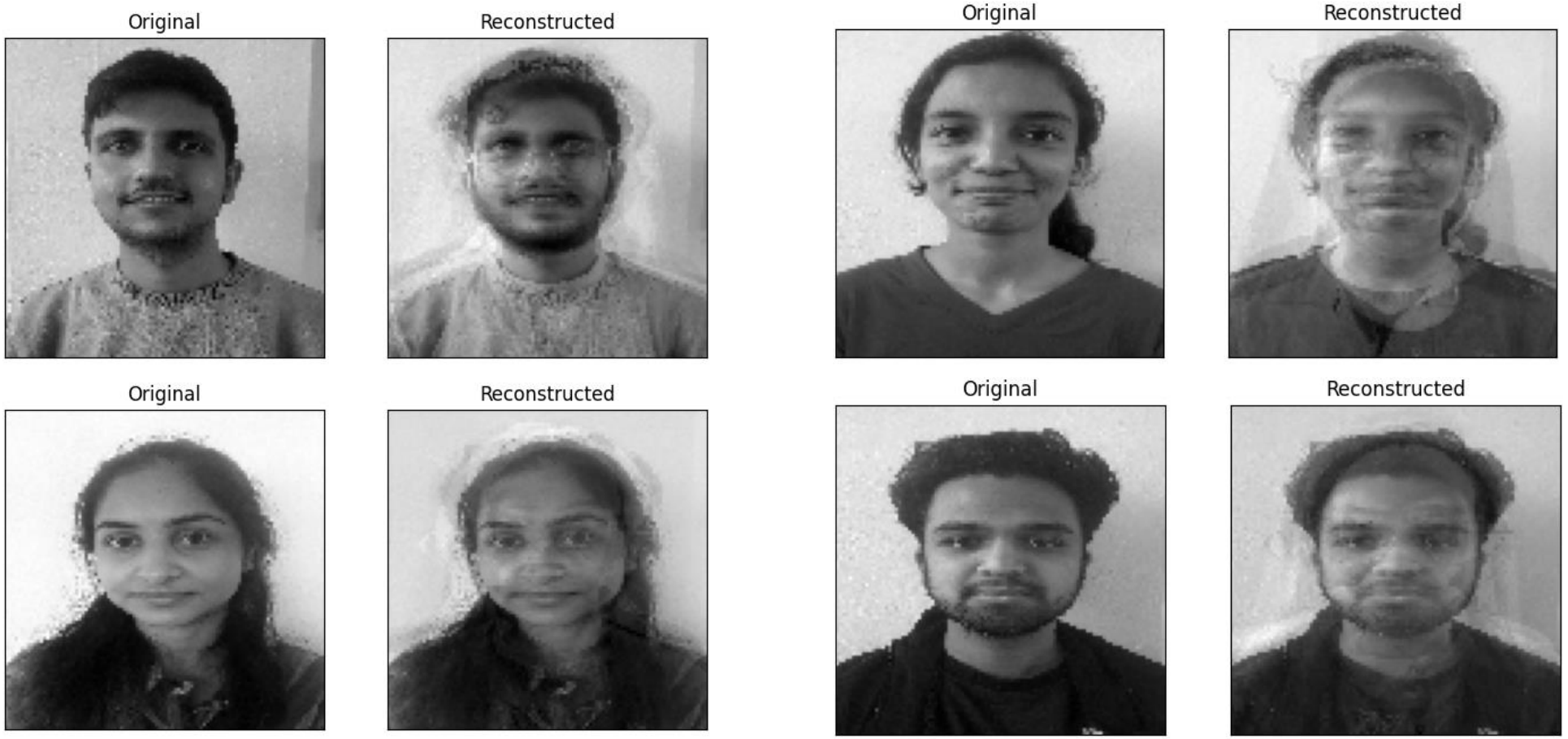
Eigen Face Analysis – [11]

- Image reconstruction for $K = 3$



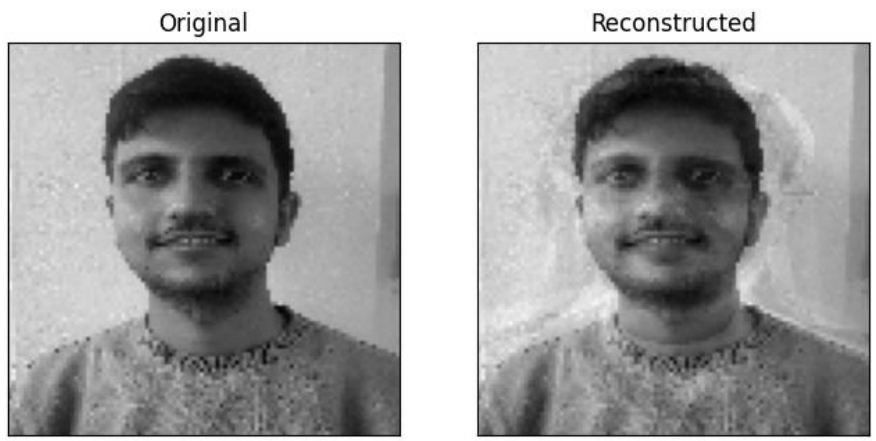
Eigen Face Analysis – [12]

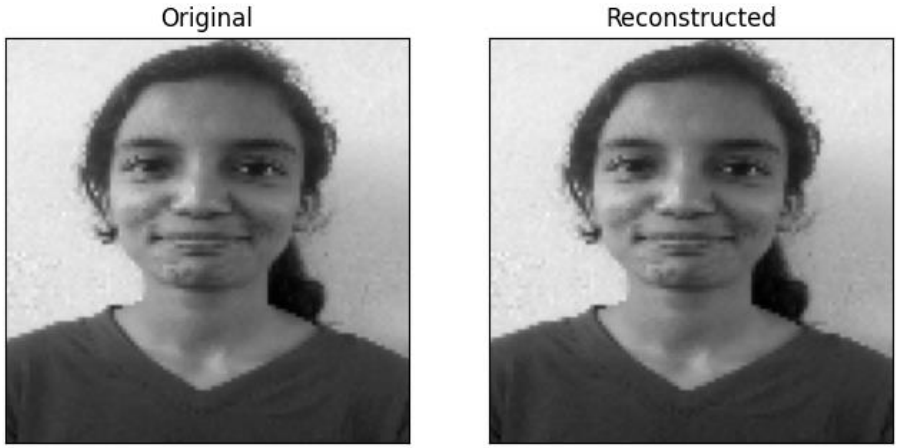
- Image reconstruction for $K = 8$

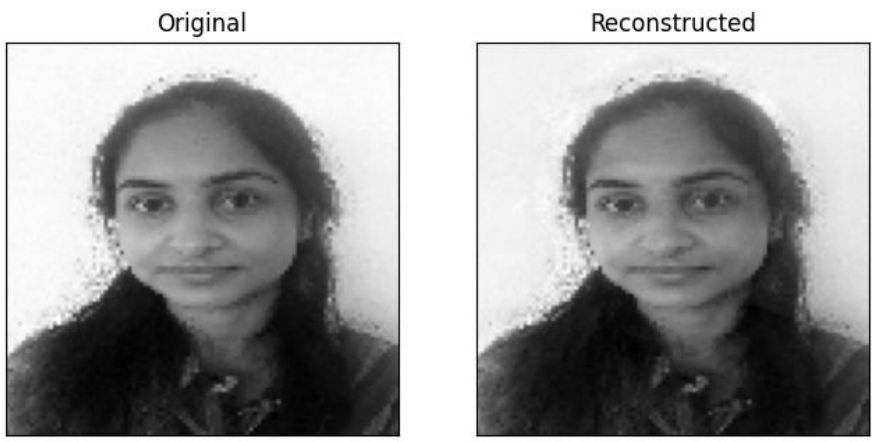


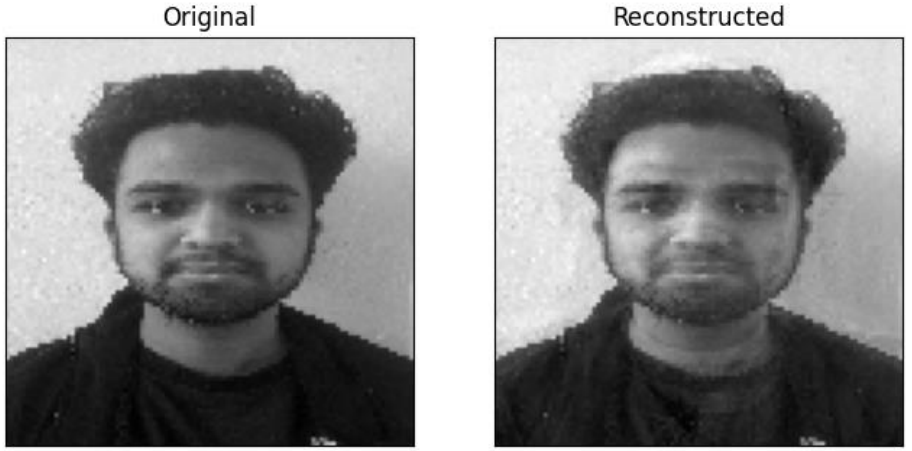
Eigen Face Analysis – [13]

- Image reconstruction for *K = 10*



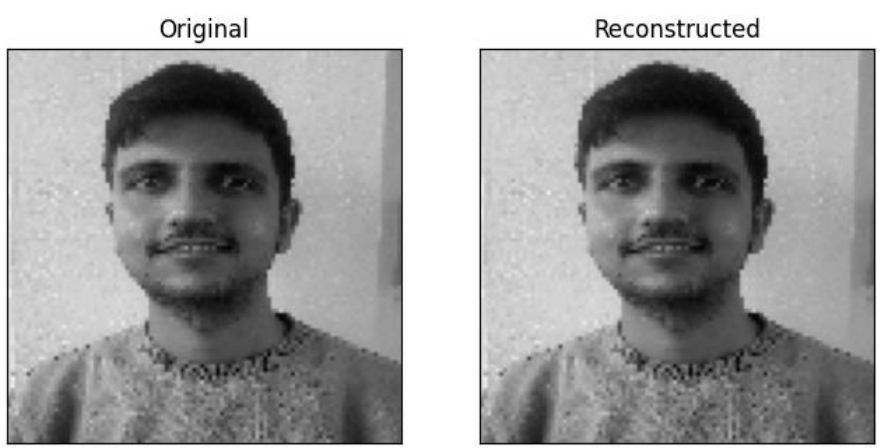


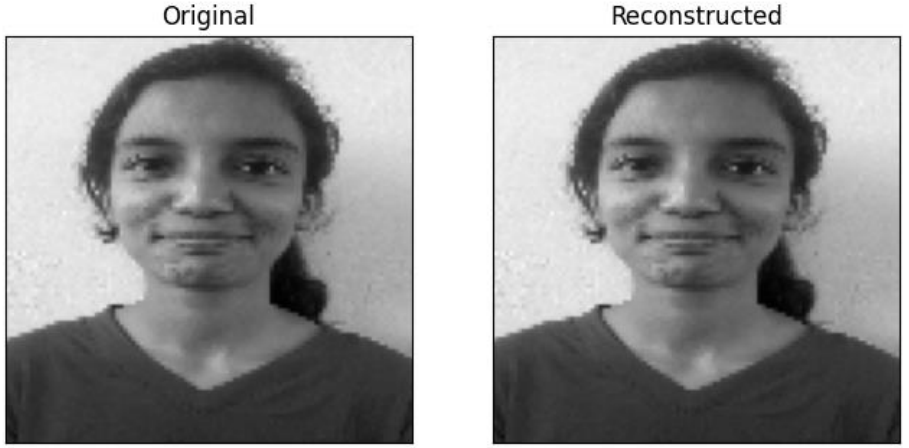


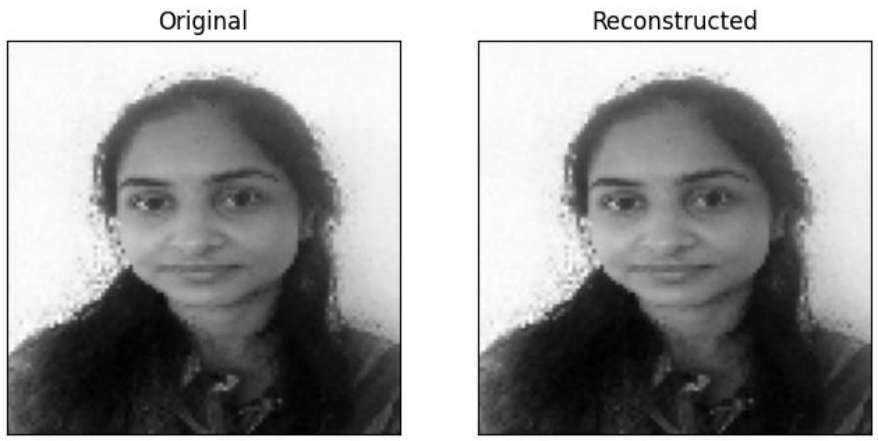


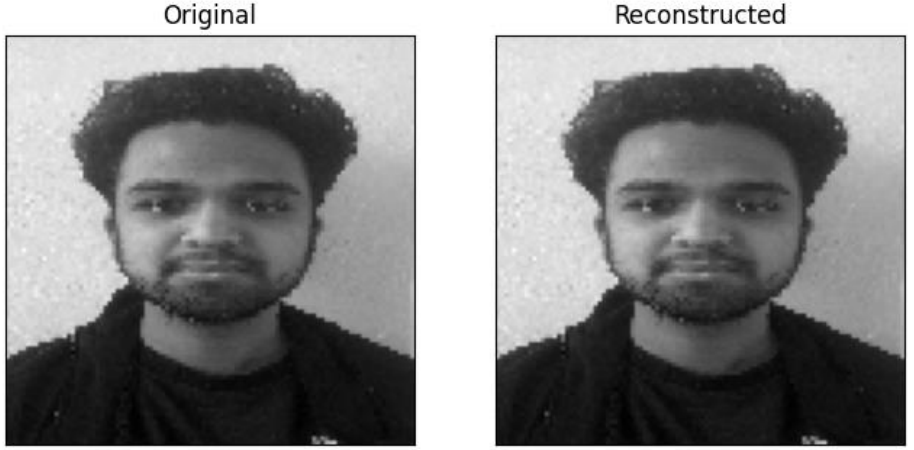
Eigen Face Analysis – [14]

- Image reconstruction for *K = 12*









Eigen Face Analysis – [15]

- After observing the above figures, we can see that:
  - At K = 3 images can not be distinguished
  - At K = 8 few images can not be properly distinguished
  - At K = 10 all images can be distinguished
  - At K = 12 all images are reconstructed same as original images
- Hence, the minimum number of Principal Components required to reconstruct image clearly is 10

Eigen Face Analysis – [16]

- Mean square Error(MSE), PSNR and SSI for $K = 3$

  Image 1: MSE = 467.9855, PSNR = 21.4285, SSIM = 0.6048
  Image 2: MSE = 620.4187, PSNR = 20.2040, SSIM = 0.5833
  Image 3: MSE = 850.7383, PSNR = 18.8328, SSIM = 0.6749
  Image 4: MSE = 154.0467, PSNR = 26.2543, SSIM = 0.8928
  Image 5: MSE = 620.5678, PSNR = 20.2029, SSIM = 0.6390
  Image 6: MSE = 1000.8754, PSNR = 18.1270, SSIM = 0.5386
  Image 7: MSE = 890.1966, PSNR = 18.6359, SSIM = 0.6003
  Image 8: MSE = 556.4168, PSNR = 20.6768, SSIM = 0.6475
  Image 9: MSE = 445.8753, PSNR = 21.6387, SSIM = 0.6875
  Image 10: MSE = 647.8260, PSNR = 20.0162, SSIM = 0.5252
  Image 11: MSE = 709.9671, PSNR = 19.6184, SSIM = 0.6411
  Image 12: MSE = 575.8585, PSNR = 20.5276, SSIM = 0.5894

Eigen Face Analysis – [17]

- Mean square Error(MSE), PSNR and SSI for $K = 8$

  Image 1: MSE = 266.7984, PSNR = 23.8690, SSIM = 0.7275
  Image 2: MSE = 110.4578, PSNR = 27.6988, SSIM = 0.8740
  Image 3: MSE = 18.1659, PSNR = 35.5382, SSIM = 0.9679
  Image 4: MSE = 10.5842, PSNR = 37.8842, SSIM = 0.9838
  Image 5: MSE = 123.4439, PSNR = 27.2161, SSIM = 0.8368
  Image 6: MSE = 100.5762, PSNR = 28.1058, SSIM = 0.8773
  Image 7: MSE = 16.6731, PSNR = 35.9106, SSIM = 0.9703
  Image 8: MSE = 253.1436, PSNR = 24.0971, SSIM = 0.7821
  Image 9: MSE = 171.7263, PSNR = 25.7824, SSIM = 0.8335
  Image 10: MSE = 305.1358, PSNR = 23.2859, SSIM = 0.7188
  Image 11: MSE = 96.1071, PSNR = 28.3032, SSIM = 0.8723
  Image 12: MSE = 160.1067, PSNR = 26.0867, SSIM = 0.7561

Eigen Face Analysis – [18]

- Mean square Error(MSE), PSNR and SSI for $K = 10$

  Image 1: MSE = 80.0341, PSNR = 29.0981, SSIM = 0.8739
  Image 2: MSE = 75.2046, PSNR = 29.3684, SSIM = 0.9002
  Image 3: MSE = 13.2418, PSNR = 36.9113, SSIM = 0.9757
  Image 4: MSE = 0.9982, PSNR = 48.1384, SSIM = 0.9981
  Image 5: MSE = 49.8841, PSNR = 31.1512, SSIM = 0.9089
  Image 6: MSE = 27.8471, PSNR = 33.6830, SSIM = 0.9374
  Image 7: MSE = 0.0057, PSNR = 70.6059, SSIM = 1.0000
  Image 8: MSE = 77.1350, PSNR = 29.2583, SSIM = 0.8763
  Image 9: MSE = 99.6227, PSNR = 28.1472, SSIM = 0.8720
  Image 10: MSE = 0.5506, PSNR = 50.7224, SSIM = 0.9988
  Image 11: MSE = 12.1076, PSNR = 37.3002, SSIM = 0.9731
  Image 12: MSE = 18.7011, PSNR = 35.4121, SSIM = 0.9341

Eigen Face Analysis – [19]

- Mean square Error(MSE), PSNR and SSI for $K = 12$

  Image 1: MSE = 0.0000, PSNR = 312.4213, SSIM = 1.0000
  Image 2: MSE = 0.0000, PSNR = 315.8577, SSIM = 1.0000
  Image 3: MSE = 0.0000, PSNR = 309.7421, SSIM = 1.0000
  Image 4: MSE = 0.0000, PSNR = 308.4583, SSIM = 1.0000
  Image 5: MSE = 0.0000, PSNR = 310.9190, SSIM = 1.0000
  Image 6: MSE = 0.0000, PSNR = 309.8100, SSIM = 1.0000
  Image 7: MSE = 0.0000, PSNR = 309.9213, SSIM = 1.0000
  Image 8: MSE = 0.0000, PSNR = 311.4866, SSIM = 1.0000
  Image 9: MSE = 0.0000, PSNR = 309.9119, SSIM = 1.0000
  Image 10: MSE = 0.0000, PSNR = 310.9385, SSIM = 1.0000
  Image 11: MSE = 0.0000, PSNR = 311.0443, SSIM = 1.0000
  Image 12: MSE = 0.0000, PSNR = 311.6517, SSIM = 1.0000