3.

$$P_0(y) = \begin{cases} (2/3)(y+1) & 0 \le y \le 1 \\ 0 & \text{o.w.} \end{cases}$$

$$P_1(y) = \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{o.w.} \end{cases}$$

$$L(\vec{y}) = \frac{P(y|H_1)}{P(y|H_0)} = \frac{3}{2(y+1)} \quad , 0 \le y \le 1$$

uniform costs and equal priors :

$$\frac{3}{2(y+1)} \ge 1 \rightarrow \frac{1}{2} \ge y \rightarrow \delta_B(y) = \begin{cases} 1 & 0 \le y \le 1/2 \\ 0 & 1/2 < y \le 1 \end{cases}$$

min Bayes risk: $r(\delta_B) = \frac{1}{2}\int_0^{1/2} \frac{2}{3}(y+1)\,dy + \frac{1}{2}\left(\frac{1}{2}\right)$

$$= \frac{1}{3}\left(\frac{y^2}{2} + y\right)\Big|_0^{1/2} + \frac{1}{4} = \frac{1}{3}\left(\frac{1}{8} + \frac{1}{2}\right) + \frac{1}{4} = \frac{11}{24}$$

4. $H_0 : Y = N - s \quad , \quad H_1 : Y = N + s$

$$P_N(n) = \frac{1}{\pi(1+n^2)} \quad , n \in \mathbb{R}$$

$\Rightarrow P_0(y) = P_N(y+s) \ , \ P_1(y) = P_N(y-s)$

$$\Rightarrow L(y) = \frac{1+(y+s)^2}{1+(y-s)^2}$$

uniform costs and equal priors :

$L(y) \ge 1 \rightarrow 1+(y+s)^2 \ge 1+(y-s)^2$

$$\rightarrow 2sy \ge -2sy \rightarrow \delta_B(y) = \begin{cases} 1 & y \ge 0 \\ 0 & y < 0 \end{cases}$$

$$r(\delta_B) = \frac{1}{2}\int_0^{\infty} \frac{1}{\pi(1+(y+s)^2)}\,dy + \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\pi(1+(y-s)^2)}\,dy$$

$$= \frac{1}{2\pi}\tan^{-1}(y+s)\Big|_0^{\infty} + \frac{1}{2\pi}\tan^{-1}(y-s)\Big|_{-\infty}^{0}$$

$$= \frac{1}{4} - \frac{1}{2\pi}\tan^{-1}(s) + \frac{-1}{2\pi}\tan^{-1}(s) - \frac{-1}{4}$$

$$= \frac{1}{2} - \frac{\tan^{-1}(s)}{\pi}$$

5.

$$P(N=n|H_i) = \frac{(m_i)^n}{n!} e^{-m_i} \quad , n = 0,1,2,\ldots \quad i = 0,1$$

$E(N|H_i) = m_i$

$$\text{LRT} : \frac{P(y|H_1)}{P(y|H_0)} \underset{0}{\overset{1}{\gtrless}} \frac{\pi_0}{\pi_1} \frac{C_{10} - C_{00}}{C_{01} - C_{11}}$$

left hand side : $\dfrac{m_1^n e^{-m_1}}{m_0^n e^{-m_0}} = \left(\dfrac{m_1}{m_0}\right)^n e^{m_0 - m_1}$

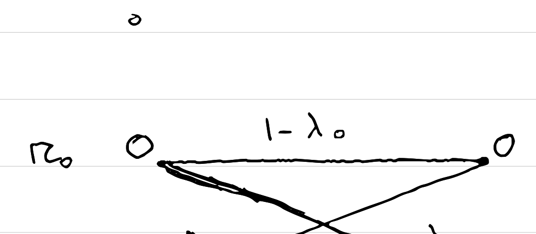
$\rightarrow$ LRT in general : $\left(\dfrac{m_1}{m_0}\right)^n e^{m_0 - m_1} \underset{0}{\overset{1}{\gtrless}} \dfrac{\pi_0}{\pi_1} \dfrac{C_{10} - C_{00}}{C_{01} - C_{11}}$

for uniform costs and equal priors :

$$\left(\frac{m_1}{m_0}\right)^n e^{m_0 - m_1} \underset{0}{\overset{1}{\gtrless}} 1 \rightarrow n\ln\left(\frac{m_1}{m_0}\right) + m_0 - m_1 \underset{0}{\overset{1}{\gtrless}} 0$$

$$\rightarrow n \underset{0}{\overset{1}{\gtrless}} (m_1 - m_0)\ln\left(\frac{m_0}{m_1}\right)$$

2.



		error	FA
(i)	always 0 :	0.7	0
	always 1 :	0.3	1
	keep :	0.13	0.2
	flip :	0.87	0.8
(ii)	always 0 :	0.7	0
	always 1 :	0.3	1
	keep :	0.6	0.2
	flip :	0.4	0.8
(iii)	0 :	0.2	0
	1 :	0.8	1
	keep :	0.6	0.6
	flip :	0.4	0.6
(iv)	0 :	0.5	0
	1 :	0.5	1
	keep :	0.5	0.5
	flip :	0.5	0.5

1. (c) & (d) if we want to choose based on $r(\delta)$ we should choose first rule since $r\_first = 0.16 < r\_second = 0.22$