ECE 7001, Stochastic Processes, Detection, and Estimation — Jan. 17, 2023
OSU, Spring 2023 — Due: Jan. 26, 2023

Problem Set 1

This problem set has two parts. The first part is designed to give you a feel for the hypothesis testing problem through simulations. The second set of problems are more classical, mathematical type problems. They will enable you to go through the details of designing optimal Bayesian decision rules for some interesting pdfs. Peculiar pdfs are considered so that you will have exposure to some of the tricky aspects of decision rule design (non-contiguous decision regions for example).

Problem 1 - MATLAB Exercise

Your task is to generate a binary decision rule for some sonar data that has been collected. You are trying to classify underwater objects from the Titanic. You are going to design the initial system which must simply distinguish between metal objects (H_1) and wooden objects (H_0). At your disposal are several data files: observations for H_1 only, observations for H_0 only and a test file containing observations from both H_1 and H_0 as well as the true source information.

On Carmen, under “Problem Sets,” you’ll find a pointer to an ascii file called `hw1dat`. This file has four columns: data from H_0, data from H_1, source information for test data, and test data.

(a) Design **two** possible decision rules. Play with the data to determine what might be good decision rules. (I found it instructive to plot approximate pdfs using the `hist` command.)

(b) For each decision rule, use the test data to determine the four possible $P_i(\Gamma_j)$.

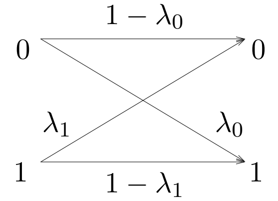
(c) Estimate the prior probabilities and compute the Bayes risk for your two decision rules. Assume uniform costs.

(d) Which rule is better and why?

(e) Comment on your findings.

Problem 2 - MATLAB Exercise

The binary communications channel is a common toy model used to study simple communications systems. The system works as follows:



Let π_1 be the prior probability that a “1” is transmitted, so that $\pi_0 = 1 - \pi_1$ is the prior probability that a “0” is transmitted. Let λ_1 be the probability that a transmitted “1” is flipped by the channel, and λ_0 the probability that a transmitted “0” is flipped by the channel.

On Carmen under "Problem Sets," you'll find the Matlab routine `binchan.m`. This Matlab file simulates the binary channel. The input parameters are π_0, λ_1 and λ_0. The outputs are transmitted bits and observed bits, respectively. Consider the following system parameters:

(a) $\pi_0 = 0.3, \lambda_1 = 0.1, \lambda_0 = 0.2$

(b) $\pi_0 = 0.3, \lambda_1 = 0.7, \lambda_0 = 0.2$

(c) $\pi_0 = 0.8, \lambda_1 = 0.5, \lambda_0 = 0.6$

(d) $\pi_0 = 0.5, \lambda_1 = 0.5, \lambda_0 = 0.5$

Compute the probability of error of the **four** possible deterministic decision rules (i.e., always choose 0, always choose 1, keep observation, reverse observation). Also determine the probability of false alarm for these four rules.

Problem 3 (Poor, Ch. 2, Pr. 2(a))

Suppose Y is a random variable that, under hypothesis H_0, has pdf

$$p_0(y) = \begin{cases} (2/3)(y+1) & 0 \le y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

and under hypothesis H_1, has pdf

$$p_1(y) = \begin{cases} 1 & 0 \le y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the Bayes rule and minimum Bayes risk for testing H_0 versus H_1 with uniform costs and equal priors.

Problem 4 (Poor, Ch. 2, Pr. 6(a))

Repeat the previous exercise for the hypothesis pair

$$\begin{array}{lcl} H_0 & : & Y = N - s \\ H_1 & : & Y = N + s \end{array},$$

where $s > 0$ is a fixed real number and N is a continuous random variable with density

$$p_N(n) = \frac{1}{\pi(1+n^2)}, \quad n \in \mathbb{R}.$$

Problem 5

The Poisson distribution of events is encountered frequency as a model of shot noise, photon counts, and other phenomena. Each time the experiment is conducted a certain number of events occur. Our observation is just this number, or count, which ranges from 0 to ∞ and obeys a Poisson distribution under both hypotheses; that is,

$$\mathbb{P}\left(N = n \mid H_i\right) = \frac{(m_i)^n}{n!} e^{-m_i}, \quad n = 0, 1, 2, \dots \text{ and } i = 0, 1,$$

where m_i is the Poisson parameter that specifies the average number of events, $E(N \mid H_i) = m_i$. Find the likelihood ratio test for general costs and priors. Then, interpret the log-likelihood ratio test for uniform costs and equal priors.