ECE 7001, Stochastic Processes, Detection, and Estimation — Jan. 26, 2023
OSU, Spring 2023 — Due: Feb. 9, 2023

Problem Set 2

Problem 1 - MATLAB Exercise

This problem uses the Titanic data from Homework 1 (i.e., `hw1dat`).

(a) Compute the theoretical ROC curve given that $f_{Y|H_0}(y \mid H_0) = \mathcal{U}(-1.5, 1.5)$ and $f_{Y|H_1}(y \mid H_1) = \frac{1}{2}\mathcal{N}(-2,1) + \frac{1}{2}\mathcal{N}(2,1)$. What is the size-0.3 NP decision rule and the resulting P_D?

(b) Using an LRT rule with the same form as part (a), we will empirically design Neyman-Pearson tests to achieve particular false-alarm rates. Varying the threshold in the LRT rule, plot the estimated ROC curve (i.e., P_D vs. P_F) using *only the first 200 points of* the H_0 and H_1 data columns. What is the size-0.3 NP decision rule and the resulting P_D?

(c) Comment on your results and any issues to empirically designing a Neyman-Pearson test with limited data.

Problem 2 (Poor, Ch. 2, Pr. 2(c))

Suppose Y is a random variable that, under hypothesis H_0, has pdf

$$p_0(y) = \begin{cases} (2/3)(y+1) & 0 \le y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

and under hypothesis H_1, has pdf

$$p_1(y) = \begin{cases} 1 & 0 \le y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find the NP rule and the corresponding detection probability for false-alarm probability $\alpha \in (0,1)$.

Problem 3 (Poor, Ch. 2, Pr. 6(c))

Repeat the previous exercise for the hypothesis pair

$$\begin{array}{rcl} H_0 & : & Y = N - s \\ H_1 & : & Y = N + s \end{array},$$

where $s > 0$ is a fixed real number and N is a continuous random variable with density

$$p_N(n) = \frac{1}{\pi(1+n^2)}, \quad n \in \mathbb{R}.$$

Problem 4 (Poor, Ch. 2, Pr. 16)

Generalize the Bayesian hypothesis testing method to $M > 2$ hypotheses. Provide extended definitions for Γ_i, C_{ij}, $\delta(y)$, $R_j(\delta)$, π_j, and $r(\delta)$ and formulate the optimal decision rule.

Problem 5 - (Poor, Ch. 3, Pr. 3)

Hint: Remember that, in the M-ary Bayes problem, minimum-error-probability implies a particular cost assignment.

Problem 6 (Poor, Ch. 2, Pr. 20)

Solve Poor, Ch. 2, Pr. 20.

Note: Some of the problems above require solving a quadratic, thus giving two possible solutions. In such cases, only one of the solutions may be valid. Clearly explain your reasoning when discarding a solution.