ECE 7001, Stochastic Processes, Detection, and Estimation — Feb. 16, 2023
OSU, Spring 2023 — Due: Mar. 2, 2023

Problem Set 3

Problem 1 - (Poor, Ch. 3, Pr. 9)

For the LMP stated in part (b), you can ignore the randomization γ since it is not needed.

Problem 2 (Poor, Ch. 3, Pr. 13)

Problem 3 (Poor, Ch. 3, Pr. 14)

Problem 4 (Poor, Ch. 3, Pr. 20)

Problem 5 (Poor, Ch. 3, Pr. 21)

Use equation (III.C.18) for P_e and recall the binary symmetric channel covered in class and in Example II.B.1 in Poor.

Problem 6

A radar system detects aircrafts based on the received iid signal $\vec{Y} = Y_1, Y_2, \ldots, Y_n$. If an aircraft is present (H_1), $Y_k \sim \mathcal{N}(\mu, \sigma^2)$ for some $\mu > 0$ and otherwise (H_0), $Y_k \sim \mathcal{N}(0, \sigma^2)$. Note that the moment generating function for a normal random variable, $\mathcal{N}(\mu, \sigma^2)$, is $g(r) = \exp\left(\mu r + \frac{1}{2}\sigma^2 r^2\right)$.

(a) Find the tightest Chernoff bound for $q = \mathbb{P}\left(Y_1 > 2\mu \mid H = H_0\right)$.

(b) Find the **log** likelihood ratio, $LLR(Y_k)$ and find the distribution of $LLR(\vec{Y})$ under H_0 and H_1.

(c) Find the tightest Chernoff bound for the probability of false alarm, P_F, for the maximum likelihood (ML) detector.

(d) Compare your results for parts (a) and (c) to find a number, $n_{\min}$, of observations, which guarantees $P_F \leqslant q$. Very briefly comment on the efficiency of non-sequential detection.