Least Cost Influence Maximization Across Multiple Social Networks

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***Abstract*—Recently, in online social networks (OSNs), the *least cost influence* (LCI) problem has become one of the central research topics. It aims at identifying a minimum number of seed users who can trigger a wide cascade of information propagation. Most of existing literature investigated the LCI problem only based on an individual network. However, nowadays users often join several OSNs such that information could be spread across different networks simultaneously. Therefore, in order to obtain the best set of seed users, it is crucial to consider the role of overlapping users under this circumstances. In this article, we propose a unified framework to represent and analyze the influence diffusion in multiplex networks. More specifically, we tackle the LCI problem by mapping a set of networks into a single one via lossless and lossy coupling schemes. The lossless coupling scheme preserves all properties of original networks to achieve high-quality solutions, while the lossy coupling scheme offers an attractive alternative when the running time and memory consumption are of primary concern. Various experiments conducted on both real and synthesized datasets have validated the effectiveness of the coupling schemes, which also provide some interesting insights into the process of influence propagation in multiplex networks.**

***Index Terms*—Coupling, influence propagation, multiple networks, online social networks (OSNs).**

I. Introduction

In the recent decade, the popularity of online social networks, such as Facebook, Google+, Myspace and Twitter etc., has created a new major communication medium and formed a promising landscape for information sharing and discovery. On average [1], Facebook users spend 7 h and 45 min per person per month on interacting with their friends; 3.2 billion likes and comments are posted every day on Facebook; 340 million tweets are sent out everyday on Twitter. Such engagement of online users fertilizes the land for information propagation to a degree which has never been achieved before in the mass media. More importantly, online social networks (OSNs) also inherit one of the major properties of real social networks—the word-of-mouth effect, in which personal opinion or decision can be reshaped or reformed through influence from friends and colleagues. Recently, motivated by the significant effect of viral marketing, OSNs have been the most attractive platforms to increase brand awareness of new products as well as strengthen the relationship between customers and companies. In general, the ultimate goal is to find the least advertising cost set of users which can trigger a massive influence.

Along with the fast development of all existing OSNs, there have been quite a number of users who maintain several accounts simultaneously, which allow them to propagate information across different networks. For example, Jack, a user of both Twitter and Facebook, learned of a new book from Twitter. After reading it, he found it very interesting and shared this book with friends in Facebook as well as Twitter. This can be done by configuring both of the accounts to allow automatically posting across different social networks. As a consequence, the product information is exposed to his friends and further spreads out on both networks. If we only focus on an individual network, the spread of the information is estimated inaccurately. Therefore considering the influence only in one network fails to identify the most influential users, which motivates us to study the problem in multiplex networks where the influence of users is evaluated based on all OSNs in which they participate.

Related works. Nearly all of the existing works studied different variants of the least cost influence problem on a single network. Kempe *et al.* [2] first formulated the influence maximization problem which asks to find a set of k users who can maximize the influence. The influence is propagated based on a stochastic process called Independent Cascade Model (IC) in which a user will influence his friends with probability proportional to the strength of their friendship. The author proved that the problem is NP-hard and proposed a greedy algorithm with approximation ratio of $(1 - 1/e)$. After that, a considerable number of works studied and designed new algorithms for the problem variants on the same or extended models such as [3]–[6]. There are also works on the linear threshold (LT) model for influence propagation in which a user will adopt the new product when the total influence of his friends surpass some threshold. Dinh *et al.* [7] proved the inapproximability as well as proposed efficient algorithms for this problem on a special case of LT model. In their model, the influence between users is uniform and a user is influenced if a certain fraction ρ of his friends are active.

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Recently, researchers have started to explore multiplex networks with works of Yagan *et al.* [8] and Liu *et al.* [9], which studied the connection between offline and online networks. The first work investigated the outbreak of information using the SIR model on random networks. The second one analyzed networks formed by online interactions and offline events. The authors focused on understanding the flow of information and network clustering but not solving the least cost influence problem. Additionally, these works did not study any specific optimization problem of viral marketing. Shen *et al.* [5] explored the information propagation in multiplex online social networks taking into account the interest and engagement of users. The authors combined all networks into one network by representing an overlapping user as a super node. This method cannot preserve the individual networks' properties.

In this article, we study the least cost influence (LCI) problem which aims at finding a set of users with minimum cardinality to influence a certain fraction of users in multiplex networks. Due to the complex diffusion process in multiplex networks, it is difficult to develop the solution by directly extending previous solutions in a single network. Additionally, studying the problem in multiplex networks introduces several new challenges: 1) how to accurately evaluate the influence of overlapping users; 2) in which network, a user is easier to be influenced; and 3) which network propagates the influence better. To answer these questions, we first introduce a model representation to illustrate how information propagate in multiplex networks via coupling schemes. By mapping multiple networks into one network, different coupling schemes can preserve partial or full properties of the original networks. After that, we can exploit existing solutions on a single network to solve the problem in multiplex networks. Moreover, through comprehensive experiments, we have validated the effectiveness of the coupling schemes, and also provide some interesting insights into the process of influence propagation in multiplex networks. Our main contributions are summarized as follows.

- We propose a model representation via various coupling schemes to reduce the problem in multiplex networks to an equivalent problem on a single network. The proposed coupling schemes can be applied for most popular diffusion models including: linear threshold model, stochastic threshold model and independent cascading model.
- We provide a scalable greedy algorithm to solve the LCI problem. In particular, the improvement factor scales up with the size of the network which allows the algorithm to run on very large networks with millions of nodes.
- We conduct extensive experiments on both real and synthesized datasets. The results show that considering multiplex networks instead of a single network can effectively choose the most influential users.

The remainder of this paper is organized as follows. In Section II, we present the influence propagation model in multiplex networks and define the problem. The lossless and lossy coupling schemes are introduced in Sections III and IV. A scalable greedy algorithm is proposed in Section V. Section VI shows the experimental results on the performance of different algorithms and coupling schemes. Finally, Section VII concludes the paper.

II. Model and Problem Definition

A. Graph Notations

We consider k networks $G^1, G^2, \ldots, G^k$, each of which is modeled as a weighted directed graph $G^i = (V^i, E^i, \theta^i, W^i)$. The vertex set $V^i = \{u'\text{s}\}$ represents the participation of $n^i = |V^i|$ users in the network G^i, and the edge set $E^i = \{(u,v)'\text{s}\}$ contains $m^i = |E^i|$ oriented connections (e.g., friendships or relationships) among network users. $W^i = \{w^i(u,v)'\text{s}\}$ is the (normalized) weight function associated to all edges in the ith network. In our model, weight $w^i(u,v)$ can also interpreted as the strength of influence (or the strength of the relationship) a user u has on another user v in the ith network. The sets of incoming and outgoing neighbors of vertex u in network G^i are denoted by N_u^{i-} and N_u^{i+}, respectively. In addition, each user u is associated with a threshold $\theta^i(u)$ indicating the persistence of his opinions. The higher $\theta^i(u)$ is, the more unlikely that u will be influenced by the opinions of his friends. Furthermore, the users that actively participate in multiple networks are referred to as *overlapping users* and can be identified using methods in [10] and [11] (note that identifying overlapping users is not the focus of this paper). Those users are considered as bridge users for information propagation across networks. Finally, we denote by $G^{1\ldots k}$ the system consisting of k networks and by U the exhaustive set of all users $U = \cup_{i=1}^k V^i$.

B. Influence Propagation Model

We first describe the LT model [7], which is a popular model for studying information and influence diffusion in a single network, and then discuss how the LT model can be extended to cope with multiplex networks. In the classic LT model, each node u can be either *active* or *inactive*: u is in an *active* state if it is selected into the seed set, or the total influence from the in-degree neighbors exceeds its threshold $\theta(u)$, i.e., $\sum_{v \in N(u)} w(v,u) \geq \theta(u)$. Otherwise, u is in an *inactive* state.

In multiplex network system, given a number of k networks, the information is propagated separately in each network and can only flows to other networks via the overlapping users. The information starts to spread out from a set of seed users S i.e., all users in S are active and the remaining users are inactive. At time t, a user u becomes active if the total influence from its active neighbors surpasses its threshold in some network, i.e., there exists i such that

$$\sum_{v \in N_u^{i-}, v \in A} w^i(v,u) \geq \theta^i(u)$$

where A is the set of active users after time $(t-1)$.

In each time step, some inactive users become activated and try to influence other users in the next time step. The process terminates until no more inactive users can be activated. If we limit the propagation time to d, then the process will stop after $t = d$ time steps. The set of active users in time d is denoted as $A^d(G^{1\ldots k}, S)$. Note that d is also the number of hops up to which the influence can be propagated from the seed set, so d is called the number of propagation hops.

C. Problem Definition

In this paper, we address the fundamental problem of viral marketing in multiplex networks: the LCI problem. The problem asks to find a seed set of minimum cardinality which influences a large fraction of users.

Definition 1 (LCI Problem): Given a system of k networks $G^{1\ldots k}$ with the set of users U, a positive integer d, and $0 < \beta \leq 1$, the LCI problem asks to find a seed set $S \subset U$ of minimum cardinality such that the number of active users after d hops according to the LT model is at least β fraction of users, i.e., $|A^d(G^{1\ldots k}, S)| \geq \beta |U|$.

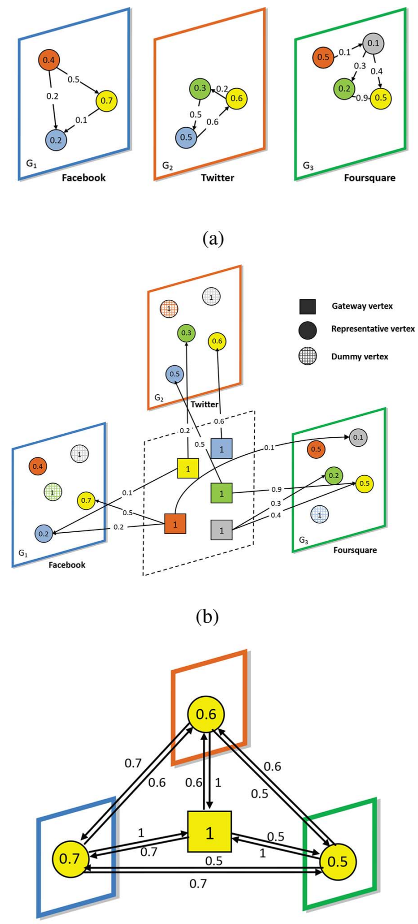
When $k = 1$, we have the variant of the problem on a single network which is NP-hard to solve [12], Dinh *et al.* [7] proved the inapproximability and proposed an algorithm for a special case when the influence between users is uniform and a user is activated if a certain fraction ρ of his friends are active. In the following sections, we will present different coupling strategies to reduce the problem in multiplex networks to the problem in a single network in order to utilize the algorithm design.

III. Lossless Coupling Schemes

Here, we present the lossless scheme to couple multiple networks into a new single network with respect to the influence diffusion process on each participant network. A notable advantage of this newly coupled graph is that we can use any existing algorithm on a single network to produce the solution in multiplex networks with the same quality.

A. Clique Lossless Coupling Scheme

In the LT model, the first issue is solved by introducing dummy nodes for each user u in networks to which it does not belong. These dummy nodes are isolated. Now, the vertex set V^i of ith network can be represented by $V^i = \{u_1^i, u_2^i, \ldots, u_n^i\}$, where $U = \{u_1, u_2, \ldots, u_n\}$ is the set of all users. u_p^i is called the *representative vertex* of u_p in network G^i. In the new representation, there is an edge from u_p^i to u_q^i if u_p and u_q are connected in G^i. Now we can union all k networks to form a new network G. The approach to overcome the second challenge is to allow nodes $u^1, u^2, \ldots, u^k$ of a user u to influence each other, e.g., adding edge (u^i, u^j) with weight $\theta(u^j)$. When u^i is influenced, u^j is also influenced in the next time step as they are actually a single overlapping user u, thus the information is transferred from network G^i to G^j. Howeverk an emerged problem is that the information is delayed when it is transferred between two networks. Right after being activated, u^i will influence its neighbors while u^j needs one more time step before it starts to influence its neighbors. It would be better if both u^i and u^j start to influence their neighbors in the same time. For this reason, new *gateway vertex* u^0 is added to G such that both u^i and u^j can only influence other vertices through u^0. In particular, all edges (u^i, v^i) $((u^j, z^j))$ will be replaced by edges (u^0, v^i) $((u^0, z^j))$. In addition, more edges are added between u^0, u^i, and u^j to let them influence each other, since the connection between gateway and representative vertices of the same user forms a clique, so we call it clique lossless coupling scheme. After forming the topology of the coupled network, we assign edge weights and vertex thresholds as following.



(a) (b) (c)

Fig. 1. Example of the *clique lossless coupling scheme*. (a). An instance of multiplex networks with 4 users. Each user is represented by vertices of the same color with different thresholds in different networks e.g., green user has thresholds of 0.3 and 0.2 in G^2 and G^3. (b). The influence of gateway vertices on representative vertices represent the influence between users in multiplex networks. (c). The connection between gateway and representative vertices of the same user.

Vertex thresholds. All dummy vertices and gateway vertices have the threshold of 1. Any remaining representative vertex u_p^i has the same threshold as u_p in G^i, i.e., $\theta(u_p^i) = \theta^i(u_p)$.

Edge weights. If there is an edge between user u and v in G^i, then the edge (u^0, v^i) has weight $w(u^0, v^i) = w^i(u, v)$. The edges between gateway and representative vertices of the same user u are assigned as $w(u^i, u^j) = \theta(u^j), \forall\, 0 \leq i, j \leq k, i \neq j$ to synchronize their state together.

A simple example of the clique lossless coupling scheme is illustrated in Fig. 1.

Next, we will show that the propagation process in the original multiplex networks and the coupled network is actually the same. Influence is alternatively propagated between gateway and representative vertices, so the problem with d hops in the multiplex networks is equivalent to the problem with $2d$ hops in the coupled network.

Lemma 1: Suppose that the propagation process in the coupled network G starts from the seed set which contains only gateway vertices $S = \{s_1^0, \ldots, s_p^0\}$, then representative vertices are activated only at even propagation hops.

Proof: Suppose that a gateway vertex u^0 is the first gateway vertex that is activated at the odd hops $2d + 1$. u^0 must be activated by some vertex u^i and u^i is the first activated vertex among vertices $u^1, u^2, \ldots, u^k$. It means that u^i is activated in hop $2d$. Since all incoming neighbors of u^i are gateway vertices, some gateway vertex becomes active in hop $2d - 1$ (contradiction). □

Lemma 2: Suppose that the propagation process on $G^{1\ldots k}$ and G starts from the same seed set S, then the following conditions are equivalent.

1) User u is active after d propagation hops in $G^{1\ldots k}$.
2) There exists i such that u^i is active after $2d-1$ propagation hops in G.
3) Vertex u^0 is active after $2d$ propagation hops in G.

Proof: We will prove this lemma by induction. Supposing it is correct for any $1 \le d \le t$, we need to prove it is correct for $d = t + 1$. Denote $A^{1\ldots k}(t)$ and $A(t)$ as the set of active users and active vertices after t propagation hops in $G^{1\ldots k}$ and G, respectively.

(1) $\Rightarrow$ (2): If user u is active at time $t + 1$ in $G^{1\ldots k}$, it must be activated in some network G^j. We have

$$\sum_{v \in N_u^{j-} \cap A^{1\ldots k}(t)} w^j(v, u) \ge \theta^j(u)$$

Due to the induction assumption, for each $v \in A^{1\ldots k}(t)$, we also have $v^0 \in A(2t)$ in G. Thus

$$\sum_{v^0 \in N_{u^j}^- \cap A(2t)} w(v^0, u^j) = \sum_{v \in N_u^{j-} \cap A^{1\ldots k}(t)} w^j(v, u) \ge \theta^j(u) = \theta(u^j).$$

This means that u^j is active after $(2(t + 1) - 1)$ propagation hops.

(2) $\Rightarrow$ (3): If there exists i such that u^i is active after $2(t + 1) - 1$ propagation hops on G, then u^i will activate u^0 in hop $2(t + 1)$

(3) $\Rightarrow$ (1): Suppose that $u^0 \notin S$ is active after $2(t + 1)$ propagation hops in G, then there exists u^j which activates u^0 before. This is equivalent to

$$\sum_{v \in N_{u^j}^-, v \in A(2t)} w(v, u^j) \ge \theta(u^j).$$

For each $v \in A(2t)$, we also have $v \in A^{1\ldots k}(t)$. Substituting this into the above inequality, we have,

$$\sum_{v \in N_u^{j-} \cap A^{1\ldots k}(t)} w^j(v, u) = \sum_{v^0 \in N_{u^j}^- \cap A(2t)} w(v^0, u^j) \ge \theta(u^j) = \theta^j(u).$$

Thus, u is active in network G^j after $t + 1$ hops. □

Next, we will show that the number of influenced vertices in the coupled network is $(k + 1)$ times the number of influenced users in multiplex networks as stated in Theorem 1.

Theorem 1: Given a system of k networks $G^{1\ldots k}$ with the user set U, the coupled network G produced by the lossless coupling scheme, and a seed set $S = \{s_1, s_2, \ldots, s_p\}$, if $A^d(G^{1\ldots k}, S) = \{a_1, a_2, \ldots, a_q\}$ is the set of active users caused by S after d propagation hops in multiplex networks, then $A^{2d}(G, S) = \{a_1^0, a_1^1, \ldots, a_1^k, \ldots, a_q^0, a_q^1, \ldots, a_q^k\}$ is the set of active vertices caused by S after $2d$ propagation hops in the coupled network.

Proof: For each user $a_i \in A^d(G^{1\ldots k}, S)$ i.e., a_i is active after d hops in $G^{1\ldots k}$, then there exists a_i^j which is active after $2d - 1$ hops in G according to the Lemma 2. As a result, all $a_i^0, a_i^1, \ldots, a_i^k$ are active after $2d$ hops. So, $B = \{a_1^0, a_1^1, \ldots, a_1^k, \ldots, a_q^0, a_q^1, \ldots, a_q^k\} \subseteq A^{2d}(G, S)$.

Let consider a vertex of $A^{2d}(G, S)$ which is given by the following.

Case 1. A gateway vertex u^0 is active after $2d$ hops in G, so vertex u must be active after d hops in $G^{1\ldots k}$. This implies $u \in A^d(G^{1\ldots k}, S)$, thus $u^0 \in B$.

Case 2. A representative vertex u^i. If u^i is active after $2d-1$ hops, then u must be active after d hops due to Lemma 2, thus $u \in A^d(G^{1\ldots k}, S)$. Otherwise, u^i is activated at hop $2d$, it must be activated by some vertex u^j, $j > 0$ since all gateway vertices only change their state at even hops. Again, $u \in A^d(G^{1\ldots k}, S)$. This results in $u^i \in B$.

From the two above cases, we also have $A^{2d}(G, S) \subseteq B$. Because $A^{2d}(G, S) = B$, the proof is completed. □

Theorem 1 provides the basis to derive the solution for LCI in multiplex networks from the solution on a single network. It implies an important algorithmic property of the *lossless coupling scheme* regarding the relationship between the solutions of LCI in $G^{1\ldots k}$ and G. The equivalence of two solutions is stated below.

Theorem 2: When the *lossless scheme* is used, the set $S = \{s_1, s_2, \ldots, s_p\}$ influences β fraction of users in $G^{1\ldots k}$ after d propagation hops if and only if $S' = \{s_1^0, s_2^0, \ldots, s_p^0\}$ influences β fraction of vertices in coupled network G after $2d$ propagation hops.

Size of the coupled network. Each user u has $k + 1$ corresponding vertices $u^0, u^1, \ldots, u^k$ in the coupled network, thus the number of vertices is $|V| = (k + 1)|U| = (k + 1)n$. The number of edges equals the total number of edges from all input networks plus the number of new edges for synchronizing. Thus, the total number of edges is $|E| = \sum_{i=1}^k |E^i| + nk(k+1)$.

B. Star Lossless Coupling Scheme

In Section III-A, we discussed the clique lossless coupling scheme, however, in this scheme, the number of edges to synchronize the state of vertices $u^0, u^1, \ldots, u^k$ is added up to $k(k + 1)$ for each user u, which results in $nk(k + 1)$ extra edges in the coupled network. In real networks, the number of edges is often linear to the number of vertices, while the number of extra edges greatly increases the size of the coupled network, especially when k is large. Therefore, we would like to design another synchronization strategy that has less additional edges.

Note that the large volume of extra edges is due to the direct synchronization between each pair of representative vertices of u in *clique lossless coupling scheme*, so we can reduce it by using indirect synchronization. In the new coupling scheme, we create one intermediate vertex u^{k+1} with

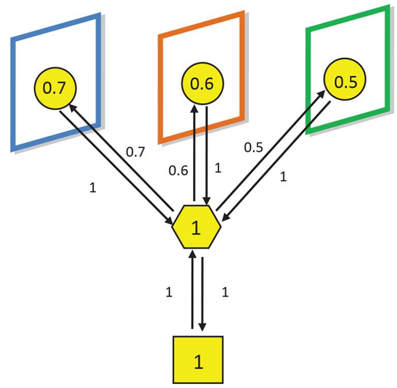


Fig. 2. Synchronization of *star lossless coupling scheme*.

threshold $\theta(u^{k+1}) = 1$ and let the active state propagate from any vertex in $u^1, u^2, \ldots, u^k$ via this vertex. Specifically, the synchronization edges are established as follows: $w(u^i, u^{k+1}) = 1$ and $w(u^{k+1}, u^i) = \theta(u^i)$ $1 \le i \le k$; $w(u^{k+1}, u^0) = w(u^0, u^{k+1}) = 1$. The synchronization strategy of *star lossless coupling scheme* is illustrated in Fig. 2. Now, the number of extra edge for each user is $2(k+1)$ and the size of the coupled network is reduced as shown in the following proposition.

Proposition 1: When the *star lossless scheme* is used, the coupled network has $|V| = (k+2)|U| = (k+2)n$ vertices and $|E| = \sum_{i=1}^{k} |E^i| + 2n(k+1)$ edges.

In *star lossless coupling scheme*, it takes two hops to synchronize the states of representative vertices of each user, which leads to delaying the propagation of influence in the coupled network. Due to the similarity between the *star lossless scheme* and the *clique lossless scheme*, we state the following property of the *star lossless scheme* without proof.

Theorem 3: When the star lossless coupling scheme is used, the set $S = \{s_1, s_2, \ldots, s_p\}$ influences β fraction of users in $G^{1\ldots k}$ after d propagation hops if and only if $S' = \{s_1^0, s_2^0, \ldots, s_p^0\}$ influences β fraction of vertices in coupled network G after $3d$ propagation hops.

C. Reduced Lossless Schemes

In all of the above coupling schemes, we create representative vertices in all networks in $G^{1\ldots k}$ to guarantee that the number of influenced vertices in the coupled network is scaled up from the number of influenced users in the original system of networks. This creates an extraordinary redundant vertices. For example, we have n users and four networks with $0.8n$, $0.6n$, $0.3n$, and $0.2n$ users, then the total number of vertices in all networks is only $1.9n$ while the number of vertices created in the *clique lossless scheme* and the *star lossless scheme* are $5n$ and $6n$, respectively. The redundant ratios of these two schemes are 260% and 315%. To overcome this redundant, we use assign weight for vertices in the coupled network and guarantee that the total weight of active vertices is scaled from the number of active users in the original system. In particular, we only create representative vertices $u^{i_1}, u^{i_2}, \ldots, u^{i_p}$ for user u where $G^{i_1}, G^{i_2}, \ldots, G^{i_p}$ are networks that u joins in. Each representative vertex is assigned weight 1, and the user vertex is assigned weight $k-p$. We have the *reduced clique lossless scheme* or the *reduced star lossless scheme* corresponding the method to synchronize the state of user and representative vertices is clique or star type. With this modification, the number of extra vertices is only n and $2n$ when clique and star synchronization is used, respectively. The number of extra edges now depends on the participants of users.

Proposition 2: When a reduced clique lossless scheme or a reduced star lossless scheme is used, the coupled network has $|V| = \sum_{i=1}^{k} |V^i| + n$ or $|V| = \sum_{i=1}^{k} |V^i| + 2n$ vertices, respectively.

The relation between the set of active vertices in a coupled network and the set of active users in original networks is similar to previous schemes. We state this relation without proof below.

Theorem 4: When the reduced clique lossless scheme (reduced star lossless scheme) is used, the set $S = \{s_1, s_2, \ldots, s_p\}$ influences β fraction of users in $G^{1\cdots k}$ after d propagation hops if and only if the total weight of active vertices caused by $S' = \{s_1^0, s_2^0, \ldots, s_p^0\}$ after $2d$ $(3d)$ hops in coupled network G is β fraction of the total weight of all vertices.

D. Extensions to Other Diffusion Models

Here, we show that we can design lossless coupling schemes for some other well-known diffusion models in each component network. As a result, top influential users can be identified under these diffusion models. In particular, we investigate two most popular stochastic diffusion models which are Stochastic Threshold and Independent Cascading models [2].

- *Stochastic Threshold model*. This model is similar to the LT model but the threshold $\theta^i(u^i)$ of each node u^i of G^i is a random value in the range $[0, \Theta^i(u^i)]$. Node u^i will be influenced when $\sum_{v^i \in N^-_{u^i}, v \in A} w^i(v^i, u^i) \ge \theta^i(u^i)$
- *Independent Cascading model*. In this model, there are only edge weights representing the influence between users. Once node u^i of G^i is influenced, it has a single chance to influence its neighbor $v^i \in N^+(u^i)$ with probability $w^i(u^i, v^i)$.

For both models, we use the same approach of using gateway vertices, representative vertices, and the synchronization edges between gateway vertices and their representative vertices. The weight of edge (u^i, u^j), $0 \le i \ne j \le k$, will be $\Theta(u^j)$ for Stochastic Threshold model and 1 for the Independent Cascading model. Once u^i is influenced, u^j will be influenced with probability 1 in the next time step. The proof for the equivalence of the coupling scheme is similar to ones for the LT model.

IV. Lossy Coupling Schemes

In the preceding coupling scheme for the LT model, a complicated coupled network is produced with large numbers of auxiliary vertices and edges. It is ideal to have a coupled network which only contains users as vertices. This network provides a compact view of the relationship between users crossing the whole system of networks. The loss of the information is unavoidable when we try to represent the information of multiplex networks in a compact single network. The goal is to design a scheme that minimizes the loss as much as possible, i.e., the solution for the problem in the coupled network is very close to one in the original system. Next, we present these schemes based on the following key observations.

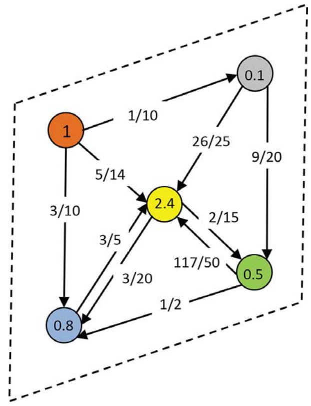


Fig. 3. Lossy coupled network using easiness parameters.

Observation 1. User u will be activated if there exists i such that: $\sum_{v \in N_u^{i-} \cap A} w^i(v,u) \geq \theta^i(u)$, where A is the set of active users. We can relax the condition to activate u with positive parameters $\alpha^1(u)$, $\alpha^2(u)$, ..., $\alpha^k(u)$, as follows:

$$\sum_{i=1}^{k} \left(\alpha^i(u) \sum_{v \in N_u^{i-} \cap A} w^i(v,u) \right) \geq \sum_{i=1}^{k} \alpha^i(u)\theta^i(u). \tag{1}$$

Note that sometimes the condition to activate u is met, but the condition (1) still needs more influence from u's friends to satisfy it. The more this need for extra influence is, the looser condition (1) is. We can reduce this redundancy by increasing the value of $\alpha^i(u)$ proportional to the value of $\sum_{v \in N_u^{i-} \cap A} w^i(v,u) - \theta^i(u)$. In the special case, if $\sum_{v \in N_u^{i-} \cap A} w^i(v,u) > \theta^i(u)$ and we choose $\alpha^i(u) \gg \alpha^j(u)$, $\forall j \neq i$, then there is no redundancy. Unfortunately, we do not know beforehand in which network user u will be activated, so we can only choose parameters heuristically.

Observation 2. When user u participates in multiple networks, it is easier to influence u in some networks more than others. The following simple case illustrates such a situation. Suppose that we have two networks. In network 1, $\theta^1(u) = 0.1$ and u has eight in-neighbors, and each neighbor v influences u with $w^1(v,u) = 0.1$. In network 2, $\theta^2(u) = 0.7$ and u has eight in-neighbors, and each neighbor v influences u with $w^2(v,u) = 0.1$. The number of active neighbors to activate u is 1 and 7 in networks 1 and 2, respectively.

Easiness. Intuitively, we can say that u is easier to be influenced in the first network. We quantify the *easiness* $\epsilon^i(u)$ that u is influenced in network i as the ratio between the total influence from friends and the threshold to be influenced: $\epsilon^i(u) = \frac{\sum_{v \in N_u^{i-}} w^i(v,u)}{\theta^i(u)}$. We can use the easiness of a user in networks as the parameters of the condition 1.

Based on above observations, we couple multiplex networks into one using parameters $\{\alpha^i(u)\}$. The vertex set is the set of users $V = \{u_1, u_2, \ldots, u_n\}$. The threshold of vertex u is set to $\theta(u) = \sum_{i=1}^{k} \alpha^i(u)\theta^i(u)$

The weight of the edge (v,u) is: $w(v,u) = \sum_{i=1}^{k} \alpha^i(u) w^i(v,u)$ where $w^i(v,u) = 0$ if there is no edge from v to u in ith network.

Then the set of edges is $E = \{(v,u) \mid w(v,u) > 0\}$. Fig. 3 illustrates the loosely coupled network of networks in Fig. 1.

Besides easiness, other metrics can be used for the same purpose. We enumerate here some other metrics.

Involvement. If a user is surrounded by a group of friends who have high influence on each other, he tends to be influenced. When a few of his friends are influenced, the whole group involving him is likely to be influenced. We estimate *involvement* of a node v in a network G^i by measuring how strongly the 1-hop neighborhood v is connected and to what extent influence can propagate from one node to another in the 1-hop neighborhood. Formally, we can define *involvement* of a node v in network G^i as: $\sigma_v^i = \sum_{x,y \in N_v^i \cup \{v\}} \frac{w^i(x,y)}{\theta_y^i}$ where $N_v^i = N_v^{i+} \cup N_v^{i-}$ is the set of all neighbors of v (both incoming and outgoing).

Average. All parameters have the same value $\alpha^i(u) = 1$

Next, we show the relationship between the solution for LCI in the lossy coupled network and the original system of networks. As discussed in the above observations, if the propagation process starts from the same set of users in $G^{1\ldots k}$ and the coupled network G, then the active state of a user in G implies its active state in $G^{1\ldots k}$. This means that, if the set of users S activates β fraction of users in G, it also activates at least β fraction of users in $G^{1\ldots k}$. It implies that if a seed set is a feasible solution in G, it is also a feasible solution in $G^{1\ldots k}$. Thus, we have the following result.

Theorem 5: When the lossy coupling scheme is used, if the set of users S activates β fraction of users in G, then it activates at least β fraction of users in $G^{1\ldots k}$.

V. Algorithms

Here, we describe a greedy algorithm and its improvement in terms of scalability in large networks. In the state of the art work, Dinh *et al.* [7] only solved the problem in a special case where the threshold is uniform and the required fraction of active nodes is the same for all nodes.

A. Improved Greedy Algorithm

Algorithm 1 Improved Greedy

Input: A system of networks $G^{1\ldots k}$, fraction β, T, R.
Output: A small seeding set S
  $G \leftarrow$ *The coupled network of* $G^{1\ldots k}$
  $C \leftarrow$ *Set of user vertices*
  $I \leftarrow \emptyset$, *Counter* $\leftarrow 0$
  Initialize a heap: $H \leftarrow \emptyset$
  for $u \in C$ **do**
    $H.push((u, f_\emptyset(u)))$
  end for
  while Number of active vertices $\leq \beta|V|$ **do**
    $Counter \leftarrow Counter + 1$
    if $Counter \,\%\, R == 0$ **then**
      Update key values of all elements in H
    else
      $A \leftarrow \emptyset$
      for $i = 1$ to T **do**
        $(u, f(u)) \leftarrow H.extract{-}max()$
      end for
      for $u \in A$ **do**

$H.push((u, f_I(u)))$
end for
end if
$(u, f(u)) \leftarrow H.extract{-}max()$
$I \leftarrow I \cup \{u\}$
end while
$S \leftarrow$ corresponded users $G^{1\ldots k}$ of nodes in I
Return S

The bottleneck of the native greedy algorithm is to identify the best node to be selected in each iteration, thus we focus on reducing the evaluating the computational cost of this step while maintaining the same quality of selected nodes. We notice that the marginal gain function $f_I(\cdot)$ is recomputed for all unselected nodes and it does not change much after a single iteration. Since we only select the best one, which indicates that nodes with higher marginal gain in previous round are necessary to be reevaluated. Therefore, we use a max heap to store the marginal gain and extra the top one to reevaluate it, if it is not of largest marginal value, it will be pushed back. Since the time to extract/push an element to the heap is $O(\log n)$, the total computation cost for each iteration is $O(T(m+n) + T \log n) = O(T(m+n))$. Normally, T is much smaller than n, so the running time is improved significantly. In addition, due to the property of the LT model, the required influence (remained threshold value after subtracting the influence of activate neighbors) to activate a node is decreasing when the seed set grows up. When a large number of nodes is selected, there are many nodes that are very easy to be activated. Thus, the marginal gain of a node can accumulate to a large value. If we just apply the proposed strategy, we will never evaluate these nodes again. Therefore, we need to do exhaustive reevaluation periodically. Combining two strategies together, we present the idea of using heavy and light iterations alternatively. In heavy iteration, we will update $f_I(\cdot)$ of all unselected nodes while only the top T nodes are reevaluated in light iteration. Since we do not want too many heavy iterations, we only use one such iteration after every R iterations. With this implementation, the running time is greatly reduced while the quality of the solution only fluctuates infinitesimally. The improved Greedy is described in Algorithm 1.

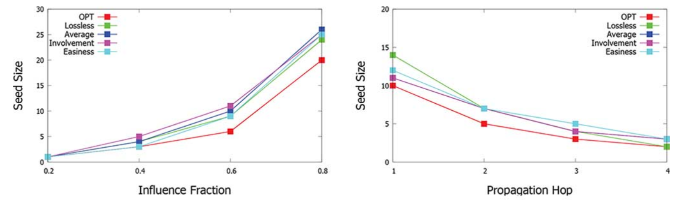
This algorithm will terminate when the number of influenced users is larger then the required fraction of total users. The complexity of this algorithm is $O((m+n)\cdot nd)$ in the worst case scenario, however, after applying the above discussed update techniques, the running time can be improved up to 700 times faster than the native greedy from experimental results.

B. Comparing With Optimal Seeding

Here, we evaluate the performance of proposed algorithm with different coupling schemes to the optimal solution. We formulate the LCI problem to a 0-1 integer linear programming (ILP) problem as follows:

$$\text{minimize} \sum_{v\in V} x_v^0$$
$$\text{subject to} \sum_{v\in V} x_v^d \geq \beta|V|$$
$$\sum_{w\in N(v)} x_u^{i-1} w_{uv} + \theta_v \cdot x_v^{i-1} \geq \theta_v \cdot x_v^i \qquad \forall v \in V, i = 1\ldots d$$
$$x_v^i \geq x_v^{i-1} \qquad \forall v \in V, i = 1\ldots d$$
$$x_v^i \in \{0,1\} \qquad \forall v \in V, i = 0\ldots d$$

where $x_v^i = 1$ if v is active in round i, otherwise, $x_v^i = 0$.



(a) (b)

Fig. 4. Seed size on synthesize network. (a) Influence fraction. (b) Propagation hop.

Since solving IP is NP-hard, we cannot run the IP on large networks. Moreover, we have to run the IP on the coupled network with the clique lossless coupling scheme, where there will be additional nodes and edges created, and the propagation hops need to be doubled (Theorem 2). Therefore, to evaluate the performance of our algorithm, we compare the result with small-size synthesize networks. For generating networks, 50 nodes are randomly chosen from a 100-users base, and the probability of connecting each pair of nodes is $p = 0.04$, which yields a coupled network of 300 users and with an expected average degree 2. Fig. 4(a) shows the obtained seed size with influence fraction from 0.2 to 0.8 under all coupling schemes. We also evaluate the impact of setting different propagation hops on seed size in Fig. 4(b) with influence fraction $\beta = 0.4$.

The optimal seeding along with the results of the improved greedy are shown in Fig. 4. As can be seen in Fig. 4(a), the seeding sizes obtained from the proposed algorithm are close to the optimal solution while varying the influenced fraction β. The same phenomenon is also shown by varying the number of propagation hops in Fig. 4(b). In particular, when the number of hops is relatively larger, the result is only one or two more than the optimal solution.

VI. Experiments

Here, we show the experimental results to compare the proposed coupling schemes and utilize these coupling schemes to analyze the influence diffusion in multiplex networks. First, we compare lossless and lossy coupling schemes to measure the tradeoff between the running time and the quality of solutions. In particular, for those different kinds of lossless coupling methods, all of them can preserve complete information of all networks. As a result, the quality of seeds are the same, the only difference would be the running time which have been theoretically proved in Propositions 1 and 2. Therefore, we only chose the clique coupling scheme to be evaluated. Second, we investigate the relationship between networks in the information diffusion to address the following questions. 1) What is the role of overlapping users in diffusing the information? 2) What do we miss when considering each network separately? 3) How and to what extent does the diffusion on one network provide a burst of information in other networks?

TABLE I
DATASETS DESCRIPTION

Networks	#Nodes	#Edges	Avg. Degree
Twitter	48277	16304712	289.7
FSQ	44992	1664402	35.99
CM	40420	175692	8.69
Het	8360	15751	1.88
NetS	1588	2742	1.73

A. Datasets

Real networks. We perform experiments on two datasets:

- *Foursquare* (*FSQ*) and *Twitter* networks [5];
- Coauthor networks in the area of Condensed Matter(CM) [13], High-Energy Theory(Het) [13], and Network Science (NetS) [14].

The statistics of those networks are described in Table I. The number of overlapping users in the first dataset FSQ-Twitter is 4100 [5]. We examine the in and out degree of overlapping nodes. For the second dataset, we match overlapping users based on authors' names. The numbers of overlapping users of the network pairs CM-Het, CM-NetS, and Het-NetS are 2860, 517, and 90, respectively. While the edge weights are provided for coauthor networks, only the topology is available for Twitter and Foursquare networks. Thus, to assign the weight of each edge, we adopt the method in [2], where each edge weight is randomly picked from 0 to 1 and then normalize it so that the total weight of incoming edges is sum up to 1 for each node. This is suitable since the influence of user u on user v tends to be small if v is under the influence of many friends. Finally, we also adopt the assignment of threshold in [2] where all thresholds are randomly chosen from 0 to 1.

Synthesized networks. We also use synthesized networks generated by Erdos–Renyi random network model [15] to test on networks with controlled parameters. There are two networks with 10 000 nodes which are formed by randomly connecting each pair of nodes with probability $p_1 = 0.0008$ and $p_2 = 0.006$. The average degrees, 8 and 60, reflect the diversity of network densities in reality. Then, we select randomly f fraction of nodes in two networks as overlapping nodes. We shall refer to $0 \leq f \leq 1$ as the *overlapping fraction*. The edge weights and node thresholds are assigned as Twitter.

Setup. We ran all of our experiments on a desktop with an Intel Xeon W350 CPU and 12 GB RAM. The number of hops is $d = 4$ and the *influenced fraction* $\beta = 0.8$, unless otherwise mentioned.

B. Comparison of Coupling Schemes

We evaluate the impact of the coupling schemes on the running time and the solution quality of the greedy algorithm to solve the LCI problem.

Solution quality. As shown in Fig. 5(a) and (b), the greedy algorithm provides larger seed sets but runs faster in lossy coupled networks than lossless coupled networks. In both Twitter-FSQ and the coauthor networks, the seed size is smallest when the lossless coupling scheme is used. It is as expected since the lossless coupling scheme reserves all the influence information which is exploited later to solve LCI. However, the seed sizes are only a bit larger using the lossy coupling schemes. In the

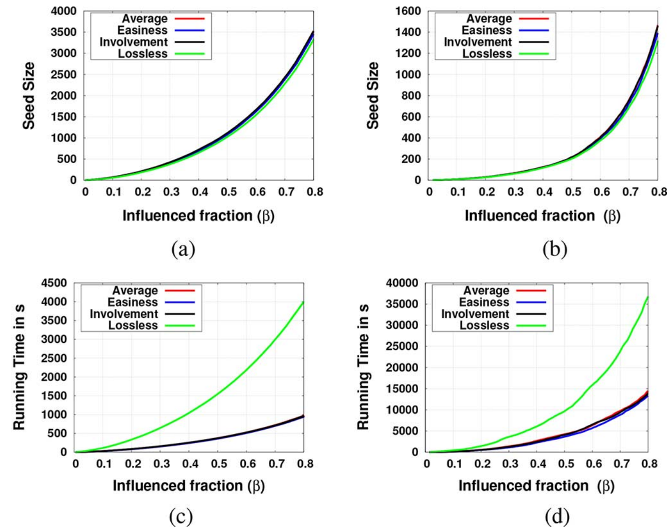


Fig. 5. Impact of coupling schemes on finding the minimum seed set. (a) Seed size: coauthor networks. (b) Seed size: FSQ-Twitter. (c). Running time: coauthor networks. (d). Running time: FSQ-Twitter.

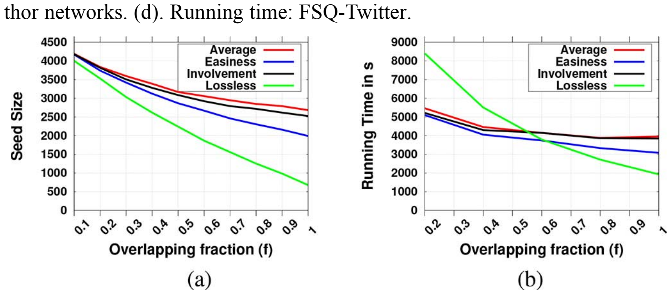


Fig. 6. Comparing coupling schemes in the synthesized networks. (a) Seed size. (b) Running time.

lossy coupling schemes, the information is only lost at overlapping users which occupy a small fraction the total number of users (roughly 5% in FSQ-Twitter and 7% in coauthor networks). Thus, the impact of the lossy coupling schemes on the solution quality is small especially when seed sets are large to influence a large fraction of users.

A closer examination reveals the relative effectiveness of the coupling methods on the seed size. That is when the seed size is significantly small, the lossess coupling outperforms all the lossy methods. For example, when the overlapping fraction $f = 0.8$, the solution using the lossless coupling is roughly 55% of that in the solution using the (lossy) Easiness, and the solution using Easiness is about 15% smaller than the other two lossy methods [Fig. 6(a)].

Running time. The greedy algorithm runs much faster in the lossy coupled networks than the lossless ones in general. As shown in Fig. 5(c) and (d), using the lossy coupling reduces the running times by a factor of 2 in FSQ-Twitter and a factor of 4 in the coauthor networks compared with the lossless coupling. The major disadvantages of the lossless coupling scheme are the redundant nodes and edges. Therefore, the lossy coupling schemes works better on networks that are sparse and the number of overlapping users is small.

However, in some other cases, the lossless coupling scheme is more efficient. As shown in Fig. 6(b), the running time in the lossless coupled networks is larger in the beginning, but gradually reduces down and beats other methods at $f = 0.4$. The larger f is, the larger the ratio between seed size in the lossless

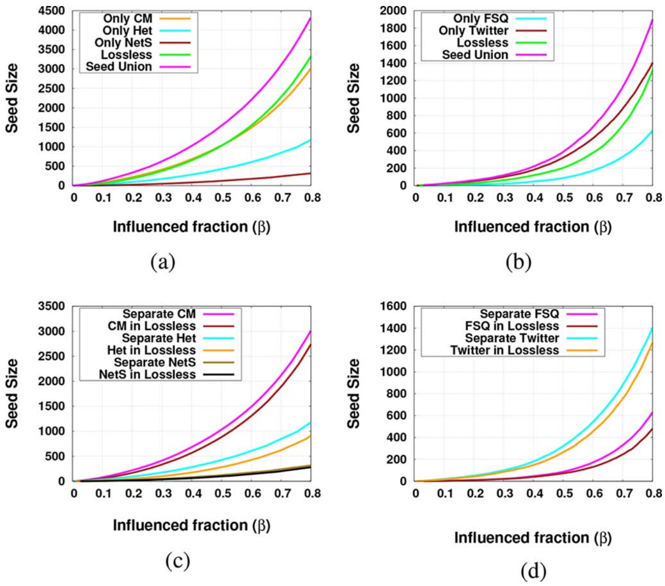


Fig. 7. Quality of seed sets with and without using the coupled network. (a) Coauthor networks. (b) FSQ and Twitter. (c) Coauthor networks. (d) FSQ and Twitter.

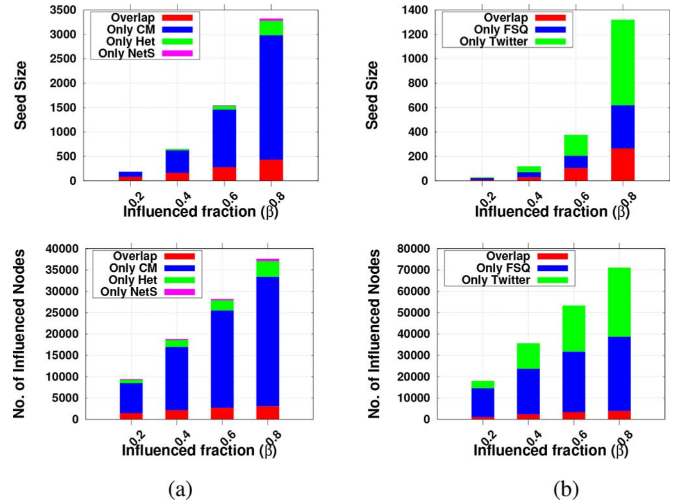


Fig. 8. Bias in selecting seed nodes. (a) Coauthor networks. (b) FSQ-Twitter.

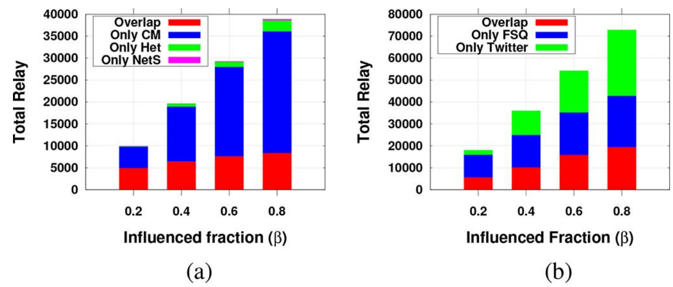


Fig. 9. Influence contribution of seed vertices from component networks. (a) Coauthor networks. (b) FSQ and Twitter.

and lossy coupled networks is. As the running time depends on the seed size, thus it reduces faster in the lossless coupled network with larger overlapping ratio.

Overall, the lossless coupling scheme returns solution with higher quality, especially when the seed set is small. However, if the constraint of running time and the memory are of priority, the lossy Easiness coupling scheme offers an attractive alternative.

C. Advantages of Using Coupled Networks

To understand the benefit of taking consideration of overlapping users and coupled network, in this part, we are going to compare the seed size with/out using coupled network. In particular, we do two comparisons on: 1) influencing a fraction β of the nodes in *all networks* by selecting seeds from each network and taking the union to compare with seeds achieved from lossless coupling scheme; 2) influencing a fraction β of the nodes in *a particular network* by only choosing seeds from that network compare to the seeds obtained from the lossless coupling scheme.

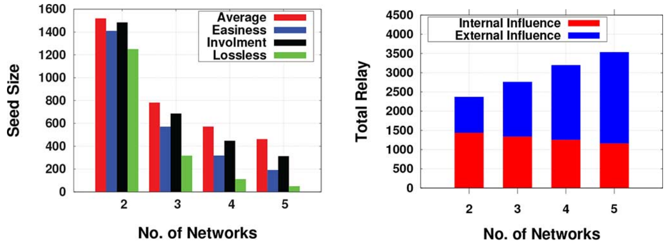
The results for the first scenario are shown in Fig. 7. The seed obtained by the lossless coupling method outperforms other methods. The size of the union set is approximately 30% and 47% larger than lossless coupling method in coauthor and FSQ-Twitter, respectively. This shows that overlapping users do propagate information through several networks and thus effectively help reduce the overall seed size.

In the second scenario, the lossless coupling scheme achieves the best result in both networks. When the network is considered as a standalone network and choose seeds individually (labeled with Only in Fig. 7), the seeds size is relatively larger than choosing from the coupled network. As shown in Fig. 7), the sizes decrease by 9%, 25%, and 17% in CM, Het, and FSQ, accordingly. This improvement is also due to the information diffusion across several networks by the overlapping users. Especially, when the network sizes are unbalanced, like Het with the smaller size of users seems to get more improvement than CM.

D. Analysis of Seed Sets

We analyze seed sets with different influenced fraction β to find out: the composition of the seed set and the influenced set; and the influence contribution of each network. As illustrated in Fig. 8, a significant fraction of the seed set is overlapping nodes although only 5% (7%) users of FSQ-Twitter (the coauthor networks) are overlapping users. With $\beta = 0.4$, the fraction of overlapping seed vertices is around 24.9% and 25% in the coauthor and FSQ-Twitter networks, respectively. As overlapping users can influence friends in different networks, they are more likely to be selected in the seed set than ones participating in only one network. Fig. 9 demonstrates the high influence contribution of the overlapping users, especially when β is small (contribute more than 50% of the total influence when $\beta = 0.2$). However, when β is large, good overlapping users are already selected, so overlapping users are not favored any more.

Additionally, there is an imbalance between the number of selected vertices and influenced vertices in each networks. In the co-author dataset, CM contributes a large number of seed vertices and influenced vertices since the size of CM is significantly larger than other networks. When $\beta = 0.8$, 76.7% of seed vertices and 80.5% of influenced vertices are from CM. In contrast, the number of seed vertices from FSQ is small but the number of influenced vertices in FSQ is much higher than Twitter. With $\beta = 0.4$, 27% (without overlapping vertices) of seed vertices belong to FSQ while 70% of influenced vertices



(a) (b)

Fig. 10. Impact of additional networks on (a) seed size and (b) the influence diffusion in a network.

are in FSQ. After the major of vertices in FSQ are influenced, the algorithm starts to select more vertices in Twitter to increase the influence fraction. This implies that it is easier for the information to propagate in one network than the other, even when we consider the overlapping between them. Moreover, we can target the overlapping users in one network (e.g., Twitter) to influence users in another network (e.g., FSQ).

E. Mutual Impact of Networks

We evaluate the mutual impact between networks when the number of network k increases. We use a user base of 10 000 users to synthesize networks for the experiment. For each network, we randomly select 4000 users from the user base and connect each pair of selected users randomly with probability 0.0025. Thus all networks have the same size and the expected average outgoing (incoming) degree of 10. The expected overlapping fraction of any network pair is 16%. We measure the seed size to influence 60% of users (6000 users) with the different number of networks [Fig. 10(a)]. When k increases from 2 to 5, the seed size decreases several times. It implies that the introduction of a new OSN increases the diffusion of information significantly.

We also compute how much new networks help the existing one to propagate the information. Using the same seed set found by the greedy algorithm to influence 60% (2400 users) of the target network (the first created network), we compute the total number of influenced vertices in that network as well as the external influence. Fig. 10(b) shows that the number of influenced vertices is raised 46% with the support of 3 new networks when k is changed from 2 to 5. In addition, the fraction of external influence is also increased dramatically from 39% when $k = 2$ to 67% when $k = 5$. It means that the majority of influence can be obtained via the support of other networks. On the other hand, these results suggest that the existing networks may benefit from the newly introduced competitor.

VII. Conclusion

In this paper, we study the LCI problem in multiplex networks. To tackle the problem, we introduced novel coupling schemes to reduce the problem to a version on a single network. Then, we designed a new metric to quantify the flow of influence inside and between networks based on the coupled network. Exhaustive experiments provide new insights to the information diffusion in multiplex networks.

In the future, we plan to investigate the problem in multiplex networks with heterogeneous diffusion models in which each network may have its own diffusion model. It is still an ongoing problem whether they can be represented efficiently, or we have better method to couple them into one network.

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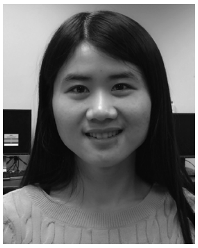

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